Exercise 1. Find the following limits

(a) $\lim\limits_{x\to 4} \dfrac{4-x}{x^2-16}$

(b) $\lim\limits_{x\to -3} \dfrac{x^2+2x-3}{x+3}$

(c) $\lim\limits_{x\to -1} \dfrac{x+1}{(2x-1)^2-9}$

(d) $\lim\limits_{x\to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3}\right)$

(e) $\lim\limits_{x\to 1} \dfrac{\sqrt{x+3}-2}{x^3-1}$

(f) $\lim\limits_{x\to 3} \dfrac{2\sqrt{x+1}-\sqrt{x+13}}{x^2-9}$

(g) $\lim\limits_{x\to 0} \dfrac{x+1-\sqrt{1-2x-x^2}}{2x}$

(h) $\lim\limits_{x\to -1} \dfrac{-x-1}{1-\sqrt{-x}}$

(i) $\lim\limits_{x\to 0} \dfrac{\sin 4x}{x}$

(j) $\lim\limits_{x\to 0} \dfrac{\sin 7x}{\sin 5x}$

(k) $\lim\limits_{x\to 0} \dfrac{2x}{\tan 3x}$

(l) $\lim\limits_{x\to 0} \dfrac{\sin 7x + \sin 3x}{\sin 5x - 4x}$

(m) $\lim\limits_{x\to 0} \dfrac{\sin^2 2x}{8x^2}$

(n) $\lim\limits_{x\to 4} \dfrac{x^2-3x-4}{\sin(x-4)}$

(o) $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{\sin 3x}$

(p) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$

Exercise 2. Find the following limits

(a) $\lim\limits_{x\to -\infty} \dfrac{2x^5-5x+6}{3x^3-2x}$

(b) $\lim\limits_{x\to \infty} \dfrac{4x^2-8}{2x^3-3x^2+x}$

(c) $\lim\limits_{x\to -\infty} \dfrac{x^5-5x^3+6}{3x^6-x}$

(d) $\lim\limits_{x\to -\infty} \dfrac{5x^3-9}{2x^2-x^3+2x}$

(e) $\lim\limits_{x\to \infty} \dfrac{-2x^6-3x^2+3}{4x^3+6}$

(f) $\lim\limits_{x\to \infty} \dfrac{4x\sqrt{x^2+7}}{9x^2-13}$

(g) $\lim\limits_{x\to -\infty} \dfrac{4x\sqrt{x^2+7}}{9x^2-13}$

(h) $\lim\limits_{x\to -\infty} \left(\sqrt{x^2+2x}-\sqrt{x^2-4x}\right)$

(i) $\lim\limits_{x\to \infty} \sqrt{2x}\left(\sqrt{x+3}-\sqrt{x}\right)$

(j) $\lim\limits_{x\to \infty} \left(\dfrac{2x+1}{2x-5}\right)^{4x}$

(k) $\lim\limits_{x\to \infty} \left(\dfrac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$

(l) $\lim\limits_{x\to \infty} \left(\dfrac{x+1}{x-2}\right)^{2x-1}$

(m) $\lim\limits_{x\to -\infty} \left(\dfrac{x^2+2}{x^2-3}\right)^{\frac{x^2}{3}}$

(n) $\lim\limits_{x\to \infty} x^2\left(\ln\dfrac{x+1}{x} + \ln\dfrac{2x+3}{2x}\right)$

(o) $\lim\limits_{x\to \infty} \left(\dfrac{x^2}{3}\ln\dfrac{2x}{2x+1}\right)$

(p) $\lim\limits_{x\to -\infty} \cos(\arctan x)$

(q) $\lim\limits_{x\to \infty} \sin(\arctan x)$

(r) $\lim\limits_{x\to \infty} \log_2\left|\dfrac{x+1}{x^2+2}\right|$

(s) $\lim\limits_{x\to \infty} \arcsin\dfrac{1-x}{x+1}$

(t) $\lim\limits_{x\to \infty} \dfrac{(x+2)\arctan x}{x}$

(u) $\lim\limits_{x\to \infty} \ln(\sin(\arctan x))$

(v) $\lim\limits_{x\to \infty} e^{\sin(\operatorname{arccot}\left(\frac{2+x}{2-x}\right))}$

