Deep Contact

Accelerating Rigid Simulation with Convolutional Networks

J. Wu

Department of Computer Science
University of Copenhagen

Master Thesis Defense, 2018

Outline

1. Introduction
   - Previous Work
   - Thesis Overview
2. Particles-Grid-Particles
   - Grid-Particle Method
   - Smoothed Particle Hydrodynamics
   - Bilinear Interpolation
3. Deep Learning Model
   - Convolutional Neural Networks
   - CNN Architecture
   - Training Configuration
4. Results and Analysis
5. Future Work

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Previous Work

- Deep Learning applied in computer simulation.
  - Deep Learning used in liquid dynamic simulation
  - Visual Interaction Networks(DeepMind)
- Speed up contact simulation.
  - Optimization for contact solver

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$$\begin{aligned}
\mathbf{w} &= \boldsymbol{J}_n\mathbf{v}^{t+1} \\
&= \underbrace{\boldsymbol{J}_n\boldsymbol{M}^{-1}\boldsymbol{J}_n^T\Delta t}_{\boldsymbol{A}}\lambda_n + \underbrace{\boldsymbol{J}_n(\Delta t\boldsymbol{M}^{-1}\mathbf{F}_{ext} + \mathbf{v}^t)}_{\boldsymbol{b}} \\
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\end{aligned}$$

$$\boldsymbol{\lambda} = LCP(\boldsymbol{A}, \boldsymbol{b})$$

Data: $N, \boldsymbol{\lambda}_{init}, \boldsymbol{A}, \boldsymbol{b}$
Result: Compute the values of $\boldsymbol{\lambda}$, the convergence rate $\boldsymbol{\theta}$
for $k = 1$ **To** N **do**
  if $k = 1$ **then**
    $\boldsymbol{\lambda} \leftarrow \boldsymbol{\lambda}_{init}$
  end
  $\boldsymbol{\lambda}_{old} \leftarrow \boldsymbol{\lambda}$
  for *all i* **do**
    $\boldsymbol{r}_i \leftarrow \boldsymbol{A}_{i*}\boldsymbol{\lambda} + \boldsymbol{b}_i$;
    $\boldsymbol{\lambda}_i \leftarrow \max(0, \boldsymbol{\lambda}_i - \frac{\boldsymbol{r}_i}{\boldsymbol{A}_{ii}})$;
  end
  $\boldsymbol{\theta}_k \leftarrow \max(|\boldsymbol{\lambda} - \boldsymbol{\lambda}_{old}|)$
end

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- It can make the simulation states be expressed by a set of matrixes, which can be accessible for deep neural networks.
- Grid image can restore the distribution of mass, linear velocity, angular velocity for deep learning neural networks, while the visualization image of simulation can only describe the position of rigid.

Grid-Particle Method

Workflow

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3. For all contacts positions, interpolated values will be generated based on label image. Then, the values will be used as starting iterate values for contact force solver. In our hypothesis, the given starting values will speed up the solver to reach convergence.

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Smoothed Particle Hydrodynamics

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$$A_S(\mathbf{x}) = \sum_i A(\mathbf{x}_i)W(\|\mathbf{x}_i - \mathbf{x}\|, h) \tag{3}$$

- **Poly6**

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$$W_{poly6}(\mathbf{r}, h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - \|\mathbf{r}\|^2)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

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- **Spicky**

$$W_{spiky}(\mathbf{r}, h) = \frac{15}{\pi h^6} \begin{cases} (h - \|\mathbf{r}\|)^3 & 0 \leq \|\mathbf{r}\| \leq h \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

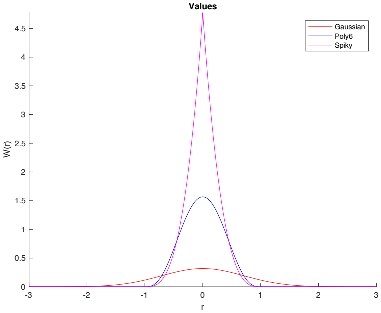

Figure: Comparation of different kernels, we set smoothing length $h = 1$ here.

