

MODERATE TURBULENCE

1.1 MGF FOR SISO SYSTEM:

$$M_{I_{ij}}(s) = \sum_{l=0}^{\infty} b_l(m_1\gamma_1, m_2\gamma_2) s^{-\frac{l\gamma_1+m_1\gamma_1}{2}} + \sum_{l=0}^{\infty} b_l(m_2\gamma_2, m_1\gamma_1) s^{-\frac{l\gamma_2+m_2\gamma_2}{2}} \quad (1.1)$$

Where,

$$b_l(m_1\gamma_1, m_2\gamma_2) = \frac{a_l(m_1\gamma_1, m_2\gamma_2)}{2} \Gamma\left(\frac{l\gamma_1 + m_1\gamma_1}{2}\right) \quad (1.2)$$

$$b_l(m_2\gamma_2, m_1\gamma_1) = \frac{a_l(m_1\gamma_1, m_2\gamma_2)}{2} \Gamma\left(\frac{l\gamma_1 + m_1\gamma_1}{2}\right) \quad (1.3)$$

1.2 PROBABILITY OF ERROR FOR M-PSK:

$$P_{MPSK}(E) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} M_Z\left(\frac{\phi\gamma}{\sin^2 \theta}\right) d\theta \quad (1.4)$$

Where,

$$\phi = \sin^2\left(\frac{\pi}{M}\right) \quad (1.5)$$

From equations (1.1) and (1.5),

$$P_{MPSK}(E) = \sum_{l=0}^{\infty} b_l(m_1\gamma_1, m_2\gamma_2) \Lambda(m_1\gamma_1) (\phi\gamma)^{-\frac{l\gamma_1+m_1\gamma_1}{2}} + \sum_{l=0}^{\infty} b_l(m_2\gamma_2, m_1\gamma_1) \Lambda(m_2\gamma_2) (\phi\gamma)^{-\frac{l\gamma_2+m_2\gamma_2}{2}} \quad (1.6)$$

Where,

$$\Lambda(m_1\gamma_1) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} (\sin \theta)^{l\gamma_1+m_1\gamma_1} d\theta \quad (1.7)$$

$$\Lambda(m_1\gamma_1) = \frac{\Gamma\left[\frac{1}{2}(1 + (l + m_1)\gamma_1)\right]}{2\sqrt{\pi}\Gamma\left[1 + \frac{1}{2}(l + m_1)\gamma_1\right]} + \quad (1.8)$$

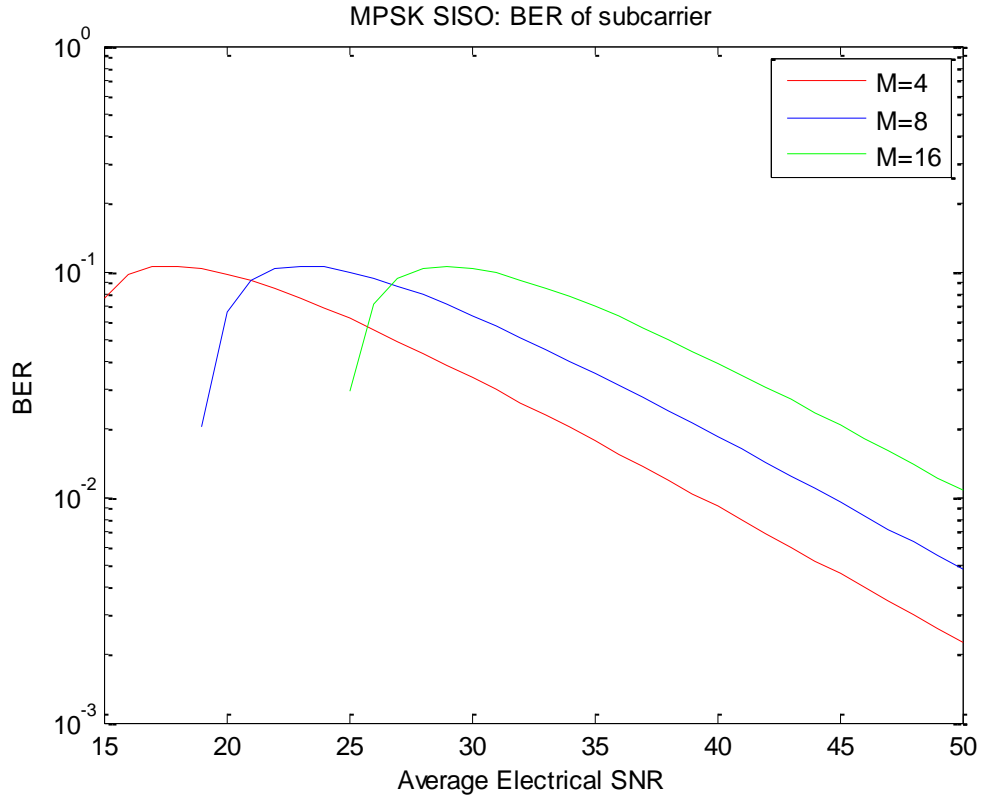
$$\frac{\cos\left[\frac{\pi}{M}\right] {}_2F_1\left[\frac{1}{2}, \frac{1}{2} - \frac{1}{2}(l + m_1)\gamma_1, \frac{3}{2}, \cos\left[\frac{\pi}{M}\right]^2\right]}{\pi}$$

