TEXAS
The University of Texas at Austin

TOPIC 1 LINEAR PROGRAMMING – SENSITIVITY ANALYSIS

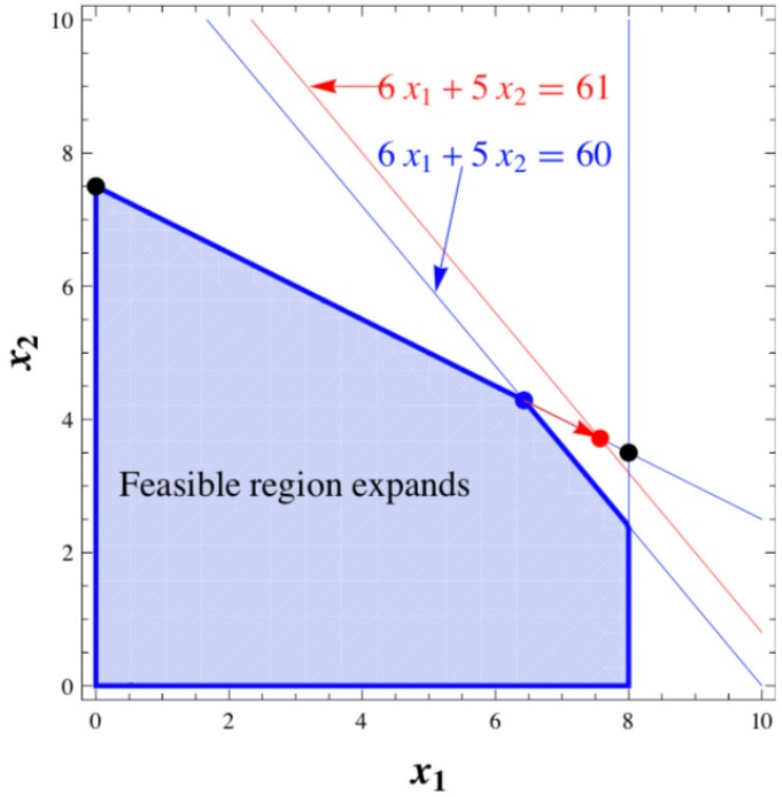
What if?

- When you solve an LP you typically choose the most likely values of input variables, such as unit costs, forecasted demands, and resource availabilities, and then find the optimal solution for these particular input values
  - This provides a single "answer"
- However, in most realistic situations, it is wishful thinking to believe that all the input values you use are exactly correct
- Therefore, it is useful – indeed, mandatory in most applied studies – to follow up the optimization step with what-if questions.
- This is called sensitivity analysis

