TOPIC 2 LINEAR PROGRAMMING – SHORT TERM FINANCING

Short Term Financing

- Corporations routinely face the problem of financing short-term cash commitments
- Linear programming can help in figuring out an optimal combination of financial assets to meet these commitments
- The decision variables depend on the list of assets to choose from and the list of liabilities to meet
- The objective is usually to minimize the net present/future value of meeting all the commitments
  - Or maximize money left over at the end of the time horizon
- The primary constraints are that all cash inflows and outflows for each time period should be sufficient to meet liabilities

Short Term Financing

- Consider the following short term financing problem (given in 1000's):

Month	Jan	Feb	Mar	Apr	May	Jun
Net cash flow	-150	-100	200	-200	50	300

- The available sources of funds are
  - a line of credit from your bank, up to \$100k, 1% interest per month
  - in Jan-Mar, you can issue a 3-month commercial paper bearing an interest of 2% for the 3-month period
  - excess funds can be reinvested at a rate of 0.3% per month

Class Participation

- Spend a few minutes thinking about what aspects of this problem are important
- How would you write equations to solve it?

x_i is money borrowed from bank at time periods 1-5, y_i is money borrowed from bond investors at time periods 1-3
Original Constraints

Add slack

Objective

Short Term Financing

- Decision Variables
  - $x_1, \dots, x_5$: total amount draw from line of credit in each month
  - $y_1, \dots, y_3$: total amount of commercial paper issued in each month
  - $z_1, \dots, z_6$: excess funds in each month
- Objective - maximize the cash balance at the end of six months
  - max z_6
- Constraints
  - Meet monthly liabilities
  - $x_i \leq 100$
  - $x_i, y_i, z_i \geq 0$

The LP

- Choose $x_1, \ldots, x_5, y_1, \ldots, y_3, z_1, \ldots, z_6$
- To maximize z_6
- Such that:

$$
\begin{aligned}
x_1 + y_1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad - z_1 &= 150 \\
x_2 + y_2 \qquad - 1.01x_1 + 1.003z_1 - z_2 &= 100 \\
x_3 + y_3 \qquad - 1.01x_2 + 1.003z_2 - z_3 &= -200 \\
x_4 - 1.02y_1 - 1.01x_3 + 1.003z_3 - z_4 &= 200 \\
x_5 - 1.02y_2 - 1.01x_4 + 1.003z_4 - z_5 &= -50 \\
-1.02y_3 - 1.01x_5 + 1.003z_5 - z_6 &= -300 \\
x_i, y_i, z_i &\geq 0 \\
x_i &\leq 100
\end{aligned}
$$

The Optimal Solution

	Jan	Feb	Mar	Apr	May	Jun
Bank		50.98				
Bonds	150	49.02	203.43			
Excess Funds			351.94			92.~~47~~ 50
Liability	-150	-100	200	-200	50	300

Slack Variables

- Sometimes it makes sense to modify an inequality constraint
  - Add a slack variable and change inequality to equality
- If you need to use the difference between the left and right side of an inequality you can eliminate a lot of tedious algebra
- Inside the simplex method, the algorithm actually creates slack variables for all the constraints anyway
  - You need not worry about extra computational cost

Class Participation

- Transform the following optimization problem to only have equality constraints (the non-negativity constraints can stay)

$$\max x_1 + x_2 + x_3$$

s.t.

$$2x_1 + x_2 - 2x_3 \geq 3$$
$$x_1 - x_3 \leq 2$$
$$x_1, x_2, x_3 \geq 0$$