(w) $\lim\limits_{x\to \infty} \ln(\sin(\operatorname{arccot}\dfrac{1}{x}))$

Exercise 3. Find the following one-sided limits

(a) $\lim\limits_{x\to 2^+} \dfrac{x^2-9}{2-x}$

(b) $\lim\limits_{x\to 5^-} \dfrac{4x^2-100}{(x-5)^2}$

(c) $\lim\limits_{x\to -1^-} \dfrac{x^2+2x+1}{x^4-1}$

(d) $\lim\limits_{x\to 3^-} \dfrac{x-\pi}{-x^2+5x-6}$

(e) $\lim\limits_{x\to 0^+} e^{\frac{1}{x}}$

(f) $\lim\limits_{x\to 0^-} \dfrac{1}{1-\pi^{\frac{1}{x}}}$

(g) $\lim\limits_{x\to 1^+} \arctan \dfrac{1}{1-x}$

(h) $\lim\limits_{x\to 1^-} 2^{\frac{1}{(x-1)^2}}$

(i) $\lim\limits_{x\to 0^+} \left(\dfrac{1}{2}\right)^{\frac{1}{x}}$

(j) $\lim\limits_{x\to 0^-} \dfrac{x}{1+e^{\frac{1}{x}}}$

Exercise 4. Find all the vertical asymptotes of the following functions

(a) $f(x) = \dfrac{x^2-9x+14}{x^2-5x+6}$

(b) $f(x) = \dfrac{x+1}{x^3-4x^2+4x}$

(c) $f(x) = \dfrac{x^3-10x^2+16x}{x^2-8x}$

Exercise 5. Examine the continuity of a function:

(a) $f(x) = \begin{cases} 5 & x < -2 \\ (\frac{1}{2})^x + 1 & -2 \le x \le 0 \\ \log_{0.5}(x+\frac{1}{2}) & 0 < x \le 1.5 \\ \frac{-2}{2x-3} & x > 1.5 \end{cases}$

(b) $f(x) = \begin{cases} \operatorname{arccot}(\ln x) & x > 0 \\ x - \pi & x \le 0 \end{cases}$

(c) $f(x) = \begin{cases} 2^x & -1 \le x \le 0 \\ -x+1 & 0 < x \le 1 \\ \log x & 1 < x \le 2 \end{cases}$

(d) $f(x) = \begin{cases} \frac{x^2-x-2}{x+1} & x < -1 \\ -3 & x = -1 \\ \frac{\pi \tan(x-1)}{x^2-1} & -1 < x < 1 \\ \frac{\pi}{2} & x = 1 \\ \arctan \frac{1}{x-1} & x > 1 \end{cases}$

Exercise 6. Find the values of parameters a, b, so that the functions are continuous:

(a) $f(x) = \begin{cases} (x-a)^2 & x < 1 \\ 2^x - 1 & x \ge 1 \end{cases}$

(b) $f(x) = \begin{cases} \frac{x^3-1}{1-x} & x \ne 1 \\ 6a^2 - a - 5 & x = 1 \end{cases}$

(c) $f(x) = \begin{cases} 2x + \cos a & x < 1 \\ b^2 & x = 1 \\ 3\ln x + 3\sqrt[3]{x} & x > 1 \end{cases}$

(d) $f(x) = \begin{cases} \arctan(\frac{\sin|x|}{x\sqrt{3}}) & x < 0 \\ \frac{\pi}{2}(1-\sqrt{a^2-1}) & x = 0 \\ \frac{1}{\pi} e^{\frac{x-1}{x^2}} - b & x > 0 \end{cases}$

(e) $f(x) = \begin{cases} \frac{2}{\pi}\arctan\left(\log_{\frac{1}{2}}|2-x|\right) & x < 2 \\ a^2 - x^2 & 2 \le x \le 3 \\ \pi^{\frac{1}{3-x}} + 6\cos b & x > 3 \end{cases}$