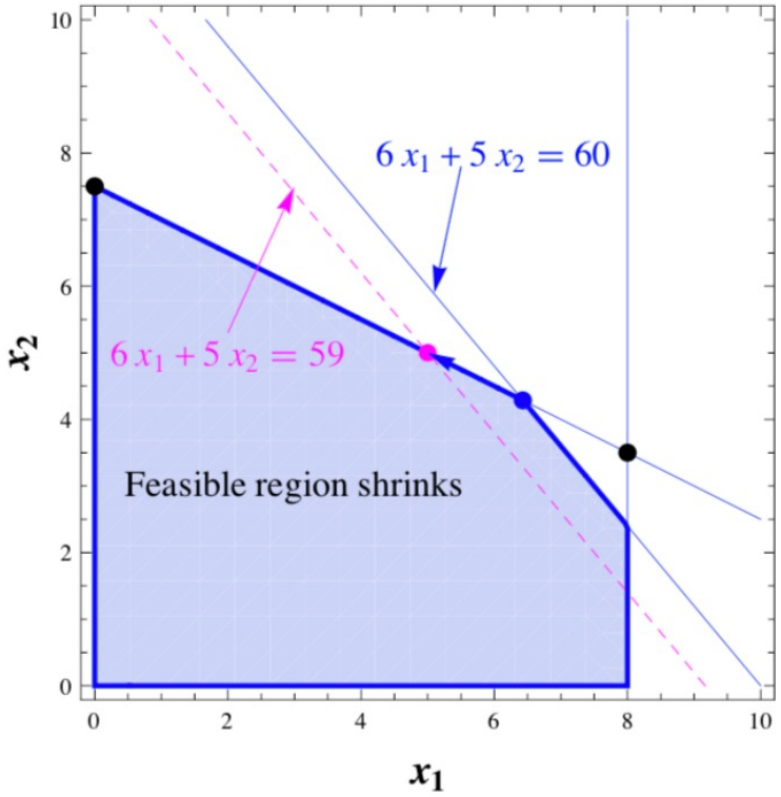
Sensitivity Analysis

- One way to get an insight into sensitivity is to ask what happens to the objective if the right-hand side (RHS) value of a constraint is increased by 1 unit
- In our OJ example, relaxing the constraint on the resources might help improve the profit (if it is a binding resource) or not help improve (if it is a non-binding resource)
- The shadow price or dual variable associated with a particular constraint is the change in the optimal value of the objective function per unit increase in the RHS value for the constraint, all other problem data remaining unchanged

Shadow Prices



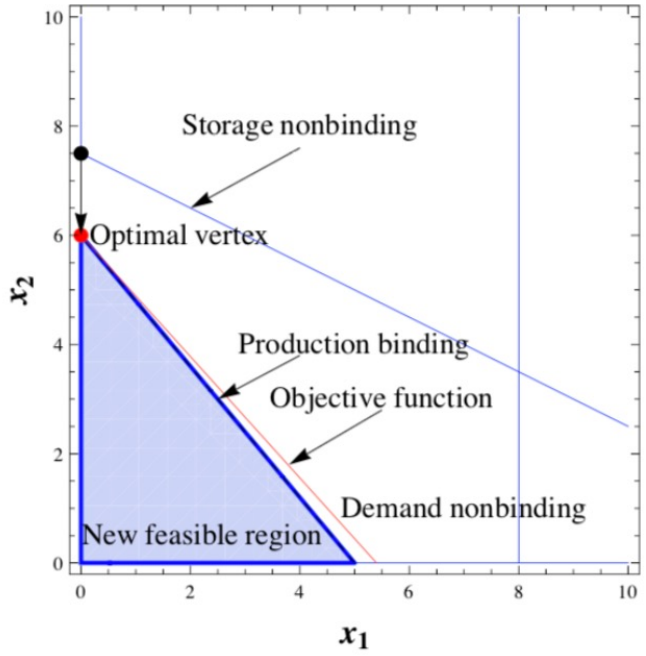
(a) Feasible region expands (relaxing a constraint).



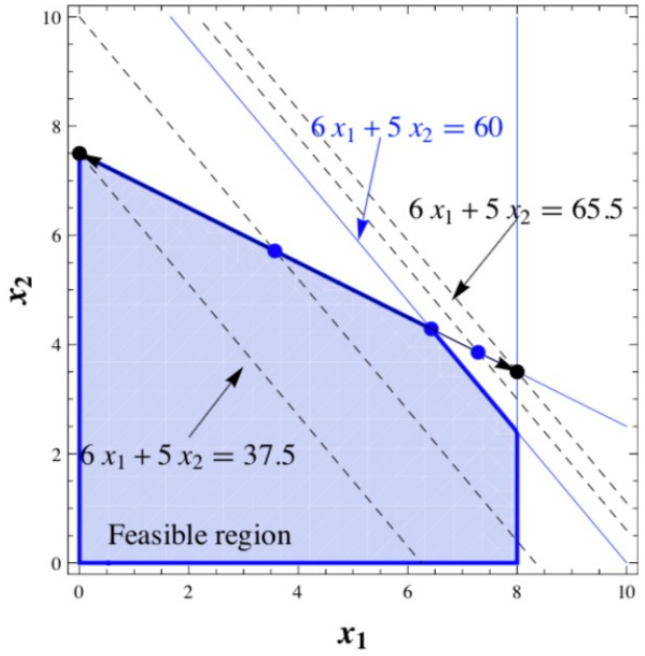
(b) Feasible region shrinks (tightening a constraint).

