

Deep Contact

Accelerating Rigid Simulation with Convolutional Networks

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Master Thesis Defense, 2018

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 - Grid-Particle Method
 - Smoothed Particle Hydrodynamics
 - Bilinear Interpolation
- 3 Deep Learning Model
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- Deep Learning applied in computer simulation.
 - Deep Learning used in liquid dynamic simulation
 - Visual Interaction Networks(DeepMind)
- Speed up contact simulation.
 - Optimization for contact solver

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Thesis Overview

Model Contact

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$$M\dot{\mathbf{u}} = \underbrace{J^T \lambda}_{\mathbf{F}_{\text{contact}}} + \mathbf{F}_{\text{ext}}$$

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$$\begin{aligned} \mathbf{w} &= J_n \mathbf{v}^{t+1} \\ &= \underbrace{J_n M^{-1} J_n^T \Delta t}_{\mathbf{A}} \lambda_n + \underbrace{J_n (\Delta t M^{-1} \mathbf{F}_{\text{ext}} + \mathbf{v}^t)}_{\mathbf{b}} \\ &= \mathbf{A} \lambda_n + \mathbf{b} \end{aligned}$$

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$$\lambda = LCP(\mathbf{A}, \mathbf{b})$$

Thesis Overview

Projected Gauss-Seidel(PGS) solver for LCP

Data: $N, \lambda_{init}, \mathbf{A}, \mathbf{b}$

Result: Compute the values of λ , the convergence rate θ

for $k = 1$ **To** N **do**

if $k = 1$ **then**

$\lambda \leftarrow \lambda_{init}$

end

$\lambda_{old} \leftarrow \lambda$

for *all* i **do**

$r_i \leftarrow \mathbf{A}_{i*} \lambda + \mathbf{b}_i$;

$\lambda_i \leftarrow \max(0, \lambda_i - \frac{r_i}{\mathbf{A}_{ii}})$;

end

$\theta_k \leftarrow \max(|\lambda - \lambda_{old}|)$

end

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- It can make the simulation states be expressed by a set of matrixes, which can be accessible for deep neural networks.
- Grid image can restore the distribution of mass, linear velocity, angular velocity for deep learning neural networks, while the visualization image of simulation can only describe the position of rigid.

Grid-Particle Method

Workflow

The whole workflow can be described as,

- ① Based on Smoothed Particle Hydrodynamics (SPH), map current state(m, v_x, v_y, ω, n_x) to a image(the number of channel is 5.), which is called feature image.

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- 3 For all contacts positions, interpolated values will be generated based on label image. Then, the values will be used as starting iterate values for contact force solver. In our hypothesis, the given starting values will speed up the solver to reach convergence.

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Smoothed Particle Hydrodynamics

Fundamentals

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$$A_S(\mathbf{x}) = \sum_i A(\mathbf{x}_i) W(\|\mathbf{x}_i - \mathbf{x}\|, h) \quad (3)$$

Smoothed Particle Hydrodynamics

Kernels

- **Poly6**

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$$W_{poly6}(\mathbf{r}, h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - \|\mathbf{r}\|^2)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \quad (4)$$

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$$W_{spiky}(\mathbf{r}, h) = \frac{15}{\pi h^6} \begin{cases} (h - \|\mathbf{r}\|)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \quad (5)$$

Smoothed Particle Hydrodynamics

Kernels

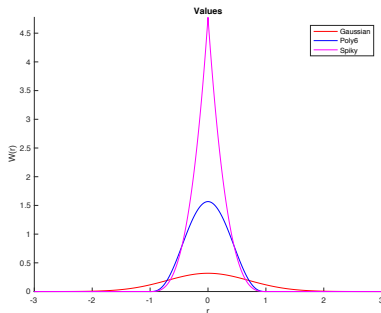


Figure: Comparison of different kernels, we set smoothing length $h = 1$ here.

