

# Improved denoising based Selective Arithmetic Mean Filtering with Wavelet thresholding

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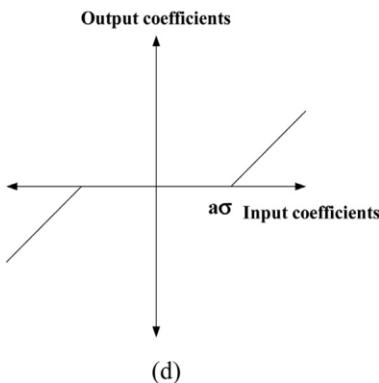
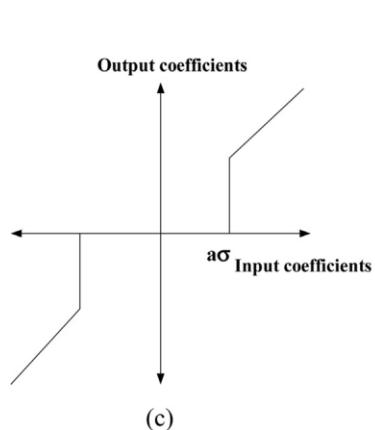
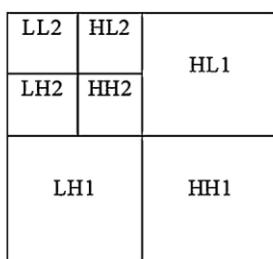
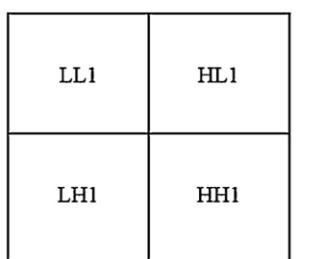
## Abstract

In this Research work, an effective selective arithmetic mean wavelet thresholding filtering technique (SAMWTFT) is proposed for restoration of images that are highly corrupted by salt and pepper noise. Method developed detects noisy pixel using difference of arithmetic filtering (AM). Detected noisy pixels are filtered out using selective arithmetic mean filtering and wavelet thresholding procedure. Computer simulation is carried out on the benchmark images and the results are compared with the state of the art studies to demonstrate the performance of the proposed technique. The results show that the SAMFT combined with wavelet thresholding provides better performance in terms of high peak signal to noise ratio (PSNR), good structural similarity (SSIM), correlation (CORR) and good visual quality even at noise density as high as 90%.

### 1. Introduction

One of the practical issues in image analysis is quality of image during acquisition process and noise is one reason for its causal. An image is often corrupted by impulse noise in its acquisition and transmission [1-3]. Salt and pepper noise is an impulse type of noise, which is also referred to as intensity spikes. It is introduced in camera sensors, faulty memory locations, or timing error in digitization process. Salt and pepper is prime noise introduced in mentioned processes. In imaging sequence also there will be degradation introduced through materials used. Among various types of noises, salt and pepper noise typically introduces error in pixel elements in the camera sensors, faulty memory locations, or timing errors in the digitization process. Small quantity of noise also significantly degrades image quality and thus denoising is often necessary step to be carried out before the image data analysis.

In the image denoising process, information about the type of noise present in the image plays a significant role. In the literature, many methods have been proposed for detecting and removing salt and pepper noise. Popular order statistics filter called standard median filter (SMF) [4] is an effective nonlinear filter for removing salt and pepper noise. This method smears details and edges during application at high noise densities [5]. Denoised image is blurred during the process because the un-noisy images are manipulated in pixel values. Various improvements have been made to the standard median filter to improve its performance on denoising, which includes size based median filter (SAMF), the noise adaptive soft-switching median filter (NASM), the extremum and median value (EM) filter, a decision based algorithm (DBA), an improved filtering algorithm for removing salt and pepper noise (IFARSPN), the decision based un-symmetric trimmed median (DBUTM) filter, the removal of wide range impulse noise (RWRIN) filter, tri-state median filter (TSMF), progressive switching median filter, and multi state median filter (MSMF) [9-18]. 50% percent of data can be retained by the mentioned algorithms better than SMF at higher noise densities.



**Fig. 1.** Image decomposition and different kinds of thresholding. (a) One level, (b) two level, (c) hard thresholding, and (d) soft thresholding.

Estimation of threshold values accurately is not easy which is a huge constraint in research. We are focusing on salt and pepper noise in this research work.

In addition, with this many algorithm have been proposed for removing high density salt and pepper noise [19–24]. A decision-based detail preserving restoration method [19] was proposed in two phases: first phase makes use of an adaptive median filter to identify noisy pixels and the second phase applies a regularization method on selected noise candidates to restore the image. A fast switching based median-mean filter (FSMMF) [20] proposed for high density salt and pepper noise removal effectively overcomes the problem of streaking effects in DBA and poor noise removal capability of MDBUTMF [21] at higher noise densities. An effective switching morphology mean filter [22] is proposed in two stages, in which the first stage detects the impulse noise and the second stage constructs an adaptive structuring element to remove the impulse noise.

Sparse Matrix representation is used to identify high density impulse noise pixels in images which is followed in SREPF [23], in which Sparse matrix representation identifies the noisy pixel and decides the processing order of noisy pixels. Then modified double Laplacian convolution is utilized for restoration. A new impulse detection and filtering method for removal of wide range impulse noise is proposed based on the minimum absolute value of four convolutions obtained by one dimensional Laplacian operators [15]. An adaptive masking weighted mean filter with consideration of contextual information (AMWMF-CI) [24] does not require any initial parameters or threshold values to be set to filter noise. The peak signal to noise ratio (PSNR) provided by improved tolerance based selective arithmetic mean filtering (ITSAMFT) [25] is bit lesser than the improved median filtering [26] at higher noise densities (91–99%). The main limitations of all these methods are the visual quality and the PSNR of the restored images are not high at higher noise densities.

In general, for the removal of salt and pepper noise various improvements have been made to the conventional median filter and were proposed as new methods. In this work improved tolerance based selective arithmetic mean filter for detection and removal of impulse noise [25] proposed by us is extended with wavelet thresholding and its performance is compared with the various existing algorithms such as SMF [4], DBA [12], DBUTM [14], IDFRWRIN [15], AMWMF-CI [24], TSAMFT, ITSAMFT, Level-2 ITSAMFT [25] and TSAMFT with wavelet thresholding. Many linear and nonlinear filtering techniques have been proposed earlier to remove impulse noise. However, the removal of impulse noise is often accomplished at the expense of blurred and distorted features of edges. Therefore, it is necessary to preserve the edges and fine details during filtering. In this paper, an effective improved tolerance based selective arithmetic mean filtering technique (ITSAMFT) combined with wavelet thresholding is proposed for restoration of images that are highly corrupted by salt and pepper noise.

The intention of the proposed filtering technique is to detect and remove the noisy pixels and restore the noise free information. In this method, a new and simple noise detection method based on the difference between the pixel of interest and arithmetic mean (AM) is presented. After detection, the tolerance based adaptive masking selective arithmetic mean filtering procedure is employed to replace the detected noisy pixels. In this work, wavelet thresholding is also performed after filtering. This helps to retain sharp edges and thus exhibits best visual quality by avoiding blurring effect even at higher noise densities and in turn produces high PSNR, SSIM and CORR values than other methods.

Section 2, the role of stationary wavelet transforms and wavelet thresholding in denoising is explained in detail. In Section 3, the proposed algorithm for improved tolerance based selective arithmetic mean filtering with wavelet thresholding is described in detail. In Section 4, experimental results are

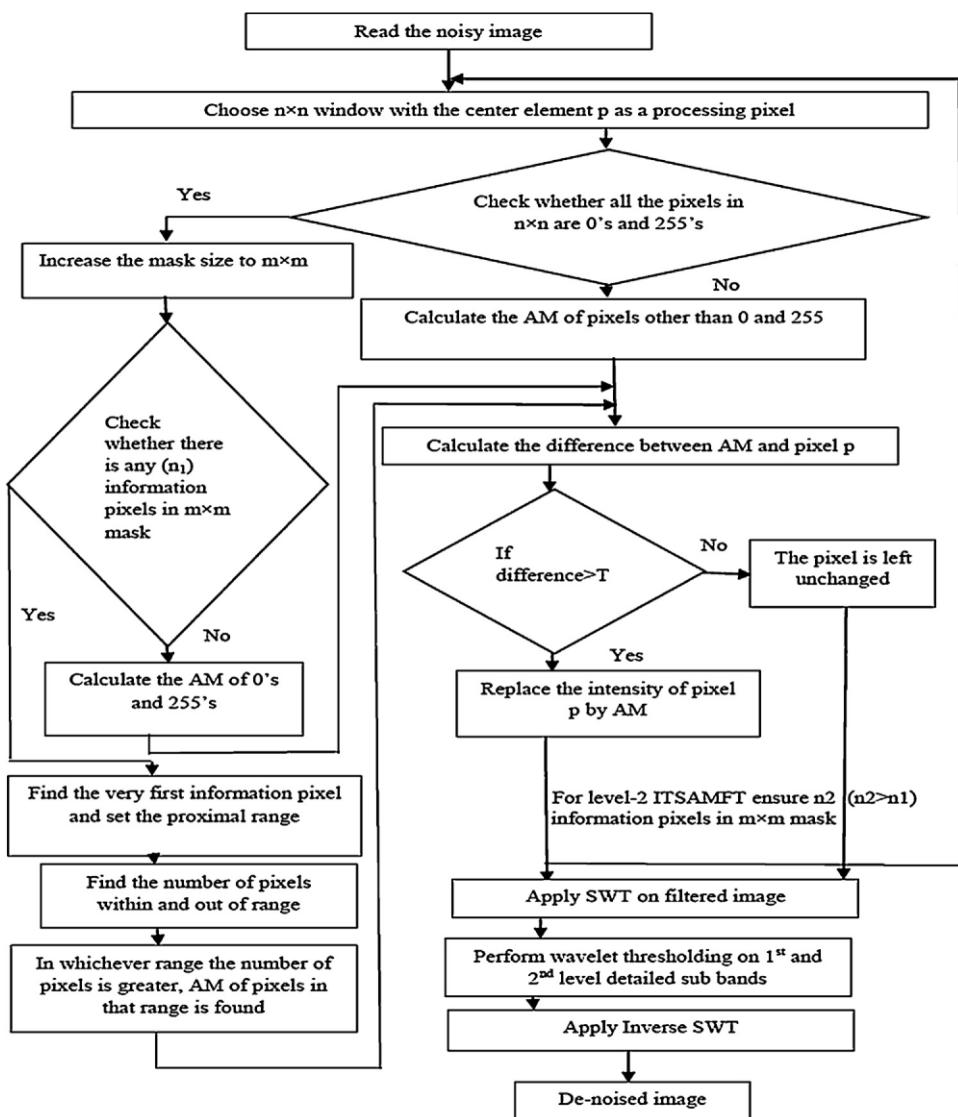
presented with discussion. The concluding remarks are given in Section 5.