and

$$\Lambda(m_2\gamma_2) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} (\sin \theta)^{l\gamma_2+m_2\gamma_2} d\theta \quad (1.9)$$

$$\Lambda(m_2\gamma_2) = \frac{\Gamma\left[\frac{1}{2}(1 + (l + m_2)\gamma_2)\right]}{2\sqrt{\pi}\Gamma\left[1 + \frac{1}{2}(l + m_2)\gamma_2\right]} + \frac{\cos\left[\frac{\pi}{M}\right] {}_2F_1\left[\frac{1}{2}, \frac{1}{2} - \frac{1}{2}(l + m_2)\gamma_2, \frac{3}{2}, \cos\left[\frac{\pi}{M}\right]^2\right]}{\pi} \quad (1.10)$$

1.3 PLOT OF BER Vs SNR FOR MPSK:



1.6 PROBABILITY OF ERROR FOR M-PAM:

$$P_{M-PAM}(E) = \frac{2}{\pi} \left(1 - \frac{1}{M}\right) \int_0^{\frac{\pi}{2}} M_Z \left(\frac{\phi\gamma}{\sin^2 \theta} \right) d\theta \quad (1.18)$$

Where,

$$\phi = \frac{3}{(M^2 - 1)} \quad (1.19)$$

From equations (1.1) and (1.11),

$$P_{M-PAM}(E) = 2 \left(1 - \frac{1}{M}\right) \sum_{l=0}^{\infty} b_l(m_1\gamma_1, m_2\gamma_2) \Lambda(m_1\gamma_1)(\phi\gamma)^{-\frac{l\gamma_1+m_1\gamma_1}{2}} \\ + \sum_{l=0}^{\infty} b_l(m_2\gamma_2, m_1\gamma_1) \Lambda(m_2\gamma_2)(\phi\gamma)^{-\frac{l\gamma_2+m_2\gamma_2}{2}} \quad (1.20)$$

Where,

$$\Lambda(m_1\gamma_1) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} (\sin \theta)^{l\gamma_1+m_1\gamma_1} d\theta \quad (1.21)$$

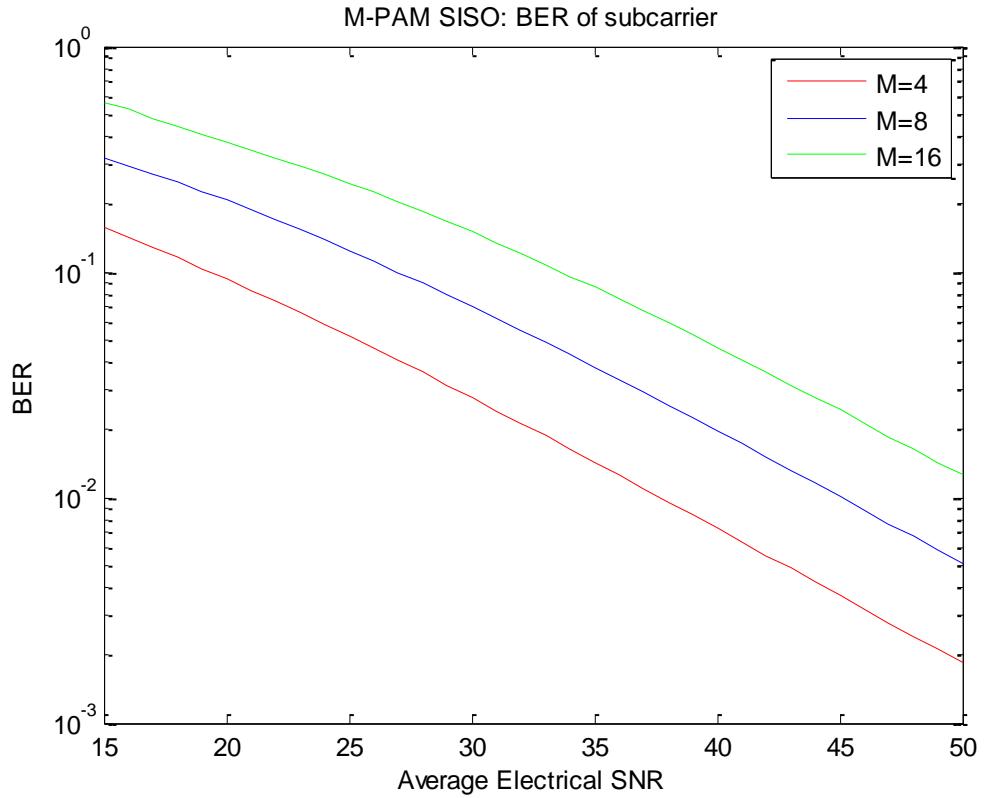
$$\Lambda(m_1\gamma_1) = \frac{\Gamma\left[\frac{1}{2}(1 + (l + m_1)\gamma_1)\right]}{2\sqrt{\pi}\Gamma\left[1 + \frac{1}{2}(l + m_1)\gamma_1\right]} \quad (1.22)$$

