AP Calculus AB Summer Enrichment Activity 2022 Name:_____
Chantilly High School

**No calculator all problems**

**To:** All students enrolled in AP Calculus AB.

**From:** AP Calculus AB teachers: Mr. Andraos (jeandraos@fcps.edu)
Mrs. Chong (AMChong@fcps.edu)
Mrs. Moldoveanu (vmoldoveanu@fcps.edu)

**Objective:** To refresh the algebraic and trigonometric concepts and skills needed to properly access the Calculus curriculum. Though optional, we strongly recommend completion prior to the school year. We will spend the first Learning Seminar going through questions on the packet.

**When:** Do not wait until the last minute to begin, but don't complete it in June and forget about it for two months. The goal of this assignment is to ensure the concepts are fresh in your mind before the first week of school so that there is a solid foundation to learn the Calculus.

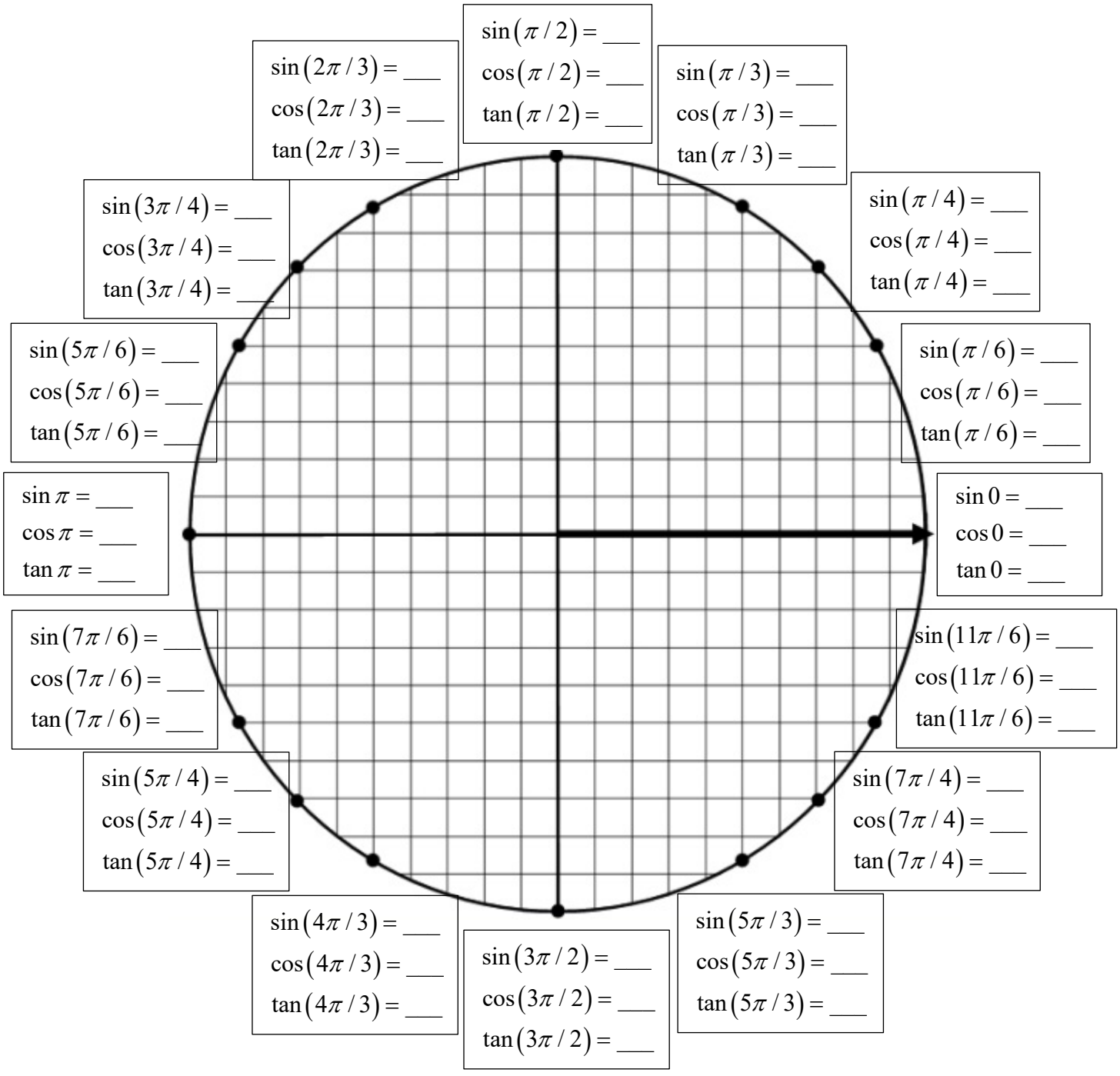
**How:** Be as resourceful as possible. Reach out to your friends or e-mail the teachers listed above if you have questions on any of the problems. Answers are posted on the last three pages, but it's more important that you know and understand the math than just getting the answer.

