# MTH 377/MTH577 Convex Optimization
# Winter Semester 2020
# Indraprastha Institute of Information Technology Delhi
# Coding Assignment
# Deadline: 15th April 2021,11:59PM

April 1, 2021

## Instructions

1. Please provide your solutions as a jupyter notebook, with all the data files in the same folder as the original notebook.
2. Please run your cells and save the outputs of the jupyter notebook before submitting it. It should be the case that the TAs can grade your notebook without running it, using the outputs saved.
3. For question requiring written solutions, please scan your solution and keep it in the same folder as the jupyter notebook.
4. Please contact Mohit Sharma or Anvit Sachadev in case of any doubts.

## Problem 1 (7 points)

Consider the following optimization problem:-

$$min_{x_1,x_2} f(x_1, x_2) = x_1^2 + \gamma\, x_2^2 - x_1\, x_2 - x_1 - x_2 \qquad (1)$$

1. (5 points)
   Suppose $\gamma = 1$. Implement your gradient descent algorithm. Specify the step size and initial point used. Report the optimal values of $x_1$, $x_2$ and f and also the number of iteration required to converge. Plot the following graphs and write your inferences:-

   - Iteration v/s Error for $\gamma = 1$ and step size = 0.1 (x-axis = iteration no., y-axis = error in that iteration).
   - Arbitrarily choose 20 different values of step size in the the interval [0,2]. Find out the number of iterations required to converge for each step size and plot step size v/s iterations graph (x-axis = step size, y-axis = iterations required to converge).
   - Arbitrarily choose 5 different values of $\gamma$ in the interval [0.5,1] and find out the number of iterations required to converge for each value of $\gamma$. Plot $\gamma$ v/s iterations graph (x-axis = $\gamma$, y-axis = iterations required to converge)

2. (2 points)
   Run your gradient descent implementation for $\gamma$ = -1 and plot Iteration v/s Error graph (x-axis = step size, y-axis = iterations required to converge). What do you observe? Explain with a valid reason.

**Note**:- Error in $k^{th}$ iteration of gradient descent is calculated as the absolute value of difference between f($x_1^k$,$x_2^k$) and f($x_1^{k-1}$,$x_2^{k-1}$) where $x_i^k$ represents the value of $x_i$ in $k^{th}$ iteration (i $\in$ {1,2}).

## Problem 2 (6 points)

Consider the following optimization problem:-

$$min_{x_1,x_2} f(x_1, x_2) = e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} + e^{-x_1-0.1} \quad (1)$$

The solution of (1) is $x_1^*$ = -0.34764, $x_2^*$ = -1.02557 $\times$ $10^{-10}$, f($x_1^*$,$x_2^*$) = 2.55927 (2)

1. (3 points)
   Implement Newton's method by using (2,1) as the initial point. Report your optimal points, optimal value of f and the update rule used in the implementation.

2. (3 points)
   Implement Newton's method by using (-2,1) as the initial point. Report your optimal points, optimal value of f and the update rule used in the implementation.

**Note**:- Your answer in both the above questions should converge to the solution points as given in (2).

## Problem 3 (7 Points)

Consider the problem of investing in $n$ assets, also known as creating a portfolio. You are required to choose a portfolio vector $w = (w_1, \ldots, w_n)$, where the $i$th entry denotes the fraction of money to be invested in the $i$th asset. Assume that investment in every asset has to be nonnegative. Suppose that $p = (p_1, \ldots, p_n)$ denotes the vector of fractional returns for all assets, where each entry of $p$ denotes the relative change in price over the consideration period. Now, assume that this fractional return vector $p$ is a random vector with some mean vector $m$ and covariance matrix $V$.

1. (2 mark) Setup the portfolio investment problem for constructing an optimal portfolio from 1 unit of wealth that minimizes the risk subject to achieving at least the target level of expected return.

2. (2 marks) Solve the above problem with $n = 20$. Feel free to start with random $m$ and $V$ and use any optimization package you feel like.

3. (3 marks) Plot the (risk, return) values of the optimal portfolio for varying target levels of return.

4. (Bonus) Does distributional assumptions used to generate $m$ make any difference to the final solution obtained?