Exploring the Robustness of Magnetic Ring Arrays Reservoir Computing with Linear Field Calibration

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Abstract. One of the challenges for reservoir computing is the robustness of the implementation in the face of fabrication error. If a system is too sensitive to fabrication error, then each manufactured reservoir becomes a unique artefact with unique computational properties. Under most circumstances, this is undesirable as it makes reproduction of results, or useful systems, complicated. This paper uses simulation to examine the properties of nano-scale magnetic ring arrays as reservoir computers under parameters corresponding to a wide variety of physically derived parameters, and investigates the effectiveness of linear field calibration to minimise the difference in unexpected behaviour of the systems.

1 Introduction

Reservoir computing seeks to exploit the complex dynamic behaviour of systems to accomplish useful computation. By providing a dynamical system with an appropriate time series input, the system will progress and can be measured to extract useful information, such as categorising the input or predicting the next value of a time series. *In materio* Reservoir Computing places an additional constraint in that the dynamical system is expected to be a physical system grounded in the real world. However, a physical dynamical system is subject to error. Error can take many forms:

– Error inherent in the limitations of the equipment used to measure the system or provide input
– Error dependent on the operating environment of the system, such as temperature
– Error due to unpredictable behaviour of the system
– Error due to limitations when fabricating the system

This paper focuses on the last of these types of error. A physical system must be fabricated in some fashion, and there will always be limitations on how

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accurately the system can be fabricated. Examples of such limitations include purity of substances used in fabrication, and the precision of equipment used in fabrication. As fabrication error is incorporated into the system when it is made, it must be accounted for in the analysis of the system. If fabrication error is not accounted for, then the behaviour of the system may depend on the specific error introduced through fabrication, which would make results difficult to reproduce. Further, if intending to deploy multiple instances of a system to perform the same task, it is necessary for multiple physical instances of the system to be capable of the same behaviour; unmitigated fabrication error may make this difficult or impossible.

While one approach to fabrication error might be to improve the fabrication process until it is negligible, this is not necessarily practical. For example, the physical device on which the work in this paper is based [17] uses systems fabricated via electron beam lithography. This process involves multiple analogue steps that are challenging to reproduce perfectly and thus can lead to fabrication errors. The first of these error prone steps is applying the layers of magnetic material and polymer to the substrate; slight variations in thickness are unavoidable, and result in variations in thickness on the finished product. The focus of the electron beam can introduce further variation; incorrect focusing results in the electron beam having a slightly warped shape depending on position, which in turn impacts how accurately it can cut shapes. The next step is removing the material that is no longer protected by the polymer, which involves wet chemicals and is therefore difficult to reproduce exactly. Finally, even removing the remaining polymer can cause errors as small particles of magnetic material can be lifted from the substrate with the polymer, producing further irregularities.

Where it is not practical to reduce the effects of fabrication error to the point where they are negligible, effects of fabrication error can be mitigated by calibration. If calibration is sufficient, each device can be individually calibrated to have uniform logical behaviour. This paper examines nano-scale magnetic ring arrays, which can be calibrated by adjusting the strength of the magnetic field they are subjected to. By applying a simple linear search it is possible to find the optimal strength of magnetic field for a given ring array that maximises its performance. This technique has been demonstrated on a physical system [17], but it has not been explored in depth to see how calibration can compensate for a wide variety of fabrication errors.

This paper investigates the effectiveness of calibration to compensate for fabrication error by simulating a wide variety of physically derived parameters. These parameters are simulated through the phenomenological level simulator of RingSim [17]. The search is carried out by PyCHARC/SpatialGA [9], a novelty search method that seeks novel behaviours of the system. By characterising the space of possible behaviours under a wide variety of parameters, it is possible to infer the size of this space and therefore how many different uncontrolled behaviours the Ring Array can exhibit within reasonable fabrication error[1].

[1] This differs from controlled behaviours, which under normal circumstances it is desirable to have as many different types of behaviour as possible.

