Rotation by 3 angles

ϕ around x

θ around y

ψ around z

Rotation Matrix

$$\boldsymbol{R}=\begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\phi\sin\theta-\cos\phi\sin\psi\cos\phi & \cos\psi\sin\theta+\sin\phi\sin\psi \\ \sin\psi\cos\theta & \sin\phi\sin\psi\sin\theta+\cos\phi\cos\psi\cos\phi & \sin\psi\sin\theta-\cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\phi\cos\theta \end{bmatrix}$$

Initial vectors

$$\vec{x_0}=(1\,0\,0)$$
$$\vec{y_0}=(0\,1\,0)$$
$$\vec{z_0}=(0\,0\,1)$$

Target vectors

$$\vec{x_1}=(A x_{1x} \quad x_{1y} \quad A x_{1z})$$
$$\vec{y_1}=(B y_{1x} \quad y_{1y} \quad B y_{1z})$$
$$\vec{z_1}=(C z_{1x} \quad -z_{1y} \quad C z_{1z})$$

where $x_{1x},\ x_{1z},\ y_{1x},\ y_{1z},\ z_{1x}$ and z_{1z} are known values,

$$\vec{x_{1u}}=(x_{1x} \quad 0 \quad x_{1z})$$
$$\vec{y_{1u}}=(y_{1x} \quad 0 \quad y_{1z})$$
$$\vec{z_{1u}}=(z_{1x} \quad 0 \quad z_{1z})$$

are unit vectors and $A,\ B\ \&\ C$ satisfy the equations:

$$(A x_{1x})^2+x_{1y}{}^2+(A x_{1z})^2=1$$
$$(B y_{1x})^2+y_{1y}{}^2+(B y_{1z})^2=1$$
$$(C z_{1x})^2+z_{1y}{}^2+(B z_{1z})^2=1$$

such that $\vec{x_1},\ \vec{y_1}$ and $\vec{z_1}$ are also unit vectors.

It is assumed that the angles a being viewed from above and in front of the object, therefore the y coordinates of $\vec{x_1}$ and $\vec{y_1}$ will be positive and the y coordinate of $\vec{z_1}$ will be negative in the coordinate system of Blender

Find the cross products of all target vectors.
Since all initial vectors are perpendicular, the target vectors will also be.

$$\vec{x}_1=\vec{y}_1\times\vec{z}_1$$
$$\vec{y}_1=\vec{z}_1\times\vec{x}_1$$
$$\vec{z}_1=\vec{x}_1\times\vec{y}_1$$

The y coordinates of each cross product will only contain A, B and C.
These are the only parts required as they eliminate x_{1y}, y_{1y} and z_{1y}.

$$\begin{aligned}
x_{1y} &= By_{1z}Cz_{1x}-By_{1x}Cz_{1z}\\
&= BC(y_{1z}z_{1x}-y_{1x}z_{1z})\\
y_{1y} &= Cz_{1z}Ax_{1x}-Cz_{1x}Ax_{1z}\\
&= AC(z_{1z}x_{1x}-z_{1x}x_{1z})\\
z_{1y} &= Ax_{1z}By_{1x}-Ax_{1x}By_{1z}\\
&= AB(x_{1z}y_{1x}-x_{1x}y_{1z})
\end{aligned}$$

Find the dot products of new target vectors.
Substibuting in the new values of the y coordinates.