Shadow Prices

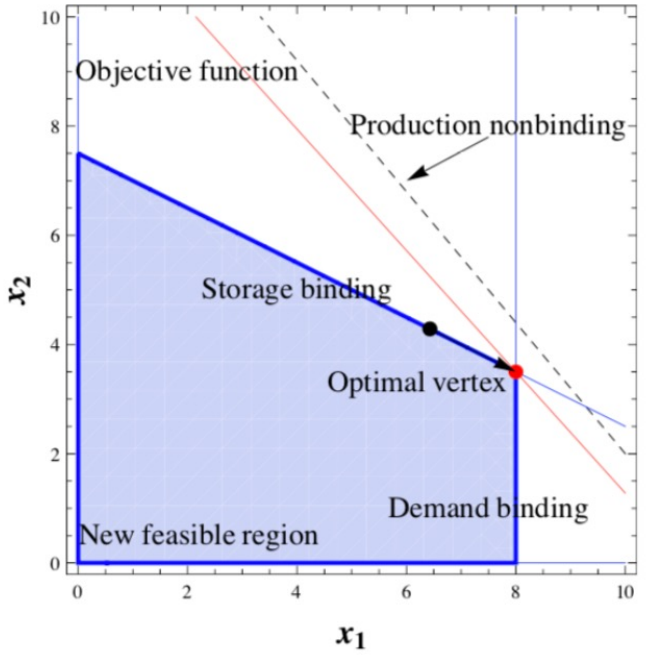
- Shadow prices remain constant over a particular range of the RHS. The range is defined by interval over which the set of binding constraints does not change
- Outside of this allowable range the shadow price may change



(a) Decreasing the rhs beyond the range.



(b) The range of the rhs for which shadow price remains constant.

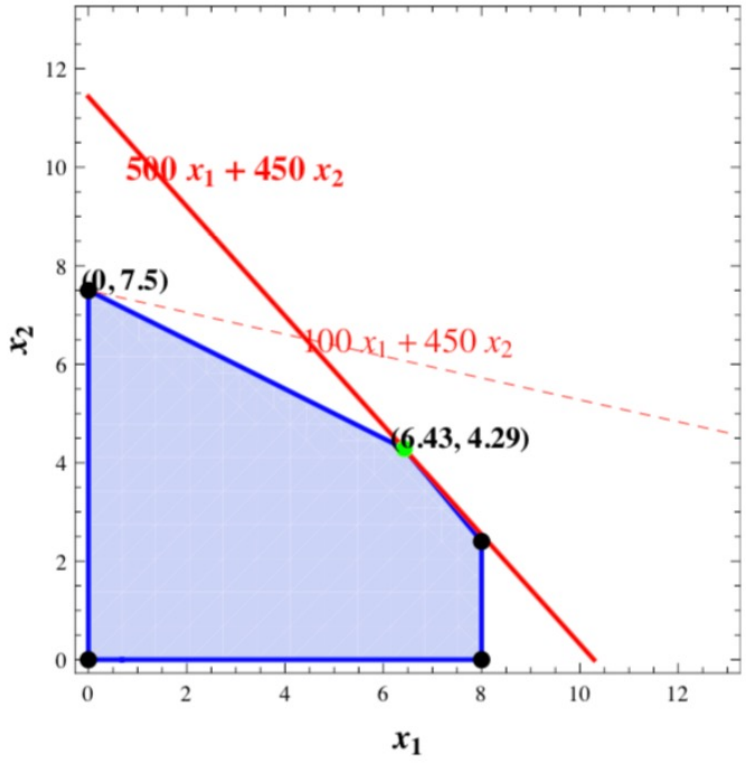


(c) Increasing the rhs beyond the range.

