



# Master Thesis

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## Deep Contact

Accelerating Rigid Simulation With Convolutional Networks

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August 6th 2018

## **Abstract**

This is a master theis from

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# Chapter 1

## Introduction

### 1.1 Motivation

### 1.2 Thesis Overview

## Chapter 2

# Rigid Body Dynamics Simulation

This chapter mainly introduces rigid body simulation to help you understand how computer simulate rigid dynamics based on traditional newton-euler equations. For more details, some contact forces solvers are described in this chapter. Afterwards, we will use one of solver to run some simulation and get the image data for the next step, grids-transfer. All the discussion about rigid simulation and contacts solver are based on 2- $D$  view.

## 2.1 Rigid dynamics Simulation

### 2.1.1 Simulation Basics

Simulating the motion of a rigid body is almost the same as simulating the motion of a particle, so I will start with particle simulation. For particle simulation, we let function  $x(t)$  describe the particle's location in world space at time  $t$ . Then we use  $v(t) = \frac{d}{d(t)}x(t)$  to denote the velocity of the particle at time  $t$ . So, the state of a particle at a time  $t$  is the particle's position and velocity. We generalize this concept by defining a state vector  $\mathbf{Y}(t)$  for a system: for a single particle,

$$\mathbf{Y}(t) = \begin{pmatrix} x_1(t) \\ v_1(t) \end{pmatrix} \quad (2.1)$$



For a system with  $n$  particles, we enlarge  $\mathbf{Y}(t)$  to be

$$\mathbf{Y}(t) = \begin{pmatrix} x_1(t) \\ v_1(t) \\ \dots \\ x_n(t) \\ v_n(t) \end{pmatrix} \quad (2.2)$$

However, to simulate the motion of particles actually, we need to know one more thing – the forces.  $F(t)$  is defined as the force acting on the particle. If the mass of the particle is  $m$ , then the changes of  $\mathbf{Y}(t)$  will be given by

$$\frac{d}{dt}\mathbf{Y}(t) = \frac{d}{dt} \begin{pmatrix} x(t) \\ v(t) \end{pmatrix} = \begin{pmatrix} v(t) \\ F(t)/m \end{pmatrix} \quad (2.3)$$

### 2.1.2 Rigid Body Concepts

Unlike a particle, a rigid body occupies a volume of space and has a particular shape. Rigid bodies are more complicated, beside translating them, we can rotate them as well. To locate a rigid body, we use  $x(t)$  to denote their translation and a rotation matrix  $R(t)$  to describe their rotation.

### 2.1.3 Rigid Body Equations of Motions

Finally, we can covert all concepts we need to define the state  $\mathbf{Y}(t)$  for a rigid body.

$$\mathbf{Y}(t) = \begin{pmatrix} x(t) \\ R(t) \\ P(t) \\ L(t) \end{pmatrix} \quad (2.4)$$

Like what is epressed in  $\mathbf{Y}(t)$ , the state of a rigid body is mainly consist by its position and orientation (describing spatial information), and its linear and angualr momentum(describe velocity information). Since mass  $M$  and bodyspace inertia tensor  $I_{body}$  are constants, we can the auxiliary quantities  $I(t)$ ,  $\omega(t)$  at any given time.

$$v(t) = \frac{P(t)}{M}I(t) = R(t)I_{body}R(t)^T \quad \omega(t) = I(t)^{-1}L(t)$$

The derivative  $\frac{d}{dt}\mathbf{Y}(t)$  is

$$\frac{d}{dt}\mathbf{Y}(t) = \frac{d}{dt} \begin{pmatrix} x(t) \\ R(t) \\ Mv(t) \\ L(t) \end{pmatrix} = \frac{d}{dt} \begin{pmatrix} v(t) \\ \omega(t) * R(t) \\ F(t) \\ \tau(t) \end{pmatrix} \quad (2.5)$$

Then, we can conclude the simulation algorithm

**Data:** this text

**Result:** how to write algorithm with L<sup>A</sup>T<sub>E</sub>X2e initialization;

```

while running the simulation world do
    read current;
    if understand then
        go to next section;
        current section becomes this one;
    else
        go back to the beginning of current section;
    end
end

```

**Algorithm 1:** How to write algorithms

## 2.2 Contact Forces Solver

## 2.3 Simulation Results

# Chapter 3

## Particle-grid-particle

The basic method for generating training data which is more accessible to learning is that we will map a discrete element method (DEM) into a continuum setting use techniques from smooth particle hydrodynamics. Given a set of bodies  $\delta$  and a set of contacts between these bodies  $C$ .

### 3.1 Grid-Based method

Traditional rigid motion simulation mainly use particle-based method. However, if we want to replace traditional contact solver with deep learning model, it is hard for cnn model to recognize the original image and do learning. Grid-based method is a good to transfer original image to a grid-cells and then use

### 3.2 Smoothed Particle Hydrodynamics

Smoothed particle hydrodynamics (SPH) was invented to simulate nonaxisymmetric phenomena in astrophysics initially. The principal

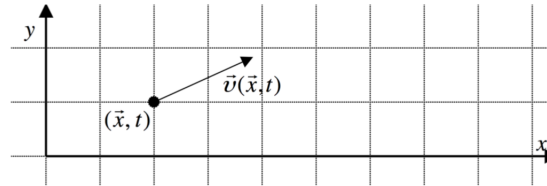


Figure 3.1: Grid description, *retrieved from MIT (2011)*

idea of SPH is to treat hydrodynamics in a completely mesh-free fashion, in terms of a set of sampling particles. It turns out that the particle presentation of SPH has excellent conservation properties. Energy, linear momentum, angular momentum, mass and velocity.

### 3.2.1 Fundamentals

At the heart of SPH is a kernel interpolation method which allows any function to be expressed in terms of its values at a set of disordered points - the particles[1]. For a field  $A(\mathbf{r})$ , a smoothed interpolated version  $A_I(\mathbf{r})$  can be defined by a kernel  $W(\mathbf{r}, h)$ ,

$$A_I(\mathbf{r}) = \int A(\mathbf{r}') W(\|\mathbf{r} - \mathbf{r}'\|, h) d\mathbf{r}' \quad (3.1)$$

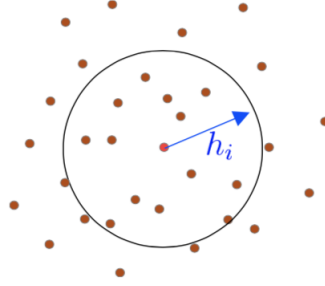


Figure 3.2: Visualization of SPH

where the integration is over the entire space, and  $W$  is an interpolating kernel with

$$\int W(\|\mathbf{r} - \mathbf{r}'\|, h) d\mathbf{r}' = 1 \quad (3.2)$$

and

$$\lim_{h \rightarrow 0} W(\|\mathbf{r} - \mathbf{r}'\|, h) d\mathbf{r}' = \delta(\|\mathbf{r} - \mathbf{r}'\|) \quad (3.3)$$

Normally, we want the kernel to be Non-negative and rotational invariant.

$$W(\|\mathbf{x}_i - \mathbf{x}_j\|, h) = W(\|\mathbf{x}_j - \mathbf{x}_i\|, h) \quad (3.4)$$

$$W(\|\mathbf{r} - \mathbf{r}'\|, h) \geq 0 \quad (3.5)$$

For numerical work, we can use midpoint rule,

$$A_I(\mathbf{x}) \approx A_S(\mathbf{x}) = \sum_i A(\mathbf{x}_i) W(\|\mathbf{x}_i - \mathbf{x}\|, h) \Delta V_i \quad (3.6)$$

Since  $V_i = m_i / \rho_i$

$$A_S(\mathbf{x}) = \sum_i \frac{m_i}{\rho_i} A(\mathbf{x}_i) W(\|\mathbf{x}_i - \mathbf{x}\|, h) \quad (3.7)$$

The default, gradient and Laplacian of  $A$  are:

$$\begin{aligned} \nabla A_S(\mathbf{x}) &= \sum_i \frac{m_i}{\rho_i} A(\mathbf{x}_i) \nabla W(\|\mathbf{x}_i - \mathbf{x}\|, h) \\ \nabla^2 A_S(\mathbf{x}) &= \sum_i \frac{m_i}{\rho_i} A(\mathbf{x}_i) \nabla^2 W(\|\mathbf{x}_i - \mathbf{x}\|, h) \end{aligned} \quad (3.8)$$

### 3.2.2 Kernels

Smoothing kernels functions are one of the most important points in SPH. Stability, accuracy and speed of the whole method depends on these fuctions. Different kernels are being used for different purposes. One possibilyty for  $W$  is a Gaussian. However, most current SPH implementations are based on kernels with finite support. We mainly introduce gaussian, poly6 and spicky kernel here. And compare the different kernels and their property.

