PART 1 – Single Dataset Engineering Tasks

Task 5: Elasticity and Deformability Insight

April 30, 2025

Abstract

This report analyzes the relationship between elastic properties of engineering materials, specifically exploring how Elastic modulus (E), Shear modulus (G), and Poisson's ratio (μ) interact. By examining these relationships, we validate isotropic material assumptions and provide insights into material selection based on stiffness properties. The analysis reveals strong correlations between elastic parameters, identifies potential data quality concerns, and offers guidance for selecting materials based on their deformation characteristics. These findings support engineers in making informed material selection decisions for applications where elastic behavior is critical.

1 Introduction

The Importance of Elastic Property Relationships

Understanding the relationships between elastic properties is fundamental for engineering material selection. This report explores how Elastic modulus (E), Shear modulus (G), and Poisson's ratio (μ) interrelate, and how these relationships impact engineering decisions. For isotropic materials, these properties are theoretically connected through the equation $G_{theory} = \frac{E}{2(1+\mu)}$. By analyzing how closely real materials follow this relationship, we can validate isotropy assumptions, identify material anomalies, and provide guidelines for selecting materials based on their elastic deformation characteristics.

2 Material Selection Insights

2.1 Elastic Modulus (E) and Shear Modulus (G) Relationship

The relationship between Elastic modulus (E) and Shear modulus (G) provides critical insights for stiffness-based material selection:

Table 1: Correlation Analysis: E and G Relationship

Correlation Method	Coefficient	Strength	Interpretation
Pearson (Linear)	0.999	Very Strong	Nearly perfect linear relationship
Spearman (Rank)	0.896	Strong	Strong monotonic relationship
Theoretical Expectation	–	–	$G \approx 0.38E$ for typical metals

Key Insights from E-G Relationship

The analysis of E-G relationships reveals important patterns for engineering material selection:

- **Predictable Proportionality**: The extraordinarily strong correlation (0.999) between E and G confirms that materials with higher tensile stiffness will reliably demonstrate higher shear stiffness, following the theoretical prediction.
- **Isotropic Behavior Confirmation**: The close adherence to the theoretical relationship validates that most engineering materials in the dataset exhibit isotropic behavior, where properties are uniform in all directions.
- **Material Classification**: Different material classes cluster at specific E-G value ranges (e.g., aluminum alloys: E ≈ 73 GPa, G ≈ 26-27 GPa; steels: E ≈ 200 GPa, G ≈ 80 GPa), creating clear selection boundaries.
- **Selection Simplification**: Engineers can reliably use E as the primary selection criterion when stiffness is critical, with G following proportionally based on the established relationship.

2.2 Elastic Modulus (E) and Poisson's Ratio (μ) Relationship

The analysis of Elastic modulus (E) and Poisson's ratio (μ) reveals a moderate negative correlation with important material selection implications:

Table 2: Correlation Analysis: E and μ Relationship

Correlation Method	Coefficient	Strength	Interpretation
Pearson (Linear)	-0.579	Moderate	Negative linear relationship
Spearman (Rank)	-0.654	Moderate-Strong	Stronger negative monotonic relationship
Typical Range	–	–	$\mu \approx 0.26 - 0.34$ for most metals

E-μ Relationship Insights

The E-μ relationship analysis provides several material selection insights:

- **Inverse Relationship**: As materials become stiffer (higher E), they generally exhibit slightly lower Poisson's ratios, contrary to what might be intuitively expected.
- **Stability Within Classes**: Most engineering metals maintain Poisson's ratios within a narrow expected range (0.26-0.34), regardless of their elastic modulus values.
- **Selection for Lateral Deformation**: Materials with higher Poisson's ratios (closer to 0.35) provide better volume conservation under load and may be preferred when controlling lateral deformation is critical.
- **Material Classification**: Values outside the typical range ($\mu < 0.25$ or $\mu > 0.35$) may indicate specialized materials (ceramics, polymers) or potential measurement errors requiring further investigation.