and

$$\Lambda(m_2\gamma_2) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} (\sin \theta)^{l\gamma_2+m_2\gamma_2} d\theta \quad (1.23)$$

$$\Lambda(m_2\gamma_2) = \frac{\Gamma\left[\frac{1}{2}(1 + (l + m_2)\gamma_2)\right]}{2\sqrt{\pi}\Gamma\left[1 + \frac{1}{2}(l + m_2)\gamma_2\right]} \quad (1.24)$$

1.7 PLOT OF BER Vs SNR FOR M-PAM:



1.8 PROBABILITY OF ERROR FOR M-QAM:

$$P_{MQAM}(E) = \xi_1 \int_0^{\pi/2} M_Z \left(\frac{\phi\gamma}{\sin^2 \theta} \right) d\theta - \xi_2 \int_0^{\pi/4} M_Z \left(\frac{\phi\gamma}{\sin^2 \theta} \right) d\theta \quad (1.25)$$

Where,

$$\phi = \frac{3}{2(M-1)}, \xi_1 = \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}} \right), \text{ and } \xi_2 = \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}} \right)^2 \quad (1.26)$$

From equations (1.1) and (1.18),

$$\begin{aligned} P_{M-PAM}(E) = & 4 \left(1 - \frac{1}{\sqrt{M}} \right) \sum_{l=0}^{\infty} b_l(m_1\gamma_1, m_2\gamma_2) \Lambda(m_1\gamma_1) (\phi\gamma)^{-\frac{l\gamma_1+m_1\gamma_1}{2}} \\ & + \sum_{l=0}^{\infty} b_l(m_2\gamma_2, m_1\gamma_1) \Lambda(m_2\gamma_2) (\phi\gamma)^{-\frac{l\gamma_2+m_2\gamma_2}{2}} - \\ & 4 \left(1 - \frac{1}{\sqrt{M}} \right)^2 \sum_{l=0}^{\infty} b_l(m_1\gamma_1, m_2\gamma_2) \Psi(m_1\gamma_1) (\phi\gamma)^{-\frac{l\gamma_1+m_1\gamma_1}{2}} \\ & + \sum_{l=0}^{\infty} b_l(m_2\gamma_2, m_1\gamma_1) \Psi(m_2\gamma_2) (\phi\gamma)^{-\frac{l\gamma_2+m_2\gamma_2}{2}} \end{aligned} \quad (1.27)$$

Where,

$$\Lambda(m_1\gamma_1) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} (\sin \theta)^{l\gamma_1+m_1\gamma_1} d\theta \quad (1.28)$$

$$\Lambda(m_1\gamma_1) = \frac{\Gamma \left[\frac{1}{2} (1 + (l + m_1)\gamma_1) \right]}{2\sqrt{\pi} \Gamma \left[1 + \frac{1}{2} (l + m_1)\gamma_1 \right]} \quad (1.29)$$

and

$$\Lambda(m_2\gamma_2) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} (\sin \theta)^{l\gamma_2+m_2\gamma_2} d\theta \quad (1.30)$$

$$\Lambda(m_2\gamma_2) = \frac{\Gamma \left[\frac{1}{2} (1 + (l + m_2)\gamma_2) \right]}{2\sqrt{\pi} \Gamma \left[1 + \frac{1}{2} (l + m_2)\gamma_2 \right]} \quad (1.31)$$

and,

$$\Psi(m_1\gamma_1) = \frac{1}{\pi} \int_0^{\frac{\pi}{4}} (\sin \theta)^{l\gamma_1+m_1\gamma_1} d\theta \quad (1.32)$$

$$\Psi(m_1\gamma_1) = \frac{i 2^{\frac{1}{2}(-1-l\gamma_1-m_1\gamma_1)} e^{-\frac{1}{2}i\pi(1+l\gamma_1+m_1\gamma_1)} (\text{Cos}[\frac{1}{2}(l+m_1)\pi\gamma_1])}{\pi(1+l\gamma_1+m_1\gamma_1)} \quad (1.33)$$

$$+ \left(\frac{i \cos \left[\frac{1}{2} \pi (-1 + l \gamma_1 + m_1 \gamma_1) \right]}{\pi (1 + l \gamma_1 + m_1 \gamma_1)} \right) \times$$

$$\text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (1 + (l + m_1) \gamma_1), \frac{1}{2} (3 + (l + m_1) \gamma_1), \frac{1}{2} \right]$$

