TEXAS
The University of Texas at Austin

TOPIC 3 NONLINEAR PROGRAMMING

Optimization

- A general optimization problem is of the form
  - $\min_x f(x)$
  - s.t. $x \in S$
- So far, we have addressed a very specific class of optimization problems
  - $f(x) = c_1x_1 + \cdots + c_nx_n$ (linear objective)
  - S is defined by $a_1x_1 + \cdots + a_nx_n \leq b$ (linear constraints)
  - S could also have integer constraints

Optimization

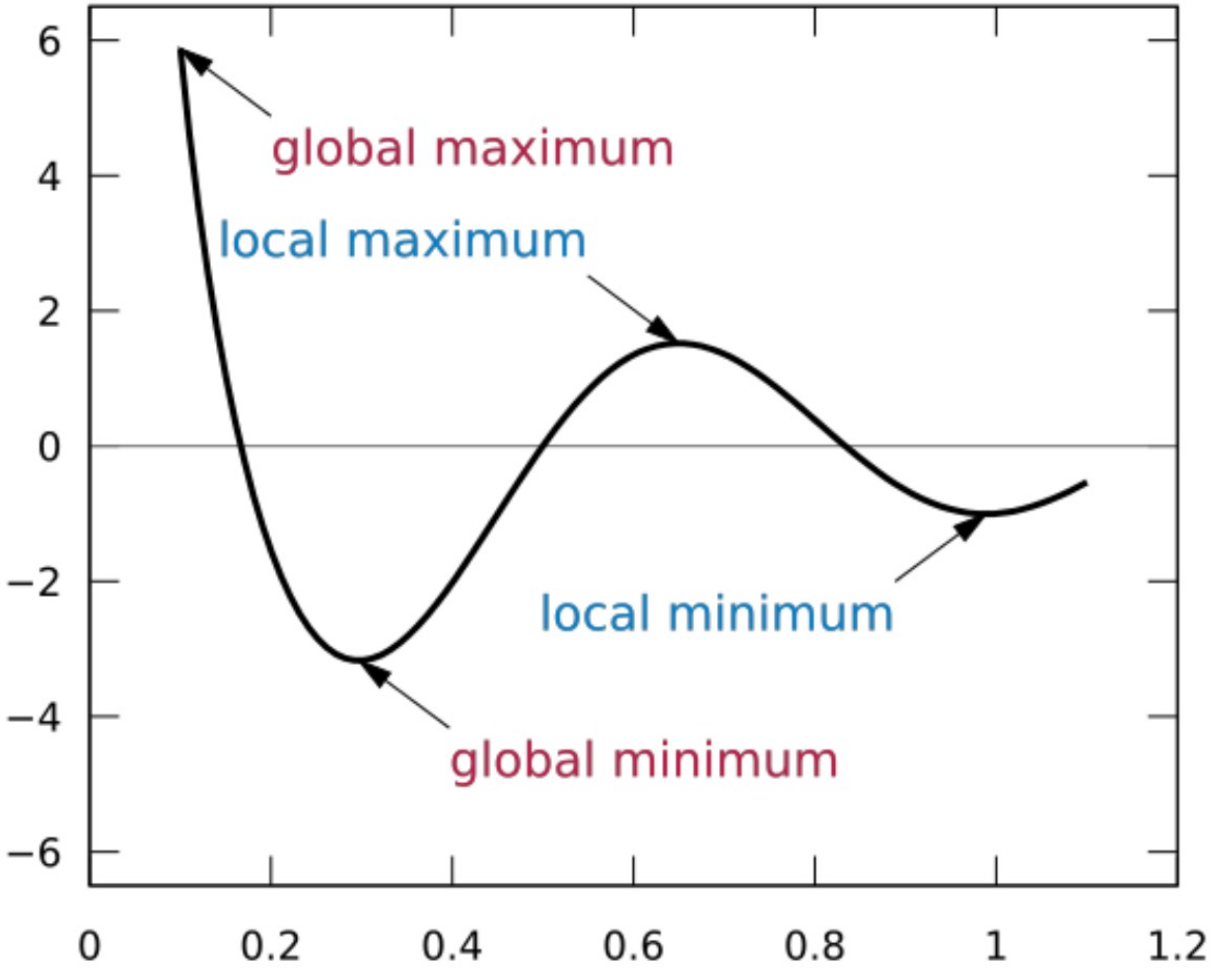
- What if $f(x)$ is some function that isn't linear
  - $f(x) = x_1^2 - x_2 + e^{x_3 x_2} - \sin(x_1 x_2^2)$
- What if S isn't defined by linear constraints
  - $x_1^2 + x_2^2 + x_3^2 \leq 1$
  - Corners?
- If either of these occur, then we call the optimization problem a nonlinear program (NLP)