Smoothed Particle Hydrodynamics

Kernels

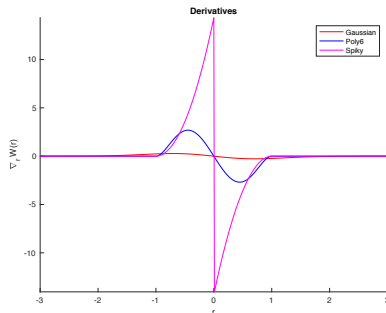


Figure: Comparison of gradient of different kernels, we set $h = 1$ here.

Mapping bodies into a grid image

Data: Given a set of bodies \mathcal{B} and the state in time t , as well as all the spacial positions of grid nodes \mathbf{x}

Result: the body grid image $\mathbf{G}_{\mathcal{B}}$

for *all* i *and* j **do**

1. Find the nearest neighbors $\mathcal{B}_{near} \subseteq \mathcal{B}$ around \mathbf{x}_{ij}
2. read the current state,

$$m_k, \mathbf{v}_k, \mathbf{q}_k, \omega_k \quad k \in \mathcal{B}_{near}$$

3. Define state vector $\mathbf{S}_k \leftarrow [m_k, v_{kx}, v_{ky}, \omega_k]$
4. Compute grid values

$$\mathbf{G}_{\mathcal{B}}(i, j) \leftarrow \sum_{k \in \mathcal{B}_{near}} W(\mathbf{x}_{ij}, \mathbf{q}_k) \mathbf{S}_k$$

end

Mapping contacts into a grid image

Data: Given a set of contacts \mathcal{C} between a set of bodies \mathcal{B} and the state in time t , as well as all the spacial positions of grid nodes \mathbf{x}

Result: the contact grid image \mathbf{G}_λ

for all i and j **do**

1. Find the nearest neighbors $\mathcal{C}_{near} \subseteq \mathcal{C}$ around \mathbf{x}_{ij}
2. read the current contact forces values and its position.

$$\mathbf{q}_k, \boldsymbol{\lambda}_k \quad k \in \mathcal{C}_{near} \quad \boldsymbol{\lambda} = [\lambda_n, \lambda_t]$$

3. Compute grid values

$$\mathbf{G}_\lambda(i, j) \leftarrow \sum_{k \in \mathcal{C}_{near}} W(\mathbf{x}_{ij}, \mathbf{q}_k) \boldsymbol{\lambda}_k$$

end

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Bilinear Interpolation

Once getting contact grid image, we need transform the grid image to a physical state, specific contact values in each contact point. We applied bilinear interpolation in our case.

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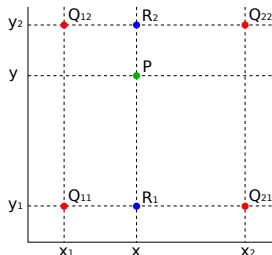


Figure: The figure shows the visualization of bilinear interpolation. The four red dots show the data points and the green dot is the point at which we want to interpolate.

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$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}), \quad (6a)$$

$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}). \quad (6b)$$

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$$f(x, y) \approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2) \quad (7)$$

SPH method

Experiments

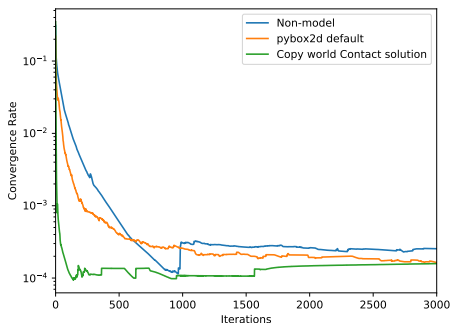


Figure: Average convergence rate for different models(not including **SPH-based model**).

