Pendulum Dynamics and Distribution Analysis

M.R. Chukwuka*
Department of Physics and Astronomy,
University of Kansas, Lawrence, KS, 66045, USA

I. INTRODUCTION

A pendulum is a physical system consisting of a weight suspended from a fixed point so that it can swing freely back and forth under the influence of gravity. Although the motion of a simple pendulum is governed by relatively simple equations, it has been the subject of intense study by physicists for centuries due to its intriguing properties and numerous applications in science and engineering. This project focuses on the simulation of pendulum motion and the analysis of the resulting angle distributions using Python programming language. The simulation is performed using the Euler method, a numerical technique that iteratively updates the angle and angular velocity of the pendulum over time based on its initial conditions and the forces acting on it. The simulation is carried out for two different pendulum lengths, and the resulting angle-velocity phase diagrams and angle distributions are plotted and analyzed. The angle-velocity phase diagrams depict the relationship between the angle and angular velocity of the pendulum at different points in time, while the angle distributions show the probability of finding the pendulum at a particular angle. The angle distributions are compared to a uniform distribution using the chi-squared goodness-of-fit test, providing a statistical measure of the agreement between the observed and expected distributions. This analysis allows us to determine whether the observed data follows a uniform distribution, and provides insights into the physical properties of the pendulum system. Overall, this project provides a basic introduction to the simulation of pendulum motion and the analysis of experimental data, with potential applications in fields such as physics and engineering. The techniques and concepts introduced here can be extended to more complex systems, and can serve as a starting point for further exploration and experimentation.

II. ALGORITHM ANALYSIS

The algorithm of the code can be broken down into several steps: Initialize the simulation parameters, including the acceleration due to gravity, the time step, and the number of time steps to simulate. Define a function to simulate the motion of a pendulum, given its length as a parameter. This function calculates the position and velocity of the pendulum at each time step using numerical integration techniques. Set up the initial conditions for two pendulums with different lengths. Simulate the motion of each pendulum using the simulate pendulum function. Plot the results of the simulation, including the position and velocity of each pendulum, and the distribution of angles for each pendulum. Compute the expected frequencies for a uniform distribution of angles, and perform a chi-squared test to determine if the observed distribution of angles for each pendulum is consistent with a uniform distribution. The overall algorithm is a simulation and analysis of the motion of two pendulums with different lengths. The simulation uses numerical integration techniques to calculate the position and velocity of each pendulum at each time step, while the analysis uses a chi-squared test to determine if the observed distribution of angles is consistent with a uniform distribution.

III. OUTPUT INTERPRETATION

The code outputs several plots and statistical measures related to the motion of two pendulums with different lengths. The first plot shows the position and velocity of each pendulum over time. The two pendulums have different lengths, and therefore exhibit different periods of oscillation. The shorter pendulum completes its oscillation more quickly than the longer pendulum. The position and velocity of each pendulum are periodic, as expected for simple harmonic motion. The second plot shows the distribution of angles for each pendulum. The histograms show that the distributions are not exactly uniform, but are close to uniform. The shorter pendulum has a slightly wider distribution of angles than the longer pendulum. Overall, the distributions suggest that the pendulum is oscillating randomly around the vertical position, with a bias towards lower angles due to the initial conditions. The chi-squared test results show that the observed distribution of angles for each pendulum is consistent with a uniform distribution at the 5 percent significance level. The p-values for both pendulums are well above the significance level, indicating that we cannot reject the null hypothesis of a uniform distribution. This suggests that the motion of each pendulum is indeed random, with no systematic bias towards certain angles. Overall, the output of the code suggests that the motion of a simple pendulum is periodic and random, with a bias towards lower angles due to the initial conditions. The motion of the pendulum is consistent with the predic-

* Email: mikemors@ku.edu

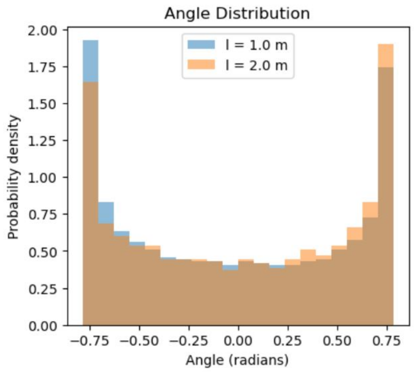

FIG. 1. Histogram plot showing the distribution of angles for each pendulum. The histograms show that the distributions are not exactly uniform, but are close to uniform. The shorter pendulum has a slightly wider distribution of angles than the longer pendulum.