Nonlinear Programming

- Why can models become nonlinear?
  - Non-constant returns to scale (diminishing returns)
  - In supply/demand models when profit is maximized
  - In measuring goodness of fit with the sum of the squared differences
  - Models where risk is measured as the variance
  - Neural Networks

- Nonlinear models are often more realistic than linear models, but they are also more difficult to solve.

Global vs Local Optimum



Solution Algorithms

- There are LOTS of algorithms to solve NLPs
- Most of them are guaranteed to find local optimums
- Very few algorithms exist that are guaranteed to find global solutions
  - Most are VERY slow
- Hopefully, the local optimum you find is good enough?

Python Packages for NLP

- There are many good-ish packages for NLP in python
  - scipy.optimize – some built in solvers to scipy
  - pyomo – part of COIN-OR
  - pysolnp – just one algorithm
- scipy.optimize is tricky because different algorithms require different syntax of objective and constraints
- pyomo is a modeling suite that has many open-source algorithms, but requires learning their language
- pysolnp is based on PhD dissertation of former student of Dantzig
- I usually just stick with scipy

Python Packages for NLP

- For scipy.optimize you must tell it which solver to use
  - BFGS: just for unconstrained problems
  - L-BFGS-B: just for box constraints (a <= x <= b)
  - SLSQP: for general constraints
- These only work well/ok for convex programs
- It's not uncommon for scipy.optimize to give garbage solutions…

Minimization or Maximization

- Almost all NLP packages only solve min problems!!
  - What if you want to maximize something?!?!
- If you want to maximize, just minimize the negative of your objective function
- Trying to make a function as big as possible is the same as trying to make the negative of that function as small as possible

Example

- A company manufactures and sells a product
  - They get to set the price
  - It costs $50/unit to manufacture
- The price drives demand through a demand function
  - $D = \alpha P^{\beta}$, where $\alpha > 0,\ \beta < -1$
  - This is called a constant elasticity of demand function

- Let's use calculus to find the price that maximizes profit

Example

- Assume $\alpha = 3777178,\ \beta = -2.154$
- Let's plug this example into python
- What if I also get to pick the number of items manufactured?

Extension

- What if the solver must pick the number to manufacture?

Another Example

- A company wants to locate a warehouse from which it will ship products to four customers.
- The locations and number of shipments are given.
- A single warehouse must be used to service all the customers.
- The company wants to determine the location of the warehouse that minimizes the total distance traveled from the warehouse to the customers.

Customer data				
	X-coordinate	Y-coordinate		Annual shipments
Customer 1	5	10		200
Customer 2	10	5		150
Customer 3	0	12		200
Customer 4	12	0		300

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Class Participation

- Formulate an NLP and solve for the optimal location of the warehouse

Constraints

- Now suppose there is a river that runs diagonally through town (Southwest to Northeast)
- The warehouse needs to be on the north side of the river for permitting
  - The y coordinate must be larger than x coordinate
- Where should they put their warehouse now?
  - Don’t worry about bridges…they deliver using a helicopter…