SPH method

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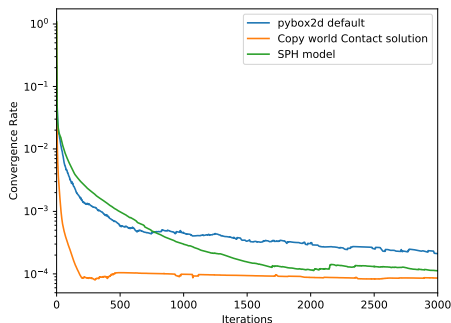


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Deep Learning Model

Convolutional Neural Networks(CNN)

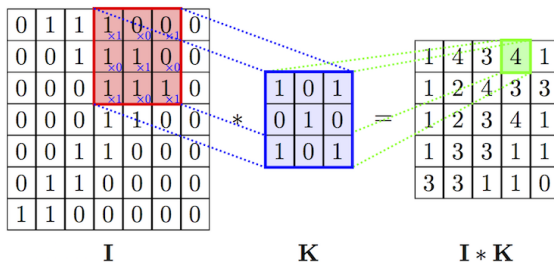


Figure: One simple example of convolution.

Deep Learning Model

Convolutional Neural Networks(CNN)

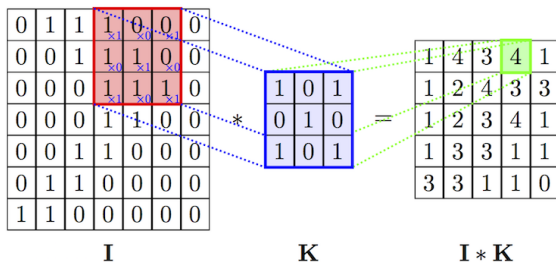
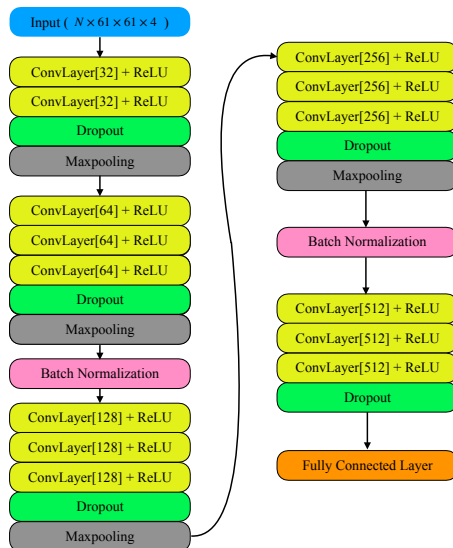


Figure: One simple example of convolution.

$$(I * K)_{xy} = \sum_{i=1}^h \sum_{j=1}^w K_{ij} \cdot I_{x+i-1, y+j-1} \quad (8)$$

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CNN Architecture



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Training Hyperparameter

Hyperparameter	Setting
Activation function	ReLU
Weight initialization	He normal
Weight regularizer	L2
Convolution border mode	Same
Stride	2
Kernel size	(3, 3)
Dropout rate	0.1
Optimizer	SGD
Initial Learning Rate	$1 \times 5 \times 10^{-3}$
Batch Size	200
Epoch	1000
Validation Rate	0.2

Table: Hyperparameter settings.

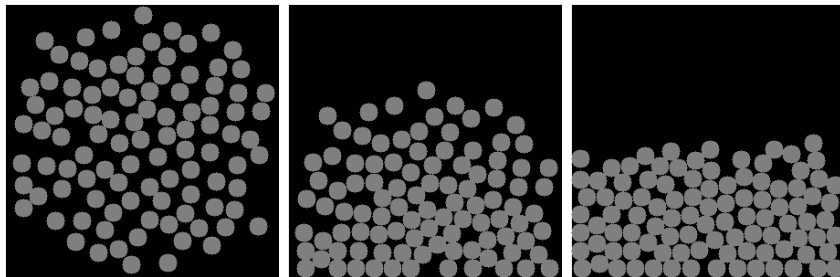
- **World Setting**

- **World Setting** the world box size is 30×30 , and there are 50 – 100 circle rigid bodies($r = 1$, all circle rigid bodies in the same size.) inside the box. Initially, the rigid circles will be located following gaussian distribution². Then, all rigid circles will fall down by gravity.

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Simulation Configuration

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- **Simulation Setting** there will be totally 600-steps simulation. For each step, $\Delta t = 0.01s$, and the number of iteration in each step will be set as fixed, 3000.