## 2. The role of stationary wavelet transform and wavelet thresholding in denoising

Wavelets are functions, suited to the expansion of non-stationary continuous signals. They form the kernel of the wavelet transform and enable a mapping of signals from the time-domain into the time-frequency domain. Their key feature is that at different frequencies, a different resolution can be obtained [28]. The discrete wavelet transform is very efficient from the computational point of view. Its only drawback is that it is translation variant. Translations of the original signal lead to different wavelet coefficients. In order to overcome this drawback and to get more complete characteristic of the analyzed signal the undecimated wavelet transform or stationary wavelet transform (SWT) was proposed in 1996 to make the wavelet decomposition time invariant [29]. This improves the power of wavelet in signal de-noising and enhancement. The general idea behind that is it does not decimate the signal. However, the filters at each level are

modified by padding them out with zeros. For stationary or redundant transform, instead of down-sampling, an up-sampling procedure is carried out before performing filter convolution at each scale [30]. Thus, it produces more precise information for the frequency localization. The redundancy of this transform facilitates the identification of salient features in a signal, especially for recognizing the noises.

By applying DWT, the image is actually decomposed into four sub-bands as shown in Fig. 1a. These four sub-bands arise from separable applications of the low pass and high pass filters of the DWT analysis filter bank. Decomposition provides sub-bands corresponding to different resolution levels and orientation. These sub-bands labeled LH1, HL1 and HH1 represent the finest scale wavelet coefficients (i.e., detail images) while the sub-band LL1 corresponds to coarse level coefficients (i.e., approximation image). The next coarse level wavelet coefficients are obtained by further decomposing the sub-band LL1 alone using similar filter bank. These results in two-level wavelet decomposition as shown in Fig. 1b. Similarly, to obtain the next level decomposition, LL2 will be used. This process continues till some final scale is reached. The decomposed image can be reconstructed using a synthesis filter bank.



**Fig. 2.** Flowchart for the proposed algorithm.

The motivation for wavelet thresholding [32-34] is that as the wavelet transform is good at energy compaction, the small coefficients are more likely due to noise and large coefficients due to important signal features [31]. These small coefficients can be thresholded without affecting the significant features of the image. Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is thresholded by comparing against a threshold. If the coefficient is smaller than the threshold, it is set to zero. Otherwise it is retained as it is or modified. Replacing the small noisy coefficients by zero and inverse wavelet transform on the thresholded result may lead to reconstruction of the essential signal characteristics with less noise. In this work we have used hard and soft thresholding.

In hard thresholding method, the absolute values of each wavelet coefficient are compared against a threshold  $h$ . If the

magnitude of the coefficients is smaller than  $h$ , then they are set to zero since they are mostly corresponding to noise. The edge relating coefficients are usually above the threshold and hence retained. The hard thresholding is represented by,

$$d_{ik}^{\text{hard}} = \begin{cases} d_{ik} & \text{if } |d_{ik}| > h \\ 0 & \text{if } |d_{ik}| \leq h \end{cases} \quad (1)$$

represents the wavelet coefficients and  $h$  is the threshold value. The function performing hard thresholding is given in Fig. 1c. The soft thresholding is generally represented by,

$$d_{ik}^{\text{soft}} = \begin{cases} \text{sign}(d_{ik})(|d_{ik}| - h) & \text{if } d_{ik} > h \\ 0 & \text{if } d_{ik} \leq h \end{cases} \quad (2)$$

where  $\text{sign}(d_{ik})$  represents the signum function. The function performing soft thresholding is given in Fig. 1d.

**Table 1**

PSNR values of restored Lena for various methods and noise levels.

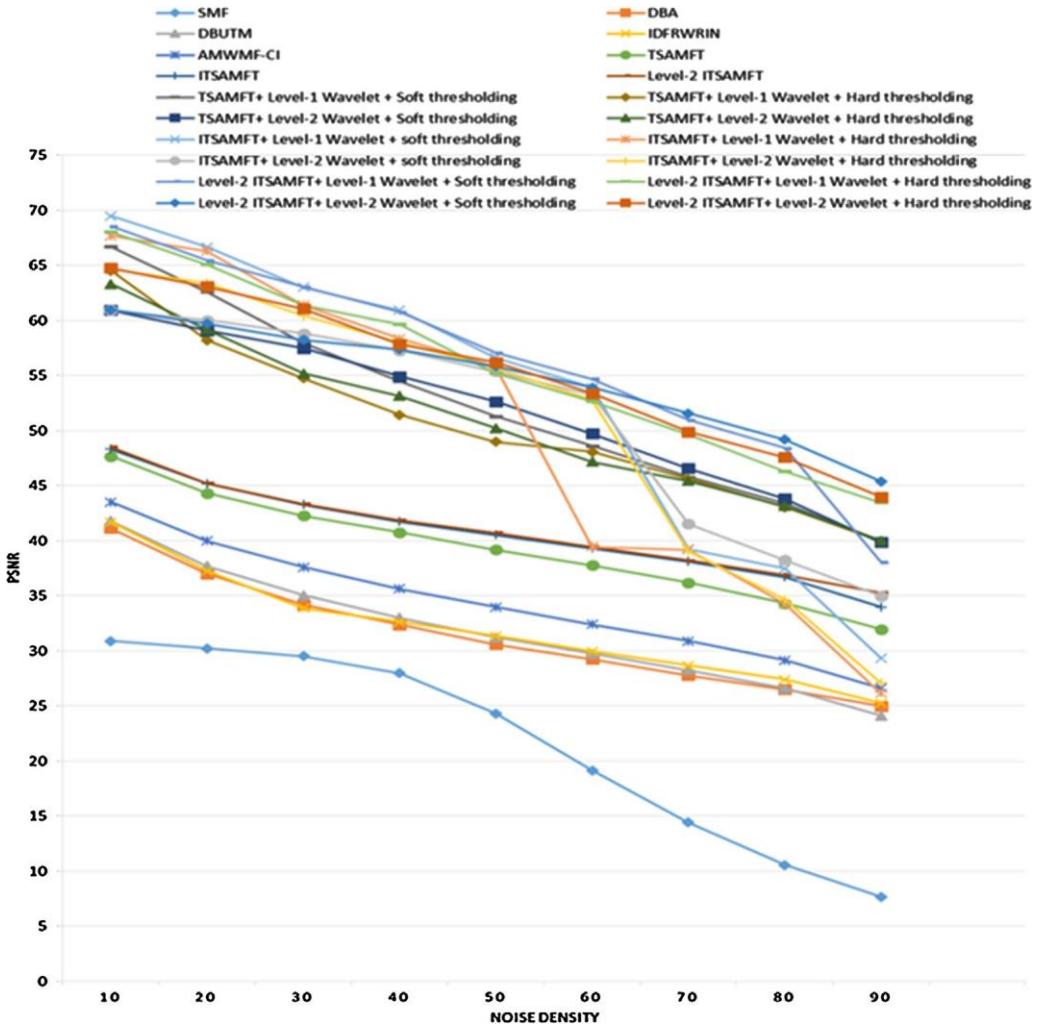
Name of the method	Noise density (%)									
	10	20	30	40	50	60	70	80	90	
PSNR(db)										
SMF	30.90	30.25	29.53	27.97	24.32	19.17	14.42	10.57	7.70	
DBA	41.15	37.02	34.21	32.38	30.62	29.27	27.77	26.51	24.99	
DBUTM	41.82	37.71	35.07	33.02	31.25	29.78	28.24	26.61	24.12	
IDFRWRIN	41.70	37.27	33.88	32.64	31.37	29.95	28.69	27.40	25.27	
AMWMF-CI	43.53	40.00	37.62	35.64	33.98	32.43	30.91	29.16	26.65	
TSAMFT	47.67	44.34	42.29	40.78	39.20	37.79	36.21	34.34	31.97	
ITSAMFT	48.33	45.21	45.28	41.73	40.53	39.33	38.13	36.72	33.99	
Level-2 ITSAMFT	48.54	45.22	43.34	41.88	40.73	39.45	38.22	36.91	35.31	
TSAMFT + Level-1 Wavelet + Soft thresholding	66.71	62.62	58.00	54.49	51.32	48.62	45.80	43.38	39.99	
TSAMFT + Level-1 Wavelet + Hard thresholding	64.52	58.22	54.76	51.45	49.00	48.08	45.61	43.04	39.97	
TSAMFT + Level-2 Wavelet + Soft thresholding	60.95	59.08	57.49	54.94	52.66	49.72	46.59	43.82	39.85	
TSAMFT + Level-2 Wavelet + Hard thresholding	63.32	59.21	55.20	53.14	50.25	47.19	45.45	43.16	40.05	
ITSAMFT + Level-1 Wavelet + Soft thresholding	<b>69.51</b>	<b>66.67</b>	<b>63.03</b>	<b>60.93</b>	56.64	53.93	39.27	37.47	29.36	
ITSAMFT + Level-1 Wavelet + Hard thresholding	67.67	66.32	61.42	58.36	55.80	39.47	39.19	34.25	26.17	
ITSAMFT + Level-2 Wavelet + Soft thresholding	60.94	60.04	58.82	57.27	55.37	53.26	41.56	38.27	35	
ITSAMFT + Level-2 Wavelet + Hard thresholding	64.66	63.42	60.42	58.03	55.67	52.79	39.05	34.66	27.11	
Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding	68.56	65.48	63.06	60.76	<b>57.08</b>	<b>54.70</b>	50.99	48.43	38.10	
Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding	68.06	65.11	61.39	59.65	55.23	52.69	49.64	46.31	43.49	
Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding	60.97	59.72	58.24	57.37	55.84	53.91	<b>51.59</b>	<b>49.18</b>	<b>45.39</b>	
Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding	64.78	63.09	61.09	57.84	56.21	53.38	49.89	47.58	43.99	

Bold represents the highest PSNR provided by the proposed method.

**Table 2**

SSIM values of restored Lena for various methods and noise levels.