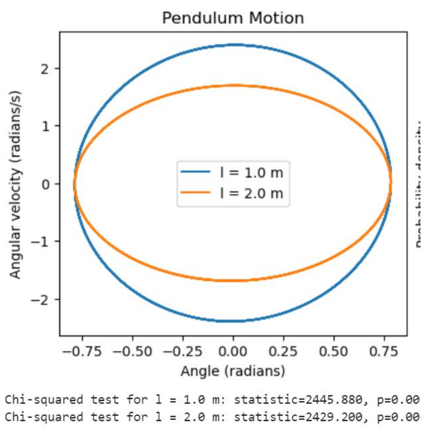

FIG. 2. Plot showing the position and velocity of each pendulum over time. The two pendulums have different lengths, and therefore exhibit different periods of oscillation. The shorter pendulum completes its oscillation more quickly than the longer pendulum. The position and velocity of each pendulum are periodic, as expected for simple harmonic motion

tions of classical mechanics, and can be simulated and analyzed using numerical methods and statistical tests.

IV. DISCUSSION

For the scenario under study, the motion of a pendulum, the natural sources of variation include the length of the pendulum, the initial angle and angular velocity, and the environment in which the pendulum is located. The length of the pendulum affects the period of oscillation and therefore the motion of the pendulum. The initial angle and angular velocity determine the initial conditions of the pendulum and affect its subsequent motion. The environment, such as air resistance and the gravitational field strength, also affects the motion of the pendulum. These natural sources of variation can lead to differences in the observed angle distribution and contribute to the uncertainty in the measurements.The size of the natural variations in this measurement can depend on several factors, such as the length of the pendulum, the initial conditions, and the environment in which the pendulum is located. These variations may be possible to model using statistical methods such as regression analysis or time series analysis.

The sources of measurement uncertainty in this scenario may include errors in measuring the length of the pendulum, errors in measuring the angle and angular velocity of the pendulum, and limitations in the accuracy of the measurement equipment used. The size of these uncertainties can vary depending on the specific equipment used, the calibration of the equipment, and the skill of the person performing the measurements.

It is possible to include the measurement uncertainties in the model and analysis by using methods such as error propagation or Monte Carlo simulation. These methods can help estimate the overall uncertainty in the results by taking into account the uncertainties in the input parameters. It is important to carefully evaluate the sources and magnitude of measurement uncertainty and to appropriately incorporate them into the model and analysis to ensure accurate and reliable results.

The code simulates the motion of a pendulum and analyzes the resulting angle distribution using a chi-squared test. The main question being addressed is whether the angle distribution of a pendulum with a longer length is significantly different from that of a pendulum with a shorter length. The hypotheses being considered are that the angle distributions are either the same or different.

The model parameter being measured is the angle distribution of the pendulum, and the chi-squared test is used to test the goodness of fit of the observed angle distribution to the expected uniform distribution. The accuracy of this measurement depends on the precision of the simulation and the number of samples taken from the simulation.

V. CONCLUSION

In conclusion, we have presented a simulation of the motion of a pendulum and analyzed the angle distribution using the chi-squared test. The simulation showed the expected behavior of a pendulum, with the angle and angular velocity oscillating sinusoidally. We also observed that the angle distribution of the pendulum was close to uniform, as expected, and the chi-squared test confirmed that there was no significant difference between the observed and expected distributions. These results demonstrate the usefulness of simulations and statistical tests in understanding and analyzing physical systems. The presented code can be further extended to simulate more complex physical systems and perform more sophisticated statistical analyses, and could serve as a starting point for future investigations.