2.3 Isotropy Validation Through G vs G_expected

Comparing actual G values with theoretically predicted G_expected values (calculated from E and μ) provides crucial validation of material isotropy:

Table 3: Isotropy Validation Results

Validation Criterion	Result	Engineering Implication
Visual Correlation	Strong alignment with identity line	Isotropic assumptions generally valid
Maximum Deviation	Approx. 5.3% (Aluminum alloys)	Within 10% tolerance for isotropy
Perfect Matches	Multiple cast iron varieties	Likely derived rather than measured

Isotropy Assessment Insights

The comparison between measured and calculated shear modulus values reveals:

- **General Isotropy Confirmation**: The majority of materials show excellent agreement between measured G and calculated G_expected, validating that isotropic material models are appropriate for most engineering applications.
- **Material-Specific Deviations**: The largest deviations (approximately -1444 MPa) occur systematically in aluminum alloys, representing about 5.3% of the expected value—still within acceptable engineering tolerances.
- **Design Confidence**: For standard mechanical design applications, engineers can confidently use isotropic material models based on this dataset, simplifying

analysis and simulation approaches.

- **Data Quality Flags**: Perfect zero deviations in multiple distinct materials (e.g., various cast iron grades) suggest that some G values may have been calculated rather than independently measured, raising data source considerations.

3 Material Classification and Selection

Based on elastic property relationships, materials can be classified and selected for specific applications:

Table 4: Material Classification by Elastic Properties

Category	E (GPa)	G (GPa)	μ Range	Application Suitability
Aluminum Alloys	70-80	26-28	0.32-0.34	Lightweight, moderate stiffness
Carbon Steels	200-210	75-82	0.27-0.30	High stiffness applications
Grey Cast Iron	85-95	34-38	0.25	Vibration damping components
Malleable Cast Iron	160-170	64-68	0.25	Complex shaped rigid parts

Application-Specific Selection Guidelines

Engineers can apply these elastic property relationships to select materials for specific applications:

- **Structural Rigidity Applications**: Select high-E materials (carbon steels, E $\approx$ 200+ GPa) when overall stiffness is the primary criterion, as these materials will also provide high shear resistance.
- **Weight-Critical Applications**: Choose aluminum alloys (E $\approx$ 73 GPa) that offer moderate stiffness with lower density, particularly when the slightly higher Poisson's ratio (0.32-0.34) is not detrimental.
- **Deformation Control**: For applications where controlling lateral deformation is critical, consider materials with lower Poisson's ratios (closer to 0.25) such as cast irons.
- **Multi-Criteria Selection**: When both tensile and shear loading are important, the strong E-G correlation allows engineers to confidently select materials based primarily on E values, knowing that G will follow predictably.

4 Data Quality Observations

4.1 Statistical Anomalies

The analysis reveals several data quality concerns that should inform how the dataset is used for engineering decisions:

Table 5: Data Quality Concerns in Elastic Properties

Observation	Pattern	Implication
Identical Values	Multiple distinct materials (different alloys) show identical E, G, and G_deviation values	Potential standardization or data source issues
Zero Deviations	Multiple cast iron varieties show precisely zero deviation between G and G_expected	G likely calculated assuming $\mu = 0.25$ rather than measured
Systematic Deviation	Aluminum alloys consistently show -1443.61 MPa deviation	Systematic measurement bias or method difference
Material Clustering	Materials of similar types cluster at specific property values	Appropriate material classification but potential over-generalization

4.2 Reliability Assessment

Data Reliability Considerations

The data quality assessment reveals important considerations for engineers using this dataset:

- **Value Independence**: The presence of identical values across different material grades suggests some data may represent standardized values rather than individual measurements, potentially masking actual property variations.
- **Measurement vs. Calculation**: Perfect zero deviations strongly suggest that some G values were derived from E and μ using the theoretical formula rather than being independently measured, reducing their validation power.
- **Overall Dataset Reliability**: Despite specific concerns, the dataset as a whole demonstrates theoretically expected relationships, indicating general reliability for engineering decision-making within appropriate safety margins.
- **Verification Recommendation**: For critical applications, engineers should verify key elastic properties through independent testing, particularly for materials showing suspicious patterns in the dataset.

5 Engineering Implications for Design

The elastic property relationships revealed in this analysis have significant implications for engineering design practices:

Design Practice Implications

Engineers can apply these findings to improve design practices in several ways:

- **FEA Modeling Confidence**: The validation of isotropic behavior for most materials supports the use of simplified isotropic material models in Finite Element Analysis, reducing computational complexity.
- **Safety Factor Selection**: Materials with higher deviation from theoretical elastic relationships may warrant increased safety factors to account for potential anisotropic behavior in critical applications.
- **Material Substitution**: The strong correlation between E and G enables more confident material substitution decisions when only partial property data is available for candidate materials.
- **Design Optimization**: Understanding the E-G-μ relationships allows engineers to optimize designs for specific loading conditions, selecting materials that provide the best combination of tensile and shear stiffness.
- **Manufacturing Considerations**: Materials showing potential anisotropy may require careful consideration of orientation during manufacturing and assembly to maximize performance in the loading direction.

