Assignment-6

April 2018

1 Problem-1

a)

We know the projection of a point X (with **euclidean coordinates in world frame** given by X) due to camera with projection matrix **P** is given by

$$Y = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

Now consider the first camera **C** with world coordinates $\overrightarrow{0}$ and aligned to world coordinate system and has calibration matrix **K** and camera matrix **P**.

The position of any point in frame of second camera X' is given by

$$X' = RX + t$$

Now obviously the coordinates of second camera's centre with respect to second camera's frame will be $\overrightarrow{0}$. So applying the above equation for seconds camera's centre ($X' = 0$) we get $X = -R^T t$, that is, the coordinates of the centre of the second camera in the coordinate system of the first camera is $-R^T t$.

First epipole :

projection of the first camera's centre in second camera frame : $e' = P' * \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Second epipole :

projection of the second's camera centre in first camera frame : $e = P * \begin{bmatrix} -R^T t \\ 1 \end{bmatrix}$

b)

For the first camera $P = K[I|\overrightarrow{0}]$ and for the second $P' = K'\begin{bmatrix}R & |-R^Tt\end{bmatrix}$.

So thus for first camera $x = K\begin{bmatrix}I & |0\end{bmatrix})\begin{bmatrix}X\\1\end{bmatrix}) = KX$

And for second camera $x' = (K'\begin{bmatrix}R & |-R^Tt\end{bmatrix})\begin{bmatrix}X\\1\end{bmatrix}) = K'(RX - R^Tt)$

Therefore,
$e' \times x'$
$= e' \times (K'(RX - R^Tt)$
$= e' \times (K'RX) - e' \times (K'R^Tt)$

Also from (a) part $e' = P' * \begin{bmatrix}0\\1\end{bmatrix} = K'\begin{bmatrix}R & |-R^Tt\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = -K'R^Tt$

So in the above equation $e' \times (K'R^Tt) = 0$ and thus

$e' \times x'$ simplifies to $e' \times (K'RX)$.
Now X can be simplified in terms of x as $X = K^{-1}x$ from the above equation
So puting X in the earlier equation

$$e' \times x' = e' \times (K'RX) = e' \times (K'RK^{-1}x)$$

Or $e' \times x' = e' \times Fx$ where $F = K'RK^{-1}$

Question 2

$\because$ the camera matrix for the first camera is $K_1 = [I|0]$, we can conclude that $R = I$ and $t = 0$, since we know that

$$\text{camera matrix} = \underbrace{K'}_{\text{camera intrinsics}} \underbrace{[R \mid t]}_{\text{camera extrinsics}}$$

camera extrinsics, camera co-od frame is translated by 't' and rotated by 'R' wrt the world frame.

$\therefore$ on comparing, the camera frame is the world frame ($t=0$ and $R=I$).

$\therefore$ the camera centre $C_1 = 0$ (origin)

By definition, the epipole is the image of C_1 in camera 2

C_1 → origin of co-od system of camera 1.

$\therefore$ the epipole is $K_2 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

↳ origin in homogenous coordinates

$$= [R \mid b] \begin{bmatrix} 0_{3\times1} \\ 1 \end{bmatrix}$$

$$= R\, 0_{3\times1} + b$$

$$= b$$

$\therefore$ b is an epipole.

Scanned by CamScanner

Question 3

In class, we derived (estimated) the rotation matrices (R) and the 3-D coordinates of the points (s) from $\tilde{w}$.

Using the same notation, if i_i and j_i represent the x and y-axes of the ith frame (respectively) and P_j represents the jth point,

we have estimates for i_i, j_i and P_j.

We also know that:

$$x_{ij} = i_i^T (P_j - t_i)$$

$$\text{and } y_{ij} = j_i^T (P_j - t_i)$$

x_{ij} and y_{ij} are know (through w)

$$\therefore \text{ we get } \quad i_i^T t_i = i_i^T P_j - x_{ij} \quad \ldots (1)$$

$$\text{and } j_i^T t_i = j_i^T P_j - y_{ij} \quad \ldots (2)$$

Using these equations, we can find the component of t_i along i_i and j_i, but not along k_i, i.e., we can estimate 2 of the 3 degrees of freedom of t_i.

Alternatively, ~~we~~ ~~have~~ if $t_i = (x_i, y_i, z_i)^T$ we have 3 variables but only 2 equations which means that translation cannot be determined.

Since the projection is orthographic, we cannot use the given information to find the distance of the camera from the object (or the origin) along the optical axis. [as the image will be the same irrespective of this distance].

Equations ① and ② are for specific points. For a given 'i', if we sum over all 'j' from 1 to N (where N is the number of points tracked) and then divide by N, we get:

$$\frac{\sum_{j=1}^{N} i_i^T t_i}{N} = \frac{i_i^T}{N} \sum_{j=1}^{n} P_j - \frac{\sum_{j=1}^{n} x_{ij}}{N}$$

∵ we had assumed the centroid of all points to be the origin, $\sum P_j = 0$

Also, $\frac{\sum_{j=1}^{N} x_{ij}}{N} = \overline{x_i}$ (by definition

∴ $LHS = \frac{N}{N} i_i^T t_i = i_i^T t_i = \overline{x_i}$

∴ we get $i_i^T t_i = \overline{x_i}$ and $j_i^T t_i = \overline{y_i}$

Hence, $t_i = \overline{x_i}\, i_i + \overline{y_i}\, j_i + \lambda k_i$

for some $\lambda \in R$.

Q 4.

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We know that image points will satisfy

$$x_1^T F_{13} x_3 = 0$$

$$x_2^T F_{23} x_3 = 0$$

(x_1, x_2, F_{13}, F_{23} known)

let $x_1 := \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$, $x_2 := \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$

$x_3 := \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$, $F_{13} = \begin{pmatrix} a^1_{11} & a^1_{12} & a^1_{13} \\ a^1_{21} & a^1_{22} & a^1_{23} \\ a^1_{31} & a^1_{32} & a^1_{33} \end{pmatrix}$

$F_{23} = \begin{pmatrix} a^2_{11} & a^2_{12} & a^2_{13} \\ a^2_{21} & a^2_{22} & a^2_{23} \\ a^2_{31} & a^2_{32} & a^2_{33} \end{pmatrix}$ (Note '2' is in subscript not square)

The first eqt^n becomes

$$x + \frac{(x_1 a^1_{12} + y_1 a^1_{22} + a^1_{32})}{(x_1 a^1_{11} + y_1 a^1_{21} + a^1_{31})} y + \frac{(x_1 a^1_{13} + y_1 a^1_{23} + a^1_{33})}{(x_1 a^1_{11} + y_1 a^1_{21} + a^1_{31})} = 0$$

The second eqt^n

$$x + \frac{(x_2 a^2_{12} + y_2 a^2_{22} + a^2_{32})}{(x_2 a^2_{11} + y_2 a^2_{21} + a^2_{31})} y + \frac{(x_2 a^2_{13} + y_2 a^2_{23} + a^2_{33})}{(x_2 a^2_{11} + y_2 a^2_{21} + a^2_{31})} = 0$$