Name of the method	Noise density (%)									
	10	20	30	40	50	60	70	80	90	
SSIM										
SMF	0.858	0.850	0.840	0.817	0.735	0.513	0.227	0.071	0.020	
DBA	0.988	0.972	0.951	0.927	0.899	0.869	0.832	0.786	0.652	
DBUTM	0.989	0.975	0.957	0.934	0.907	0.874	0.839	0.791	0.710	
IDFRWRIN	0.989	0.973	0.954	0.930	0.904	0.877	0.844	0.805	0.735	
AMWMF-CI	0.991	0.981	0.969	0.954	0.936	0.915	0.889	0.851	0.784	
TSAMFT	0.9990	0.9979	0.9964	0.9947	0.9918	0.9883	0.9823	0.9708	0.9397	
Level-1 ITSAMFT	0.9994	0.9987	0.9979	0.9968	0.9956	0.9937	0.9902	0.9833	0.9390	
Level-2 ITSAMFT	0.9995	0.9988	0.9978	0.9969	0.9955	0.9938	0.9911	0.9859	0.9771	
TSAMFT + Level-1 Wavelet + Soft thresholding	0.9249	0.9184	0.9097	0.9113	0.9156	0.9057	0.9033	0.8810	0.8515	
TSAMFT + Level-1 Wavelet + Hard thresholding	0.9175	0.9198	0.9141	0.9203	0.9198	0.9017	0.8987	0.8773	0.8497	
TSAMFT + Level-2 Wavelet + Soft thresholding	0.9086	0.9082	0.9061	0.9053	0.9024	0.9000	0.8976	0.8839	0.8448	
TSAMFT + Level-2 Wavelet + Hard thresholding	0.9148	0.9189	0.9179	0.9053	0.9022	0.9161	0.8984	0.8817	0.8457	
ITSAMFT + Level-1 Wavelet + Soft thresholding	0.9274	0.9232	0.9209	0.9170	0.9172	0.9150	0.9920	0.9855	0.9291	
ITSAMFT + Level-1 Wavelet + Hard thresholding	0.9240	0.9224	0.9192	0.9144	0.9162	0.9937	0.9913	0.9852	0.9210	
ITSAMFT + Level-2 Wavelet + Soft thresholding	0.9086	0.9078	0.9077	0.9080	0.9042	0.9037	0.9857	0.9838	0.9391	
ITSAMFT + Level-2 Wavelet + Hard thresholding	0.9153	0.9101	0.9119	0.9072	0.9109	0.9030	0.9903	0.9849	0.9264	
Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding	0.9248	0.9177	0.9196	0.9171	0.9159	0.9137	0.9132	0.9023	0.9760	
Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding	0.9211	0.9219	0.9208	0.9138	0.9194	0.9162	0.9071	0.9073	0.8970	
Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding	0.9094	0.9086	0.9081	0.9063	0.9060	0.9036	0.9019	0.9019	0.8934	
Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding	0.9167	0.9127	0.9107	0.9125	0.9072	0.9034	0.9090	0.9000	0.8943	



**Fig. 3.** Comparison graph of PSNR at various noise levels for different methods.

Several approaches have been proposed for threshold selection. A common approach is to compute the standard deviation in each sub-band and set the threshold to some multiple of standard deviation ( $\sigma$ ) in corresponding sub-band [35]. In this work we have considered threshold as  $2\sigma$ .

### 3. The proposed improved tolerance based selective arithmetic mean filtering with wavelet thresholding

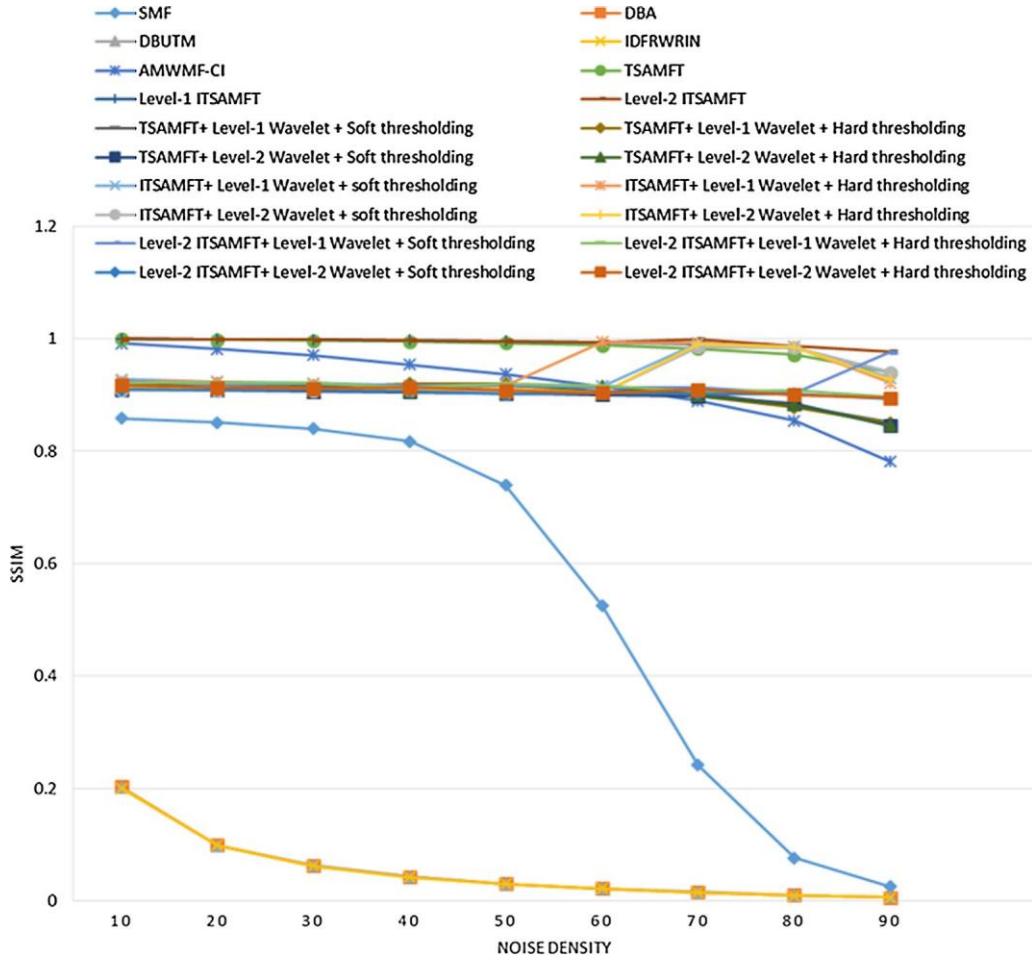
The proposed method consists of two steps. The first step detects the noisy pixel based on the difference between the pixel of interest and arithmetic mean (AM). In the second step noisy pixels are filtered out by using the tolerance based adaptive masking arithmetic mean filtering procedure. In this methodology, the following procedures are followed for noise detection and removal.

For Level-1 ITSAMFT, initially we processed with a mask of size  $n \times n$ . In this arithmetic mean value (AM) of the pixels other than 0 and 255 are determined. Then the difference between the AM and pixel  $p$  in the mask are calculated. If the difference is greater than the tolerance ( $T$ ), then the intensity of  $p$  is considered as corrupted pixel and is replaced by the AM else the pixel value is considered as uncorrupted and left unchanged. In the mask of size  $n \times n$ , if none of the pixels are other than 0 and 255 the mask around the pixel of interest is increased to size  $m \times m$  ( $m > n$ ). If all the pixels in that mask have values of 0 and 255 then the AM of pixels are estimated. Then the difference between the pixel of interest and

AM are calculated. If the difference is greater than the tolerance ( $T$ ), then the pixel of interest is considered as corrupted pixel and is replaced by AM else the pixel value is considered as uncorrupted and is left unchanged.

If  $n_1$  number of pixels in the  $m \times m$  mask are informative, i.e. other than 0 and 255 the subsequent process is followed. In  $m \times m$  mask the very first information pixel is chosen and the proximal range is set. Here the range that we considered is information pixel intensity proximal value. By exhaustive experimentation, here proximal value is identified as 10. Next, the number of pixels within and out of the considered range is found and thereby sum of the pixels within and out of that range are calculated. In whichever range the number of pixels is greater, the AM of pixels in that range is found. Then the difference between the pixel of interest and the AM is calculated. If the calculated difference is greater than the tolerance then that pixel is considered as corrupted pixel and is replaced by AM, else that pixel is considered as informative and left unchanged.

Similarly for Level-2 ITSAMFT the above procedure is followed on the filtered image except that the number of informative pixels in the  $m \times m$  mask is considered as  $n_2$  ( $n_2 > n_1$ ). After first and second level filtering, SWT is applied on the filtered images respectively. When the SWT is applied on the filtered images, the images are decomposed into approximation and detail sub-bands. For second level decomposition, SWT is applied on the approximation sub-band of first level decomposed image. After that soft and hard



**Fig. 4.** Comparison graph of SSIM at various noise levels for different methods.

thresholding is performed on the detailed sub-bands and inverse wavelet transform is applied for the reconstruction of the restored image. Finally the PSNR, SSIM and CORR values are computed to analyze the performance of the proposed denoising techniques.

In general, for the removal of salt and pepper noise various improvements have been made to the median filter and were proposed as new methods. The major contribution of our proposed algorithm is as follows: based on the difference

between the pixel of interest and the AM the noisy pixels are detected. After detection, we employed the tolerance based adaptive masking selective arithmetic mean filtering procedure to replace the detected noise pixels. In this we also made use of wavelet thresholding after filtering. This helps to retain sharp edges and thus exhibits best visual quality by avoiding blurring effect even at higher noise densities and in turn produces high PSNR, SSIM and CORR values than other methods.

**Table 3**

Correlation values of restored Lena for various methods and noise levels.

Name of the method	Noise density (%)								
	10	20	30	40	50	60	70	80	90
CORR									
TSAMFT	0.9989	0.9976	0.9958	0.9958	0.9899	0.9845	0.9756	0.9553	0.8984
Level-1 ITSAMFT	0.9973	0.9965	0.9951	0.9934	0.9909	0.9858	0.9771	0.9544	0.9019
Level-2 ITSAMFT	0.9979	0.9968	0.9953	0.9932	0.99	0.9855	0.9758	0.9551	0.8986
TSAMFT + Level-1 Wavelet + Soft thresholding	0.9924	0.9915	0.9905	0.9891	0.9866	0.9825	0.9746	0.9572	0.8965
TSAMFT + Level-1 Wavelet + Hard thresholding	0.9961	0.9952	0.9937	0.9919	0.9886	0.9836	0.9746	0.9558	0.9007
TSAMFT + Level-2 Wavelet + Soft thresholding	0.9992	0.9983	0.9972	0.9958	0.9942	0.9913	0.9868	0.9518	0.9011
TSAMFT + Level-2 Wavelet + Hard thresholding	0.9975	0.997	0.9963	0.9955	0.994	0.9921	0.9881	0.9645	0.7869
ITSAMFT + Level-1 Wavelet + Soft thresholding	0.9982	0.9975	0.9975	0.9954	0.9939	0.9916	0.9868	0.9544	0.6810
ITSAMFT + Level-1 Wavelet + Hard thresholding	0.9926	0.9921	0.9914	0.9907	0.9898	0.9882	0.9851	0.9675	0.8563
ITSAMFT + Level-2 Wavelet + Soft thresholding	0.9963	0.9957	0.9949	0.9938	0.9924	0.9903	0.9858	0.9576	0.7132
ITSAMFT + Level-2 Wavelet + Hard thresholding	0.9992	0.9983	0.9972	0.996	0.996	0.9912	0.9869	0.9787	0.9631
Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding	0.9975	0.997	0.9962	0.9953	0.9941	0.9922	0.9889	0.9829	0.9675
Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding	0.9982	0.9974	0.9965	0.9955	0.994	0.9915	0.9977	0.9805	0.9639
Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding	0.9926	0.992	0.9914	0.9907	0.9898	0.9884	0.9865	0.9821	0.9715
Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding	0.9963	0.9957	0.9948	0.9938	0.9926	0.9904	0.987	0.9806	0.967

Salt and pepper noise can also be successfully removed by arithmetic mean filtering (AMF) technique. But the image will suffer from blurring effect. In arithmetic mean filtering method, each pixel is considered to calculate the mean and also every pixel is replaced by a calculated mean. The AMF apply the same process uniformly across the image without discriminating noisy and noise free pixels. That means, affected noisy pixels are also considered to calculate the mean, which in most cases produces distortion. And unaffected information pixels are also replaced by the calculated mean. Therefore we have proposed a modified algorithm known as "improved tolerance based selective arithmetic mean filtering technique (ITSAMFT)", in which the noisy pixels are not considered for calculating the mean and the unaffected information pixels are not disturbed. If all the pixels are noisy in the considered sub window, the size of the sub window is increased to next level and

scanned for the presence of information pixels. Again, if there is no information pixels in the increased sub window, in such occasion only, 0's and 255's are considered for calculating AM. Otherwise, the noisy pixels are not considered for calculating the mean. This is the reason for the ITSAMFT to work on denoising. The flowchart of the above proposed filtering algorithm is given in Fig. 2.

