EE1101 Signals and Systems
Tutorial 6

1. An ideal highpass filter with cutoff frequency 250 rad/s is an LTI system whose frequency response is:

$$H(j\omega) = \begin{cases} 1, & |\omega| \geq 250 \\ 0, & \text{otherwise} \end{cases}$$

When the input to this system is a signal $x(t)$ with fundamental period $\frac{\pi}{7}$ and Fourier series coefficients a_k, it is found that the output is identical to $x(t)$. For what values of k is it guaranteed that a_k= 0?

2. Consider the following signal

$$x(t) = 2 + \sum_{k=1}^{3} 3 \sin\left(\frac{k\pi}{2}\right) \cos(100k\pi t).$$

Plot the magnitude and phase spectrum of $y(t) = x(t) \cos 100\pi t$. [Hint: Use properties of Fourier series.]

3. An ideal lowpass filter with cutoff frequency 500 Hz is an LTI system H_l whose frequency response is:

$$H_l(j\omega) = \begin{cases} 1 & , |\omega| \leq 2\pi.500\text{rad/s} \\ 0 & , \text{otherwise} \end{cases}$$

   (a) What is the response of this filter to the input signal $x(t) = \cos(2\pi \cdot 750t) + \sin(2\pi \cdot 1500t)$?

   (b) What is the response of this filter to the input signal $x(t) = \cos(2\pi \cdot 150t) + \sin(2\pi \cdot 1500t)$?

   (c) What is the response of this filter for a periodic square wave with period 4.5 ms? The square wave oscillates between +1 V and -1 V with 50% duty cycle and is an even function of time.

4. An LTI system has an impulse response $h(t) = \delta(t) - e^{-t}u(t)$. For the following signals input to the system, find the output using the Fourier series analysis.

   (a) $x(t) = \cos(3\pi t) + \frac{\pi}{3}$

   (b) $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$

   (c) $x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-2n)$.

5. Suppose a continuous-time periodic signal is the input to an LTI system. The signal has a Fourier Series representation

$$x(t) = \sum_{k=-\infty}^{\infty} \alpha^{|k|} e^{jk\frac{\pi}{4}t}$$

where α is a real number between 0 and 1 and the frequency response of the system is

$$H(j\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$$

How large must W be in order for the output of the system to have atleast 90% of the average energy per period of $x(t)$?

6. Consider a causal continuous-time LTI system whose input $x(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{d}{dt}y(t) + 4y(t) = x(t).$$

Find the Fourier series representation of the output $y(t)$ for the input $x(t) = \cos(2\pi t)$.

7. Determine the Fourier series coefficients of the following continuous-time signals with a fundamental period of $T = 1/2$.

   (a) $x(t) = \cos(4\pi t)$.

   (b) $y(t) = \sin(4\pi t)$.

(c) $z(t) = x(t)y(t)$, using properties of Fourier series.

(d) $z(t)$, by direct expansion. Compare this result with that of part (c).

8. Consider the input and outputs of the two systems:

(a) $x(t) = \sin(10t)$ and $y(t) = 5\cos(10t + \pi/6)$.

(b) $x(t) = 1/4\sin(10t)$ and $y(t) = cos(5t)$.

Are the systems LTI? (Use the concept of Fourier series and LTI systems to answer this question without any math)

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