(a) Time Step=0

(b) Time Step=200

(c) Time Step=400

Figure: Visualization for experiment simulation

Results and Analysis

SPH parameters

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- the smooth length h should be less than the minimum distance between two contact points d , $h \leq d$
- For a given d , $h \geq \frac{\sqrt{2}}{2}d \approx 0.71d$

Results and Analysis

SPH parameters(**Poly6**)

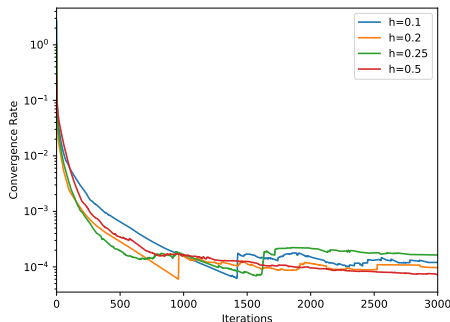


Figure: The grid size d is set 0.25. $h = 0.1, 0.2, 0.25, 0.5$ is tested respectively. This figure shows different coverage rate based on different h value. The kernel is **Poly6**

Results and Analysis

SPH parameters(**Poly6**)

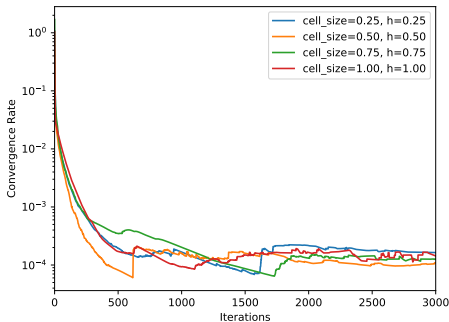


Figure: Coverage rate for different d value. The kernel is **Poly6**

Results and Analysis

SPH parameters(**Spiky**)

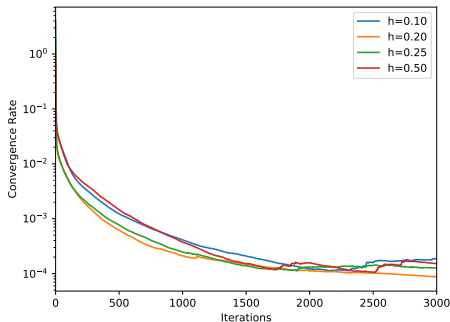


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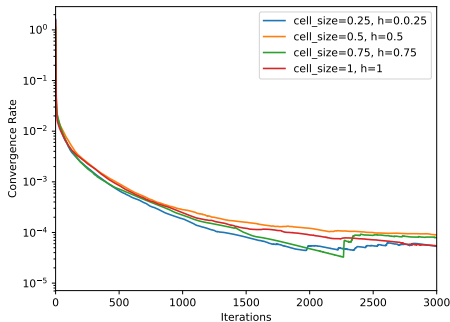


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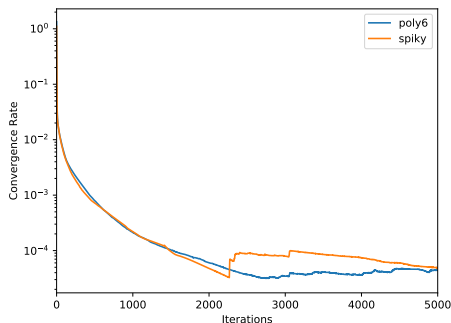


Figure: Coverage rate for kernel **Poly6** and **Spiky**. $h_{\text{poly6}} = d_{\text{poly6}} = 0.5$, while $h_{\text{spiky}} = d_{\text{spiky}} = 0.25$

Results and Analysis

SPH parameters

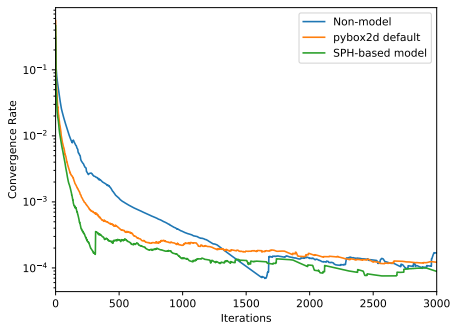


Figure: Coverage rate for models(different initial values for λ).