#### 4. Experimental results and discussion

In this experiment the performance of the proposed denoising algorithm is analyzed on gray scale Lena and Baboon images with a size of 512 × 12 pixels. In the simulation, Lena and Baboon images are corrupted by salt and pepper noise. The noise levels are broadly varied from 10% to 90% with increments of 10%, and the restoration performance are quantitatively measured by peak signal-to-noise ratio (PSNR), structural similarity index [27] (SSIM) and correlation (CORR) as given below in Eqs. (1)–(3) and qualitatively accessed by histogram.

$$\text{Peak signal to noise ratio (PSNR)}: \text{PSNR} = 10 \log_{10} \frac{\sum_{ij} 255^2}{\text{MSE}} \quad (3)$$

Mean square error:

$$\text{MSE} = \frac{1}{MN} \sum_{ij} (y_{ij} - x_{ij})^2$$

where  $y_{ij}$  and  $x_{ij}$  denote the pixel values of the restored image and the original image respectively.  $M \times N$  is the size of the image.

Structural similarity index:

$$\text{SSIM}(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (4)$$

where  $\mu_x$  and  $\mu_y$  represent the mean of the original and restored images,  $\sigma_x^2$  and  $\sigma_y^2$  represent the variance of  $x$  and  $y$ , and  $\sigma_{xy}$  represent the covariance of  $x$  and  $y$ .  $C_1 = (k_1 L)^2$  and  $C_2 = (k_2 L)^2$  are two variables to stabilize the division with weak denominator;  $L$  is the dynamic range of the pixel values (typically this is 2<sup>#bits per pixel</sup> 1);  $k_1 = 0.01$  and  $k_2 = 0.03$  by default.

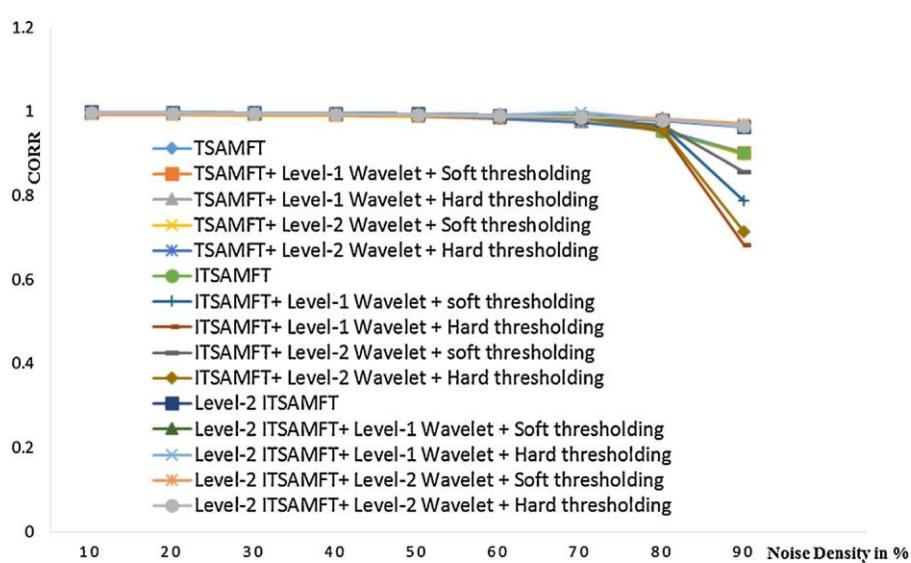
Correlation:

$$\text{corr} = \frac{\sum_{ij} (y_{ij} - \mu_y)(x_{ij} - \mu_x)}{\sqrt{\sum_{ij} (y_{ij} - \mu_y)^2} \sqrt{\sum_{ij} (x_{ij} - \mu_x)^2}} \quad (5)$$

where  $y_{ij}$  and  $x_{ij}$  denote the pixel values of the restored image and the original image, respectively.  $M \times N$  is the size of the image.

The PSNR and SSIM are estimated for Lena and Baboon images and a comparison of performance with various algorithms including SMF [4], DBA [12], DBUTM [14], IDFRWRIN [15], AMWMF-CI [24], TSAMFT, ITSAMFT and Level-2 ITSAMFT [25] are presented in Tables 1 and 2 and Tables 5 and 6 respectively. And the PSNR and SSIM results for Lena image are also shown in Figs. 3–4. Our results have been compared with the PSNR and SSIM values of SMF, DBA, DBUTM, IDFRWRIN and AMWMF-CI methods specified in [24]. The computed correlation value of the proposed method is compared with the TSAMFT with wavelet thresholding for Lena and Baboon images in Tables 3 and 7. And the correlation results for Lena image are also shown in Fig. 5.

From the denoising results of Lena and Baboon, it is observed that while wavelet thresholding is applied after tolerance based selective arithmetic mean filtering and improved tolerance based selective arithmetic mean filtering, there is a drastic improvement in PSNR values for all noise densities by maintaining good values of SSIM. For all noise levels that is from low to high, our methods achieve higher PSNR and best visual quality than other methods. It is observed that soft thresholding preserves image details very well for a wide range of noise density.



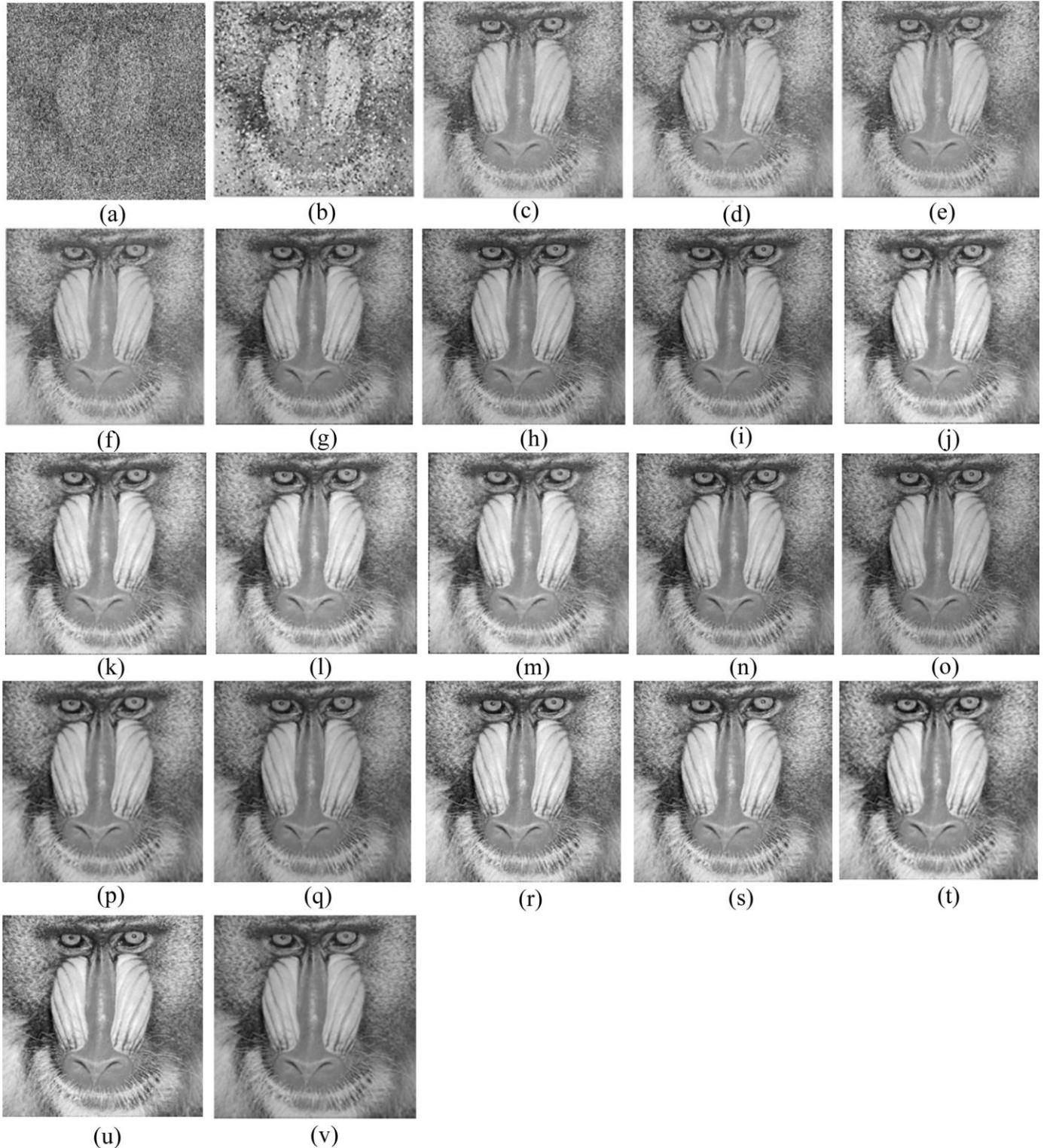
**Fig. 5.** Comparison graph of correlation at various noise levels for different methods.

