# Deep Contact

## Accelerating Rigid Simulation with Convolutional Networks

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Master Thesis Defense, 2018

# Outline

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# Previous Work

- Deep Learning applied in computer simulation.
- Speed up contact simulation.

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1. Introduction
   - Previous Work
   - Thesis Overview
2. Particles-Grid-Particles
   - Grid-Particle Method
   - Smoothed Particle Hydrodynamics
   - Bilinear Interpolation
3. Deep Learning Model
   - Convolutional Neural Networks
   - CNN Architecture
   - Training Configuration
4. Results and Analysis
5. Future Work

# Thesis Overview

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$$\begin{aligned}
\mathbf{w} &= \boldsymbol{J}_n\mathbf{v}^{t+1} \\
&= \underbrace{\boldsymbol{J}_n\boldsymbol{M}^{-1}\boldsymbol{J}_n^T\Delta t}_{\boldsymbol{A}}\boldsymbol{\lambda}_n + \underbrace{\boldsymbol{J}_n(\Delta t\boldsymbol{M}^{-1}\mathbf{F}_{ext} + \mathbf{v}^t)}_{\boldsymbol{b}} \\
&= \boldsymbol{A}\boldsymbol{\lambda}_n + \boldsymbol{b}
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$$\boldsymbol{\lambda} = LCP(\boldsymbol{A}, \boldsymbol{b})$$

**Data:** $N, \boldsymbol{\lambda}_{init}, \boldsymbol{A}, \boldsymbol{b}$
**Result:** Compute the values of $\boldsymbol{\lambda}$, the convergence rate $\boldsymbol{\theta}$

```
for k = 1 To N do
    if k = 1 then
        λ ← λ_init
    end
    λ_old ← λ
    for all i do
        r_i ← A_{i*} λ + b_i ;
        λ_i ← max(0, λ_i − r_i / A_ii) ;
    end
    θ_k ← max(|λ − λ_old|)
end
```

$$\boldsymbol{r}_i \leftarrow \boldsymbol{A}_{i*}\boldsymbol{\lambda} + \boldsymbol{b}_i \qquad \boldsymbol{\lambda}_i \leftarrow \max\left(0, \boldsymbol{\lambda}_i - \frac{\boldsymbol{r}_i}{\boldsymbol{A}_{ii}}\right) \qquad \boldsymbol{\theta}_k \leftarrow \max(|\boldsymbol{\lambda} - \boldsymbol{\lambda}_{old}|)$$

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# Grid-Particle Method

## Workflow

The whole workflow can be described as,

1. Based on Smoothed Particle Hydrodynamics(SPH), map current state($m, v_x, v_y, \omega, n_x$) to a image(the number of channel is 5.), which is called feature image.

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3. For all contacts positions, interpolated values will be gener- ated based on label image. Then, the values will be used as starting iterate values for contact force solver. In our hypoth- esis, the given starting values will speed up the solver to reach convergence.

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# Smoothed Particle Hydrodynamics

## Fundamentals

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$$A_S(\mathbf{x}) = \sum_i A(\mathbf{x}_i)W(\|\mathbf{x}_i - \mathbf{x}\|, h) \tag{3}$$

- **Poly6**

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$$W_{poly6}(\mathbf{r}, h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - \|\mathbf{r}\|^2)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

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$$W_{spiky}(\mathbf{r}, h) = \frac{15}{\pi h^6} \begin{cases} (h - \|\mathbf{r}\|)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

# Smoothed Particle Hydrodynamics

## Kernels

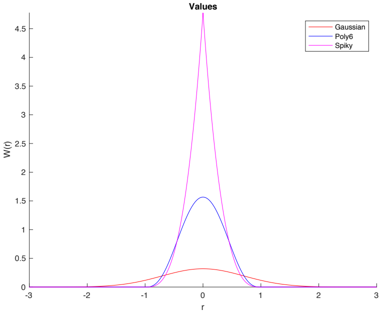


Figure: Comparation of different kernels, we set smoothing length $h = 1$ here.

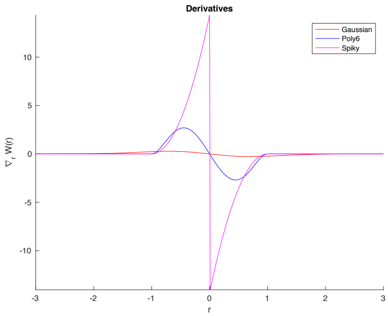


Figure: Comparation of gradient of different kernels, we set $h = 1$ here.