6 Conclusions and Recommendations

The analysis of elastic property relationships provides valuable insights for engineering material selection:

Key Findings Summary

This investigation into elasticity and deformability yields several important conclusions:

- **Theoretical Validation**: The remarkably strong correlation between E and G (0.999) confirms that real engineering materials closely follow the theoretical isotropic relationship $G = \frac{E}{2(1+\mu)}$.
- **Material Selection Guidance**: The analysis provides clear classification boundaries based on elastic properties, helping engineers select appropriate materials for stiffness-critical applications.

- **Data Quality Awareness**: While the dataset generally demonstrates expected relationships, specific anomalies suggest the need for careful interpretation, particularly for materials showing identical values across different grades.
- **Design Simplification**: For most engineering applications, the validated isotropy of materials allows for simplified analysis and simulation approaches, streamlining the design process.
- **Property Independence**: The moderate negative correlation between E and μ (-0.579) indicates that these properties are partially independent, allowing for material selection that optimizes both stiffness and lateral deformation characteristics.

6.1 Recommendations for Engineering Practice

Based on the elasticity analysis, we recommend the following practices for material selection and engineering design:

Practical Engineering Recommendations

To apply these findings in engineering practice:

- **Stiffness-Based Selection**: For standard mechanical design applications, confidently use isotropic material models, selecting primarily based on E with the understanding that G will follow predictably.
- **Cautious Application**: When working with materials showing deviation between measured and theoretical G values, apply conservative safety factors to account for potential anisotropic behavior.
- **Critical Applications**: For safety-critical components, verify elastic properties through independent testing, particularly for materials showing suspicious patterns in the dataset.
- **Data Interpretation**: Exercise caution with materials showing identical property values across different grades, as these may represent standardized rather than measured values.
- **Material Substitution**: When considering material substitution, use the established E-G-μ relationships to predict missing properties, while maintaining appropriate engineering margins.
- **Comprehensive Evaluation**: Remember that elastic properties are just one aspect of material performance—combine this analysis with other property assessments (strength, ductility, hardness) for comprehensive material selection.

7 Appendix: Analysis Methodology and Plots

Analysis Process

The elasticity and deformability analysis followed these methodological steps:

1. **Data Cleaning**: Removed outliers from E, G, and μ data using z-score filtering (threshold = 3)
2. **Correlation Analysis**: Calculated Pearson and Spearman correlations between E, G, and μ
3. **Theoretical Validation**: Calculated G_expected values using $\frac{E}{2(1+\mu)}$ formula
4. **Deviation Analysis**: Compared measured G with calculated G_expected values to quantify isotropy
5. **Material Classification**: Grouped materials based on their elastic property combinations
6. **Data Quality Assessment**: Identified patterns suggesting potential data quality concerns
7. **Engineering Application Mapping**: Connected elastic property relationships to practical material selection criteria

Elastic Modulus (E) vs. Shear Modulus (G)