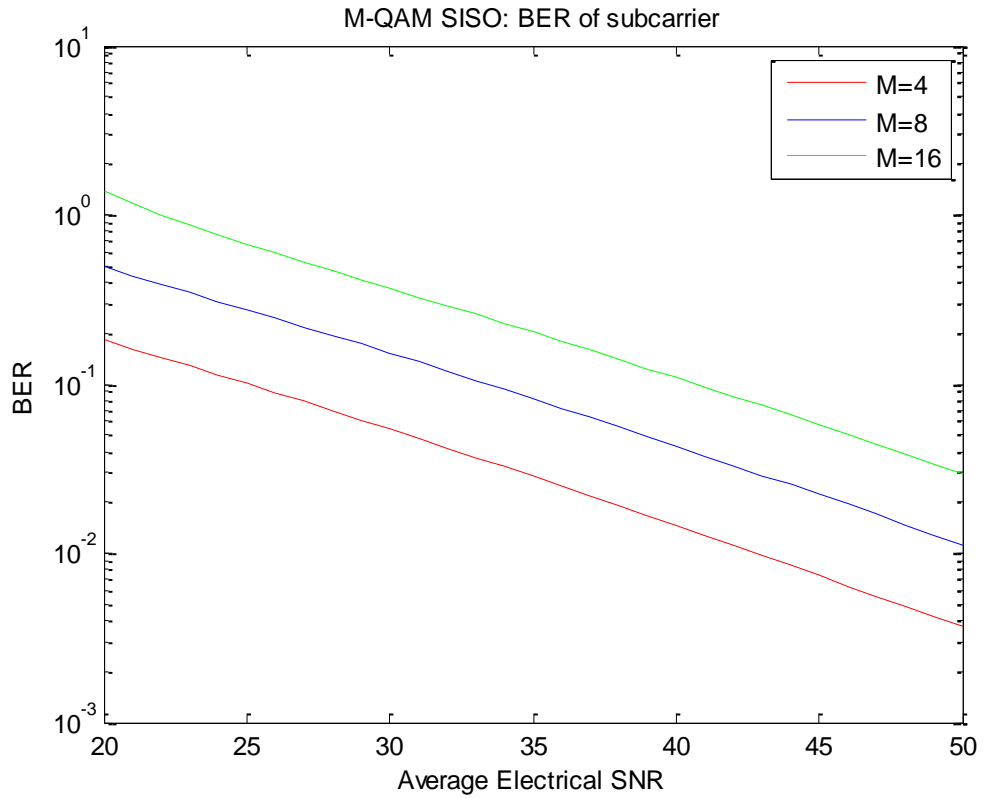
and

$$\Psi(m_2 \gamma_2) = \frac{1}{\pi} \int_0^{\frac{\pi}{4}} (\sin \theta)^{l \gamma_2 + m_2 \gamma_2} d\theta \quad (1.34)$$

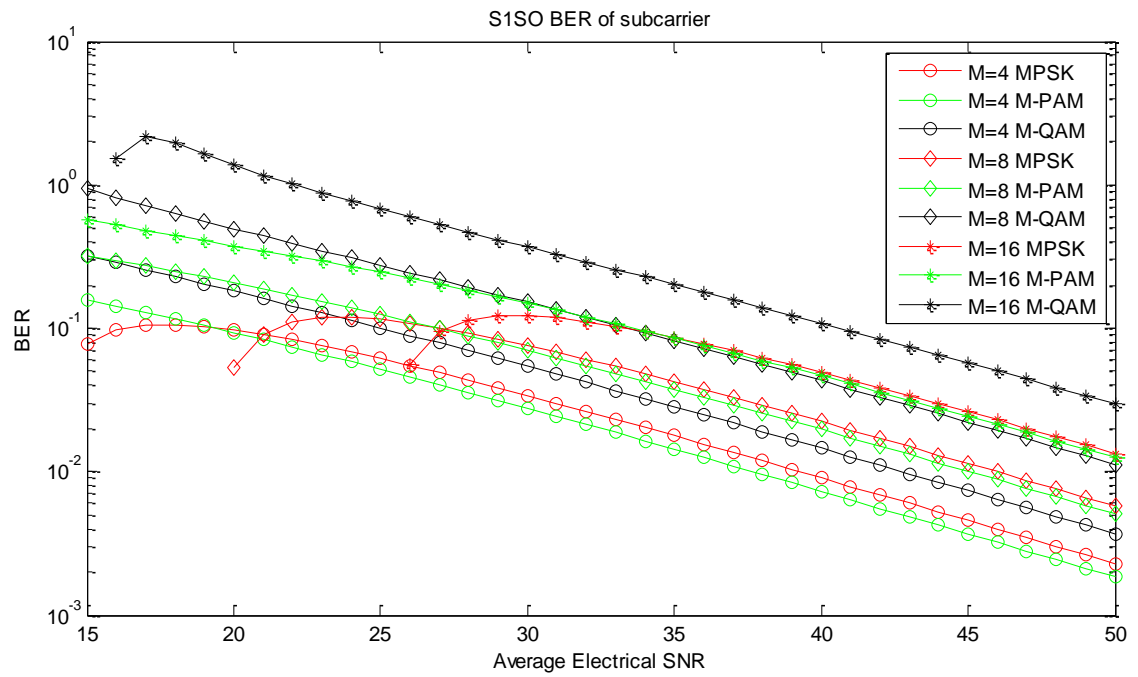
$$\Psi(m_2 \gamma_2) = \frac{i 2^{\frac{1}{2}(-1 - l \gamma_2 - m_2 \gamma_2)} e^{-\frac{1}{2} i \pi (1 + l \gamma_2 + m_2 \gamma_2)} (\cos \left[\frac{1}{2} (l + m_2) \pi \gamma_2 \right])}{\pi (1 + l \gamma_2 + m_2 \gamma_2)} + \left(\frac{i \cos \left[\frac{1}{2} \pi (-1 + l \gamma_2 + m_2 \gamma_2) \right]}{\pi (1 + l \gamma_2 + m_2 \gamma_2)} \right) \times \quad (1.35)$$

