

Optimization Project 1: Linear Programming – Marketing Budget Allocation

Group Members:

Sankalp Kulkarni (Sk57277), Anurag Sahu (as229468), Jahnavi Angati(ja54632), Jyotis Joy(jj37257)

Project Overview:

As we are aware, the recent CMO survey shows that the share of the marketing budget is now around 11% of total budget allocation in the organizations. However, the return on this investment depends on the effective allocation of the marketing budget across different channels. Like we have recently seen in the case of P&G and Netflix, it is prudent to allocate the budget across different marketing channels effectively. Therefore, we are going to formulate a budget allocation strategy for our marketing organization in this project.

Problem definition and the project outline:

Simply put, as of now, we do not know what the budget allocation for each media channel should be to get the best returns. This report defines and answers the following problems at hand:

1. **Objective:** How much amount should we invest in the different media channels that we are currently leveraging for our marketing activities?
2. **Budget Variations:** Will the budget allocation differ if we consider estimated return on investments (referred as ROI from now on) recommended by the two consultancies?
 - a. If yes, then how much is the difference?
3. **What-If scenarios:** We know that verification of ROI estimates might take time and ROIs can vary based on new developments in the market and hence we want to be ready with all foreseeable scenarios which directly impact our objective:
 - a. Quantifying the change in our objective if ROI of consultancy 1 turned out to be correct and we are in the process of allocating budget as per ROI suggested by consultancy 2
 - b. Vice-versa of (a)
 - c. If ROI expectations are not met/changes, then how will it change our optimal budget allocation for each channel?
 - d. Do we need the upper bound of \$3M on budget allocation for each media channel?
4. **Re-investment strategy:** Since we are planning to re-invest half of the returns, what should be the optimal allocation for each month of next year when all constraints are kept the same?
5. **Budget Stability:** Finally, we examine the stability of our budget allocation:
 - a. As per our definition, stable is defined as when change in budget allocation is not more than \$1M for each month.
 - b. If it is not stable, then what is causing the instability and how can we mitigate it?

Solution:

1. In order to tackle this problem, we are defining our objective function which will provide us with an optimal solution.

- a. Objective: Maximize ROI on marketing investment by optimal budget allocation across channels.

- b. Mathematically,

$$\text{Maximize } \sum_{i=1}^{10} x_i \times \text{ROI}_i$$

Where, $x(i)$ = Marketing Budget Allocated to the Platform

- c. For example, In our current scenario (using ROI expectations of 2nd consultancy) :

$$\begin{aligned} \text{Maximize} = & 0.049(\text{Print}) + 0.023(\text{TV}) + 0.024(\text{SEO}) + 0.039(\text{AdWords}) + \\ & 0.044(\text{Facebook}) + 0.046(\text{LinkedIn}) + 0.026(\text{Instagram}) + 0.019(\text{Snapchat}) + \\ & 0.037(\text{Twitter}) + 0.026(\text{Email}) \end{aligned}$$

2. Constraints: We are designing the solution keeping in mind the below constraints mentioned by our boss:

- a. Total marketing budget is \$10M.

$$\sum_{i=1}^{10} x_i \leq \$10M$$

- b. For each media platform, maximum budget allocation should not exceed \$3M.

$$x_p \leq 3M$$

- c. The total amount used in social media (consisting of Facebook, LinkedIn, Instagram, Snapchat, Twitter) should be at least twice of SEO and AdWords.

In this case and given current order of platforms in the file, we can write it mathematically as:

$$x_{\text{Facebook}} + x_{\text{LinkedIn}} + x_{\text{Instagram}} + x_{\text{Snapchat}} + x_{\text{Twitter}} \geq 2 \times (x_{\text{SEO}} + x_{\text{AdWords}})$$

- d. The amount invested in the print and TV should not be more than the amount spent on Facebook and Email.

In this case and given current order of platforms in the file, we can write it mathematically as:

$$x_{\text{Print}} + x_{\text{TV}} \leq x_{\text{Facebook}} + x_{\text{Email}}$$

Assessing optimal allocation:

First Consultancy:

1. Our first task in hand is to identify optimal budget allocation as per the ROI estimates provided by the first consultancy as below:

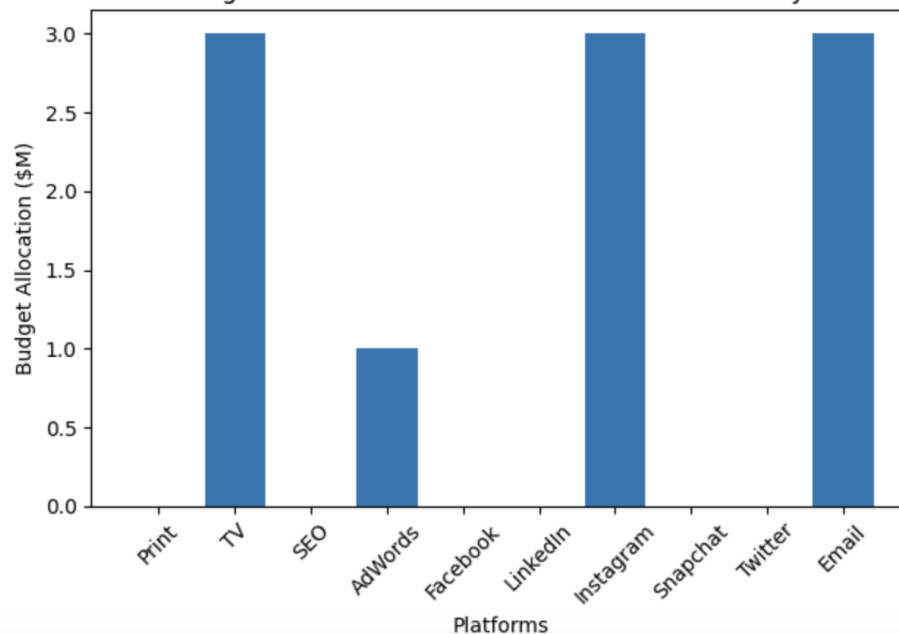
	Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
0	ROI	0.031	0.049	0.024	0.039	0.016	0.024	0.046	0.026	0.033	0.044

2. We leverage Gurobi software to help us provide the optimal solution.

Optimal ROI: 456.00 K

Platform	Is Budget Allocated	Amount to be Invested (\$M)
TV	Yes	\$3.00 M
Instagram	Yes	\$3.00 M
Email	Yes	\$3.00 M
AdWords	Yes	\$1.00 M
Twitter	No	\$0.00 M
Snapchat	No	\$0.00 M
SEO	No	\$0.00 M
Print	No	\$0.00 M
LinkedIn	No	\$0.00 M
Facebook	No	\$0.00 M

Budget Allocation for Media Channels : Consultancy 1



3. As per the given setup, if the first ROI expectations are assumed to be true, we will end up with **actual ROI of \$456,000** on our marketing investment of \$10M. (Return of ~4.5% on invested amount)
4. In this optimal solution, we will end up allocating \$3M for TV, Email and Instagram each and \$1M for AdWords.

5. Insights:

- This allocation of \$10M budget is in line with the expected ROI provided by the first consultancy to us.
- TV, Email and Instagram all are expected to have the higher ROIs (>4%) while AdWords being the closest contender with 3.9% ROI which also reflects in this allocation.

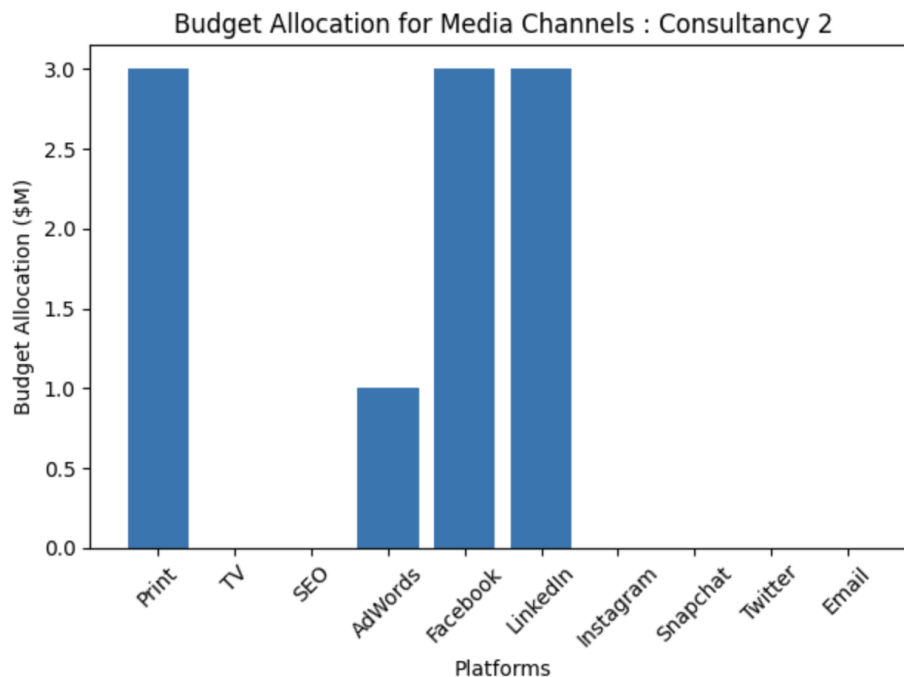
Second Consultancy:

1. While taking the second opinion from another consultancy, we keep all the constraints the same and then re-run the analysis to check for maximum objective i.e. maximum ROI in \$.

	Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
1	Second Firms ROI Estimate	0.049	0.023	0.024	0.039	0.044	0.046	0.026	0.019	0.037	0.026

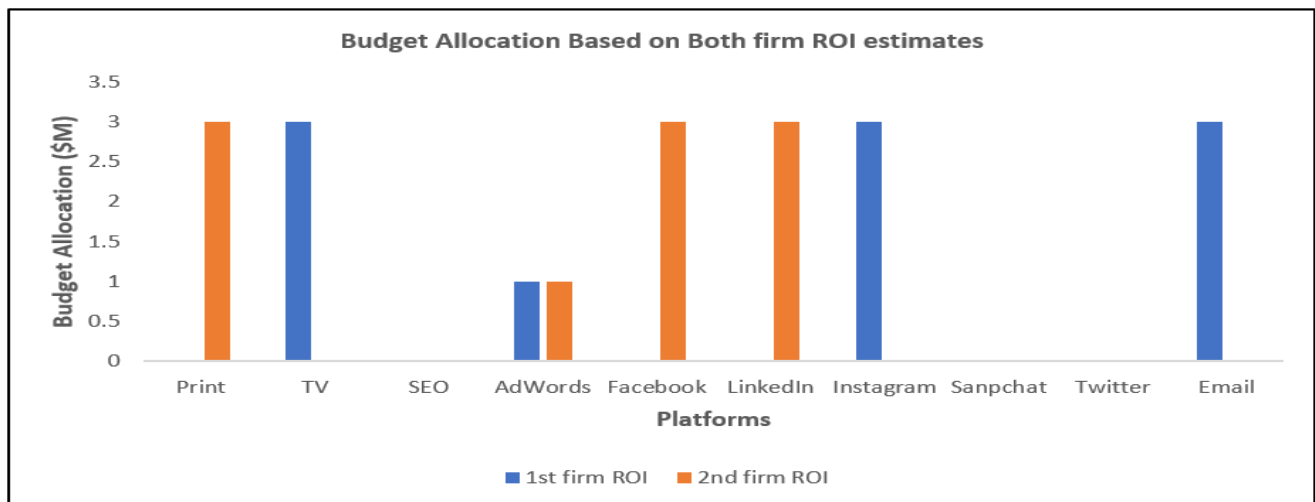
Optimal ROI: 456.00 K

Platform	Is Budget Allocated	Amount to be Invested (\$M)
Print	Yes	\$3.00 M
LinkedIn	Yes	\$3.00 M
Facebook	Yes	\$3.00 M
AdWords	Yes	\$1.00 M
Twitter	No	\$0.00 M
TV	No	\$0.00 M
Snapchat	No	\$0.00 M
SEO	No	\$0.00 M
Instagram	No	\$0.00 M
Email	No	\$0.00 M



2. As per our re-assessment based on the alternate ROI expectations, we get 2 major observations:
 - a. The main objective i.e., **maximum possible ROI remains unchanged at \$456,000.**
 - b. **The allocation does not remain the same.**
3. In this new allocation we will be allocating \$3M each in Print, Facebook and LinkedIn and \$1M for AdWords.
4. Insights:
 - The updated budget allocation aligns with the recommendations from the second ROI table, where Print, Facebook, and LinkedIn are identified as the platforms with the highest expected ROI, each exceeding 4%. Remarkably, AdWords maintains its fourth position with an identical expected ROI as advised by the Consultancy 1.

Allocation Comparison:



- Notably, **significant shifts in allocation have occurred across six platforms.**
- TV, Email, and Instagram have seen their allocations reduced to zero, while Print, Facebook, and LinkedIn have experienced a substantial increase, receiving a total of \$3 million, compared to their initial allocation of zero in the first allocation round.

Key Point/Recommendations :

- Overall, between the two optimal results, while our ROI amount remains unchanged, we would have to trade carefully and choose the best platform to invest.
- Certain platforms such as Print media, TV will take considerable time to de-invest from them if the results are not promising and if we receive the new findings where we do not find them lucrative enough or if we find some other platform picking up as we move ahead in the year and want to use remaining fund for those new platforms.
- This aspect will have to be carefully considered before finalizing the fund allocation.

Assessing “What-If Scenarios”:

Although both the optimal solutions hint towards a positive ROI, we should consider the dynamic nature of market forces and should assess the impact of different scenarios on our main objective. This assessment can easily be executed using the available data and this data-driven approach will help us to deal with the possible uncertainty.

1. What if we assume that the first consultancy is right and their ROI data is accurate but we decide to allocate a budget based on our allocation from the second consultancy?