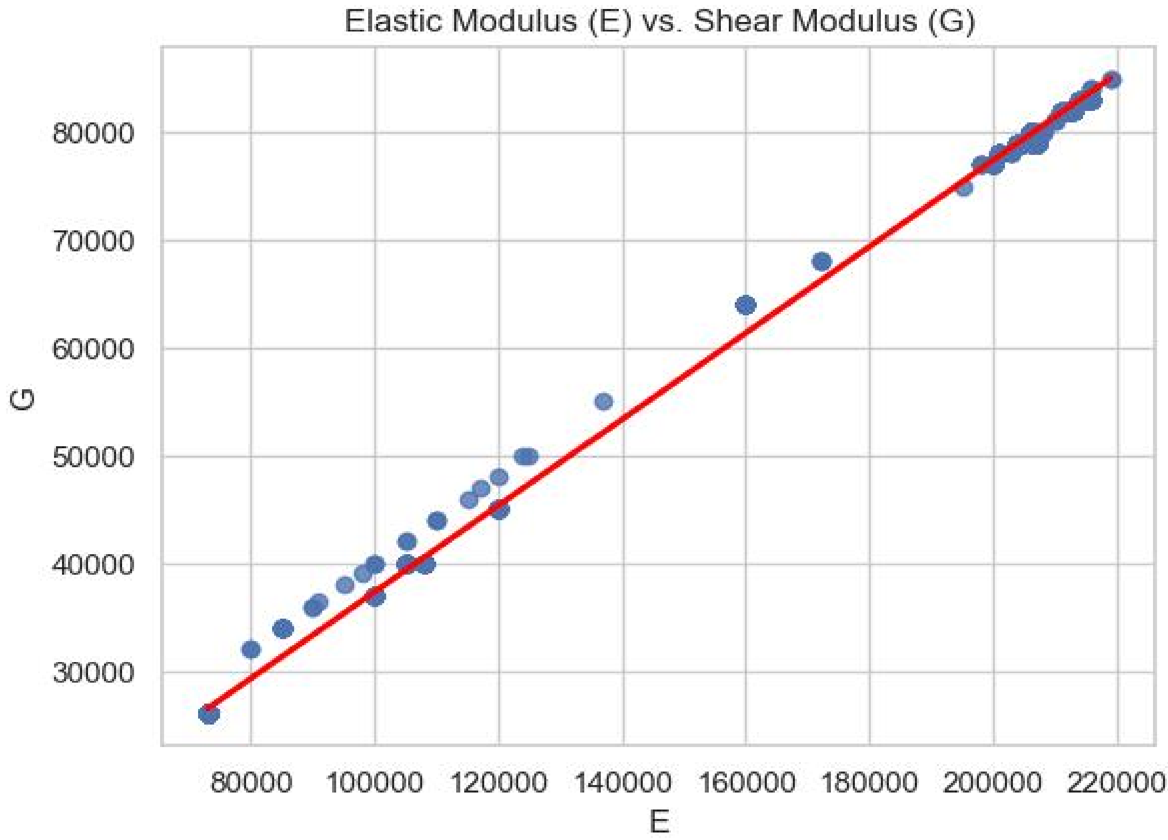

Figure 1: Scatter plot showing the relationship between Elastic Modulus (E) and Shear Modulus (G)

Axes & Units:

- **X-axis (E)**: Elastic Modulus, likely in MPa (range: 80,000–220,000 MPa)
- **Y-axis (G)**: Shear Modulus, likely in MPa (range: 30,000–80,000 MPa)
- **Note:** Metals like steels fall near E $\approx$ 200 GPa, G $\approx$ 80 GPa

Trend Observed:

- Positive correlation: Consistent with isotropic elasticity theory.
- Theoretical slope $\approx$ 0.38 (shown as red dashed line).
- Tight clustering at high E/G values indicates stiff, isotropic metallic behavior.

Potential Deviations:

- **Above trend line:** Possibly overestimated G or anisotropic effects.
- **Below trend line:** Could indicate low Poisson's ratio or experimental error.

Table 6: Material Classification Based on Elastic Properties

E (GPa)	G (GPa)	Likely Materials	Typical Applications
80–120	30–45	Aluminum alloys	Aerospace, lightweight structures
180–220	70–85	Carbon steels, Ti alloys	Load-bearing mechanical components
¿200	¡60	Composites (e.g., CFRP)	Directionally stiff or anisotropic designs

Engineering Insights

- **Isotropy Validation:** Compute Poisson's ratio $\mu = \frac{E}{2G} - 1$. Valid range for metals: 0.25–0.35.
- **Outliers:** $\mu < 0.2$ or $\mu > 0.4$ imply anisotropy or data error.
- **Design Implications:**
  - High E & G: Use for stiff structural elements (e.g., machine bases, shafts).
  - Low E & G: Good for lightweight parts with moderate stiffness needs (e.g., covers, panels).
  - Investigate anomalous points before selection — may be due to composites or poor measurements.

Elastic Modulus (E) vs. Poisson's Ratio (mu)