# Mapping bodies into a grid image

**Data:** Given a set of bodies $\mathcal{B}$ and the state in time $t$, as well as all the spacial positions of grid nodes $\mathbf{x}$

**Result:** the body grid image $\boldsymbol{G}_{\mathcal{B}}$

**for** *all i and j* **do**

1. Find the nearest neihbors $\mathcal{B}_{near} \subseteq \mathcal{B}$ around $\mathbf{x}_{ij}$
2. read the current state,

$$m_k, \boldsymbol{v}_k, \boldsymbol{q}_k, \omega_k \quad k \in \mathcal{B}_{near}$$

3. Define state vector $\boldsymbol{S}_k \leftarrow [m_k, v_{kx}, v_{ky}, \omega_k]$
4. Compute grid values

$$\boldsymbol{G}_{\mathcal{B}}(i,j) \leftarrow \sum_{k \in \mathcal{B}_{near}} W(\mathbf{x}_{ij}, \boldsymbol{q}_k)\boldsymbol{S}_k$$

**end**

# Mapping contacts into a grid image

**Data:** Given a set of contacts $\mathcal{C}$ between a set of bodies $\mathcal{B}$ and the state in time $t$, as well as all the spacial positions of grid nodes $\mathbf{x}$

**Result:** the contact grid image $\boldsymbol{G}_\lambda$

**for** *all i and j* **do**

1. Find the nearest neihbors $\mathcal{C}_{near} \subseteq \mathcal{C}$ around $\mathbf{x}_{ij}$
2. read the current contact forces values and its position.

$$\boldsymbol{q}_k, \boldsymbol{\lambda}_k \quad k \in \mathcal{C}_{near} \quad \boldsymbol{\lambda} = [\lambda_n, \lambda_t]$$

3. Compute grid values

$$\boldsymbol{G}_\lambda(i,j) \leftarrow \sum_{k \in \mathcal{C}_{near}} W(\mathbf{x}_{ij}, \boldsymbol{q}_k)\boldsymbol{\lambda}_k$$

**end**

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# Bilinear Interpolation

Once getting contact grid image, we need transform the grid image to a physical state, specific contact values in each contact point. We applied bilinear interpolation in our case.

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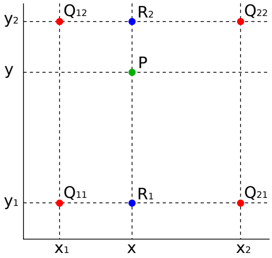


Figure: The figure shows the visualization of bilinear interpolation. The four red dots show the data points and the green dot is the point at which we want to interpolate.

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$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}). \tag{6b}$$

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After getting the two values in $x-$direction $f(x, y_1)$ and $f(x, y_2)$, we can combine these values to do interpolation in $y-$ direction.

$$f(x, y) \approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2) \tag{7}$$

# SPH method
## Experiments

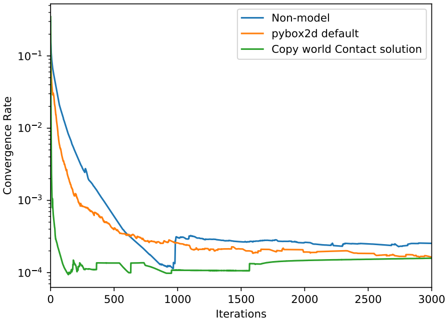
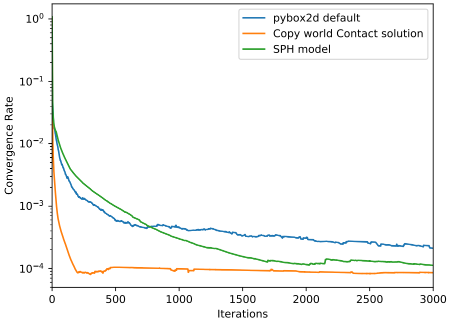



Figure: Average convergence rate for different models(not including **SPH-based model**).

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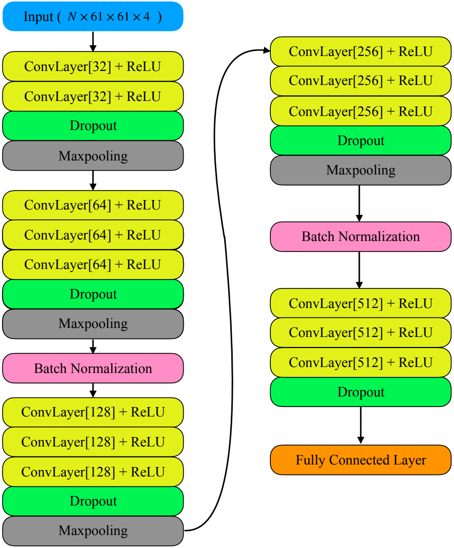
# CNN Architecture



# Outline

# Training Hyperparameter

| Hyperparameter | Setting |
| --- | --- |
| Activation function | ReLU |
| Weight initilization | He normal |
| Weight regularizer | L2 |
| Convolution border mode | Same |
| Stride | 2 |
| Kernel size | $(3, 3)$ |
| Dropout rate | 0.1 |
| Optimizer | SGD |
| Initial Learning Rate | $1 \times 5 \times 10^{-3}$ |
| Batch Size | 200 |
| Epoch | 1000 |
| Validation Rate | 0.2 |

Table: Hyperparameter settings.