In this simulation, **we end up falling short of our optimal value by \$204,000**. i.e. we will attain suboptimal returns. This shortage is mostly driven by the platforms like Print, Facebook and LinkedIn as according to the first consultancy, these platforms will not yield better returns which is an opposite view to that of consultancy 2. Since the second consultancy expects these platforms to be better investment avenues, we allocated \$3M each on these platforms and found that **we will only be able to attain ~56% of our potential ROI**.

Output:

```
#Reading the expected ROI of 1st consultancy
obj = np.array(roi_data.iloc[0][0:])
obj

array([0.031, 0.049, 0.024, 0.039, 0.016, 0.024, 0.046, 0.026, 0.033,
       0.044])

#Subtracting the optimal solution from solution with allocation as per 2nd consultancy
diff_1 = m.objVal-sum((obj*m1.x))

print("The objective will get lowered by: {:.2f} ".format(diff_1))

The objective will get lowered by: 204000.00
```

2. What if we assume that the second consultancy is right, and their ROI data is accurate but we decide to allocate a budget based on our allocation as per first consultancy?

In this simulation, **we end up falling short of our optimal value by \$192,000**. i.e., we will attain suboptimal returns. This shortage is mostly driven by the platforms like TV, Instagram and Email as according to the second consultancy, these platforms will not yield better returns which is an opposite view to that of consultancy 1. Since the first consultancy expects these platforms to be better investment avenues, we allocated \$3M each on these platforms and found that **we will only be able to attain ~58% of our potential ROI**.

Output:

```
#Reading the expected ROI of 2nd consultancy
obj1 = np.array(roi_data.iloc[1][0:])
obj1

array([0.049, 0.023, 0.024, 0.039, 0.044, 0.046, 0.026, 0.019, 0.037,
       0.026])
```

```
#Subtracting the optimal solution from solution with allocation as per 1st consultancy
diff_2 = m1.objVal-sum((obj1*m.x))

print("The objective will get lowered by:{:.2f} ".format(diff_2))

The objective will get lowered by:192000.00
```

3. What if we eliminate the \$3 million maximum investment constraint for each channel?

In this simulation, removing this constraint leads to an approximate **\$0.9 million increase** in overall returns, suggesting improved outcomes. However, a closer examination of platform-specific allocation reveals a more nuanced perspective.

Without this constraint, our \$10 million budget would primarily be allocated to just two platforms: TV and Email (1st consultancy) or Print and Facebook (2nd consultancy).

This presents several organizational challenges.

Few of them are listed below:

- Firstly, these platforms cater to distinct audience segments, limiting our overall reach by focusing on only two out of the available ten platforms.
- Secondly, if either of these platforms fails to meet expectations, our ROI could suffer significantly.
- Additionally, considering the significant influence of Social Media in the digital era, it's important to note that we can leverage insights and strategies from one Social Media platform and apply them to others. However, with the focus solely on TV and Email, we miss out on potential learnings and opportunities for growth that newer platforms may offer. This could have a detrimental effect in the long run, especially considering the evolving landscape of digital marketing.

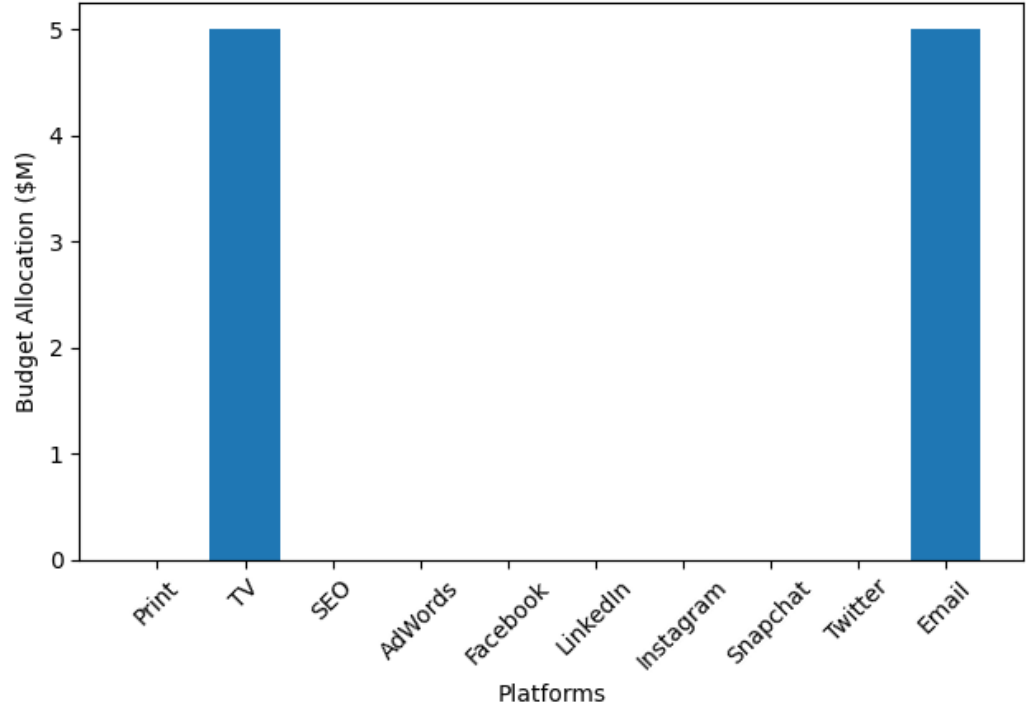
In conclusion, **while the removal of the third constraint may seem beneficial from a pure data perspective, maintaining the third constraint is essential for a holistic approach.** By adhering to this constraint, we can diversify our marketing investments, reducing dependency on just a couple of platforms and better positioning ourselves for long-term success.

