CS 763 Assignment 6

Due Date: 19 April, 23:00

April 10, 2018

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1. Let us consider a point in real world $\mathbf{X}$. And the transformation between two cameras is described by rotation matrix $\mathbf{R}$ and a translation vector $\mathbf{t}$ such that $\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{t}$. Assume that the first camera is aligned to world co-ordinate and situated at origin and camera matrix is $\mathbf{K}$. The camera calibration matrix of the second camera is $\mathbf{K}'$.

   (a) What are the expressions for the epipoles, $\mathbf{e}$ and $\mathbf{e}'$ in terms of one or more of the following: $\mathbf{P}$, $\mathbf{P}'$, $\mathbf{R}$, and $\mathbf{t}$, where $\mathbf{P}$ and $\mathbf{P}'$ camera projection matrix.

   (b) An alternate formulation of the fundamental matrix, $\mathbf{F} = [\mathbf{e}']_\times \mathbf{K}'\mathbf{R}\mathbf{K}^{-1}$. Using the results from the earlier parts, show that $\mathbf{e}' \times x' = \mathbf{F}x$ where x' is the image of $\mathbf{X}$ in the second camera coordinate system.

   Note: $[\mathbf{e}']_\times$ denotes skew symmetric.

2. Let K_1 and K_2 be two camera matrices. We know that the fundamental matrix corresponding to these camera matrices is of the following from:

$$F = S_b R \tag{1}$$

   where S_b is the matrix

$$S_b = \begin{pmatrix} 0 & b_y & -b_z \\ -b_y & 0 & b_x \\ b_z & -b_x & 0 \end{pmatrix} \tag{2}$$

   Assume that $K_1 = [I|0]$ and $K_2 = [\mathbf{R}|\mathbf{b}]$, where $\mathbf{R}$ is a 3×3 (non-singular) matrix. Prove that the last column of K_2, denoted by $\mathbf{b}$, is one of the epipoles.

3. Recall that in class, we discussed about Structure from Motion (SfM) and learnt that we can find the global positions as well as the rotation of the frames using the Factorization method. Your task is to propose a method to find the translation T_i of the camera between frame $i - 1$ and i. Note that we assumed orthographic projection while performing SfM.

4. Suppose there are three images I_1, I_2 and I_3, and if $\mathbf{F}_{13}$ describes the fundamental matrix between images I_1 and I_3 and $\mathbf{F}_{23}$ describes the fundamental matrix between images I_2 and I_3. Now, let's assume that you are given points $\mathbf{x}_1$ and $\mathbf{x}_2$ in images I_1 and I_2 respectively. Derive the expression for corresponding point $\mathbf{x}_3$ in image I_3.