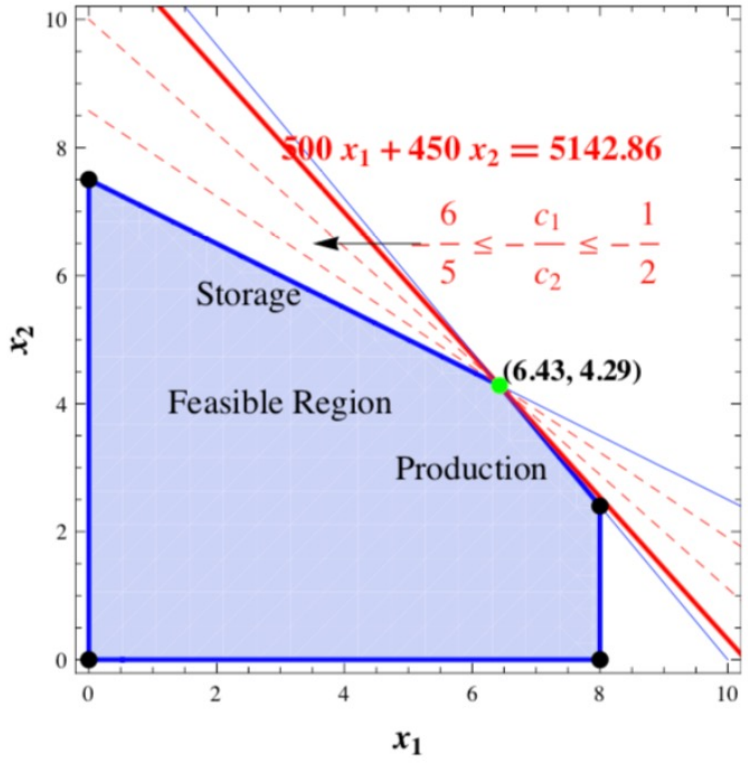
Sensitivity Analysis

- One can ask various other sensitivity questions. For example what happens when the profits on 6-Oz glasses increase? That is, what happens when c_1 changes? Currently c_1=500.
- How long will the current optimal production decision (6.43, 4.29) continue to remain optimal? What is the range for the values of c_1?
- What happens when the slope (or the value of c_1) is changed beyond the range? Where does the optimal solution move?

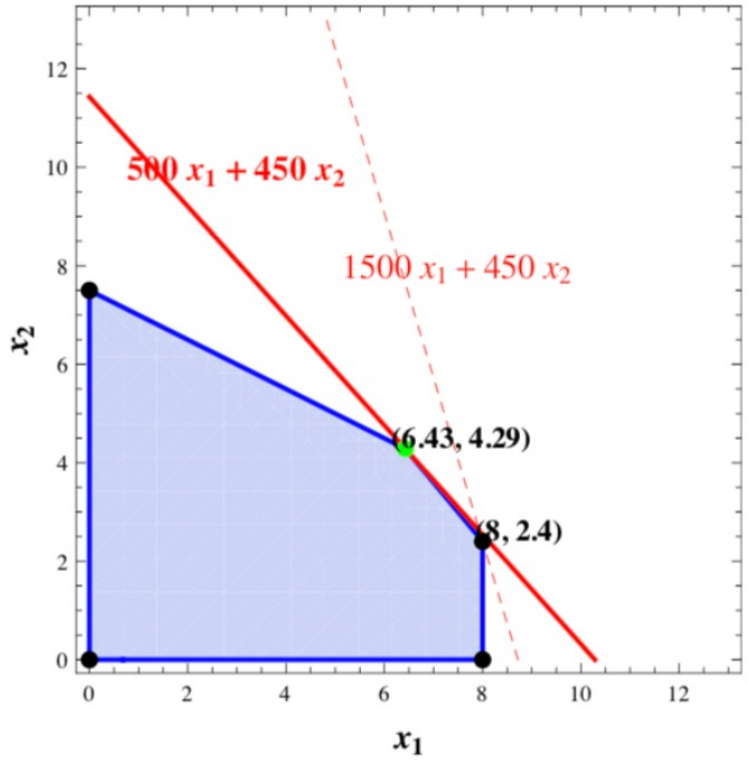
Sensitivity Analysis



(a) When the slope is increased beyond the range.



(b) When the slope is within the range.



(c) When the slope is reduced beyond the range.

Sensitivity Analysis

- Let’s answer these questions with python!