First consultancy:

Optimal ROI: 465.00 K

Platform	Is Budget Allocated	Amount to be Invested (\$M)
TV	Yes	\$5.00 M
Email	Yes	\$5.00 M
Twitter	No	\$0.00 M
Snapchat	No	\$0.00 M
SEO	No	\$0.00 M
Print	No	\$0.00 M
LinkedIn	No	\$0.00 M
Instagram	No	\$0.00 M
Facebook	No	\$0.00 M
AdWords	No	\$0.00 M

Budget Allocation for Media Channels (1st Consultancy): Without upper bound

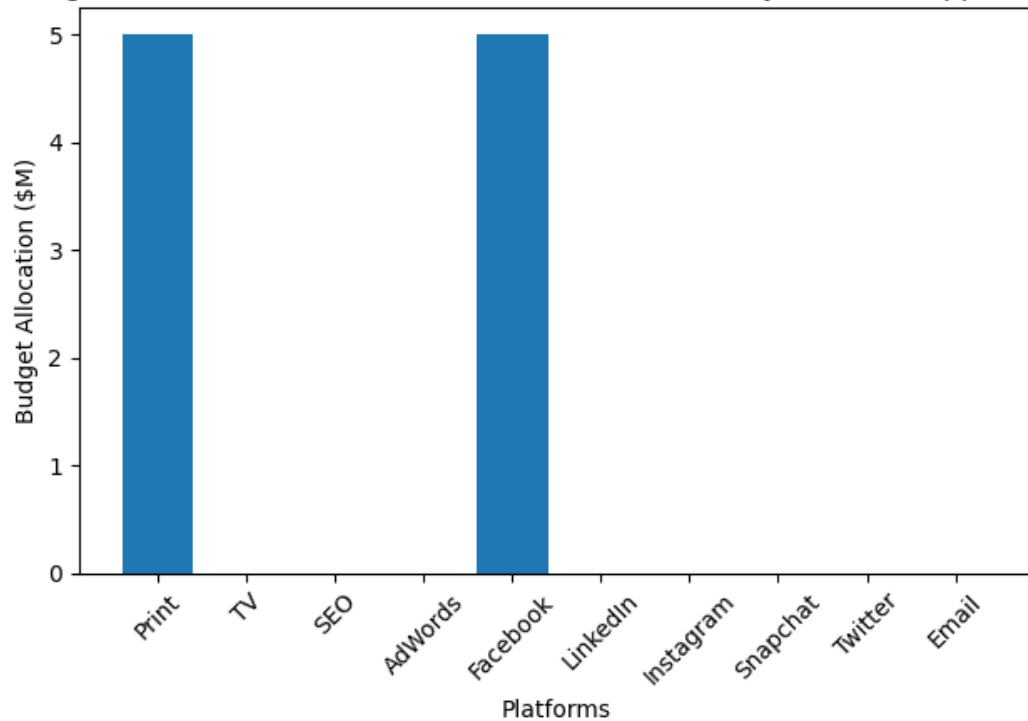


Second consultancy:

Optimal ROI: 465.00 K

Platform	Is Budget Allocated	Amount to be Invested (\$M)
Print	Yes	\$5.00 M
Facebook	Yes	\$5.00 M
Twitter	No	\$0.00 M
TV	No	\$0.00 M
Snapchat	No	\$0.00 M
SEO	No	\$0.00 M
LinkedIn	No	\$0.00 M
Instagram	No	\$0.00 M
Email	No	\$0.00 M
AdWords	No	\$0.00 M

