Time Value of Money

READ TIME VALUE OF MONEY

Time Value of Money – FV, PV, interest or discount rate, time

When you save money, you expect that money to grow over time. At a bank, the client earns interest on savings.

A bank earns interest when it lends to a borrower to buy a house so the client expects to pay the mortgage rate.

We have a present value (PV), a future value (FV), a rate of interest or return (I/YR), and a time period or number of years (N).

First make sure your calculator is on 1 period per year and not 12 periods per year. To do this type "1" and hold down the orange key (second function) while pressing on the PMT key (second function is P/YR).

If the calculator is not on 1 period per year, you will get the wrong answer.

Examples: N = 1

1) Suppose you put $200 into a bank account at the annual risk-free interest rate of 19%. How much money will this deposit be worth in one year?

FV = $200 * 1.19 = $238 (PV = -200, I = 19, N = 1, FV=238)

2) A one-year investment is certain to pay $200. If the annual risk-free interest rate is 3.2%, what is the present value of the payment on the day of investment?

PV = 200/1.032 = $193.8 (FV = +200, I = 3.2, N = 1, PV=-193.8)

3) One year from now you will need a $1,000 deposit to rent a new apartment. You now have $906. If you invest your $906 for one year, what is the lowest annual interest rate that will enable you to meet your goal?

$906 * (1 + r) = $1,000 Annual interest rate = r = (1000/906) – 1 = 10.38%

(FV = 1000, PV = -906, N = 1, I=10.38)

Examples

(1) You invest $162 today. What will be the value of your investment 4 years from now if the annual risk-free interest rate is 6%? (Try changing years to 30 and then

FV = 162 * 1.06^4 = $204.52 (PV = -162, I = 6, N = 4, FV=204.52)

(2) An investment will pay $16,600 in 10 years. What is the present value of this investment today if the annual risk-free interest rate is 9%? What about a discount rate of 10%?

PV @r = 9% = 16600/(1.09)^10 = $7012.02 (FV = 16,600, I = 9, N = 10, PV=-7012.02)

PV @r = 10% = 16600/(1.10)^10 = $6400.02 (FV = 16,600, I = 10, N = 10, PV=-6400.02)

(3) The risk-free rate is 7% and you can either get (i) $3,150 today or (ii) $7,000 9 years from now. Which option would you prefer and why?

Option (i) PV = $3,150

Option (ii) PV = 7000 / (1.07)^9 = $3807.54 (FV = 7000, I = 7, N = 9, PV=-3807.54)

You would prefer option (i) / (ii)

Time Value of Money – What if we add a constant CF or PMT

You are applying the concept of the power of compounding.

The "power of compounding" is that when you reinvest the principal and the interest at the end of the period, you earn not only more return on the original investment, called the principal, but you also earn interest on interest.

Now we will introduce a new concept. Some problems in finance have multiple cash flows over multiple periods. If the cash flow is always constant or the same, we can call that a payment (PMT).

Now, when we practice problems you might sometimes need to think of a recurring cash flow over multiple periods and use the PMT key in your financial calculator.

Examples – can you recognize the constant CF or PMT?

(1) How much must be invested today at 8% interest to accumulate enough to retire a $10,000 debt due in 7 years from now today?

Investment = PV = 10,000 / 1.08^7 = $5,834.90

(2) Given an 11% rate of return, what is the amount that must be put into an investment account at the end of each of the next ten years in order to accumulate $60,000?

N = 10; I/YR = 11; PV = 0; FV = 60,000

Annual Investment = PMT = $3,588.09

(3) Company XYZ's earnings per share is expected to grow from $3 to $4.5 over the next 8 years. What is the rate of growth in XYZ's earnings?

N = 8; PMT = 0; PV = -3; FV = 4.5

Growth rate = I/YR = 5.2%

Examples

(4) An investor just won the lottery and will receive $50,000 per year at the end of each of the next 20 years. At a 10% interest rate, what is the PV of the winnings?

