

The order of growth running time for priority queue with N items of a d-ary heap would be  $\log_{\text{base } d} \text{ of } (N)$  for insert,  $d \cdot \log_{\text{base } d} \text{ of } (N)$  for delMax and 1 for max. Let us first understand heap sort time complexity.

Heap Sort has  $O(n \log n)$  time complexities for all the cases; best, average and worst. The height of a complete binary tree containing n elements is  $\log(n)$ . To fully heapify an element whose subtrees are already max-heaps, comparing the element with its left and right children needs to occur. Then it continues pushing that element downwards until it reaches a point where both its children are smaller than aforementioned element. So In worst case, we will need to move an element from the root to the leaf making multiple  $\log(n)$  comparisons and swaps in order to accomplish our task. The worst case complexity of the build heap step is  $n/2 \cdot \log(n)$  which is thus  **$n \log n$** . For our case it would be  **$2 \cdot n \log n$**  with the leading coefficient being 2 attained from the given table.

```
void heapSort(int arr[], int n)
{
    for (int i = n / 2 - 1; i >= 0; i--)
        heapify(arr, n, i);

    for (int i=n-1; i>=0; i--)
    {
        swap(arr[0], arr[i]); <--Swaps mentioned in passage

        heapify(arr, i, 0);
    }
}
```

As for the delMax function we have a worst case time complexity of  **$d \cdot \log_{\text{base } d} \text{ of } (N)$** . We can see so below since the complexity was obtained from the table given.

```
int delMax(int hole)
{
    int keyItem = array[hole];
    array[hole] = array[currentSize - 1];
    currentSize--;
    sink( hole );
    return keyItem;
}
```