Budget Allocation for Media Channels (2nd consultancy): Without upper bound



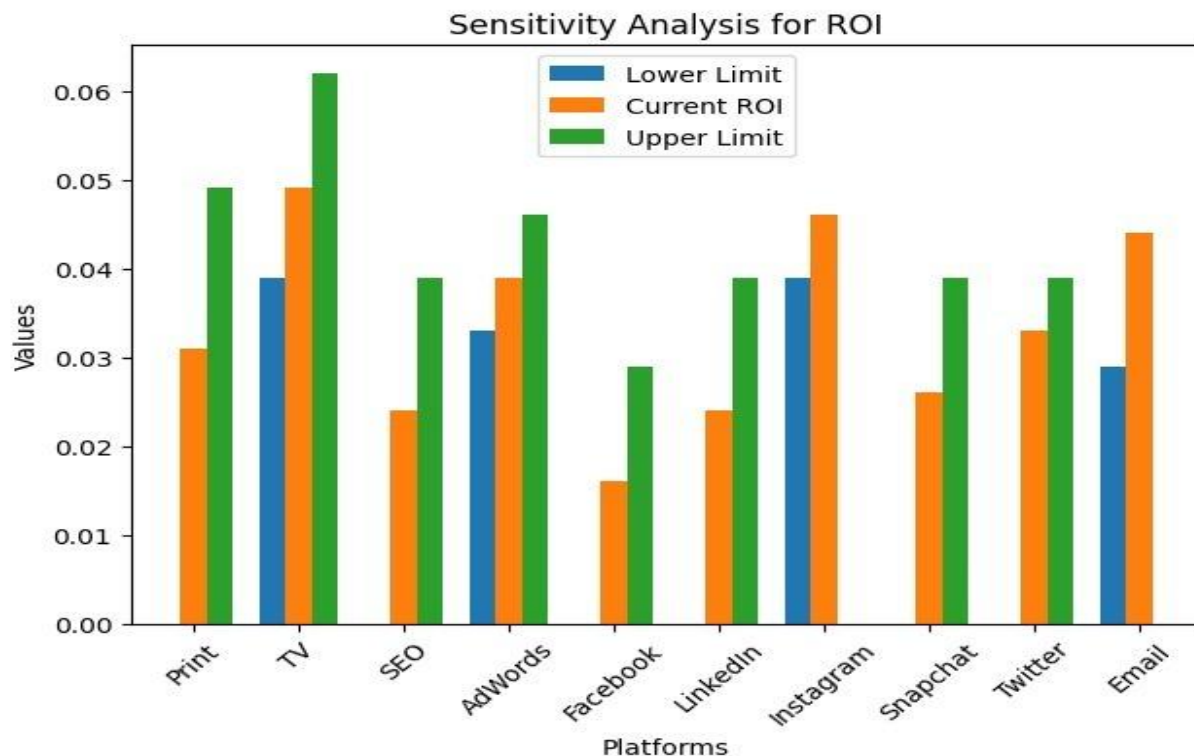
Sensitivity Analysis :

Next, we have to keep in mind that we would want to know the upper and lower threshold values in case of actual ROI returns turned out to be different than the expected ROI for each of these platforms while other ROI values remain constant.

Knowing the upper and lower levels will help us in the decision-making and we can take corrective actions in time, if needed. Hence, we have to check the range of expected ROI that we can observe from the platforms and which will not alter our optimal return value of \$456K.

The table below provides a clear overview of how sensitive the optimal allocation is to changes in expected ROI for each platform, holding other ROIs constant. It helps in understanding the range within which ROI can fluctuate while maintaining the same optimal allocation.

Platform	Lower Limit	Curr ROI	Upper Limit
Print	-inf	0.031	0.049
TV	0.039	0.049	0.062
SEO	-inf	0.024	0.039
AdWords	0.033	0.039	0.046
Facebook	-inf	0.016	0.02899999999999998
LinkedIn	-inf	0.024	0.039
Instagram	0.039	0.046	inf
Snapchat	-inf	0.026	0.039
Twitter	-inf	0.033	0.039
Email	0.02899999999999998	0.044	inf



**The absent limits mean they are tending to infinity towards either of the end (+/-)*

For instance, platforms like "Print," "SEO," "Facebook," "LinkedIn," "Snapchat," and "Twitter" can experience a decrease in ROI (independently) without affecting the allocation, while "Instagram" and "Email " have lower thresholds such that if respective ROI decreases beyond those values while other ROIs remain constant then we might not attained the expected optimal returns.