PMT = $50,000; I/YR = 10; N = 20; FV = 0

Present value = PV = $425,678.19

(5) If 10,000 is invested today in an account that earns interest at a rate of 9.5%, what is the value of the equal withdrawals that can be taken out of he account at the end of each of the next 5 years if the investor plans to deplete the account at the end of the time period (i.e. account balance reaches 0)?

N = 5; I/YR = 9.5; PV = -10,000; FV = 0

Annual withdrawals = PMT = $2,604.36

Time Value of Money – What if the CF is not even?

If cash flows are flat or equal, we can call those payments and use PMT. Some problems in finance though do not have equal Cash Flows.

For example, the company might be considering a project that entails a cash outlay at year zero. Financial executives project future Cash Flows to be received in future years, and those are almost always different from year to year. The value of the project today has to take account of different differing cash flows.

Or, with stock prices, you buy the stock price today. Future sales are hopefully projected to grow (unless we are in a recession) so that future cash flows are projected to be higher and different in the future. Stockholders are the owners of those Cash Flows.

Thus sometimes we will need to use what is called the Cash Flow and NPV mode.

Examples

(6) An investment you made has the following cashflows:

Year 1	Year 2	Year 3	Year 4	Year 5
-200	-100	100	150	350

Risk-free rate = 4%

(i) Assume these cashflows happen at the end of each year; for example, you make a payment of $200 one year from now. What is the present value of this investment?

CF0 = 0; CF1 = -200; CF2 = -100; CF3 = 100; CF4 = 150; CF5 = 350; I/YR = 4%

PV = NPV = $220.03

Annual Percentage Rate (APR) vs. Effective Annual Rate (EAR)

Sometimes rates are quoted as "Annual," "APR," or "Nominal."

If the rate is compounded more frequently than annually, the real rate is higher.

This is called the Effective Annual Rate, EAR, or just Effective Rate.

In the U.S., interest is paid on bonds semiannually but the rate is quoted as an APR. Everyone investing in U.S. bonds knows this or should know this.

Also, credit card rates are quoted on a nominal or effective basis, but some people, if they are not financially savvy, may not completely understand that when they don't pay bills off or pay on time, the real rate or effective rate is higher than the quoted rate (and late fees are adding to it.)

Examples

(1) You deposit some money in a savings account offering an annual percentage rate of 2%, compounded monthly. What is the effective annual rate (EAR) that your investment will earn?

EAR = (1+2%/12)^(1*12) – 1 = (1 + .1667%)^12 – 1 = 2.02%

If you don't want to use parentheses, use this order:

.02/12 = …. Add 1 = ….. Raise to the power of 12 = … subtract 1 = … times 100

(2) A friend tells you that his credit card is charging an annual percentage rate of 18% compounded monthly. You tell him you will compute the effective rate or EAR. What is that rate?

EAR = (1+18%/12)^(1*12) – 1 = (1 + 1.5%)^12 – 1 = 19.56%

.18/12 = …. Add 1 = ….. Raise to the power of 12 = … subtract 1 = … times 100

Examples

(3) A bond has an 8% coupon rate (with interest paid semi-annually), a face value of $1,000, and matures in 10 years. If the bond is priced to yield 6% (you can treat this as the annual discount rate), what is the bond's current price (i.e. Present Value)?

* Note: (i) A 8% semi-annual bond pay two interests per year with each worth 8%/2 = 4% of face value

(ii) The face value of a bond is what you receive by the time the bond matures (i.e. in 10 years)

FV = 1000

PMT = (.08*1000/2) = 40

I = (6/2) = 3

N = (10*2) = 20

PV = -1,148.77 (or just 1,148.77)

Loans called Annuities, Interest vs. Principal, Perpetuities

Loans where interest and principal are paid in even payments over time are called annuities. A good example is a mortgage on a house.

The principal or balance goes down over time, so the interest as a proportion of the total payment at the beginning is high.