**Figs. 6 and 7** shows the visual qualities of the filtered images using various algorithms for both Lena and Baboon images corrupted by 70% impulse noise. Similarly, **Figs. 8 and 9** shows 90% impulse noise. It is observed that the proposed Level-2 ITSAMFT + Level-1 Wavelet decomposition + Soft thresholding,

Level-2 ITSAMFT + Level-1 Wavelet decomposition + Hard thresholding, Level-2 ITSAMFT + Level-2 Wavelet decomposition + Soft thresholding and Level-2 ITSAMFT + Level-2 Wavelet decomposition + Hard thresholding methods provides better performance in terms of image quality at higher noise density



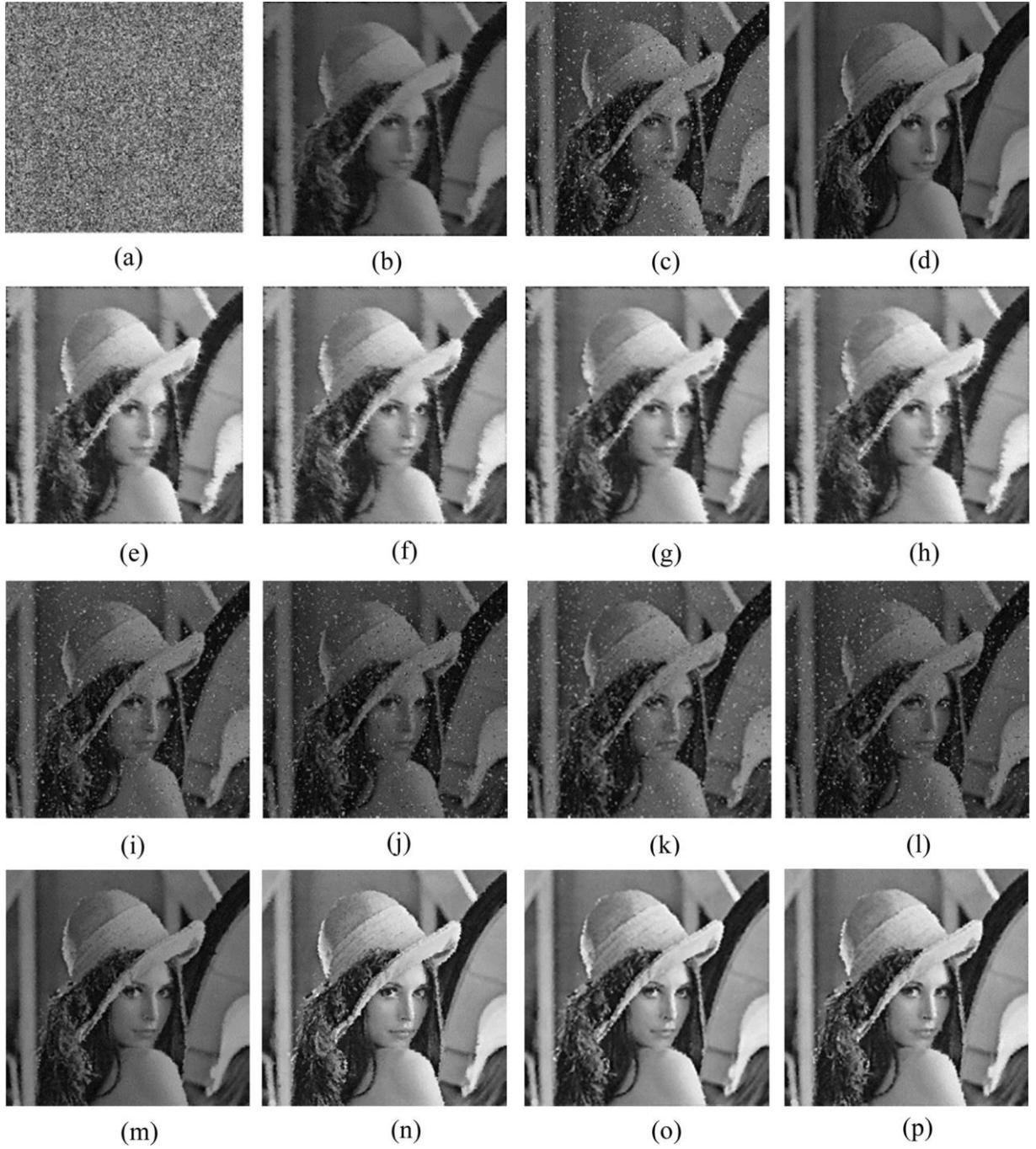
**Fig. 6.** Lena with 70% noise and restoration results for various methods. (a) Lena with 70% salt-pepper noise, restoration results obtained using (b) SMF, (c) DBA, (d) DBUTM, (e) RWRIN, (f) AMWMMF-CI, (g) TSAMFT, (h) ITSAMFT, (i) Level-2 ITSAMFT, (j) TSAMFT + Level-1 Wavelet + Soft thresholding, (k) TSAMFT + Level-1 Wavelet + Hard thresholding, (l) TSAMFT + Level-2 Wavelet + Soft thresholding, (m) TSAMFT + Level-2 Wavelet + Hard thresholding, (n) ITSAMFT + Level-1 Wavelet + Soft thresholding, (o) ITSAMFT + Level-1 Wavelet + Hard thresholding, (p) ITSAMFT + Level-2 Wavelet + Soft thresholding, (q) ITSAMFT + Level-2 Wavelet + Hard thresholding, (r) Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding, (s) Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding, (t) Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding, (u) Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding, and (v) original Lena.



**Fig. 7.** Baboon with 70% noise and restoration results for various methods. (a) Baboon with 70% salt-pepper noise, restoration results obtained using (b) SMF, (c) DBA, (d) DBUTM, (e) RWRIN, (f) AMWMF-CL, (g) TSAMFT, (h) ITSAMFT, (i) Level-2 ITSAMFT, (j) TSAMFT + Level-1 Wavelet + Soft thresholding, (k) TSAMFT + Level-1 Wavelet + Hard thresholding, (l) TSAMFT + Level-2 Wavelet + Soft thresholding, (m) TSAMFT + Level-2 Wavelet + Hard thresholding, (n) ITSAMFT + Level-1 Wavelet + Soft thresholding, (o) ITSAMFT + Level-1 Wavelet + Hard thresholding, (p) ITSAMFT + Level-2 Wavelet + Soft thresholding, (q) ITSAMFT + Level-2 Wavelet + Hard thresholding, (r) Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding, (s) Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding, (t) Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding, (u) Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding, and (v) original Baboon.

90%. Moreover, from Fig. 8e-h (Fig. 9e-h) and Fig. 8i-l (Fig. 9i-l), one can observe some blurring effect and some impulse spots remained in the resultant image at 90% noise density. But at the same time for same noise density, results in Figs. 8m-p and 9m-p

reveal that the proposed Level-2 ITSAMFT + Level-1/2 Wavelet decomposition + Soft/Hard thresholding methods exhibits best visual quality. Similarly from Fig. 5 it is interesting to note that one of the proposed Level-2 ITSAMFT + Level-1/2 Wavelet



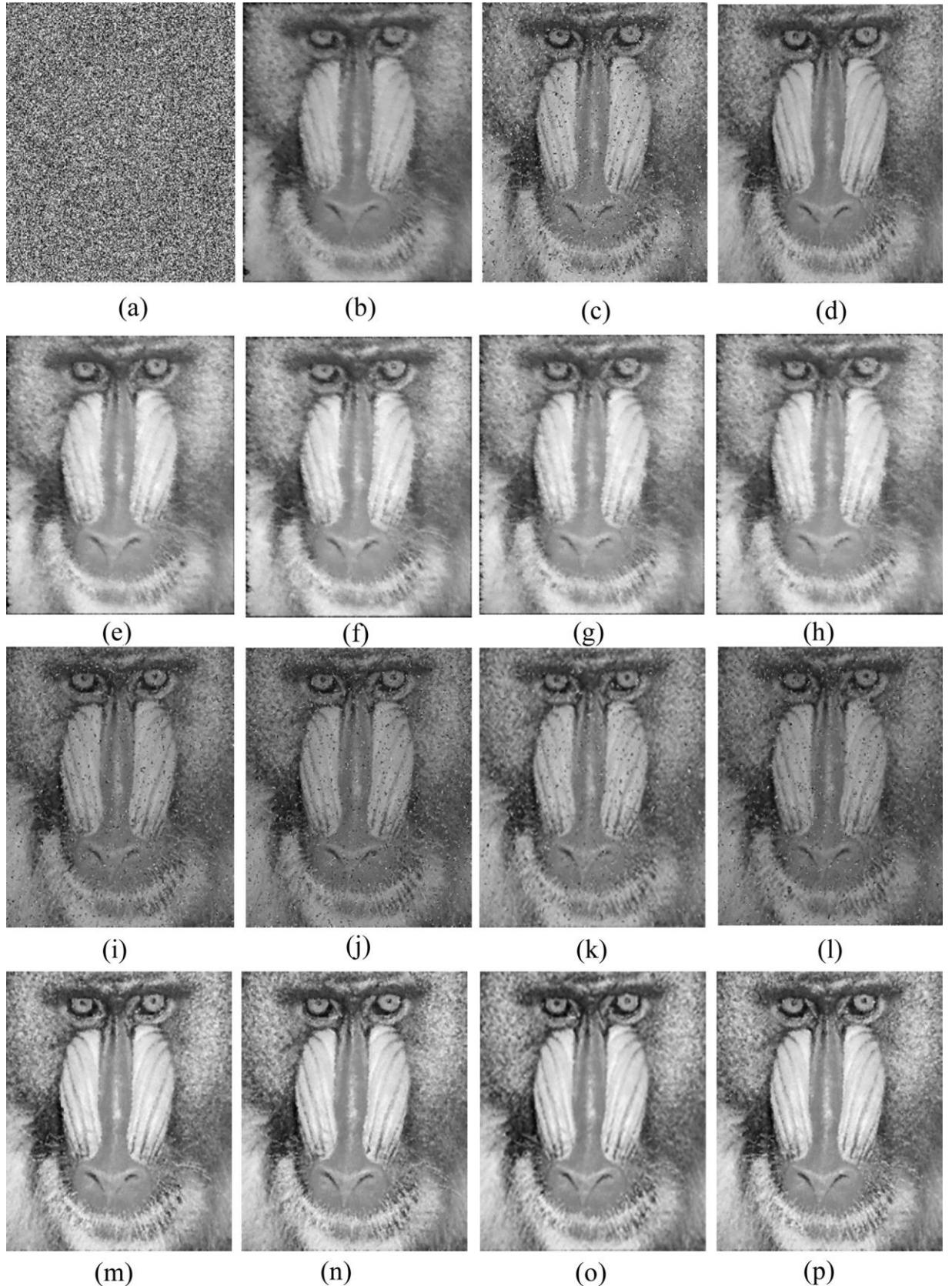
**Fig. 8.** Lena with 90% noise and restoration results for various methods. (a) Lena with 90% salt-pepper noise, restoration results obtained using (b) TSAMFT, (c) ITSAMFT, (d) Level-2 ITSAMFT, (e) TSAMFT + Level-1 Wavelet + Soft thresholding, (f) TSAMFT + Level-1 Wavelet + Hard thresholding, (g) TSAMFT + Level-2 Wavelet + Soft thresholding, (h) TSAMFT + Level-2 Wavelet + Hard thresholding, (i) ITSAMFT + Level-1 Wavelet + Soft thresholding, (j) ITSAMFT + Level-1 Wavelet + Hard thresholding, (k) ITSAMFT + Level-2 Wavelet + Soft thresholding, (l) ITSAMFT + Level-2 Wavelet + Hard thresholding, (m) Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding, (n) Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding, (o) Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding, and (p) Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding.

decomposition+Soft/Hard thresholding methods exhibits higher correlation values than other methods especially at higher noise densities.