$$\text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} (1 + (l + m_2) \gamma_2), \frac{1}{2} (3 + (l + m_2) \gamma_2), \frac{1}{2} \right]$$

1.7 PLOT OF BER Vs SNR FOR M-QAM:



1.8 PLOT OF MPSK, M-PAM, M-QAM:



1.9 MGF FOR MIMO SYSTEM:

$$M_Y(s) = \sum_{k=0}^{n_T n_R} \binom{n_T n_R}{k} \sum_{i=0}^{\infty} \sum_{j=0}^i \frac{\tilde{c}_j(m_1 \gamma_1, m_2 \gamma_2) \tilde{c}_{i-j}(m_2 \gamma_2, m_1 \gamma_1)}{2\Gamma((n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2)} \Gamma\left(\frac{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2}{2}\right) s^{-\frac{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2}{2}} \quad (1.36)$$

Where,

$$\tilde{c}_0(m_1 \gamma_1, m_2 \gamma_2) = [\tilde{b}_0(m_1 \gamma_1, m_2 \gamma_2)]^{n_T n_R - k} \quad (1.37)$$

$$\tilde{c}_0(m_2 \gamma_2, m_1 \gamma_1) = [\tilde{b}_0(m_2 \gamma_2, m_1 \gamma_1)]^{n_T n_R - k} \quad (1.38)$$

$$\tilde{c}_m(m_1 \gamma_1, m_2 \gamma_2) = \frac{1}{m \tilde{b}_0(m_1 \gamma_1, m_2 \gamma_2)} \quad (1.39)$$

$$\sum_{l=0}^m (l(n_T n_R - k) - m + l) \tilde{b}_0(m_1 \gamma_1, m_2 \gamma_2) \tilde{c}_{m-l}(m_1 \gamma_1, m_2 \gamma_2) \tilde{c}_m(m_2 \gamma_2, m_1 \gamma_1) = \frac{1}{m \tilde{b}_0(m_2 \gamma_2, m_1 \gamma_1)} \sum_{l=0}^m (l(n_T n_R - k) - m + l) \times \quad (1.40)$$

$$\tilde{b}_0(m_2 \gamma_2, m_1 \gamma_1) \tilde{c}_{m-l}(m_2 \gamma_2, m_1 \gamma_1)$$

1.10 PROBABILITY OF ERROR FOR MIMO M-PSK:

$$P_{MPSK}(E) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} M_{Y^2} \left(\frac{\phi \gamma}{n_T^2 n_R \sin^2 \theta} \right) d\theta \quad (1.41)$$

Where,

$$\phi = \sin^2 \left(\frac{\pi}{M} \right) \quad (1.42)$$

From equations (1.36) and (1.41),

$$P_{MPSK} = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sum_{k=0}^{n_T n_R} \binom{n_T n_R}{k} \sum_{i=0}^{\infty} \sum_{j=0}^i \frac{\tilde{c}_j(m_1 \gamma_1, m_2 \gamma_2) \tilde{c}_{i-j}(m_2 \gamma_2, m_1 \gamma_1)}{2\Gamma((n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2)} \Lambda_{ij}(n_T n_R, M) \Gamma\left(\frac{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2}{2}\right) \left(\frac{\phi \gamma}{n_T^2 n_R}\right)^{-\frac{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2}{2}} \quad (1.43)$$