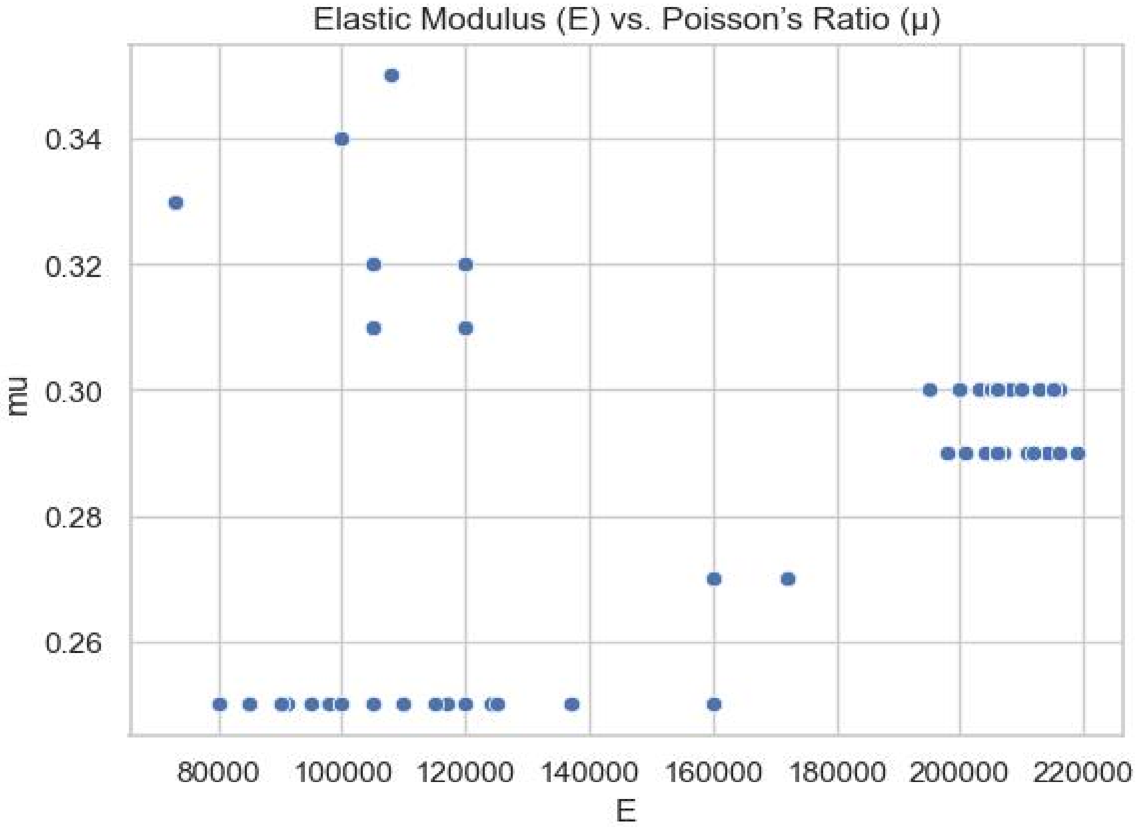

Figure 2: Scatter plot showing the relationship between Elastic Modulus (E) and Poisson's Ratio (mu)

Axes & Data Ranges:

- **X-axis (E)**: Elastic Modulus (80–220 GPa, typical for metals)
- **Y-axis (mu)**: Poisson's Ratio (0.26–0.34, mostly within expected range for isotropic metals)

Key Observation:

- Nearly constant Poisson's Ratio (≈ 0.30) across all values of E.
- Confirms classical elasticity theory: μ is material-dependent but does not vary systematically with E.

Outlier Interpretation:

- $\mu < 0.25$: May indicate brittle ceramics or anisotropic microstructures.
- $\mu > 0.35$: Possible inclusion of ductile polymers or data measurement issues.

Table 7: Material Classification Based on and E Values

Range	E Range (GPa)	Material Type	Design Implications
0.28–0.32	80–120	Aluminum Alloys	Vibration damping, lightweight frames
0.29–0.33	180–220	Steels, Ti Alloys	High stiffness, structural components
<0.25	Any	Brittle Ceramics	Avoid in tension-critical applications
> 0.35	Any	Polymers/Elastomers	Use in energy-dissipating parts

Engineering Insights

- **Isotropy Validation:** Nearly flat vs. E distribution confirms isotropic elasticity.
- **Data Reliability:** Cluster near 0.3 supports consistent and accurate dataset.
- **Design Guidelines:**
  - For stiffness-driven parts, choose high-E with mu ~ 0.3.
  - For energy-absorbing components, prioritize materials with mu → 0.35.

Actual vs. Theoretical Shear Modulus (G)