The quality assessment values of restored Lena and Baboon images for various methods at different tolerance values and noise levels are presented in Tables 4 and 8. Comparison graphs for PSNR, SSIM and CORR at various noise levels for different tolerance values and methods are shown in Figs. 10–12 for Lena image. Tolerance is determined based on experimentation. From Fig. 10 it is clear that

for low tolerance values, all methods give enhanced PSNR for high noise densities. For large tolerance values, some noisy pixels are not replaced. This results in decrease in the PSNR for the images which are corrupted by high density salt and pepper noise. Therefore based on the desired denoising performance, tolerance can be fixed by experimentation.

Another effective tool for the image quality assessment is histogram. The histogram of an image indicates the dynamic range of the image, i.e., how well the image is spread out. Dynamic



**Fig. 9.** Baboon with 90% noise and restoration results for various methods. (a) Baboon with 90% salt-pepper noise, restoration results obtained using (b) TSAMFT, (c) ITSAMFT, (d) Level-2 ITSAMFT, (e) TSAMFT + Level-1 Wavelet + Soft thresholding, (f) TSAMFT + Level-1 Wavelet + Hard thresholding, (g) TSAMFT + Level-2 Wavelet + Soft thresholding, (h) TSAMFT + Level-2 Wavelet + Hard thresholding, (i) ITSAMFT + Level-1 Wavelet + Soft thresholding, (j) ITSAMFT + Level-1 Wavelet + Hard thresholding, (k) ITSAMFT + Level-2 Wavelet + Soft thresholding, (l) ITSAMFT + Level-2 Wavelet + Hard thresholding, (m) Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding, (n) Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding, (o) Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding, and (p) Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding.

**Table 4**

Quality assessment values of restored Lena for various methods at different tolerance values and noise levels.

Name of the method	Quality assessment values	Tolerance value														
		0	10	20	30	40	0	10	20	30	40	0	10	20	30	40
Noise density																
30%																
TSAMFT	PSNR	42.29	42.37	40.66	38.87	38.19	37.70	37.83	37.16	36.09	35.48	31.97	32.10	32.41	32.89	33.16
	SSIM	0.9964	0.9965	0.9967	0.9959	0.9961	0.9883	0.9886	0.9869	0.9845	0.9835	0.9397	0.9396	0.9387	0.9314	0.9176
	CORR	0.9958	0.9958	0.9949	0.9925	0.9898	0.9845	0.9846	0.9763	0.9617	0.9519	0.8984	0.9028	0.8921	0.8507	0.8024
TSAMFT + L1 WT + ST	PSNR	58.00	56.68	52.57	47.59	45.31	48.62	49.50	48.35	46.63	43.78	39.99	40.69	41.19	43.51	43.31
	SSIM	0.9097	0.9158	0.9237	0.9182	0.9256	0.9057	0.8997	0.8994	0.8992	0.9054	0.8515	0.8458	0.8549	0.8330	0.8224
	CORR	0.9951	0.9951	0.9949	0.9937	0.9929	0.9858	0.9858	0.9800	0.9719	0.9679	0.9019	0.9017	0.8893	0.8637	0.8214
TSAMFT + L2WT + ST	PSNR	57.49	57.13	54.32	51.04	48.93	49.72	50.27	49.66	49.14	46.93	39.85	40.77	42.17	44.03	45.33
	SSIM	0.9061	0.9058	0.9081	0.9083	0.9080	0.9000	0.8957	0.8979	0.8978	0.8980	0.8448	0.8455	0.8456	0.8312	0.8180
	CORR	0.9905	0.9905	0.9901	0.9896	0.9891	0.9825	0.9821	0.9791	0.9754	0.9721	0.8965	0.9026	0.8993	0.8801	0.8542
Level-1 ITSAMFT	PSNR	43.28	43.18	40.86	39.04	38.26	39.33	39.35	37.21	35.67	34.99	33.99	33.90	32.99	32.16	31.80
	SSIM	0.9979	0.9974	0.9978	0.9974	0.9976	0.9937	0.9938	0.9934	0.9930	0.9927	0.9390	0.9366	0.9388	0.9379	0.9383
	CORR	0.9972	0.9972	0.9962	0.9941	0.9917	0.9913	0.9912	0.9893	0.9855	0.9816	0.6492	0.6507	0.6593	0.6661	0.6652
Level-1	PSNR	63.03	61.35	52.70	47.21	45.09	53.93	53.62	41.30	42.41	37.79	29.36	29.34	27.70	27.33	27.32
ITSAMFT + L1WT + ST	SSIM	0.9209	0.9230	0.9123	0.9185	0.9180	0.9150	0.9174	0.9909	0.9156	0.9873	0.9291	0.9305	0.9301	0.9280	0.9279
	CORR	0.9963	0.9963	0.9959	0.9951	0.9941	0.9921	0.9923	0.9913	0.9901	0.9883	0.7869	0.7963	0.7937	0.7997	0.8054
Level-1	PSNR	58.82	58.50	54.56	50.76	48.45	53.26	44.51	49.05	40.20	43.13	35.00	35.33	34.4	33.64	33.40
ITSAMFT + L2WT + ST	SSIM	0.9077	0.9075	0.9063	0.9095	0.9103	0.9037	0.9802	0.9044	0.9792	0.9118	0.9391	0.9384	0.9380	0.9385	0.9396
	CORR	0.9914	0.9914	0.9911	0.9906	0.9900	0.9903	0.9884	0.9878	0.9868	0.9863	0.8563	0.8539	0.8504	0.8355	0.8651
Level-2 ITSAMFT	PSNR	43.34	43.30	41.11	39.12	38.41	39.45	39.49	37.72	35.96	35.26	35.31	35.42	34.54	33.35	32.73
	SSIM	0.9978	0.9979	0.9979	0.9975	0.9974	0.9938	0.9939	0.9934	0.9933	0.9933	0.9771	0.9770	0.9769	0.9766	0.9747
	CORR	0.9972	0.9972	0.9963	0.9942	0.9921	0.9912	0.9913	0.9896	0.9864	0.9829	0.9631	0.9628	0.9610	0.9566	0.9517
Level2	PSNR	63.06	63.45	52.79	47.62	45.36	54.70	54.78	48.00	42.36	40.81	38.10	44.12	42.47	35.60	37.46
ITSAMFT + L1WT + ST	SSIM	0.9196	0.9169	0.9232	0.9163	0.9178	0.9137	0.9084	0.9188	0.9286	0.9171	0.9760	0.8973	0.8930	0.9754	0.9005
	CORR	0.9962	0.9963	0.9959	0.9951	0.9943	0.9922	0.9922	0.9916	0.9900	0.9888	0.9675	0.9670	0.9672	0.9639	0.9634
Level-2	PSNR	58.24	58.53	55.34	50.74	48.47	53.91	53.93	49.99	45.44	43.46	45.39	45.51	43.79	40.55	38.82
ITSAMFT + L2WT + ST	SSIM	0.9081	0.9082	0.9084	0.9100	0.9100	0.9036	0.9055	0.9048	0.9064	0.9085	0.8934	0.8931	0.8960	0.8978	0.9060
	CORR	0.9914	0.9914	0.9912	0.9906	0.9900	0.9884	0.9885	0.9878	0.9870	0.9859	0.9715	0.9715	0.9697	0.9681	0.9674

**Table 5**

PSNR values of restored Baboon for various methods and noise levels.

Name of the method	Noise density (%)									
	10	20	30	40	50	60	70	80	90	
	PSNR(db)									
SMF	21.26	21.13	20.88	20.46	19.34	16.89	13.43	10.25	7.58	
DBA	31.60	27.99	25.78	24.03	22.79	21.60	20.65	19.69	18.56	
DBUTM	31.85	28.37	26.21	24.49	23.06	21.83	20.75	19.64	18.31	
IDFRWRIN	32.01	28.36	26.04	24.39	23.09	22.02	21.00	19.96	18.81	
AMWMF-CI	33.35	30.10	28.02	26.36	24.97	23.70	22.62	21.49	20.08	
TSAMFT	45.19	41.82	39.69	37.95	36.52	35.14	33.73	32.30	30.55	
ITSAMFT	45.76	42.50	40.36	39.00	37.54	36.32	34.95	33.59	31.50	
Level-2 ITSAMFT	45.52	42.52	40.46	38.95	37.52	36.25	35.01	33.63	32.02	
TSAMFT + Level-1 Wavelet + Soft thresholding	52.82	50.18	49.16	47.18	46.79	45.09	44.05	41.24	37.92	
TSAMFT + Level-1 Wavelet + Hard thresholding	52.26	50.36	48.70	46.68	47.05	45.27	43.40	40.94	37.89	
TSAMFT + Level-2 Wavelet + Soft thresholding	52.77	51.09	49.15	47.19	45.75	45.55	43.16	41.17	37.99	
TSAMFT + Level-2 Wavelet + Hard thresholding	51.58	49.94	47.99	46.24	46.38	45.49	43.47	40.86	37.98	
ITSAMFT + Level-1 Wavelet + Soft thresholding	52.80	53.60	50.54	<b>52.90</b>	48.99	38.38	38.76	32.98	26.12	
ITSAMFT + Level-1 Wavelet + Hard thresholding	55.30	54.46	52.40	51.43	47.71	46.68	35.38	30.66	25.35	
ITSAMFT + Level-2 Wavelet + Soft thresholding	47.62	47.25	46.51	45.45	45.63	39.66	36.63	31.98	30.76	
ITSAMFT + Level-2 Wavelet + Hard thresholding	49.30	47.20	46.42	46.80	46.33	47.03	30.92	30.39	25.34	
Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding	54.82	54.11	<b>53.66</b>	51.45	<b>50.52</b>	<b>47.57</b>	44.67	41.03	<b>39.02</b>	
Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding	<b>55.66</b>	<b>55.30</b>	50.26	52.69	49.19	44.69	<b>45.05</b>	40.38	38.01	
Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding	46.96	47.18	46.59	44.77	45.03	42.64	41.84	<b>41.43</b>	37.37	
Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding	54.99	50.35	52.22	49.62	48.45	44.56	44.27	40.07	37.83	

Bold represents the highest PSNR provided by the proposed method.