This link has examples and explanations:
https://drive.google.com/file/d/1YxGLa0QTupbVhOfl0gpSSTBZQFDE0OWv/view?usp=sharing

AP Calculus AB Summer Enrichment Activity 2022 Name:_____
Chantilly High School

**No calculator all problems**

**Part 1a – Fill in the following Unit Circle values.**
*Objective: Have all unit circle values memorized for the three basic trigonometric functions*



**Part 1b – Unit Circle Values (of other trig functions)** **No Calculator**

*Objective: Evaluate cosecant, secant, and cotangent of a specified angle.*

Reciprocal Trig Identities:

$\csc\theta =$ $\sec\theta =$ $\cot\theta =$

Evaluate the following:

**1.** $\csc\frac{2\pi}{3} =$ **2.** $\sec\frac{5\pi}{6} =$ **3.** $\cot\frac{\pi}{6} =$

**4.** $\csc\frac{7\pi}{6} =$ **5.** $\sec\pi =$ **6.** $\cot\frac{3\pi}{4} =$

**7.** $\csc\frac{5\pi}{3} =$ **8.** $\sec\frac{11\pi}{6} =$ **9.** $\cot\frac{4\pi}{3} =$

**Part 1c – Inverse Trig Values** **No Calculator**

*Objective: Evaluate inverse sine, inverse cosine, and inverse tangent.*

Determine the domain and range of the following:

| $y = \arcsin x$ | $y = \arccos x$ | $y = \arctan x$ |
|---|---|---|
| Domain: | Domain: | Domain: |
| Range: | Range: | Range: |

Evaluate the following. Be mindful of the range since the output may only have one answer.

**10.** $\arcsin\left(\frac{1}{2}\right) =$ **11.** $\arccos\left(\frac{1}{2}\right) =$ **12.** $\arctan(1) =$

**13.** $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ **14.** $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ **15.** $\tan^{-1}\sqrt{3} =$

**16.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ **17.** $\arccos\left(-\frac{1}{2}\right) =$ **18.** $\arctan\left(-\frac{\sqrt{3}}{3}\right) =$

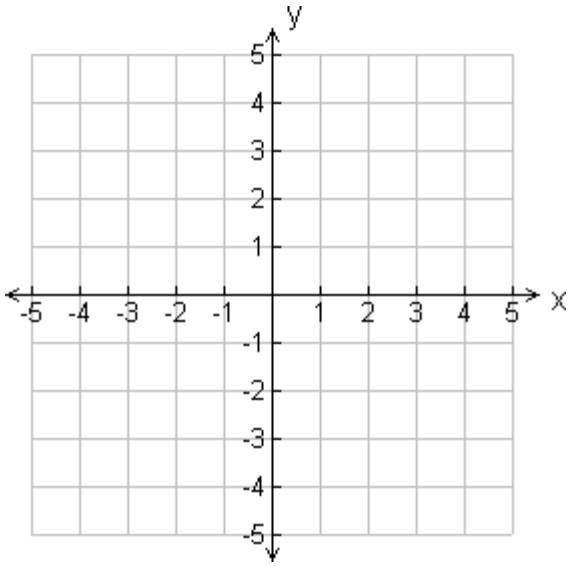
**19.** $\arcsin(-1) =$ **20.** $\cos^{-1}(-1) =$ **21.** $\arctan(-1) =$