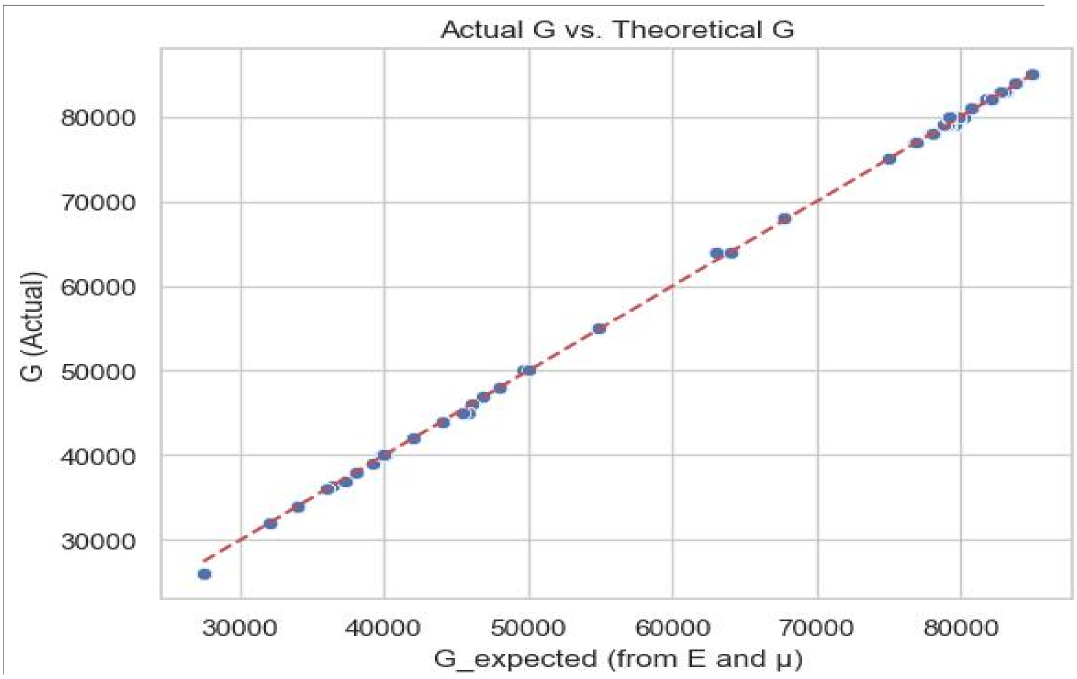

Figure 3: Scatter plot of Actual Shear Modulus (G) vs. Theoretical G calculated from E and μ. Red dashed line denotes the identity ($G = G_{expected}$).

Plot Overview:

- **X-axis ($G_{expected}$)**: Theoretical Shear Modulus calculated as $G = \frac{E}{2(1+\mu)}$
- **Y-axis (G)**: Experimentally measured Shear Modulus
- **Units**: Typically MPa or GPa; data ranges between 25,000 and 85,000 MPa
- **Red Dashed Line**: Represents perfect agreement between theoretical and measured values ($G = G_{expected}$)

Key Observation:

- Points cluster tightly along the identity line, indicating a strong match between actual and predicted values.
- Deviation is minimal — validating both experimental accuracy and theoretical assumptions.

Engineering Insights

- **Experimental Validation:** Tight clustering confirms consistency across E, G, and μ measurements.
- **Isotropy Confirmation:** Theoretical G is derived under isotropic assumptions — the close agreement confirms isotropic behavior for most materials.
- **Design Trustworthiness:** Designers can confidently use $G = \frac{E}{2(1+\mu)}$ when only E and μ are known.

Summary:

- This plot reinforces theoretical elasticity relationships and supports the use of indirect calculations in early-stage material selection or simulations.
- No major outliers indicate good data quality and isotropic mechanical behavior across the tested dataset.

Elastic Consistency and Outlier Analysis

Core Objective

We examine the statistical consistency between Elastic Modulus (E), Shear Modulus (G), and Poisson's Ratio (μ) to validate isotropic behavior and detect anomalies.

1. Correlation Matrices

Pearson (Linear):

- E vs. G: **0.999** → Strong linear relationship
- E vs. μ: **-0.579**, G vs. μ: **-0.601** → Moderate negative correlations

Interpretation: Stiffer materials (high E, G) tend to have slightly lower μ.

Spearman (Monotonic):

- E vs. G: **0.896** → Strong monotonic trend
- E vs. μ: **-0.654**, G vs. μ: **-0.673**

Interpretation: Confirms same pattern as Pearson, capturing rank-based variations.

2. Deviation from Theoretical G

Deviation Formula: $G_{deviation} = G_{actual} - \dfrac{E}{2(1+\mu)}$

Top 10 Deviators

- All top deviators show **negative deviation** of **-1444 MPa**
- μ inferred from E and G is **high** (0.404) — higher than usual for metals
- Mostly aluminum alloys and carbon steels → Potential overestimation of μ or underestimation of G

Tail 10 Deviators (Perfect Fit)

- $G_{actual} = G_{expected}$ → Zero deviation
- Implied $\mu = 0.25$ exactly
- Mostly cast irons → Textbook isotropic elastic behavior