**Table 6**

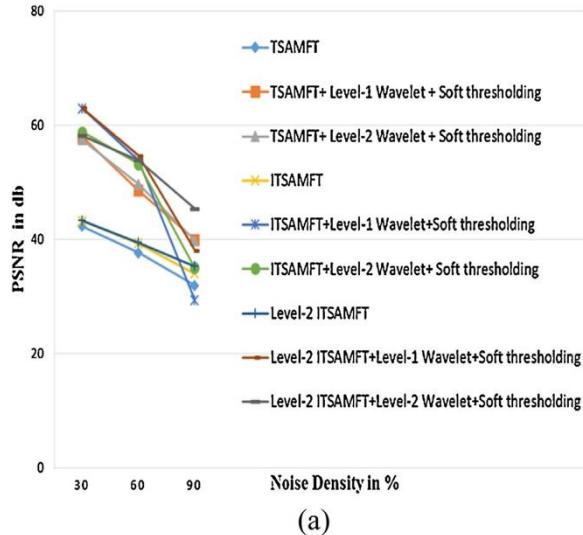
SSIM values of restored Baboon for various methods and noise levels.

Name of the method	Noise density (%)									
	10	20	30	40	50	60	70	80	90	
	SSIM									
SMF	0.478	0.472	0.463	0.443	0.400	0.297	0.158	0.061	0.019	
DBA	0.967	0.926	0.874	0.814	0.751	0.682	0.606	0.515	0.400	
DBUTM	0.969	0.930	0.882	0.827	0.762	0.690	0.608	0.519	0.396	
IDFRWRIN	0.970	0.930	0.883	0.826	0.768	0.700	0.629	0.546	0.423	
AMWMF-CI	0.977	0.949	0.915	0.877	0.828	0.772	0.703	0.612	0.476	
TSAMFT	0.9982	0.9957	0.9929	0.9896	0.9846	0.9768	0.9659	0.9491	0.9063	
ITSAMFT	0.9985	0.9967	0.9942	0.9920	0.9881	0.9829	0.9752	0.9604	0.9061	
Level-2 ITSAMFT	0.9984	0.9966	0.9943	0.9917	0.9881	0.9834	0.9753	0.9623	0.9375	
TSAMFT + Level-1 Wavelet + Soft thresholding	0.9824	0.9815	0.9784	0.9758	0.9693	0.9614	0.9461	0.9269	0.8883	
TSAMFT + Level-1 Wavelet + Hard thresholding	0.9837	0.9818	0.9789	0.9771	0.9665	0.9588	0.9493	0.9305	0.8892	
TSAMFT + Level-2 Wavelet + Soft thresholding	0.9823	0.9807	0.9786	0.9758	0.9708	0.9599	0.9499	0.9322	0.8869	
TSAMFT + Level-2 Wavelet + Hard thresholding	0.9839	0.9824	0.9805	0.9772	0.9690	0.9575	0.9478	0.9295	0.8885	
ITSAMFT + Level-1 Wavelet + Soft thresholding	0.9835	0.9819	0.9809	0.9770	0.9762	0.9829	0.9771	0.9662	0.8978	
ITSAMFT + Level-1 Wavelet + Hard thresholding	0.9840	0.9826	0.9808	0.9783	0.9758	0.9706	0.9772	0.9635	0.8943	
ITSAMFT + Level-2 Wavelet + Soft thresholding	0.9605	0.9593	0.9589	0.9581	0.9560	0.9637	0.9632	0.9575	0.9050	
ITSAMFT + Level-2 Wavelet + Hard thresholding	0.9722	0.9718	0.9704	0.9676	0.9648	0.9590	0.9699	0.9587	0.8866	
Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding	0.9825	0.9814	0.9797	0.9783	0.9753	0.9714	0.9665	0.9568	0.9293	
Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding	0.9840	0.9819	0.9818	0.9769	0.9754	0.9713	0.9635	0.9534	0.9282	
Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding	0.9610	0.9598	0.9587	0.9586	0.9562	0.9556	0.9531	0.9458	0.9343	
Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding	0.9841	0.9839	0.9812	0.9792	0.9754	0.9735	0.9637	0.9550	0.9289	

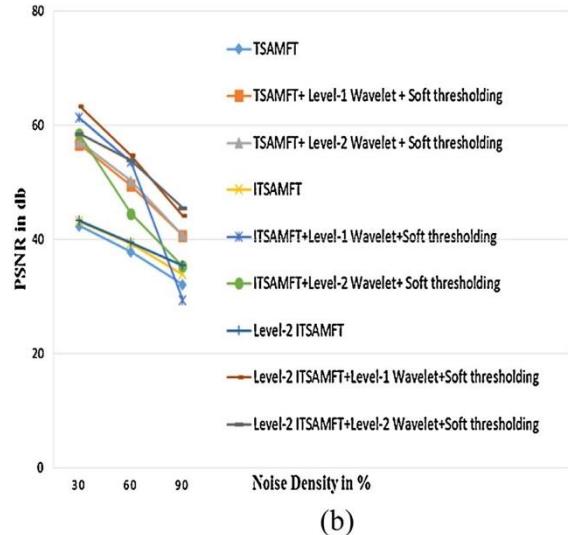
**Table 7**

Correlation values of restored Baboon for various methods and noise levels.

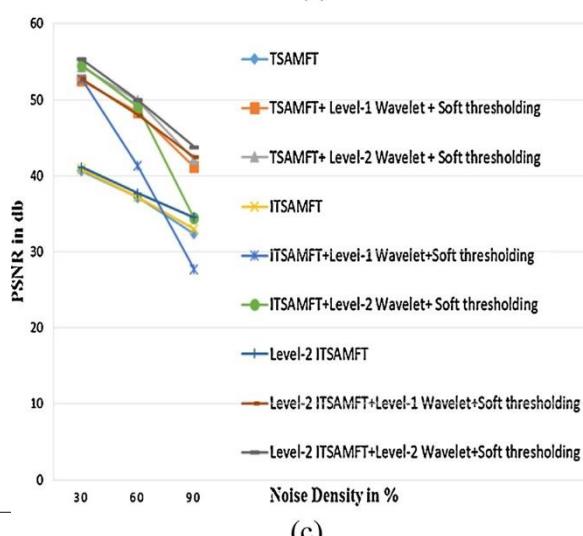
Name of the method	Noise density (%)									
	10	20	30	40	50	60	70	80	90	
	CORR									
TSAMFT	0.9980	0.9952	0.9920	0.9875	0.9815	0.9714	0.9552	0.9252	0.8460	
ITSAMFT	0.9985	0.9967	0.9947	0.9923	0.9884	0.9828	0.9728	0.9336	0.6365	
Level-2 ITSAMFT	0.9985	0.9968	0.9947	0.9921	0.9885	0.9829	0.9747	0.9584	0.9263	
TSAMFT + Level-1 Wavelet + Soft thresholding	0.9957	0.9938	0.9913	0.9876	0.9822	0.9739	0.9580	0.9258	0.8510	
TSAMFT + Level-1 Wavelet + Hard thresholding	0.9964	0.9944	0.9912	0.9876	0.9810	0.9714	0.9570	0.9264	0.8542	
TSAMFT + Level-2 Wavelet + Soft thresholding	0.9955	0.9939	0.9914	0.9877	0.9819	0.9735	0.9569	0.9296	0.8518	
TSAMFT + Level-2 Wavelet + Hard thresholding	0.9964	0.9943	0.9915	0.9876	0.9813	0.9718	0.9516	0.9241	0.8507	
ITSAMFT + Level-1 Wavelet + Soft thresholding	0.9959	0.9949	0.9934	0.9916	0.9891	0.9851	0.9785	0.9484	0.7834	
ITSAMFT + Level-1 Wavelet + Hard thresholding	0.9970	0.9958	0.9941	0.9920	0.9890	0.9841	0.9758	0.9398	0.6864	
ITSAMFT + Level-2 Wavelet + Soft thresholding	0.9764	0.9753	0.9743	0.9730	0.9715	0.9692	0.9644	0.9458	0.8357	
ITSAMFT + Level-2 Wavelet + Hard thresholding	0.9873	0.9859	0.9843	0.9824	0.9799	0.9760	0.9690	0.9375	0.6963	
Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding	0.9960	0.9949	0.9934	0.9917	0.9891	0.9852	0.9793	0.9667	0.9353	
Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding	0.9971	0.9957	0.9940	0.9919	0.9890	0.9844	0.9767	0.9627	0.9310	
Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding	0.9764	0.9754	0.9743	0.9730	0.9714	0.9690	0.9657	0.9585	0.9380	
Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding	0.9970	0.9957	0.9941	0.9919	0.9888	0.9846	0.9767	0.9628	0.9307	



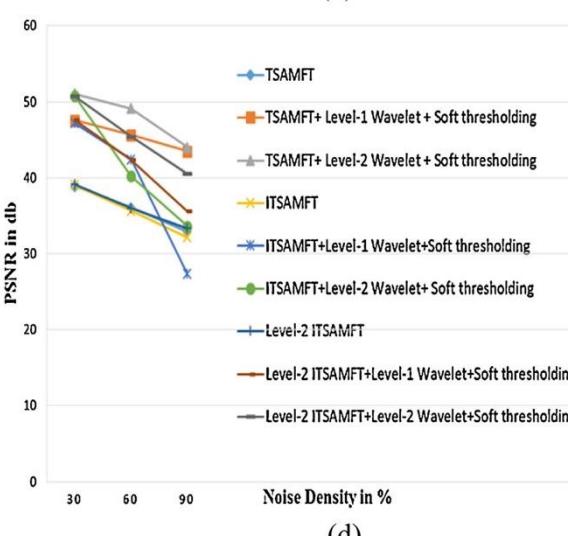
(a)



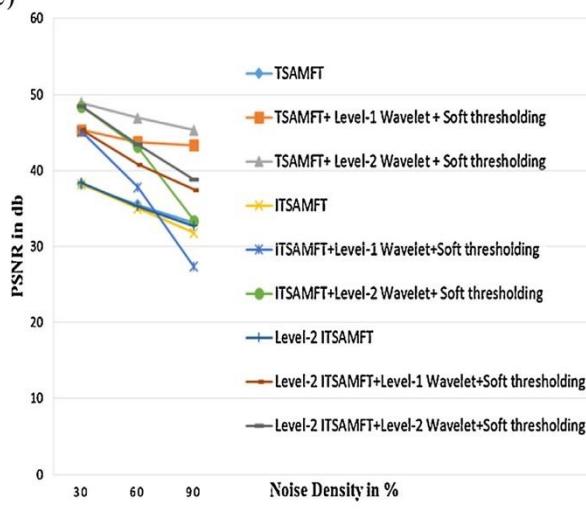
(b)



(c)