**Part 2 – Piecewise functions** **No Calculator**

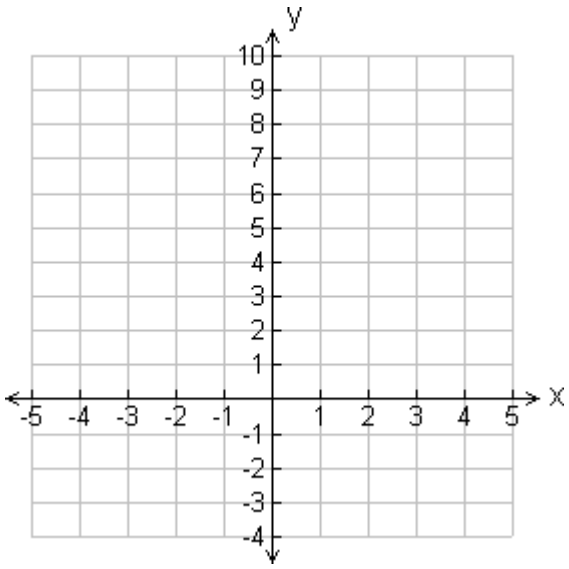
*Objective: Be able to visually and analytically understand the characteristics of a piecewise function.*

#22 – 23 Graph the following functions

**22.** $f(x)=\begin{cases}-x^2 & x\le 2\\ x-4 & x>2\end{cases}$

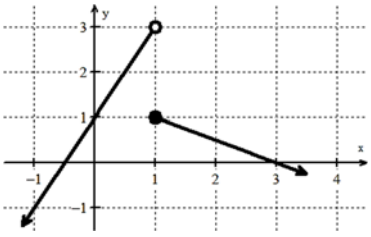



**23.** $f(x)=\begin{cases}-3x+1 & x<-1\\ (x-1)^2 & -1\le x\le 3\\ 7 & x>3\end{cases}$

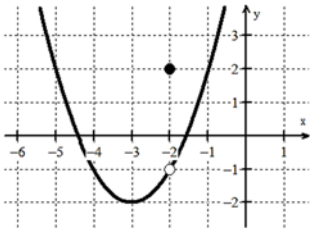



#24 – 25 Write the equation of each piecewise function. (Hint: for quadratics, use vertex form).

**24.**




**25.**




**Part 3 – Transformations**

*Objectives: to know the different types of transformations and reflections, and to be able to apply them when graphing a parent function.*

Describe in words what the following would do to the graph of *f(x)*.

**26.** $f(x)-4$

**27.** $f(x-4)$

**28.** $-f(x+2)$

**29.** $5f(x)+3$

**30.** $f(2x)$

**31.** $|f(x)|$

**Part 4 – Rational Functions** **No Calculator**

*Objectives: Find key elements (asymptotes, holes, intercepts, domain & range) of a rational function and graph.*

Determine the key elements of each function, then use to graph. Clearly denote all asymptotes & holes.