# Simulation Configuration

- **World Setting**

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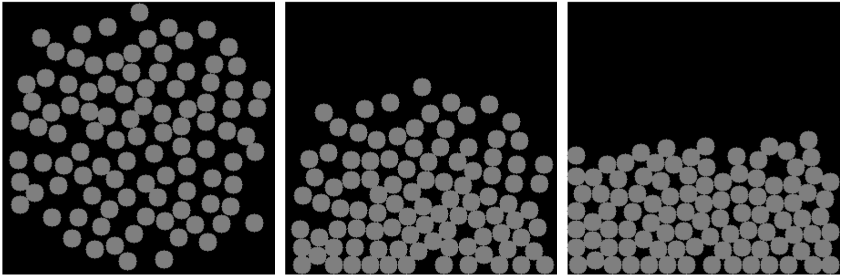
- **Simulation Setting** there will be totally 600-steps simulation. For each step, $\Delta t = 0.01s$, and the number of iteration in each step will be set as fixed, 3000.



(a) Time Step=0 (b) Time Step=200 (c) Time Step=400

Figure: Visualization for experiment simulation

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- the smooth length $h$ should be less than the minimum distance between two contact points $d$, $h \leq d$
- For a given $d$, $h \geq \frac{\sqrt{2}}{2} d \approx 0.71d$

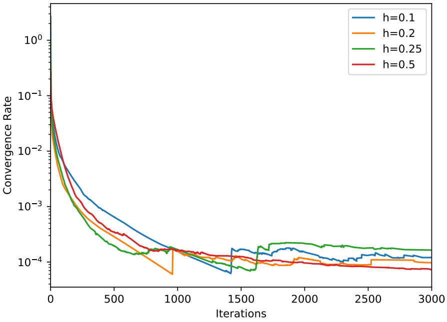



Figure: The grid size $d$ is set 0.25. $h = 0.1, 0.2, 0.25, 0.5$ is tested respectively. This figure shows different coveragence rate based on different $h$ value. The kernel is **Poly6**

# Results and Analysis

## SPH parameters(**Poly6**)

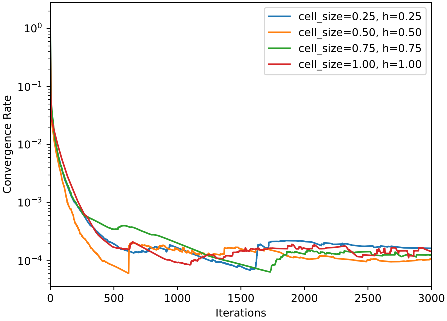
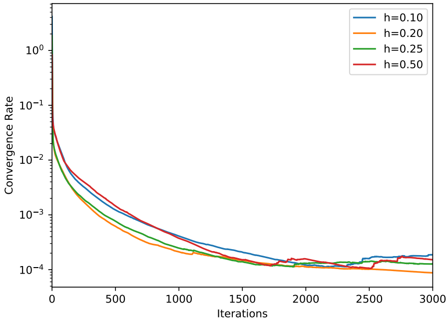



Figure: Coveragence rate for different *d* value. The kernel is **Poly6**

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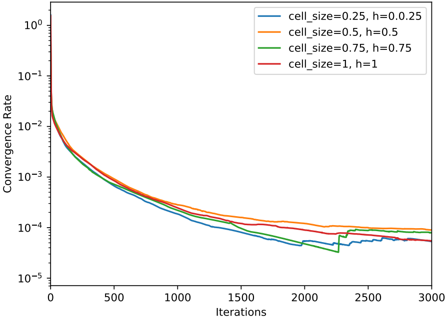


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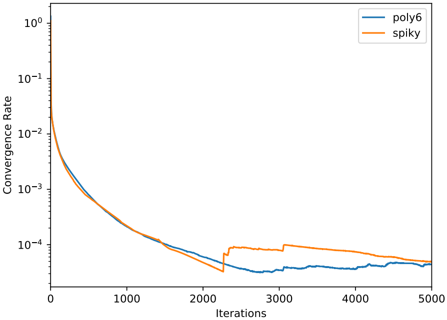