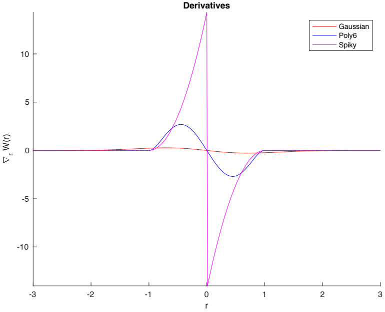

Figure: Comparation of gradient of different kernels, we set $h = 1$ here.

Mapping bodies into a grid image

Data: Given a set of bodies $\mathcal{B}$ and the state in time t, as well as all the spacial positions of grid nodes $\mathbf{x}$

Result: the body grid image $\boldsymbol{G}_{\mathcal{B}}$

for *all i and j* **do**

1. Find the nearest neihbors $\mathcal{B}_{near} \subseteq \mathcal{B}$ around $\mathbf{x}_{ij}$
2. read the current state,

$$m_k, \boldsymbol{v}_k, \boldsymbol{q}_k, \omega_k \quad k \in \mathcal{B}_{near}$$

3. Define state vector $\boldsymbol{S}_k \leftarrow [m_k, v_{k_x}, v_{k_y}, \omega_k]$
4. Compute grid values

$$\boldsymbol{G}_{\mathcal{B}}(i,j) \leftarrow \sum_{k \in \mathcal{B}_{near}} W(\mathbf{x}_{ij}, \boldsymbol{q}_k)\boldsymbol{S}_k$$

end

Mapping contacts into a grid image

Data: Given a set of contacts $\mathcal{C}$ between a set of bodies $\mathcal{B}$ and the state in time t, as well as all the spacial positions of grid nodes $\mathbf{x}$

Result: the contact grid image $\boldsymbol{G}_\lambda$

for *all i and j* **do**

1. Find the nearest neihbors $\mathcal{C}_{near} \subseteq \mathcal{C}$ around $\mathbf{x}_{ij}$
2. read the current contact forces values and its position.

$$\boldsymbol{q}_k, \boldsymbol{\lambda}_k \quad k \in \mathcal{C}_{near} \quad \boldsymbol{\lambda} = [\lambda_n, \lambda_t]$$

3. Compute grid values

$$\boldsymbol{G}_\lambda(i,j) \leftarrow \sum_{k \in \mathcal{C}_{near}} W(\mathbf{x}_{ij}, \boldsymbol{q}_k)\boldsymbol{\lambda}_k$$

end

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Bilinear Interpolation

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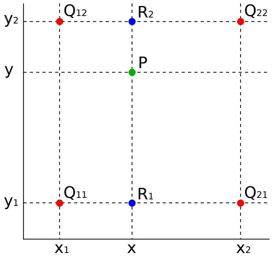

Figure: The figure shows the visualization of bilinear interpolation. The four red dots show the data points and the green dot is the point at which we want to interpolate.

Bilinear

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$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}), \tag{6a}$$

$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}). \tag{6b}$$

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After getting the two values in $x-$direction $f(x,y_1)$ and $f(x,y_2)$, we can combine these values to do interpolation in $y-$ direction.

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After getting the two values in $x-$direction $f(x, y_1)$ and $f(x, y_2)$, we can combine these values to do interpolation in $y-$ direction.

$$f(x, y) \approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2) \tag{7}$$

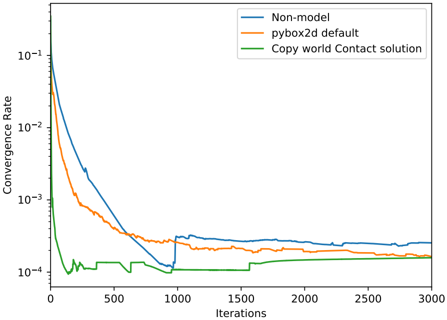
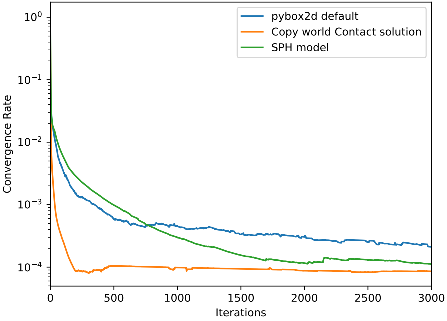


Figure: Average convergence rate for different models(not including **SPH-based model**).