Results and Analysis

CNN Training

Results and Analysis

CNN Training

- **Input Size**, the input will be $61 \times 61 \times 4$. Since the original world is 30×30 and grid size is $d = 0.5$, the generated grid would be 61×61 . There would 4 channels $[m, v_x, v_y, \omega]$

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- **Output Size**, output size depends on the label size. The original label image would be $[\lambda_n, \lambda_t]$, so the label image size would be $61 \times 61 \times 2$, which should be flattened as the actual training label. The label size would be $61 \times 61 \times 2$

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- **Weights Number**, the total weights number is 64,498,866.
- **Training Environment**, GPU(*GeForce GTX 1080 Ti, 11 Gbps GDDR5X memory*) held by Image Section, DIKU.

Results and Analysis

Simulation on CNN

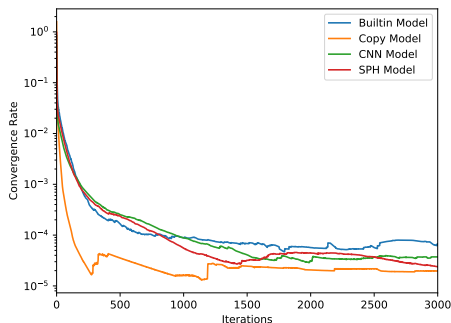


Figure: The final result. Add the final CNN solution to compare with other methods.

Results and Analysis

Simulation on CNN

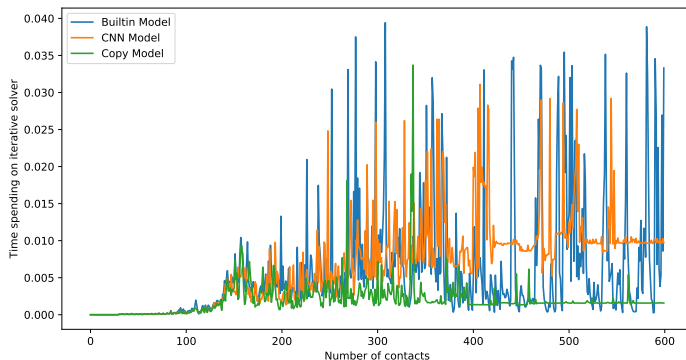


Figure: Time spent for contact solver iteration

Results and Analysis

Simulation on CNN

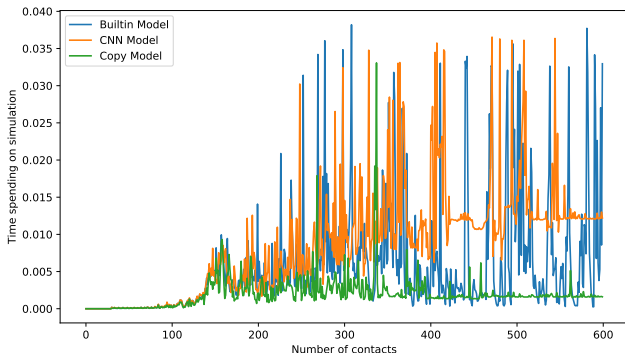


Figure: Time spend for the whole contact solution(including warm starting calculation).

Results and Analysis

Conclusion

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- **CNN-Model** actually makes the iterative solver converge faster. But it is not a great improvement to built-in warm starting.
- **CNN-Model** performs very similar to **SPH-Model**, which means the CNN predicts contact image well.
- Due to the limitation of the SPH-based method, **CNN-Model** just gets a small improvement compared with **Builtin-Model**. And it cannot perform as good as **Copy-Model**.

① Grid-Particles Method

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 - SPH

① Grid-Particles Method

- **SPH** For the SPH-based method, it is still far away from perfect. It performs not much better than warm starting. The probable attempt will including trying more new kernels, using new data and algorithm for nearest neighbor searching.

① Grid-Particles Method

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③ More Shapes Experiments

Thanks! I

- Thanks for Kenny's supervision
- Thanks for Lukas and Lucian's co-operation on initial work.