$$0=\vec{y}_1\cdot\vec{z}_1$$
$$0=\vec{z}_1\cdot\vec{x}_1$$
$$0=\vec{x}_1\cdot\vec{y}_1$$

$$\begin{aligned}
\vec{x}_1\cdot\vec{y}_1 &= Ax_{1x}By_{1x}+BC(y_{1z}z_{1x}-y_{1x}z_{1z})AC(z_{1z}x_{1x}-z_{1x}x_{1z})+Ax_{1z}Bz_{1z}\\
0 &= ABC^2(y_{1z}z_{1x}-y_{1x}z_{1z})(z_{1z}x_{1x}-z_{1x}x_{1z})+AB(x_{1x}y_{1x}+x_{1z}y_{1z})\\
0 &= AB(C^2(y_{1z}z_{1x}-y_{1x}z_{1z})(z_{1z}x_{1x}-z_{1x}x_{1z})+(x_{1x}y_{1x}+x_{1z}y_{1z}))\\
0 &= C^2(y_{1z}z_{1x}-y_{1x}z_{1z})(z_{1z}x_{1x}-z_{1x}x_{1z})+(x_{1x}y_{1x}+x_{1z}y_{1z})\\
-(x_{1x}y_{1x}+x_{1z}y_{1z}) &= C^2(y_{1z}z_{1x}-y_{1x}z_{1z})(z_{1z}x_{1x}-z_{1x}x_{1z})\\
C^2 &= \frac{(x_{1x}y_{1x}+x_{1z}y_{1z})}{(y_{1z}z_{1x}-y_{1x}z_{1z})(x_{1z}z_{1x}-x_{1x}z_{1z})}
\end{aligned}$$

Since these are all known values, C can now be solved.
Repeat the process for the other dot products

From $\vec{y}_1\cdot\vec{z}_1$

$$B^2=\frac{x_{1x}z_{1x}+x_{1z}z_{1z}}{(x_{1x}y_{1z}-x_{1z}y_{1x})(y_{1z}z_{1x}-y_{1x}z_{1z})}$$

From $\vec{z}_1\cdot\vec{x}_1$

$$A^2=\frac{y_{1x}z_{1x}+y_{1z}z_{1z}}{(x_{1z}y_{1x}-x_{1x}y_{1z})(x_{1z}z_{1x}-x_{1x}z_{1z})}$$

Substitute values into Matrix Rotation

$$\vec{x}_0 \cdot R = \vec{x}_1$$

$$\begin{pmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \end{pmatrix} = \begin{pmatrix} A x_{1x} & x_{1y} & A x_{1z} \end{pmatrix}$$

$$-\sin\theta = A x_{1z}$$

$$\theta = \sin^{-1}(-A x_{1z})$$

$$\frac{\cos\theta\sin\psi}{\cos\theta\cos\psi} = \frac{x_{1y}}{A x_{1x}}$$

$$\tan\psi = \frac{x_{1y}}{A x_{1x}}$$

$$\psi = \tan^{-1}\left(\frac{x_{1y}}{A x_{1x}}\right)$$

$$\vec{y}_0 \cdot R = \vec{y}_1$$

$$\begin{pmatrix} \cos\psi\sin\phi\sin\theta - \cos\phi\sin\psi\cos\phi & \sin\phi\sin\psi\sin\theta + \cos\phi\cos\psi\cos\phi & \cos\theta\sin\phi \end{pmatrix} = \begin{pmatrix} B y_{1x} & y_{1y} & B y_{1z} \end{pmatrix}$$

$$\cos\theta\sin\phi = B y_{1z}$$

$$\vec{z}_0 \cdot R = \vec{z}_1$$

$$\begin{pmatrix} \cos\psi\sin\theta + \sin\phi\sin\psi & \sin\psi\sin\theta - \cos\psi\sin\phi & \cos\phi\cos\theta \end{pmatrix} = \begin{pmatrix} C z_{1x} & -z_{1y} & C z_{1z} \end{pmatrix}$$

$$\cos\theta\sin\phi = C z_{1z}$$

$$\frac{\cos\theta\sin\phi}{\cos\theta\cos\phi} = \frac{B y_{1z}}{C z_{1z}}$$

$$\tan\phi = \frac{B y_{1z}}{C z_{1z}}$$

$$\phi = \tan^{-1}\left(\frac{B y_{1z}}{C z_{1z}}\right)$$