SPH method
Experiments



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Deep Learning Model

Convolutional Neural Networks(CNN)

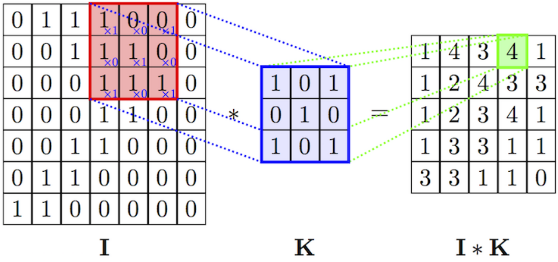

Figure: One simple example of convolution.

Deep Learning Model

Convolutional Neural Networks(CNN)

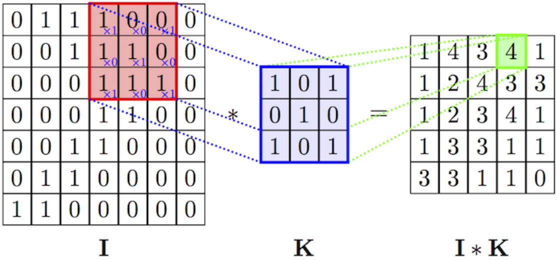

Figure: One simple example of convolution.

$$(\boldsymbol{I} * \boldsymbol{K})_{xy} = \sum_{i=1}^{h} \sum_{j=1}^{w} \boldsymbol{K}_{ij} \cdot \boldsymbol{I}_{x+i-1,y+j-1} \tag{8}$$

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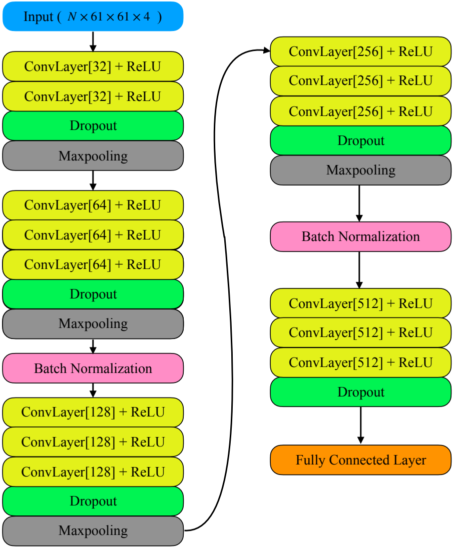
CNN Architecture



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Loss function

Firstly, we define a filter funtion,

$$g(x) = \begin{cases} 0, & x = 0 \\ 1, & x \neq 0 \end{cases} \tag{9}$$

Then, we can update the loss function.

$$L = \frac{1}{N} \sum_{i}^{N} g(\hat{y}_i)(y_i - \hat{y}_i)^2 \tag{10}$$

Learning Rate Scheduling

Data: *epoch*
Result: learning rate η
if $epoch < 100$ **then**
  $\eta = 5 \times 10{-}3$
end
if $100 < epoch < 300$ **then**
  $\eta = 2 \times 10^{-3}$
end
if $300 < epoch < 500$ **then**
  $\eta = 1 \times 10^{-3}$
end
if $epoch > 300$ **then**
  $\eta = 2 \times 10^{-4}$
end

Algorithm 1: Learning Rate Scheduling

Training Hyperparameter

Hyperparameter	Setting
Activation function	ReLU
Weight initilization	He normal
Weight regularizer	L2
Convolution border mode	Same
Stride	2
Kernel size	$(3, 3)$
Dropout rate	0.1
Optimizer	SGD
Initial Learning Rate	$1 \times 5 \times 10^{-3}$
Batch Size	200
Epoch	1000
Validation Rate	0.2

Table: Hyperparameter settings.

Simulation Configuration

- **World Setting**

Simulation Configuration

- **World Setting** the world box size is 30×30, and there are $50 - 100$ circle rigid bodies($r = 1$, all circle rigid bodies in the same size.) inside the box. Initially, the rigid circles will be located following gaussian distribution2. Then, all rigid circles will fall down by gravity.

