Question 1

2024-08-14

Q1 ~ Probability Practice

Question- Part A

It is provided that random clickers will click either one with equal probability which means they are mutually exclusive and exhaustive:

P(Yes | Random Clicker) = 0.5 P(No | Random Clicker) = 0.5

It is also provided that the expected probability of random clickers is 30%. Therefore,

P(Random Clicker) = 0.3

P(Truthful Clicker) = 1 - P(Random Clicker) = 1 - 0.3 = 0.7

After trial period, probabilities of survey result is: P(Yes) = 0.65 P(No) = 0.35

Q) What fraction of people who are truthful clickers answered yes?

Using rule of total probability: P(Yes) = (P(Yes | Truthful Clicker) * P(Truthful Clicker)) + (P(Yes | Random Clicker) * P(Random Clicker))

P(Yes | Truthful Clicker) = (P(Yes) - (P(Yes | Random Clicker) * P(Random Clicker))) / P(Truthful Clicker)

Therefore, P(Yes | Truthful Clicker) =((0.65-(0.5*0.3))/0.7

Hence, Fraction of people who are truthful clickers and answered yes are 71.428%

Question- Part B

Let say D stands for having Disease and nD stands for not having disease Also, +ve stands for positive and -ve stands for negative

Therefore, it is provided that: P(D) = 0.000025 => P(nD) = 1 - 0.000025 = 0.99975

Here, Sensitivity is 0.993

Therefore, P(+ve | D) = 0.993 P(-ve | D) = 1 - 0.993 = 0.007

Here, Specificity is 0.9999

Therefore, P(-ve | nD) = 0.9999 P(+ve | nD) = 1 - 0.9999 = 0.0001

Q) Suppose someone tests positive. What is the probability that they have the disease?

Using Bayes Rule, we can calculate joint probabilities i.e, test being positive given Disease and test being positive given not having Disease:

P(+ve , D) = P(+ve | D) * P(D) P(+ve, nD) = P(+ve | nD) * P(nD)

P(D | +ve) = (P(D) * P(+ve | D))/(P(D) * P(+ve | D) + P(nD) * P(+ve | nD))

P(D | +ve) = (0.000025 * 0.993) / ((0.000025 * 0.993) + (0.99975 * 0.0001))

P(D | +ve) = 0.1989183 = 19.89%

I have been assuming that the entire population went for the testing which is not possible. Therefore, there must be lesser false positives in our denominator

Hence, the probability of someone having disease given that they tested positive will be greater than 19.89%