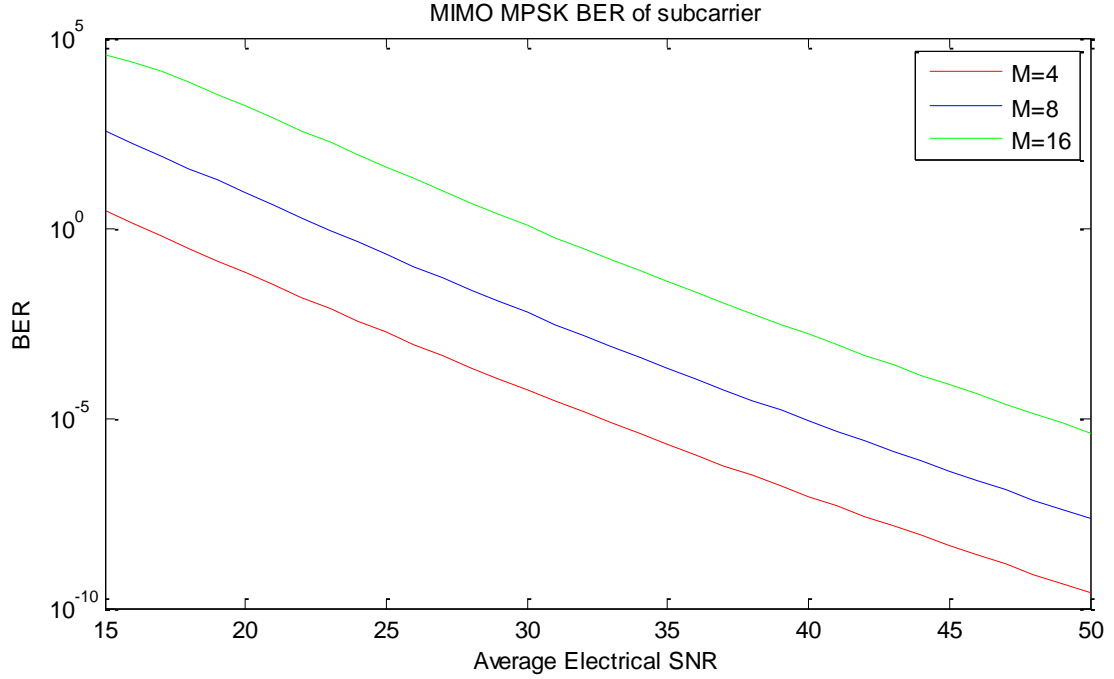
Where,

$$\Lambda_{ij}(n_T n_R, M) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \sin \theta^{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2} d\theta \quad (1.44)$$

$$\Lambda_{ij}(n_T n_R, M) = \frac{\sqrt{\pi} \Gamma\left(\frac{1 + (n_T n_R - k)m_1 \gamma_1 + j\gamma_1 + km_2 \gamma_2 + (i-j)\gamma_2}{2}\right)}{2\Gamma\left(1 + \frac{(n_T n_R - k)m_1 \gamma_1 + j\gamma_1 + km_2 \gamma_2 + (i-j)\gamma_2}{2}\right)} - \cos\left(\frac{M-1}{M}\pi\right) \quad (1.45)$$

$$2F1\left[\frac{1}{2}, \frac{1 - (n_T n_R - k)m_1 \gamma_1 - j\gamma_1 - km_2 \gamma_2 - (i-j)\gamma_2}{2}; \frac{3}{2}; \cos^2\left(\frac{M-1}{M}\pi\right)\right]$$

1.11 PLOT OF MIMO MPSK:



1.14 PROBABILITY OF ERROR FOR M-PAM:

$$P_{M-PAM}(E) = \frac{2}{\pi} \left(1 - \frac{1}{M}\right) \int_0^{\frac{\pi}{2}} M_{Y^2} \left(\frac{\phi \gamma}{n_T^2 n_R \sin^2 \theta} \right) d\theta \quad (1.49)$$

Where,

$$\phi = \frac{3}{(M^2 - 1)} \quad (1.50)$$

From equations (1.36) and (1.48),

$$P_{M-PAM}(E) = 2 \left(1 - \frac{1}{M}\right) \sum_{k=0}^{n_T n_R} \binom{n_T n_R}{k} \sum_{i=0}^{\infty} \sum_{j=0}^i \frac{\tilde{c}_j(m_1 \gamma_1, m_2 \gamma_2) \tilde{c}_{i-j}(m_2 \gamma_2, m_1 \gamma_1)}{2\Gamma((n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2)} \Lambda_{ij}(n_T n_R, M) \quad (1.51)$$