Figure: Coveragence rate for kernel **Poly6** anf **Spiky**. $h_{\text{poly6}} = d_{\text{poly6}} = 0.5$, while $h_{\text{spiky}} = d_{\text{spiky}} = 0.25$

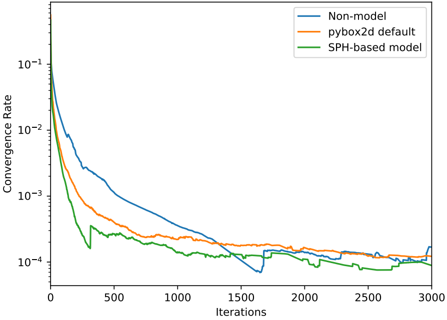


Figure: Coveragence rate for models(different initial values for $\boldsymbol{\lambda}$).

- **Input Size**, the input will be $61 \times 61 \times 4$. Since the orginal world is $30 \times 30$ and grid size is $d = 0.5$, the generated grid would be $61 \times 61$. There would 4 channels$[m, v_x, v_y, \omega]$

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- **Weights Number**, the total weights number is $64,498,866$.
- **Training Environment**, GPU(*GeForce GTX 1080 Ti, 11 Gbps GDDR5X memory*) held by Image Section, DIKU.

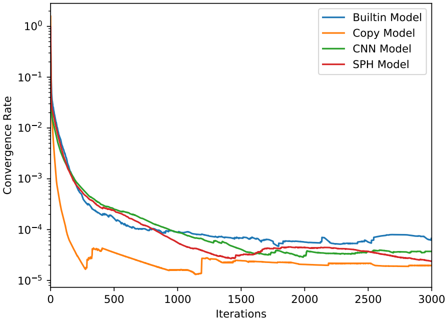


Figure: The final result. Add the final CNN solution to compare with other methods.

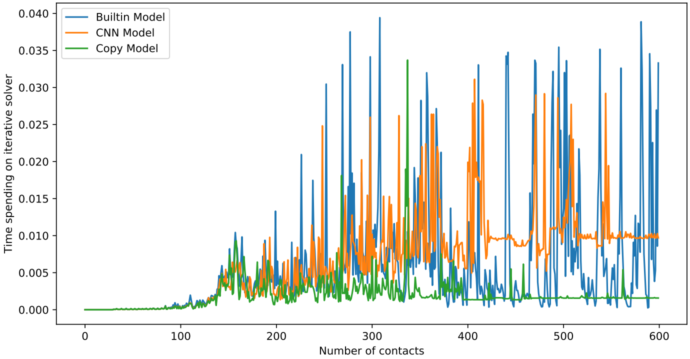


Figure: Time spent for contact solver iteration

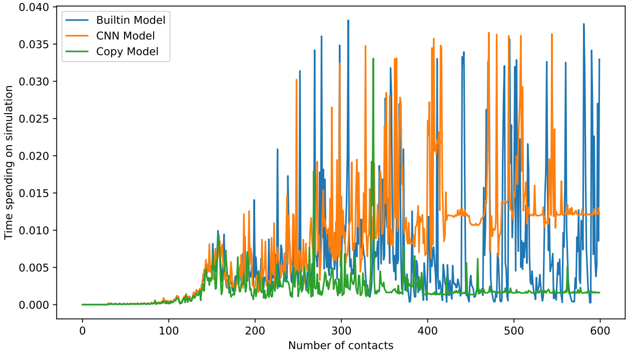


Figure: Time spend for the whole contact solution(including warm starting calculation).

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- **CNN-Model** actually makes the iterative solver converge faster. But it is not a great improvement to buit-in warm starting.
- **CNN-Model** performs very similar to **SPH-Model**, which means the CNN predicts contact image well.
- Due to the limitation of the SPH-based method, **CNN-Model** just gets a small improvement compared with **Builtin-Model**. And it cannot perform as good as **Copy-Model**.

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   - **SPH** For the SPH-based method, it is still far away from perfect. It performs not much better than warm starting. The probable attempt will including trying more new kernels, using new data and algorithm for nearest neighbor searching.
   - **Interpolation Method** It might lose some essential informa- tion when it was interpolated back to particles. So exploring another interpolation method would be helpful to this project.
2. **Deep Learning Model**, More learning models can be explored, like Recurrent Neural Networks(RNN), Long short-term memory(LSTM).
3. **More Shapes Experiments**

# For Further Reading I


A. Author.
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Some Press, 1990.

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On this and that.
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