*So far we have discussed time value of money problems with a fixed number of periods, perhaps as long as 30. But what if the periods go on **forever**? Then we apply a formula that is called a **perpetuity or growing perpetuity**.*

- *Example: Preferred stockholders get paid after bond investors but before regular stockholders. These investors get paid a dividend **forever**.*
- *Another example: Some companies might disappear after 100 years as the economy changes, but often they evolve and become a different type of business. Thus, when we value the stock of a company, we assume a perpetuity formula in the future year 5 or 10.*

Annuity Application to Mortgage

To buy a house, which costs $250,000 you make a down payment of $50,000, and take out a fixed-rate amortizing mortgage of $200,000 for 15 years, with an annual percentage rate (APR) of 4.25%, compounded monthly. What will be your required monthly payment?

* Note: APR is basically a simple annual rate in mortgage terms

Interest rate per month = 4.25% / 12 = 0.35417%

Solve for PMT in Calculator: PV = 200,000 N = (15*12) = 180 I/YR = (4.25/12) = .35417

PMT = -$1504.56 (or just $1504.56)

Example of Minimum Payment to Prevent Growing Perpetuity

(1) A friend of yours has a credit card balance of $500 with an APR of 15% compounded monthly. Because it is not secured by a house like a mortgage loan is, the credit card company requires a minimum monthly payment, usually 2% of the loan balance. Otherwise, if your peer paid nothing, the debt would turn into growing perpetuity. The debt would grow and never be paid off.

Your peer will not charge anymore and thinks it is ok to go with the minimum payment of .02*500 = 10. You are concerned and say, let me tell you how many months / years that will take to pay off.

PV = ___ I = ___ PMT = ___ now solve: N = ___ months /12 = _____ years

(PV = -500, I = 15/12 = 1.25, PMT = 10, N=78.96 or about 79 months; /12 = 6.58 or about 7 years)

(2) The friend asks, "what if I want to pay it off in 1 or 2 years. How much should I pay per month?"

PV = ___ I = ___ N = ___ now solve: PMT = ___ months

(PV = -500, I = 15/12 = 1.25, N = 12, PMT = $45.13)

(PV = -500, I = 15/12 = 1.25, N = 24, PMT = $24.24)

Examples: Annuities vs Perpetuities

(1) You are offered an investment that will pay you $30,000 income per year forever, beginning one year from now. Assuming an annual risk-free interest rate of 7%, what would you expect to pay for this offer?

PV = 30,000 / 7% = $428,571

(2) You may purchase an annuity that will pay you $400,000 in income per year starting one year from now and continuing for a total of 24 years, or 24 payments. Assuming an annual risk-free interest rate of 3%, what is a fair price for this annuity?

(PMT = 400,000, I = 3, N = 24, PV=-6,774,216.85)

Examples: Annuity and Perpetuity

(3) To buy a house, you take out a fixed-rate amortizing mortgage of $200,000 for 15 years, with an annual percentage rate (APR) of 10.8%, compounded monthly. What will be your required monthly payment?

* Note: APR is basically a simple annual rate in mortgage terms

PV = -200,000 I =(10.8/12) = .9 N= (15*12) =180 PMT = 2,284.14

(4) You are planning to donate money to fund a community sporting event every year indefinitely, beginning one year from now. Assume the event costs $9,800 in the first year, and this amount will grow each year by 1.3%. If the annual interest rate is 1.9%, how much money should you donate today?

Money to donate = PV = 9800/(1.9% - 1.3%) = $1,633,333

Examples: Annuities vs Perpetuities

(5) Company XYZ's preferred stock stocks are expected to pay a $9 annual dividend forever. If the required return on equivalent investments is 11%, what is a share of XYZ's preferred stock worth?

Price = PV = 9/11% = $81.82

(6) Company XYZ's preferred stock stocks are expected to pay a $9 annual dividend at the end of the year (CF1). If the required return on equivalent investments is 11% and the dividend is expected to increase forever at 5%, what is a share of XYZ's preferred stock worth? Compare to the flat perpetuity.

Price = PV = = 9/(11% - 5%) = $150

Analysis: Compare to example 7: $150 is almost double. Though the first year CF at $9 is the same and the discount rate is the same, the stock value is nearly double because the dividend is expected to increase forever. That is a growing perpetuity is more valuable than a constant or zero growth perpetuity.