#### Poly6

The kernel is also known as the 6th degree polynomial kernel.

$$W_{poly6}(\mathbf{r}, h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - \|\mathbf{r}\|^2)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \quad (3.9)$$

Then, the gradient of this kernel function can be

$$\nabla W_{poly6}(\mathbf{r}, h) = -\frac{945}{32\pi h^9} \begin{cases} \mathbf{r}(h^2 - \|\mathbf{r}\|^2)^2 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \quad (3.10)$$

The laplacian of this kenel can be expressed by,

$$\nabla^2 W_{poly6}(\mathbf{r}, h) = -\frac{945}{16\pi h^9} \begin{cases} (h^2 - \|\mathbf{r}\|^2)(3h^2 - 7\|\mathbf{r}\|^2) & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \quad (3.11)$$

## Spicky

The kernel proposed by Desbrum[2]

$$W_{spiky}(\mathbf{r}, h) = \frac{15}{\pi h^6} \begin{cases} (h - \|\mathbf{r}\|)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \quad (3.12)$$

Then, the gradient of spiky kernel can be described by,

$$\nabla W_{spiky}(\mathbf{r}, h) = -\frac{45\mathbf{r}}{\pi h^6 \|\mathbf{r}\|} \begin{cases} (h - \|\mathbf{r}\|)^2 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \quad (3.13)$$

The laplacian of spiky can be expressed by,

$$\nabla^2 W_{spiky}(\mathbf{r}, h) = \frac{90}{\pi h^6} \begin{cases} h - \|\mathbf{r}\| & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \quad (3.14)$$

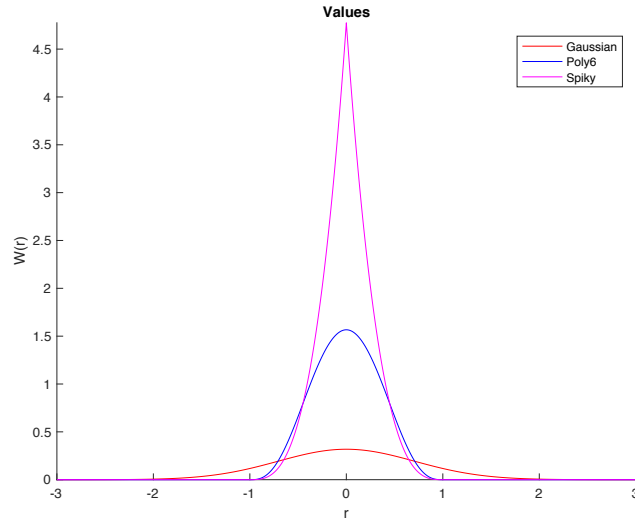


Figure 3.3: Comparison of different kernels, we set smoothing length  $h = 1$  here.

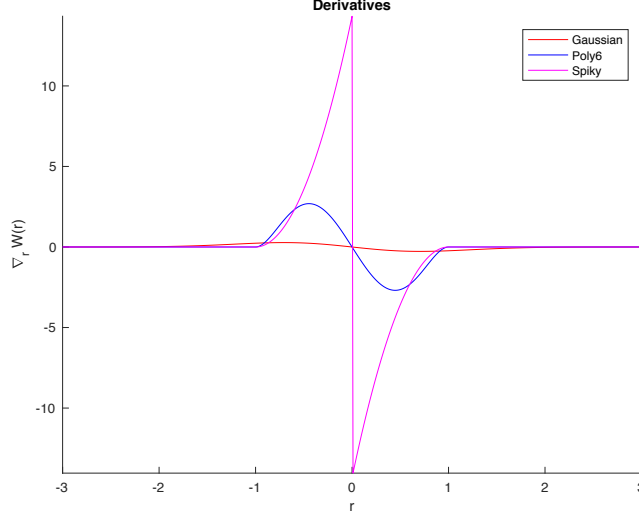


Figure 3.4: Comparison of gradient of different kernels, we set  $h = 1$  here.