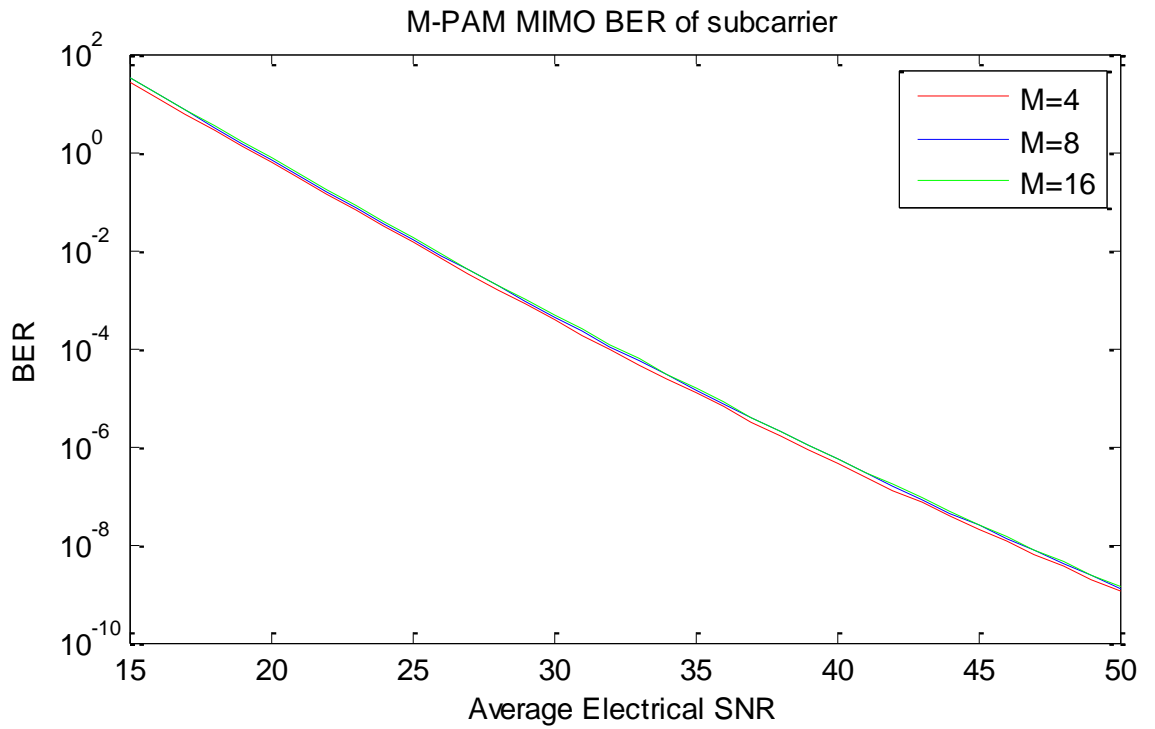
$$\Gamma\left(\frac{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2}{2}\right) \left(\frac{\phi \gamma}{n_T^2 n_R}\right)^{\frac{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2}{2}}$$

Where,

$$\Lambda_{ij}(n_T n_R, M) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} (\sin \theta)^{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2} d\theta \quad (1.52)$$

$$\Lambda_{ij}(n_T n_R, M) = \frac{\Gamma\left[\frac{1}{2}(1 - km_1 \gamma_1 + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2)\right]}{2\sqrt{\pi} \Gamma\left[\frac{1}{2}(2 - km_1 \gamma_1 + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2)\right]} \quad (1.53)$$

1.15 PLOT OF MIMO M-PAM:



1.16 PROBABILITY OF ERROR FOR M-QAM:

$$P_{MQAM} = \xi_1 \int_0^{\pi/2} M_{Y^2} \left(\frac{\phi \gamma}{n_T^2 n_R \sin^2 \theta} \right) d\theta - \xi_2 \int_0^{\pi/4} M_{Y^2} \left(\frac{\phi \gamma}{n_T^2 n_R \sin^2 \theta} \right) d\theta \quad (1.54)$$

Where,

$$\phi = \frac{3}{2(M-1)}, \xi_1 = \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}} \right), \text{ and } \xi_2 = \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}} \right)^2 \quad (1.55)$$

From equations (1.36) and (1.54),

$$\begin{aligned}
P_{M-PAM} &= 4 \left(1 - \frac{1}{\sqrt{M}}\right) \sum_{k=0}^{n_T n_R} \binom{n_T n_R}{k} \\
&\sum_{i=0}^{\infty} \sum_{j=0}^i \frac{\tilde{c}_j(m_1 \gamma_1, m_2 \gamma_2) \tilde{c}_{i-j}(m_2 \gamma_2, m_1 \gamma_1)}{2\Gamma((n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2)} \Lambda_{ij}(n_T n_R, M) \\
&\left(\frac{\phi \gamma}{n_T^2 n_R}\right)^{-\frac{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2}{2}} - \\
&4 \left(1 - \frac{1}{\sqrt{M}}\right)^2 \sum_{k=0}^{n_T n_R} \binom{n_T n_R}{k} \sum_{i=0}^{\infty} \sum_{j=0}^i \frac{\tilde{c}_j(m_1 \gamma_1, m_2 \gamma_2) \tilde{c}_{i-j}(m_2 \gamma_2, m_1 \gamma_1)}{2\Gamma((n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2)} \\
&\psi_{ij}(n_T n_R, M) \left(\frac{\phi \gamma}{n_T^2 n_R}\right)^{-\frac{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2}{2}}
\end{aligned} \tag{1.56}$$