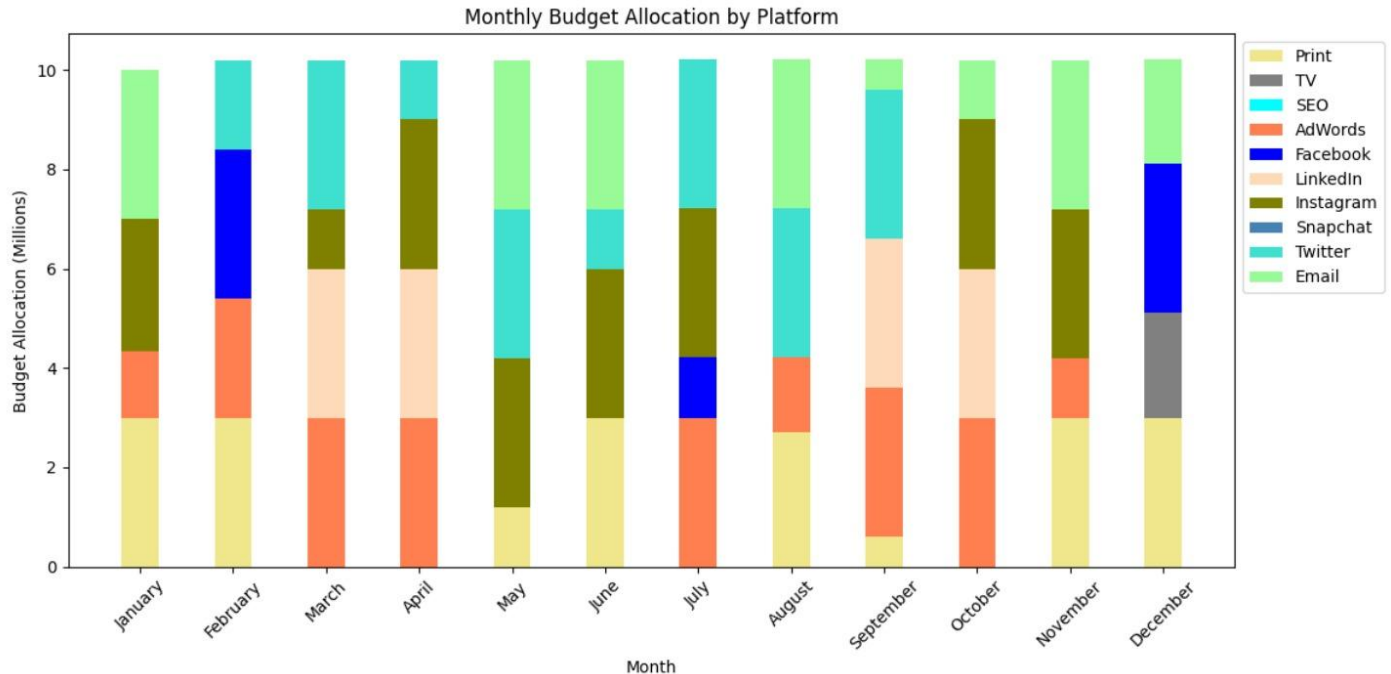
Using the first ROI as the starting point, we can vary the expected ROIs for each platform in their respective range in the table above. For these ranges of values, the optimal solution will remain the same. Beyond these limits, the allocation may need to be adjusted to achieve the desired outcome.

Analyzing re-investment strategy:

In this section, we analyze the optimal monthly budget allocation for our marketing activities, taking into consideration the reinvestment of half of the return and adhering to the specified constraints.

The allocation strategy is dynamic and varies from month to month to maximize ROI while staying within budget limits.

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Total Budget
January	3.000000	0.000000	0.0	1.333333	0.000000	0.0	2.666667	0.0	0.000000	3.000000	10.000000
February	3.000000	0.000000	0.0	2.395500	3.000000	0.0	0.000000	0.0	1.791000	0.000000	10.186500
March	0.000000	0.000000	0.0	3.000000	0.000000	3.0	1.199429	0.0	3.000000	0.000000	10.199429
April	0.000000	0.000000	0.0	3.000000	0.000000	3.0	3.000000	0.0	1.199707	0.000000	10.199707
May	1.196177	0.000000	0.0	0.000000	0.000000	0.0	3.000000	0.0	3.000000	3.000000	10.196177
June	3.000000	0.000000	0.0	0.000000	0.000000	0.0	3.000000	0.0	1.201481	3.000000	10.201481
July	0.000000	0.000000	0.0	3.000000	1.207644	0.0	3.000000	0.0	3.000000	0.000000	10.207644
August	2.709695	0.000000	0.0	1.500000	0.000000	0.0	0.000000	0.0	3.000000	3.000000	10.209695
September	0.607204	0.000000	0.0	3.000000	0.000000	3.0	0.000000	0.0	3.000000	0.607204	10.214408
October	0.000000	0.000000	0.0	3.000000	0.000000	3.0	3.000000	0.0	0.000000	1.197045	10.197045
November	3.000000	0.000000	0.0	1.182065	0.000000	0.0	3.000000	0.0	0.000000	3.000000	10.182065
December	3.000000	2.108393	0.0	0.000000	3.000000	0.0	0.000000	0.0	0.000000	2.108393	10.216785



Key Takeaways:

- As we progress through the year, the allocation strategy continues to adapt.
- Different platforms receive varying proportions of the budget, reflecting changing ROI values and constraints.
- The dynamic allocation approach allows us to respond to fluctuations in ROI and market conditions.
- It ensures that we make the most of our budget while adhering to all the constraints and reinvestment strategy.
- Despite the changing allocation patterns, the primary objective remains to maximize ROI and each month's allocation is determined with this objective in mind.