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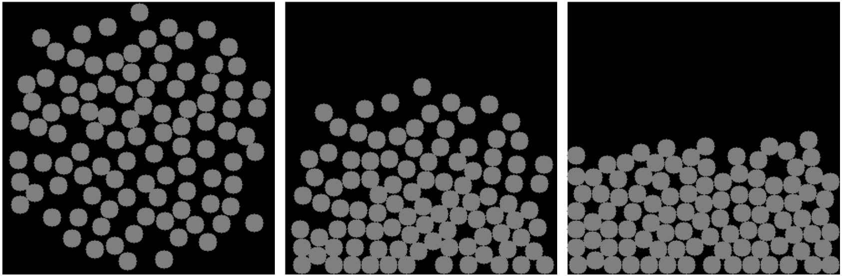
- **Simulation Setting** there will be totally 600-steps simulation. For each step, $\Delta t = 0.01s$, and the number of iteration in each step will be set as fixed, 3000.



(a) Time Step=0 (b) Time Step=200 (c) Time Step=400

Figure: Visualization for experiment simulation

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- For a given d, $h \geq \frac{\sqrt{2}}{2} d \approx 0.71d$

Results and Analysis

SPH parameters(**Poly6**)

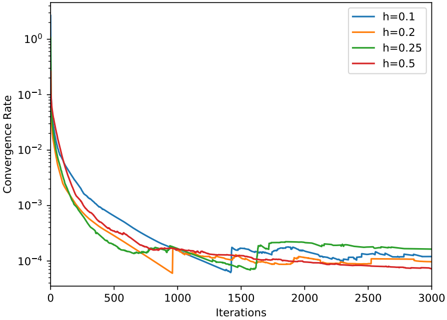

Figure: The grid size d is set 0.25. $h = 0.1, 0.2, 0.25, 0.5$ is tested respectively. This figure shows different coveragence rate based on different h value. The kernel is **Poly6**

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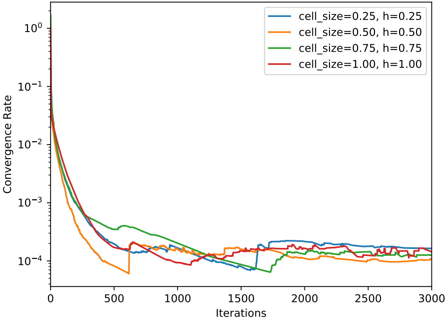

Figure: Coveragence rate for different d value. The kernel is **Poly6**

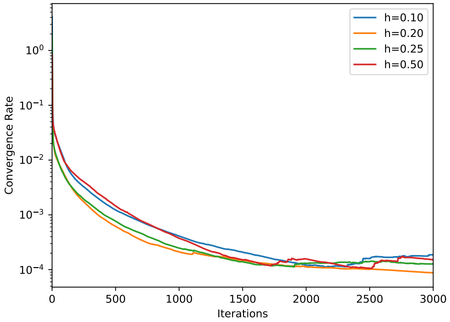

Figure: The grid size d is set 0.25. $h = 0.1, 0.2, 0.25, 0.5$ is tested respectively. This figure shows different coveragence rate based on different h value. The kernel is **Spiky**

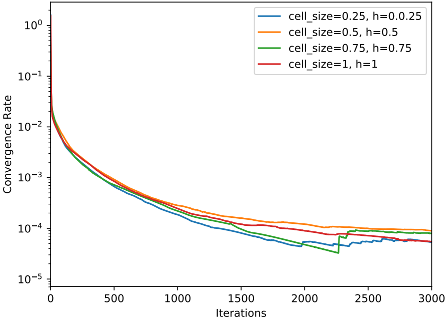

Figure: Coveragence rate for different *d* value. The kernel is **Spiky**

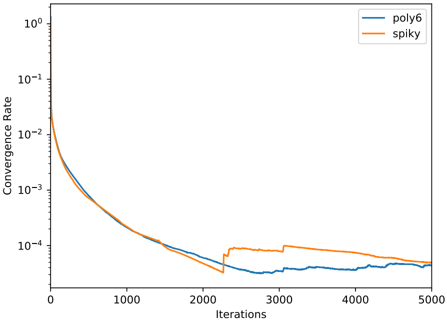

Figure: Coveragence rate for kernel **Poly6** anf **Spiky**. $h_{\text{poly6}} = d_{\text{poly6}} = 0.5$, while $h_{\text{spiky}} = d_{\text{spiky}} = 0.25$