**32.** $f(x) = \dfrac{6-2x}{x^2-3x}$

a) Horizontal asymptote: _____

b) Vertical asymptote: _____

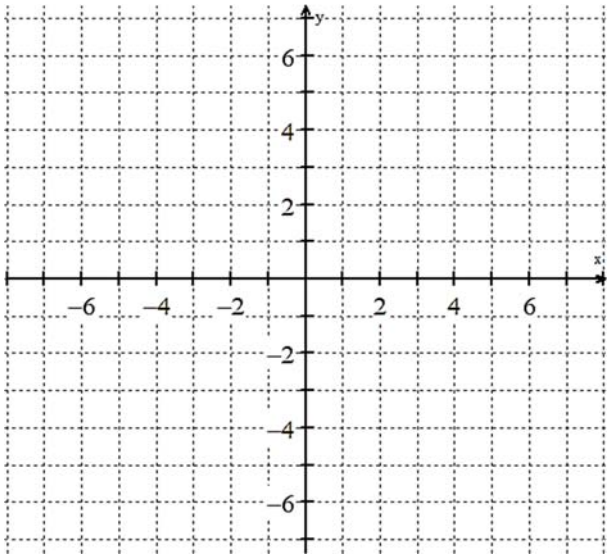
c) Coordinates of hole(s): _____

d) *y*-intercept (ordered pair): _____

e) *x*-intercept (ordered pair): _____

f) domain (interval notation): _____

g) range (interval notation): _____




**33.** $g(x) = \dfrac{3x^2+6x-24}{x^2-4}$

a) Horizontal asymptote: _____

b) Vertical asymptote: _____

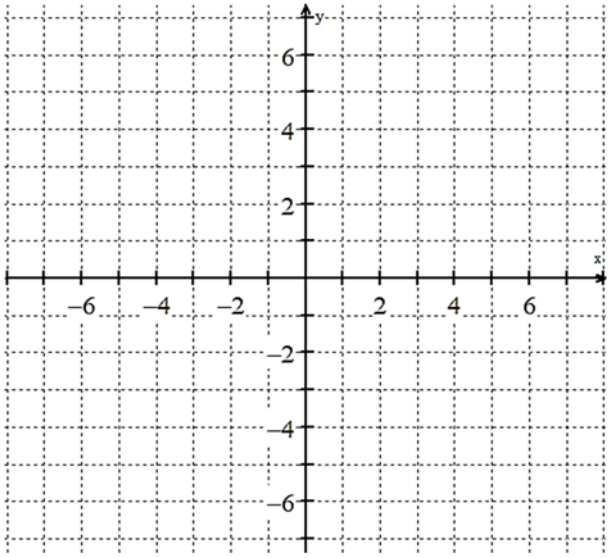
c) Coordinates of hole(s): _____

d) *y*-intercept (ordered pair): _____

e) *x*-intercept (ordered pair): _____

f) domain (interval notation): _____

g) range (interval notation): _____




**34.** Find the equation of the oblique asymptote of $g(x) = \dfrac{-2x^2+5x+3}{2x-1}$ .

**Part 5 – Exponents and Logarithms** **No Calculator**

*Objectives: Know the properties of exponents and logarithms, and be able to simplify expressions or solve equations involving exponents and logarithms.*

Evaluate/simplify the following:

**35.** $x^8\left(3x^3\right)^4$

**36.** $\dfrac{(2x^5)^4}{(4x^9)^3}$

**37.** $\left(\dfrac{4x^5}{3y^6}\right)^{-2}$

**38.** $\log_2 32$

**39.** $\log_5 125$

**40.** $\log\dfrac{1}{100}$

**41.** $3^{\log_3 11}$

**42.** $\ln e$

**43.** $\log_3 6 - 3\log_3 2 + \log_3 12$

Solve:

**44.** $8^{9x-2} = 64$

**45.** $\log_7\left(k-3\right) = 2$

**46.** $\log(-x-4) + \log 3 = 1$

**47.** Rewrite $\dfrac{1}{\sqrt[3]{x^4}}$ as a rational exponent expression with no denominator.

**48.** Rewrite $\dfrac{3}{2}x^{-7/5}$ in radical form, with no negative exponents.

**Part 6 – Factoring** **No Calculator**

*Objectives: Be able to factor polynomial expressions and solve equations requiring factoring.*

Factor the following:

**49.** $x^2+10x-24$

**50.** $2x^2-3x-20$

**51.** $12x^2-5x-2$

**52.** $x^4-81$

**53.** $x^3+8$

**54.** $4x^3-10x^2-6x$

Mega Factoring: i) Factor the GCF ii) Simplify what remains.

**55.** $(x+4)^6(7x-2)^2+(x+4)^5(7x-2)^3$

**56.** $16(2x-1)^3(3x+2)^8+20(2x-1)^4(3x+2)^7$

Solve by factoring.

**57.** $4x^2-21x-18=0$

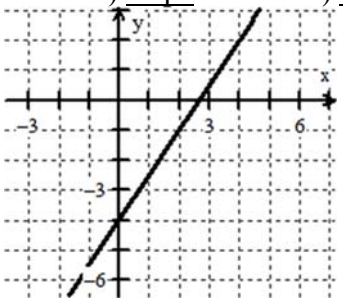
**58.** $x^3+3x^2-5x-15=0$

**Part 7 – Slopes/Equations of Lines** **No Calculator**

*Objectives: Be able to determine the slope and the equation of a line given certain properties.*

**59.** Find the i) slope and the ii) equation of the following lines:

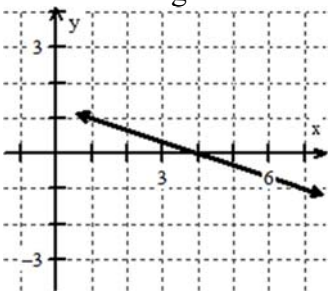
a)




i) slope: ____

ii) equation: _____

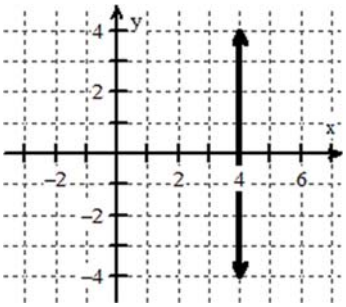
b)




i) slope: ____

ii) equation: _____

c)




i) slope: ____

ii) equation: _____

**60.** In point slope form, write the equation of the line that goes through the points (–4, 8) and (2, –6)

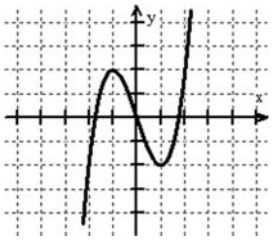
**61.** In point-slope form, write the equation of the line that is parallel to $y = -3x + 7$ and goes through (8, –1).

**62.** In slope-intercept form, write the equation of the line whose *x*-intercept is 9 and is perpendicular to $y = \frac{3}{4}x - 2$.

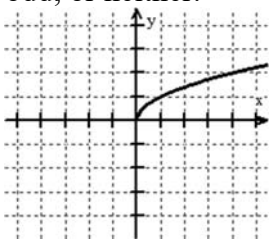
**Part 8 – Miscellaneous** **No Calculator**

#63 – 65 Determine if the function is even, odd, or neither.

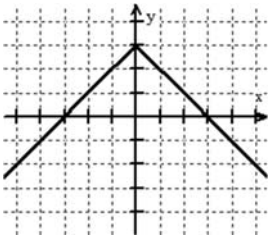
**63.**




**64.**




**65.**




**66.** Solve the system:

$x^2 + y^2 - 17x - y + 30 = 0$

$y = x + 1$

**67.** Simplify: $\dfrac{\frac{1}{b} + \frac{1}{2}}{\frac{b^2 - 4}{6}}$

**Answers**

Some answers to the first page:

$\sin(\pi/6)=\frac{1}{2},\ \cos(\pi/6)=\frac{\sqrt{3}}{2},\ \tan(\pi/6)=\frac{\sqrt{3}}{3}\quad \sin(2\pi/3)=\frac{\sqrt{3}}{2},\ \cos(2\pi/3)=-\frac{1}{2},\ \tan(2\pi/3)=-\sqrt{3}$

Part 1b

Reciprocal Trig Identities: $\csc\theta=\frac{1}{\sin\theta},\ \sec\theta=\frac{1}{\cos\theta},\ \cot\theta=\frac{1}{\tan\theta}$

**1.** $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$ **2.** $-\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$ **3.** $\sqrt{3}$

**4.** –2 **5.** –1 **6.** –1

**7.** $-\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$ **8.** $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$ **9.** $\frac{\sqrt{3}}{3}$

| $y=\arcsin x$ | $y=\arccos x$ | $y=\arctan x$ |
|---|---|---|
| Domain: [–1, 1] | Domain: [–1, 1] | Domain: $(-\infty,\infty)$ |
| Range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ | Range: [0, π] | Range: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ |

**10.** $\frac{\pi}{6}$ **11.** $\frac{\pi}{3}$ **12.** $\frac{\pi}{4}$

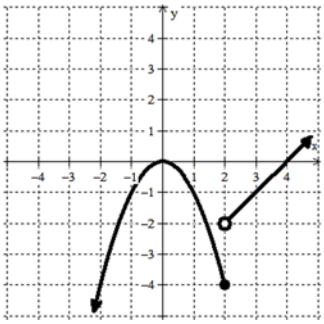
**13.** $-\frac{\pi}{3}$ **14.** $\frac{\pi}{6}$ **15.** $\frac{\pi}{3}$

**16.** $-\frac{\pi}{4}$ **17.** $\frac{2\pi}{3}$ **18.** $-\frac{\pi}{6}$

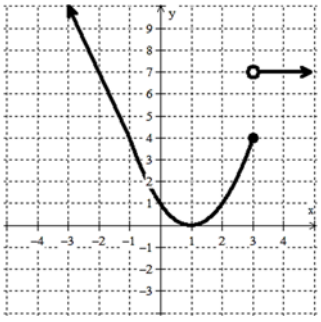
**19.** $-\frac{\pi}{2}$ **20.** π **21.** $-\frac{\pi}{4}$