Note:

It is crucial to emphasize that this allocation strategy requires continuous monitoring and adjustment as ROI and market dynamics evolve. Regular optimization ensures that we remain on track to achieve our ROI targets.

The provided allocation data serves as a foundation for our ongoing marketing strategy, allowing us to adapt and thrive in a competitive environment.

Budget Stability

In the context of our marketing budget allocation strategy, the term "stability" carries a specific and vital meaning. A stable budget refers to a monthly allocation plan where the monthly changes in spending for each advertising platform do not exceed \$1 million. This definition of stability ensures a controlled and manageable degree of fluctuation in budget allocations from month to month. The concept of a stable budget bears significant implications for our marketing strategy and financial management. A stable budget provides a level of predictability and consistency that is invaluable in navigating the dynamic landscape of marketing.

Now, let's turn our attention to the question at hand: Is the allocation we have determined considered stable? To answer this, we will evaluate whether the monthly changes in spending for each platform fall within the \$1 million limit as defined for stability.

Is the allocation we found stable?

Regrettably, the allocation we have determined does not meet the criteria of a stable budget.

A thorough analysis of the provided budget allocation reveals that it does not guarantee that the monthly change in spending for each platform remains within the stipulated limit of \$1 million, as evident from the data presented in the table.

For instance, consider the 'Print' media allocation. It remains consistent at a spending level of \$3 million from January to March, displaying minimal change. However, in April, it abruptly drops to \$0, resulting in a change exceeding \$1 million. This pattern of fluctuation is not unique and is observed across various platforms.

Potential stability modeling strategies

In light of this, it becomes imperative to model and enforce stability constraints within our budget allocation without the need to initiate a new optimization model.

The objective is to maintain consistency and predictability in our monthly spending, aligning with the stability criteria.

Design of the Solution:

To model this stability constraint effectively, we propose the following approach:

A. Number of Decision Variables:

- a. Introduce one additional decision variable for each platform for each month, resulting in a total of 120 decision variables. $[12 \text{ (for months)} * 10 \text{ (platforms)} = 120]$

B. Number of Constraints:

- a. Implement constraints for each month-platform combination, ensuring that the change in spending from one month to the next does not exceed \$1 million.
- b. For example, February's spending on 'Print' minus January's spending on 'Print' should be less than or equal to \$1 million.
- c. In addition to these new constraints, it is important to note that the existing constraints, such as
 - i. Total monthly spend not exceeding \$10 million,
 - ii. Constraints related to social media and specific platform combinations, and
 - iii. The \$3 million limit per channel per month, should remain in place.

These collectively shape our budget allocation strategy.

By incorporating these stability constraints into our existing optimization model, we aim to strike a balance between maximizing ROI and ensuring budget stability, thus providing a more robust framework for effective marketing budget management.

Conclusion:

Summary of Findings

1. We are expecting to achieve **a return of \$456K (~4.5%) on our budget of \$10M** by using either of the expected ROI from the two consultancies.
2. Although **the return value remains unchanged, the allocation of funds varies significantly** if we use expectations of one consultancy over another. We need to trade this carefully and allocate funds strategically.
3. Cross match of ROI expectations and funds allocation between the two consultancies will hamper our returns significantly in both cases by more than 40%.
4. While we can expect **~\$0.9M more** on our returns by removing the investment cap on each platform, it will restrict our presence and might hurt our growth on other platforms in the future.
5. We have identified the upper and lower threshold values for each platform which we need to keep track of for the in-time course correction/decision-making to achieve our intended ROI amount.
6. Our current re-investment strategy will lead us to an unstable budget allocation and we have proposed a potential solution for mitigating this scenario. If needed, we are in a position to start the project.

Concluding remarks:

In conclusion, this report provides a comprehensive data-driven framework for optimizing the allocation of a \$10 million marketing budget. Through rigorous analysis of ROI estimates, we have derived optimal budget allocations that align with both consultancy recommendations. The flexibility of our approach allows us to adapt to varying market conditions and maximize ROI while complying with budgetary constraints.

Furthermore, we have highlighted the importance of budget stability and proposed strategies for achieving it. By embracing these insights, our marketing department is well-equipped to assist the Chief Marketing Officer in making informed decisions, ensuring that our budget allocations are not only financially sound but also aligned with our organizational goals.

This report serves as a valuable tool in enhancing our marketing strategy, driving profitability, and ultimately contributing to the company's success.