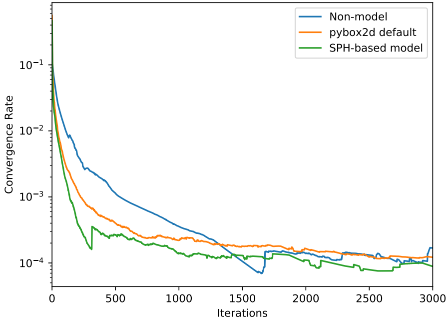

Figure: Coveragence rate for models(different initial values for $\boldsymbol{\lambda}$).

- **Input Size**, the input will be $61 \times 61 \times 4$. Since the orginal world is 30×30 and grid size is $d = 0.5$, the generated grid would be 61×61. There would 4 channels$[m, v_x, v_y, \omega]$

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- **Output Size**, output size depends on the label size. The orginal label image would be $[\lambda_n, \lambda_t]$, so the label image size would be $61 \times 61 \times 2$, which should be flattened as the actual training label. The label size would be $61 \times 61 \times 2$

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- **Weights Number**, the total weights number is $64, 498, 866$.
- **Training Environment**, GPU(*GeForce GTX 1080 Ti, 11 Gbps GDDR5X memory*) held by Image Section, DIKU.

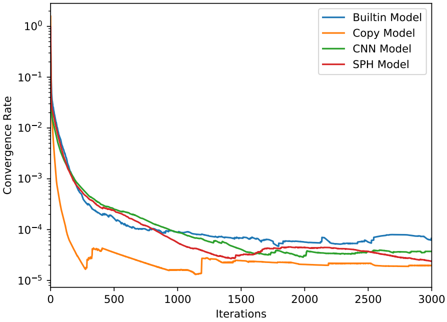

Figure: The final result. Add the final CNN solution to compare with other methods.

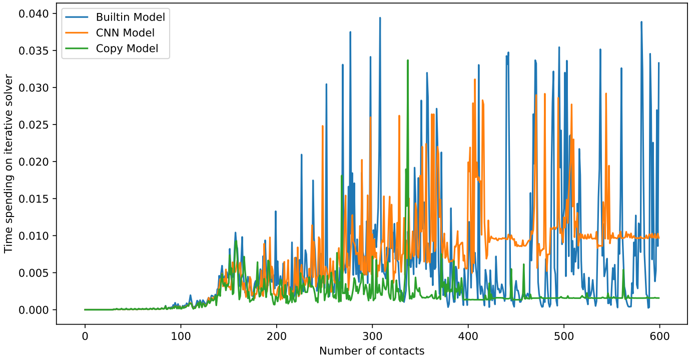

Figure: Time spent for contact solver iteration

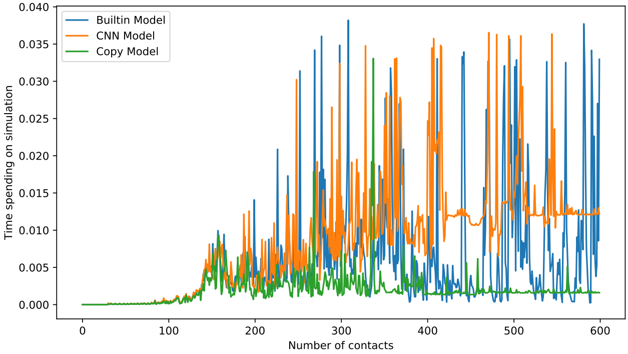

Figure: Time spend for the whole contact solution(including warm starting calculation).

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- **CNN-Model** performs very similar to **SPH-Model**, which means the CNN predicts contact image well.
- Due to the limitation of the SPH-based method, **CNN-Model** just gets a small improvement compared with **Builtin-Model**. And it cannot perform as good as **Copy-Model**.

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3. **More Shapes Experiments**

Thanks

- Thanks for Kenny's supervision
- Thanks for Lukas and Lucian's co-operation on initial work.