The main purpose of the search of behaviours under calibration is to show under which circumstances the same behaviours can be achieved. The sensitivity of the ring array to fabrication errors is not directly explored by this search. This is in part due to the current method of utilising the ring array requiring calibration [17], which makes it difficult to extract the effects of fabrication errors in isolation as the correct input values for a given use are dependent on the effects of fabrication error. However, the search also yields some information on the sensitivity of the ring array by identifying areas where calibration is insufficient to provide useful behaviours.

1.1 Organisation

Section 2 gives an overview of relevant concepts as well as the current state of the art of the magnetic ring array system evaluated in this paper. Section 3 outlines the details of the design of the experiment, with a particular focus on the measures used. The experiment itself is presented in Sect. 4, and the results follow in Sect. 5. An evaluation of the results in presented in Sect. 6, with a detailed discussion of how the results can be interpreted. Finally, conclusions are given in Sect. 7.

2 Background

Error in a system can be attributable to a number of factors that increase the uncertainty in using a system. Uncertainty [13] can be characterised as one of two types: aleatory uncertainty, which represents the inherent randomness of processes, and epistemic uncertainty, which represents the effect of incomplete knowledge of a system. While aleatory uncertainty is normally unavoidable, epistemic uncertainty can be reduced by a better understanding of the system under consideration [13].

Fabrication error of a physical system falls under the category of epistemic uncertainty. While it may not be practical, a system can be modelled at a higher fidelity that encompasses the effect of fabrication error; for example, using a microscope to detect physical defects and compensate for them. However, identifying and compensating for defects in this manner is a time consuming and difficult process and relies upon understanding physical phenomena that may be difficult to model [17]. An alternative to this approach is to calibrate the system [8], by applying some modification to the input or output such that a known or optimal result can be reached. This allows the epistemic uncertainty of the system to be reduced without having to identify the precise nature of defects in the system.

Reservoir computing [15] takes the behaviour of a dynamical system and attempts to extract useful computation from it. The Echo State Network (ESN) [12] is one of the archetypal artificial reservoirs, consisting of a neural network where the hidden layer consists of sparse, random connections.

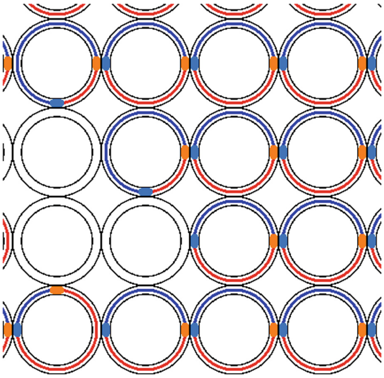

Fig. 1. Simulated segment of Nano-scale Magnetic Ring Array Reservoir under consideration

In materio Reservoir Computing takes the basic idea of reservoir computing and applies it to physical dynamical systems. These can be any physical system that exhibits dynamical behaviours, with examples as diverse as origami-based robotic feedback systems [1] or magnetic thin film arrays [5].

This paper investigates the Nano-scale Magnetic Ring Array reservoir, as defined by Dawidek et al. [7]. The mano-scale magnetic ring array utilises the phenomena of constrained domain walls to obtain reservoir properties. In the presence of a sufficiently strong magnetic field, domain walls nucleate in pairs on the rings. These domain walls can then be driven around the rings by a rotating magnetic field, exhibiting stochastic pinning behaviour at the junctions between rings. If a domain wall is pinned and comes into contact with its counterpart, it annihilates. By modulating the strength of the rotating magnetic field it is possible to provide an input to the system, and by measuring the electrical resistance of the ring array it is possible to measure some of the properties of the Domain Wall population using techniques such as anisotropic magnetoresistance [2].