### 3.2.3 Grid size and smoothing length

The grid should be also fine enough to capture the variation in our simulation. In our case, it is reasonable to have a grid fine enough such that no two contact points are mapped into the same cell.

Smoothing length,  $h$ , is one of the most important parameters that affects the whole SPH method by changing the kernel value results and neighbor searching results. Too small or too big values might cause lose essential information in the simulation.

### 3.2.4 Neighbor Search

Neighbor search is one of the most crucial procedures in SPH method considering all interpolation equations,  $A(\mathbf{r})$ , needs the neighbor list for every particle (refer to equation 3.8). A naive neighbor searching approach would end up with a complexity of  $O(n^2)$ . The complexity is not good enough since it is impossible to reach any interactive speed when the particle count increases. With an efficient nearest neighbor searching (NNS) algorithm, it is possible to have a signifi-

cant performance increase since it is the most time consuming procedure in SPH computation. In order to decrease the complexity, we choose to use  $k - d$  tree data structure to store the particle spatial information and then do the nearest neighbor searching.

### **Hierarchical Tree**

Using an adaptive hierarchy tree search is proposed by Paiva[3] to find particle neighbors. Since the simulation takes place in two dimensions,  $k - d$  tree data structure was used in this approach.

An octree structure has been adapted from Macey (b) Octree Demo. The hierarchy tree is formed with a pre-defined height. The simulation box is divided recursively into eight pieces, nodes. The nodes at height = 1 are called leaves. Each node has a surrounding box for particle query which is used to check if the particle is inside the node. Each parent node, contains the elements that are divided through its children. The particle is being checked at each level of the tree if it's intersecting the node. If it does, descend one level down in that node. After reaching to leaves, bottom level, particle has been checked if its within the search distance,  $h$ . If the particle is in the range, add it to the neighbor list. Neighbor searching is done when the whole tree has been traversed. The search distance set to be smoothing length in this implementation.

The complexity of this tree search method is  $O(n \log(n))$ ,  $n$  being the number of particles. The performance of this algorithm is worse than Spatial Hashing method. In addition, the results obtained using this NNS algorithm wasn't accurate and stable for this implementation. Therefore as mentioned above, Spatial Hashing method was preferred.

## **3.3 Grid to particle**

After getting the grid image for simulation state in  $t$ , we will use the grid cells as input and renew the grid image based on trained model. Once the grid image in  $t + \delta t$  have been renewed, the next step is to transfer the grid to particles which will instore all information including velocity,



### 3.3.1 interpolation

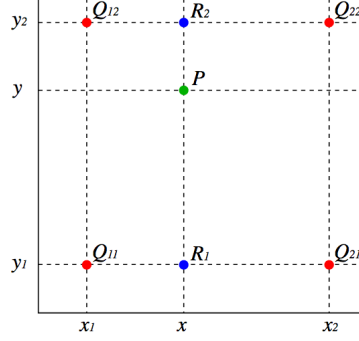


Figure 3.5: Comparison of gradient of different kernels, we set  $h = 1$  here.

We first do linear interpolation in the  $x$ -direction. This yields

$$\begin{aligned} f(x, y_1) &\approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}), \\ f(x, y_2) &\approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}). \end{aligned} \quad (3.15)$$

$$\begin{aligned} f(x, y) &\approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2) \\ &= \frac{y_2 - y}{y_2 - y_1} \left( \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}) \right) \\ &\quad + \frac{y - y_1}{y_2 - y_1} \left( \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}) \right) \\ &= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \left( \begin{aligned} &f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{21})(x - x_1)(y_2 - y) \\ &+ f(Q_{12})(x_2 - x)(y - y_1) + f(Q_{22})(x - x_1)(y - y_1) \end{aligned} \right) \\ &= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \begin{bmatrix} x_2 - x & x - x_1 \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} f(Q_{11}) & f(Q_{12}) \\ f(Q_{21}) & f(Q_{22}) \end{bmatrix} \begin{bmatrix} y_2 - y \\ y - y_1 \end{bmatrix} \end{aligned} \quad (3.16)$$

## 3.4 Conclusion

# Chapter 4

## Deep Learning For Simulation

4.1 Convolutional Neural Networks

4.2 CNN Constructure

4.3 Traing Results

4.4 Simulation based on Trained model

# Chapter 5

## Impletement

# References

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