Part 2

**22.**



**23.**



**24.** $f(x)=\begin{cases}2x+1, & x<1\\ -\frac{1}{2}x+\frac{3}{2}, & x\geq 1\end{cases}$

**25.** $f(x)=\begin{cases}(x+3)^2-2, & x\neq -2\\ 2, & x=-2\end{cases}$

Part 3

**26.** Vertical shift down 4

**27.** Horizontal shift right 4

**28.** Reflection over *x*-axis, Horizontal shift left 2

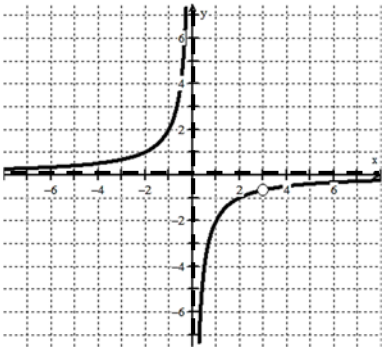
**29.** Vertical Stretch by 5, Vertical shift up 3

**30.** Horizontal compression by 1/2

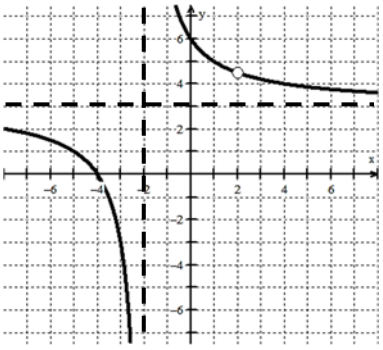
**31.** Reflects all negative *y*-values over the *x*-axis

Part 4

**32.** a) $y = 0$
b) $x = 0$
c) $\left(3, -\frac{2}{3}\right)$
d) n/a
e) n/a
f) $(-\infty, 0) \cup (0,3) \cup (3, \infty)$
g) $\left(-\infty, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, 0\right) \cup (0, \infty)$



**33.** a) $y = 3$
b) $x = -2$
c) $\left(2, \frac{9}{2}\right)$
d) $(0,6)$
e) $(-4,0)$
f) $(-\infty, -2) \cup (-2,2) \cup (2, \infty)$
g) $(-\infty, 3) \cup \left(3, \frac{9}{2}\right) \cup \left(\frac{9}{2}, \infty\right)$



**34.** $y = -x + 2$

Part 5

**35.** $81x^{20}$

**36.** $\frac{1}{4x^7}$

**37.** $\frac{9y^{12}}{16x^{10}}$

**38.** 5

**39.** 3

**40.** –2

**41.** 11

**42.** 1

**43.** 2

**44.** $x = \frac{4}{9}$

**45.** $k = 52$

**46.** $x = -\frac{22}{3}$

**47.** $x^{-4/3}$

**48.** $\frac{3}{2\sqrt[5]{x^7}}$

Part 6

**49.** $(x+12)(x-2)$

**50.** $(2x+5)(x-4)$

**51.** $(3x-2)(4x+1)$

**52.** $(x^2+9)(x+3)(x-3)$

**53.** $(x+2)(x^2-2x+4)$

**54.** $2x(x-3)(2x+1)$

**55.** $(x+4)^5(7x-2)^2(8x+2)$

**56.** $4(2x-1)^3(3x+2)^7(22x+3)$

**57.** $x=-\frac{3}{4},6$

**58.** $x=-3,\ \pm\sqrt{5}$

Part 7

**59.** **a)** i) $\frac{3}{2}$

ii) $y=\frac{3}{2}x-4$

**b)** i) $-\frac{1}{3}$

ii) $y-0=-\frac{1}{3}(x-4)$ or $y=-\frac{1}{3}x+\frac{4}{3}$

**c)** i) undefined

ii) $x=4$

**60.** $y-8=-\frac{7}{3}(x+4)$ or $y+6=-\frac{7}{3}(x-2)$

**61.** $y+1=-3(x-8)$

**62.** $y=-\frac{4}{3}x+12$

**63.** odd

**64.** neither

**65.** even

**66.** (3, 4) and (5, 6)

**67.** $\frac{3}{b^2-2b}$ or $\frac{3}{b(b-2)}$