Where,

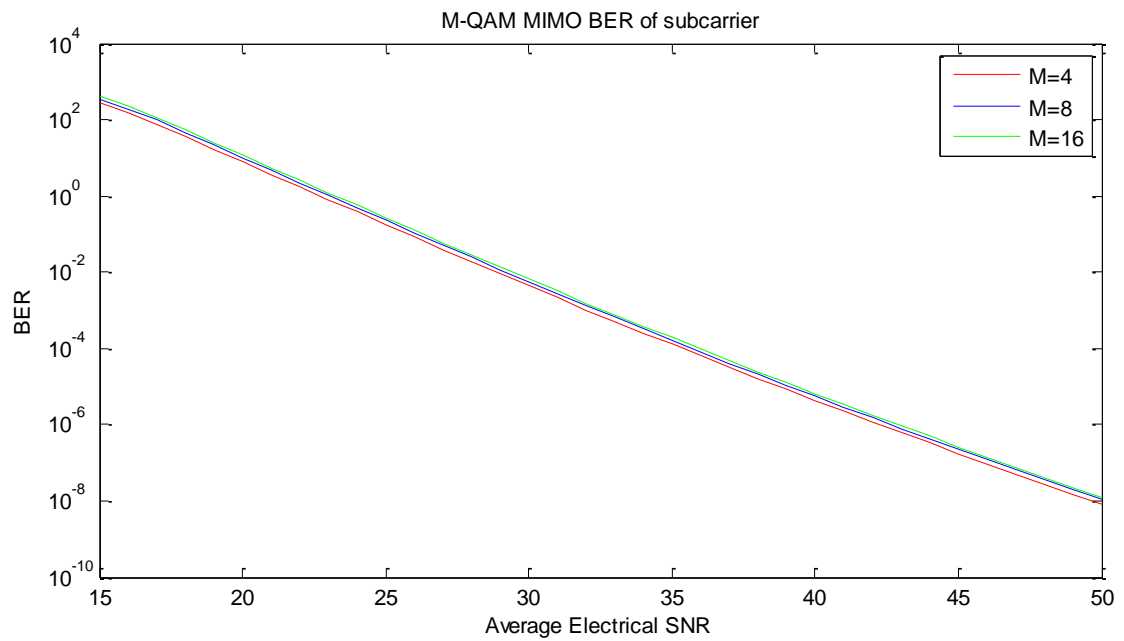
$$\Lambda_{ij}(n_T n_R, M) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} (\sin \theta)^{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2} d\theta \tag{1.57}$$

$$\Lambda_{ij}(n_T n_R, M) = \frac{\Gamma\left[\frac{1}{2}(1 - km_1 \gamma_1 + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2)\right]}{2\sqrt{\pi} \Gamma\left[\frac{1}{2}(2 - km_1 \gamma_1 + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2)\right]} \tag{1.58}$$

$$\psi_{ij}(n_T n_R, M) = \frac{1}{\pi} \int_0^{\frac{\pi}{4}} (\sin \theta)^{(n_T n_R - k)m_1 \gamma_1 + km_2 \gamma_2 + j\gamma_1 + (i-j)\gamma_2} d\theta \tag{1.57}$$

$$\begin{aligned}
\psi_{ij}(n_T n_R, M) &= \frac{2^{\frac{1}{2}(-1 + km_1 \gamma_1 - m_1 n_T n_R \gamma_1 - i\gamma_2 - km_2 \gamma_2 + j(-\gamma_1 + \gamma_2))}}{(\pi(-1 - j\gamma_1 + km_1 \gamma_1 - m_1 n_T n_R \gamma_1 - i\gamma_2 + j\gamma_2 - km_2 \gamma_2))} \times \\
&e^{-\frac{1}{2}i\pi(1 - km_1 \gamma_1 + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2)} \\
&2F1\left[\frac{1}{2}, \frac{1}{2}(1 - km_1 \gamma_1 + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2), \frac{1}{2}(3 - km_1 \gamma_1 \right. \\
&\quad \left. + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2), \frac{1}{2}\right] \times \\
&(\cos\left[\frac{1}{2}\pi(-1 - km_1 \gamma_1 + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2)\right] \\
&+ i \sin\left[\frac{1}{2}\pi(-1 - km_1 \gamma_1 + m_1 n_T n_R \gamma_1 + j(\gamma_1 - \gamma_2) + i\gamma_2 + km_2 \gamma_2)\right])
\end{aligned} \tag{1.58}$$

1.8 PLOT OF MIMO M-QAM:



1.8 PLOT OF MIMO MPSK, M-PAM, M-QAM:

