

AIDI 1000: AI Algorithms and Mathematics – Assignment - 2

Due Date : March 05, 2023, 11:59 PM

Note : Submit only one pdf file showing all your work. (File name : *Assignment_2_firstname_lastname.pdf*)

1. (25 points) There are three types of coins which have different probabilities of landing heads when tossed:

- Type A coins are fair, with probability 0.5 of heads
- Type B coins are bent and have probability 0.6 of heads
- Type C coins are bent and have probability 0.9 of heads

Suppose I have a drawer containing 5 coins: 2 of type A, 2 of type B, and 1 of type C. I reach into the drawer and pick a coin at random. Without showing you the coin I flip it once and get heads (i.e. event D). What is the probability it is type A (i.e. $P(H=A|D)$)? Type B (i.e. $P(H=B|D)$)? Type C (i.e. $P(H=C|D)$)? Fill out table below with your answers.

hypothesis	prior	likelihood	posterior
H	$P(H)$	$P(D H)$	$P(H D)$
A			
B			
C			

2. (25 points - Written) Consider the following dataset of four rows and three features (Malicious, Viagara, Meet) with class labels (ham and spam). Suppose we see a message having these features
- $M_5 = (\text{Malicious} = \text{'yes'}, \text{Viagara} = \text{'no'}, \text{Meet} = \text{'yes'})$
- , What is the probability that it is a spam or ham? Using Naive Bayes algorithm.

S.No	Malicious	Viagara	Meet	class
M_1	yes	yes	yes	spam
M_2	no	no	yes	ham
M_3	yes	no	yes	spam
M_4	no	yes	no	ham

3. (30 points - Written) Use the below gradient descent algorithm to find the
- \mathbf{x}
- value that minimizes the function
- $f(\mathbf{x}) = 6x_1^2 - 3x_1x_2 + 2x_2^2$
- . Choose the starting point as
- $(x_1^{(0)}, x_2^{(0)}) = (2, 3)$
- and use the
- η_k
- as 0.1 and
- ϵ
- as 0.0001 Consider the following gradient descent algorithm (which is a variant of the directional search algorithm) that aims to find the
- x
- value that minimizes the function
- $f(x)$
- .

- Guess $x^{(0)}$, set $k \leftarrow 0$
- while $\|\nabla f(x^{(k)})\| \geq \varepsilon$ do
- $x^{(k+1)} = x^{(k)} - \eta_k \nabla f(x^{(k)})$
- $k \leftarrow k + 1$
- end while
- return $x^{(k)}$

Perform only 3 iterations of this algorithm and report the values of (x_1, x_2) after performing gradient descent each time

Hint : since the $f(x)$ is in two variable you need to apply partial differentiation

4. (20 points - Written) Find the eigen values and eigen vectors for the following matrix. $\begin{bmatrix} 2 & 5 \\ -1 & 2 \end{bmatrix}$