While it is possible to model magnetic rings using general purpose micromagnetic simulators [16], such an approach is computationally too expensive for systems with a large number of rings. To address this, RingSim [17] was developed, which is a phenomological model of the behaviour of Domain Walls in the magnetic ring array. RingSim uses a number of physically derived parameters to characterise the behaviour of the Domain Walls, and by varying these parameters it is possible to model the behaviour of a wide variety of physical systems. As such, RingSim is a useful model for exploring fabrication error.

Both the hardware implementation of the magnetic ring array and RingSim are designed with calibration in mind [17], and in both cases a search is conducted on the magnetic field strength to maximise the response of the physical or simulated ring array. The only constraints on this search are the strength of magnetic field that can be produced by the equipment.

CHARC [6] is a novelty search algorithm that seeks to explore a behaviour space. Novelty search [14] algorithms seek to find new or novel behaviours, in contrast to optimising search algorithms that seek to optimise for a given problem. The goal of exploration as opposed to optimisation is useful for characterising a behaviour space, i.e. the set of behaviours that can be expressed by the various configurations of a system considered by CHARC. CHARC, by default, characterises the behaviour of reservoirs by three measures:

– Kernel rank (KR): the degree to which different inputs produce different outputs [4].
– Generalisation rank (GR): the degree to which similar inputs produce similar outputs [4].
– Linear Memory Capacity (LMC): the ability of a system to reproduce inputs [11].

3 Design

3.1 Search Method

To explore the behaviour of the Ring Array as characterised by RingSim, PyCHARC was used. PyCHARC is an evolution of the CHARC novelty search with an emphasis on extensibility. PyCHARC was used to conduct a novelty search over the characterisation of RingSim as given by KR, GR and LMC. As the Ring Array provides only a single output in its native configuration, the Ring Array was set up with time multiplexed outputs where multiple measurements of the ring array over the rotation of the magnetic field were used as input to the readout layer of the reservoir.

One of the differences between PyCHARC and CHARC is the ability to use different types of search algorithms to explore the behaviour space. The original search algorithm used by CHARC is based on the Microbial Genetic Algorithm [10], which, although elegantly simple, suffers from poor parallelisability. This lack of parallelisability limits how many input parameters CHARC can explore.

To counter this, these experiments used a new approach called SpatialGA. SpatialGA explores by first partitioning the behaviour space and identifying partitions that have high levels of diversity from the initial population, as characterised by the density of individuals in the partition. Once the most diverse partition is identified, multiple individuals are generated by applying crossover and mutation operators to the individuals in that partition, with the hope of finding more individuals within the partition. Once the number of individuals within the partition exceeds a threshold, it is divided into sub-partitions and the process starts over[2].

[2] For brevity some details, such as penalty terms on selecting the partition to explore that ensure that all areas of the behaviour space are explored rather than focusing on a infinitesimally small but interesting area, are omitted.

Algorithm 1. Pseudocode for Kernel Rank (KR)

$input\_length := system.washout + kr\_input\_size$
$input\_signal := [uniform(0.0, 1.0)$ **for** each **input** to system, **for** $input\_length]$
$output := system.run(input\_signal)$
remove first $system.washout$ elements from $output$
$KR := matrix\_rank(output)$
return KR

Algorithm 2. Pseudocode for Generalisation Rank (GR)

$input\_length := system.washout + gr\_input\_size$
$input\_signal := [uniform(0.4, 0.6)$ **for** each **input** to system, **for** $input\_length]$
$output := system.run(input\_signal)$
remove first $system.washout$ elements from $output$
$GR := matrix\_rank(output)$
return GR

3.2 Measures

While KR, GR, and LMC are "standard" measures of the behaviour of reservoirs, we are aware that there are multiple divergent implementations of these measures. The specific versions of the measures used in this work are defined here.

Kernel Rank (KR) is a measure of the ability of a reservoir to produce different outputs for different inputs. It is calculated by supplying the system under consideration with a uniform random input stream between 0 and 1, and calculating the rank of the matrix of outputs (with a threshold on SVD values of 0.01). A pseudocode implementation is given in Algorithm 1.

Generalisation Rank (GR) is a measure of the ability of a reservoir to generalise similar inputs to similar outputs. It is calculated in a similar manner to Kernel Rank, only the random input stream is over the much smaller range of 0.4 to 0.6. A pseudocode implementation is given in Algorithm 2.

Linear Memory Capacity (LMC) is a measure of how much linear memory can be stored within the reservoir. This is calculated by attempting to construct a readout layer for the reservoir that can reproduce the last n inputs. The LMC is defined as a measure of how well the outputs of the reservoir correspond to the inputs that it is supposed to represent. A pseudocode implementation is given in Algorithm 3.

3.3 Ring Array Calibration

As in prior work [17], the Ring Array requires calibration to maximise its ability to conduct useful work. In physical systems, this is in part due to the fabrication error causing intra-device variation.

The Ring Array takes a single input in the form of a global rotating magnetic field. The magnetic field drives the domain walls in the system, causing them to rotate with it unless they become pinned at a junction.

Algorithm 3. Pseudocode for Linear Memory Capacity (LMC)

```
def output_signal(input_signal, system):
return [ [ input_signal shifted back by x + 1 ] for x in range(system.outputs) ]

input_length := system.washout + lmc_input_length + system.outputs
train_input_signal = [uniform(0.0, 1.0) for input_length]
system.train( train_input_signal broadcast to all inputs of system, output_signal(train_input_signal) )
test_input := [uniform(0.0, 1.0) for input_length]
test_output := output_signal(test_input)
predictions := system.run( test_input_signal broadcast to all inputs of system )
remove first system.washout values from test_input, predictions, test_output
LMC := 0
inpvar = variance(test_input)
for each output of system do
    covar := covariance(predictions, test_output)
    pvar := variance(predictions)
    mc := (covar ** 2) / (inpvar * pvar)
    if mc ¿= min_memory_capacity then
        LMC += mc
    end if
end for
return LMC
```

Calibration is accomplished by using a simple linear search over the available ranges of magnetic field strengths, retraining the output layer for each new field strength, and picking the magnetic field with the best response. If necessary, the process can be adapted into a depth-first search to refine the calibration further. The physical implementation used in the work of Vidamour et al. [18] was constrained to a maximum magnetic field strength of approximately 60 Oe.

Simulation differs from physical systems in that any magnetic field can be simulated, even if it is not something that is practical to realise in a physical system. This allows us to explore across a wider range of magnetic fields and conduct analysis on both realistic magnetic fields and unrealistic magnetic fields.

An additional effect of simulation is that parameters can also be consolidated: in a physical system, multiple physical parameters affect the probability of events. For example, the probability of pinning at a junction is influenced by edge roughness, any imperfections, temperature, material, and potentially other parameters. RingSim simplifies these parameters to a description of the required energy barriers [17], which are still represented by a large number of "physically derived" parameters. This paper explores a further simplification that directly exposes the probabilities of pinning to PyCHARC for exploration.

Table 1. Parameters used in RingSim for Experiment 1

Parameter	Range	Explanation
E0 E0D H0 H0D	$[1.0 \times 10^{-19}, 3.0 \times 10^{-19}]$ $[0.5 \times 10^{-21}, 1.5 \times 10^{-21}]$ $[80, 90]$ $[10, 20]$	Characterises properties of the geometry of the rings
ER ERD	$[20, 30]$ $[0.5, 3.0]$	Characterisation of the minimum propagation field of the system
PCE PCH	$[0.5, 1.0]$ $[0.5, 1.0]$	Characterisation of behaviour of two domain walls occupy one junction
alpha	$[1.0, 2.0]$	Value characterising how energy barrier varies with magnetic field

4 Experiments

Two primary experiments were conducted for this work. Both experiments used PyCHARC/SpatialGA to search over input parameters for RingSim. These configurations of RingSim were then calibrated using a linear search, and the calibrated system was measured to determine its KR, GR and LMC. These measurements were then fed back into PyCHARC/SpatialGA to inform the algorithm on where to explore next.

The two experiments differed in the parameters that were used as follows:

- Experiment 1: The default set of 9 parameters that characterises the energy barriers required, which control the probabilities of pinning at junctions. Parameters for this experiment are given in Table 1. The ranges of these parameters were initially selected to cover the full range of parameters that have been observed in fitting RingSim parameters to various physical devices, such as the device used in [17]. These ranges were then expanded to those shown in Table 1 to cover a range of values that are plausible given the currently fabricated devices, to allow PyCHARC/SpatialGA to search over a wider area.
- Experiment 2: An alternate set of 3 parameters that exposes the probabilities derived from the energy barriers. Parameters for this experiment are given in Table 2. Parameters for this experiment were selected to roughly correspond to the parameters of Experiment 1.

For each experiment, PyCHARC/SpatialGA was run for 250 generations with a target population of 25 individuals per region, to find configurations spanning as much of the behaviour space as possible. For each of the measures KR, GR and LMC, an optimisation pass was run to optimise the transformation function between the logical inputs of the system, which are specified between 0 and 1

Table 2. Parameters used in the simplified form of RingSim for Experiment 2

Parameter	Range	Explanation
BP PD	$[0,1]$ $[0,0.5]$	Distribution of the probability of pinning at junction.
DE	$[1,3]$	Modifier for when two domain walls occupy a junction.

and the simulated magnetic field, $f(x) = ax+b$. This optimisation was restricted so that the image of the function was between 15 Oe and 65 Oe.[3]

The purpose of the second experiment is to determine if reducing the number of dimensions is possible. This is desirable for a number of reasons; Firstly, reducing the parameter set results in a more robust search as there are fewer dimensions of the input space of PyCHARC.

The second reason for why a reduction in dimensionality is desirable is more subtle: if the number of input dimensions can be reduced, then this implies that there is a many-to-one relationship between the default set of parameters and the reduced set. Given that the differences in behaviour are the result of fabrication error, in the absence of correlation between parameters one would assume that their relative independence. This in turn implies that the principles of the Central Limit Theorem [3] can be applied to some extent, and therefore that extreme behaviour due to fabrication error is less likely.

5 Results

5.1 Presentation of Results

The results of the experiment are presented as a graph matrix. Each graph in the matrix is a plot of one parameter against one measure. For each parameter, the individuals found by PyCHARC were placed in bins by the value of the parameter. The plots show a line representing the median measure score for individuals in each of these parameter bins against the value of the measure after calibration, and a region showing the 90% range of the individuals found. This allows the reader to judge the spread of values for the given parameter, which shows how other parameters can affect the plotted parameter.

5.2 Results of Experiment 1

Figure 2 shows the results of the exploration, broken down by the effects of each input dimension. There are two broad categories of effect seen.

In the case of H0, E0, PCE, PCH and alpha, calibration results in the following outcomes:

[3] These limits are somewhat arbitrary, but approximately reflect the limitations of the current physical implementation with regard to sustained magnetic fields.

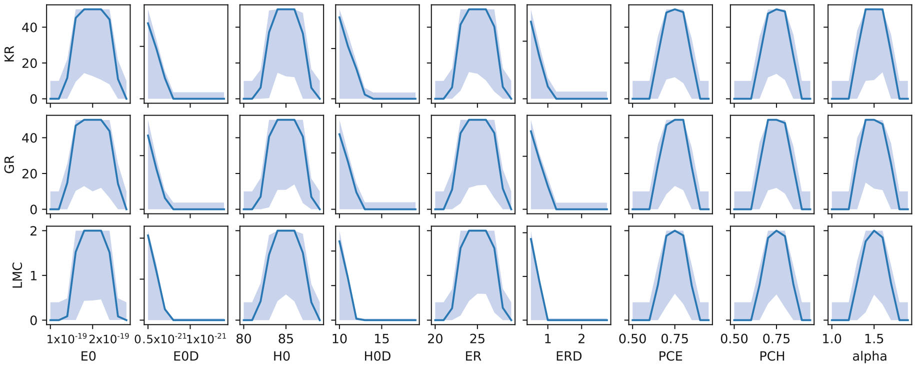

Fig. 2. Results Matrix for Experiment 1. Each graph plots one of the RingSim model parameters against one of the three metrics used by PyCHARC, showing the median and 90% range of individuals found by PyCHARC.

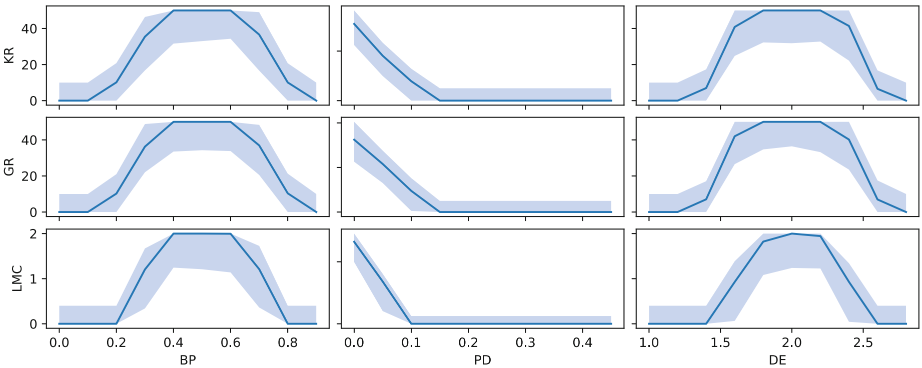

Fig. 3. Results Matrix for Experiment 2. Each graph plots one of the simplified RingSim model parameters against one of the three metrics used by PyCHARC, showing the median and 90% range of individuals found by PyCHARC.

1. Calibration cannot find a sufficiently weak magnetic field, resulting in domain walls not ever pinning at junctions, and therefore no interesting response.
2. Calibration cannot find a sufficiently strong magnetic field, resulting in domain walls always pinning at junctions, and therefore no interesting response.
3. Calibration succeeds, resulting in the reservoir exhibiting interesting behaviour.
4. Another parameter causes calibration to fail.

For these parameters, the transitions between the three outcomes are relatively abrupt.

In the case of the distribution parameters, H0D, E0D, and ERD calibration results in the following outcomes:

1. Calibration cannot find a magnetic field strength that is sufficient for the majority of the Ring Array due to variation within the Ring array being too high. This results in the output being degraded significantly.
2. Calibration finds a magnetic field strength that is sufficient for the majority of the Ring Array, resulting in the reservoir exhibiting interesting behaviour.
3. Another parameter causes calibration to fail.

In contrast to the first set of parameters, the distribution parameters result in a more gradual decline as the effectiveness of calibration decreases.

5.3 Results of Experiment 2

Figure 3 shows the results of the second experiment, using the further simplified model. The results are broadly the same as the first experiment, albeit with a lower number of dimensions. The most notable difference is that as there are fewer parameters, the chance of another parameter interfering with the calibration are substantially lower, resulting in a tighter distribution. However, other than this the behaviours exhibited are broadly the same, as PyCHARC/SpatialGA attempts to explore with respect to the evaluated measures rather than the input parameters.

6 Evaluation

There are multiple conclusions that can be drawn from these results. The most important one is that it is not necessary to explore a large number of parameters in simulations, if these parameters are later combined. While there is a potential difference in the distribution of the parameters, novelty search is capable of identifying unique behaviours regardless of the number of parameters.

One downside to simplifying the model in this way is that as the abstraction of the parameters increases, it may become difficult to determine what is physically realisable. However, provided there is some understanding of the relation between the abstract parameters and the physical system it should be possible to derive bounds for the abstract parameters that keep the exploration within realistic bounds.

Calibration on the magnetic ring array reservoir was in general very effective at finding a consistent optimal magnetic field strength, assuming such a magnetic field was possible within the bounds of the exploration. However, it was less effective when the distribution of parameters increased. This is likely due to the properties of individual junctions within the ring array becoming so divergent that a single global magnetic field was unable to compensate for their differences, resulting in some junctions always pinning or never pinning domain walls.

Given the results of calibration, it can be stated that the magnetic ring array reservoir is somewhat robust to fabrication error. This is especially true in the case that the fabrication error is uniform across the ring array; for example, a small amount of contamination of the material or an imperfection in the design.

Calibration is less effective when individual junctions with the ring array have substantially different properties. However, given that for previous experiments with real devices, the distribution parameters are between 1 and 4 orders of magnitude smaller that the base values [17], the results here suggest that for any realistic manufacturing process calibration should perform well.

The consistency of results with calibration may seem surprising; however, Experiment 2 does shed some light on why it was so successful. Assuming that the distribution of probability values within the system is relatively tight, and therefore that there exists a magnetic field that has similar effects over the entire ring array, there are only two remaining parameters. Therefore, the system only has two degrees of freedom with respect to its characterisation. As the system is a reservoir, it contains a trained linear output layer, and the calibration process effectively provides a trained linear input layer. These two linear layers allow calibration plus training to counteract the two degrees of freedom the parameters expose, leading to consistent results providing the optimal magnetic field is within the range of fields searched.

Experiment 2 also lends some credence to the idea that as in the real system multiple parameters are combined to single probability values, there is a lower chance of experiencing extreme behaviours. This can be seen to an extent with the spread of values shown in the Fig. 2 graphs; if multiple values contribute to a behaviour, then it is possible for one anomalous value to be compensated for by another. The nature of this experiment did not allow us to verify the degree to which the physical parameters are independent within the actual fabrication process, and so it is not possible to make a definitive claim that this is the case. However, if the parameters have a degree of independence then this allows a further degree of robustness for physical systems by virtue of multiple things having to go wrong for extreme behaviour to manifest.

These results also show that multiple sets of parameters can yield useful results, which suggests that it will be possible in future to use heterogeneous ring arrays, where different rings in the array have different properties. By allowing substantially different ring properties within an array, these results show that not all rings will be driven with their optimum magnetic field. There are a number of implications of this:

- Rings with relatively high energy barriers could be used as memory when the magnetic field drops too low, causing the domain walls in the rings to enter a pinned state.
- Rings with relatively low energy barriers could be used to restore portions of the ring array to a known state using a stronger than required magnetic field to cause domain walls to move with high probability, causing nucleation of domain walls in adjacent empty rings.

Hence, even though these behaviours are not useful for a global uniform ring array, it may be possible to exploit them in a heterogeneous ring array. This also highlights the usefulness of novelty search based approaches, as they can identify behaviours that would not be found with optimisation based techniques.

7 Conclusion

This paper explored the robustness of magnetic ring array reservoir computing with a calibration step by applying a novelty search approach to ring array parameters. The paper found that the behaviour of the ring array was relatively robust after calibration provided that the ring parameters resulted in a desired magnetic field that lay within the range of values obtainable. In the case that this was not possible, then the ring array would either abruptly fail or gradually degrade, depending on whether the failing was due to a global effect across the ring array or due to too much variance within the ring array. However, for the latter to cause a substantial problem would require an order of magnitude increase in variance over currently used manufacturing techniques.

The paper also demonstrated that simplifying a model to the minimum required still produces useful results with novelty search. While this may not be true with other types of search, due to the simplification having the potential to change the distribution of parameters, as novelty search seeks different behaviours it is able to largely overcome this difference.

Finally, this paper also exhibited one of the features of novelty search, in being able to capture non-optimal behaviours. In particular, the non-optimal behaviours found, while not useful for the homogeneous ring arrays considered in this paper, may have use in future work exploring the properties of heterogeneous ring arrays.

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