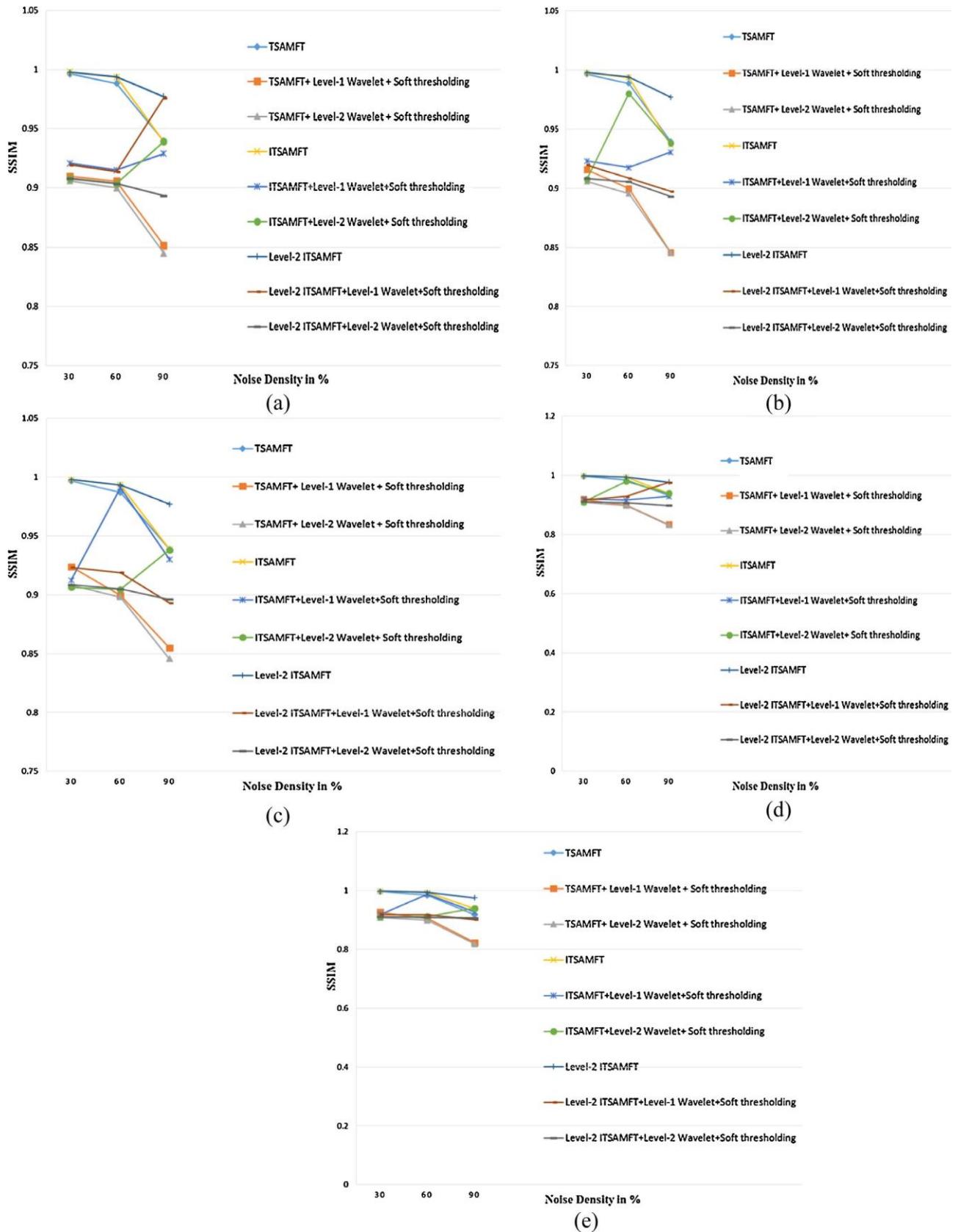


(d)

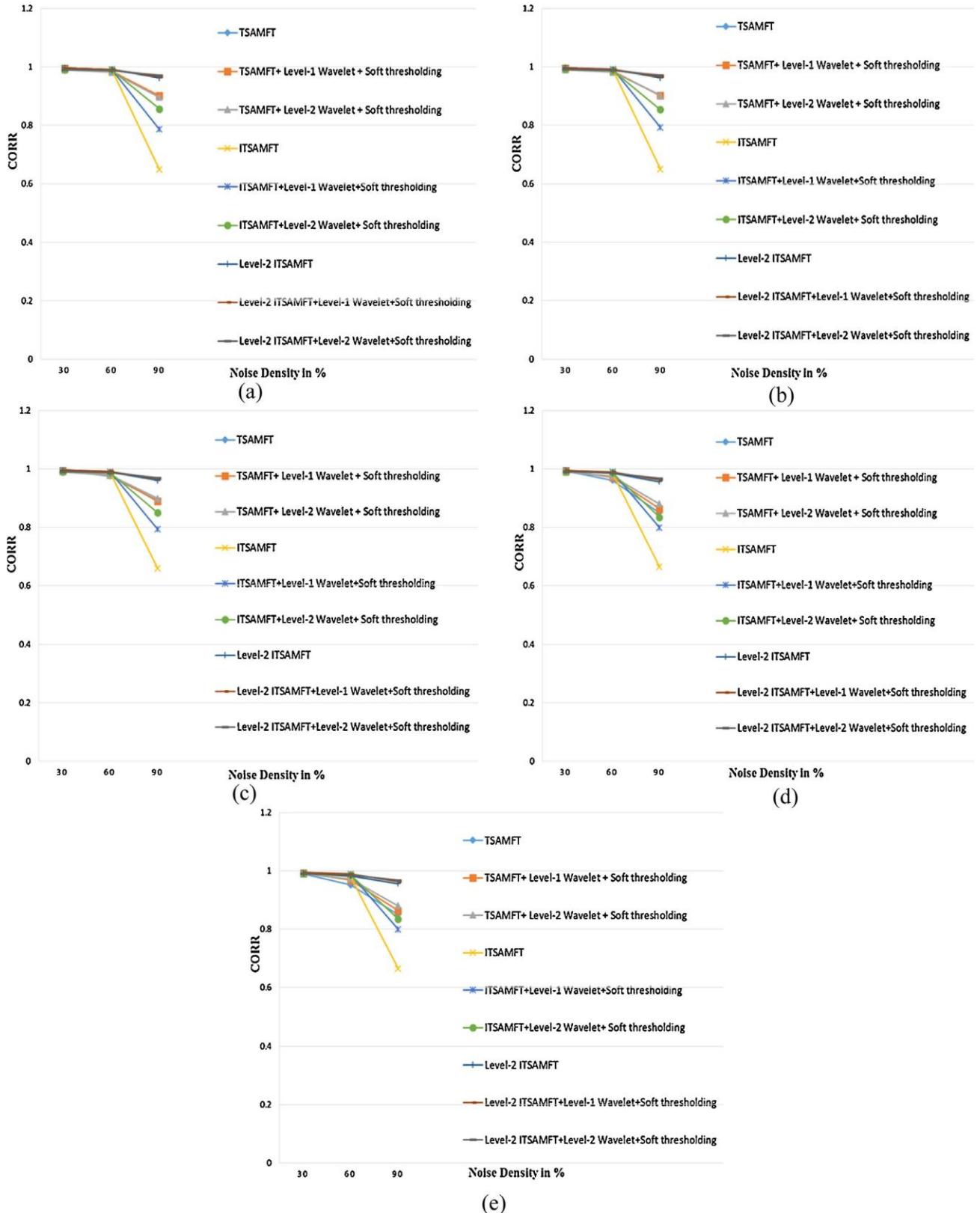


(e)

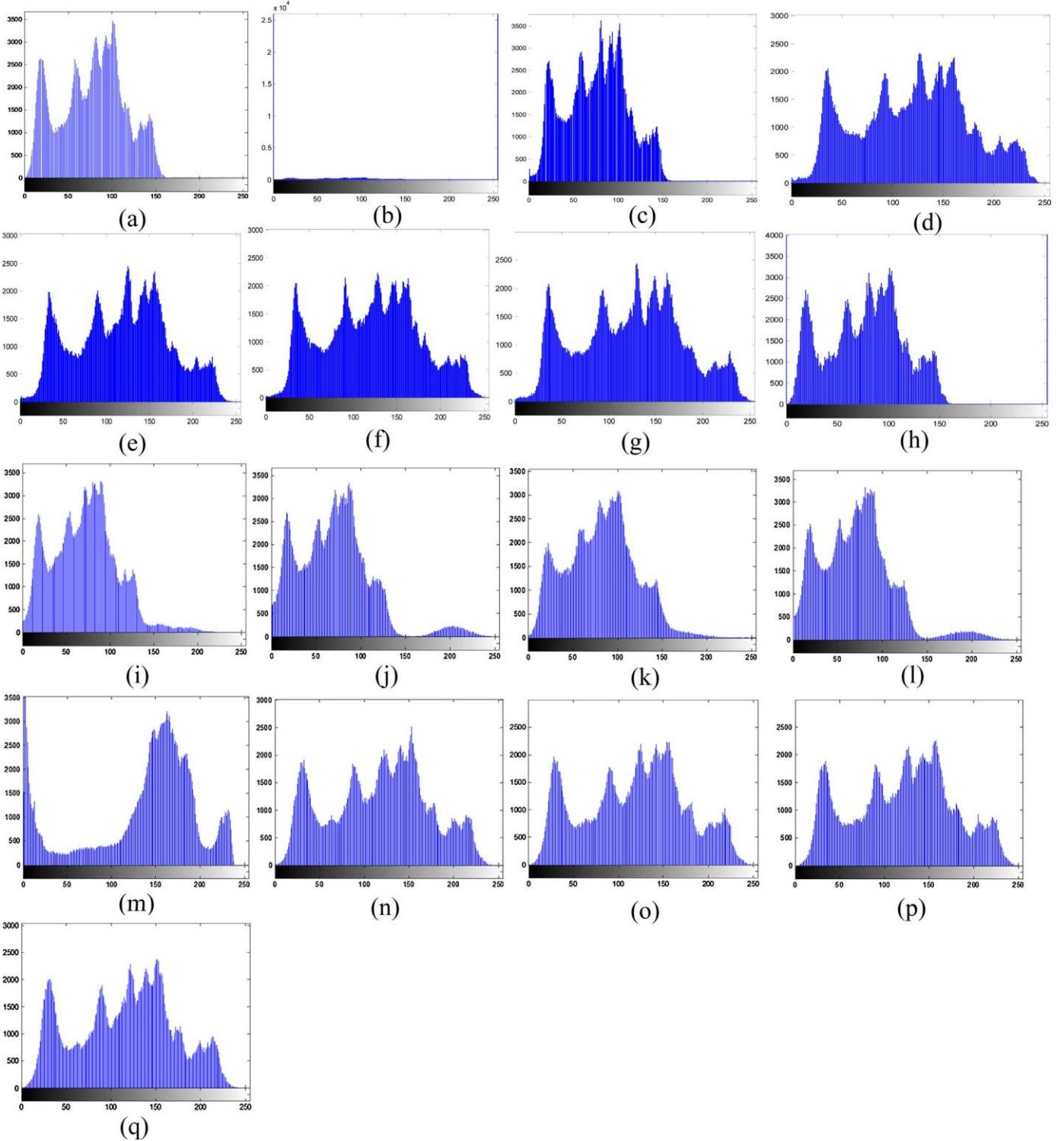




**Fig. 11.** Comparison graph of SSIM at various noise levels for different tolerance values and methods. (a) For tolerance = 0, (b) tolerance = 10, (c) tolerance = 20, (d) tolerance = 30, and (e) tolerance = 40.



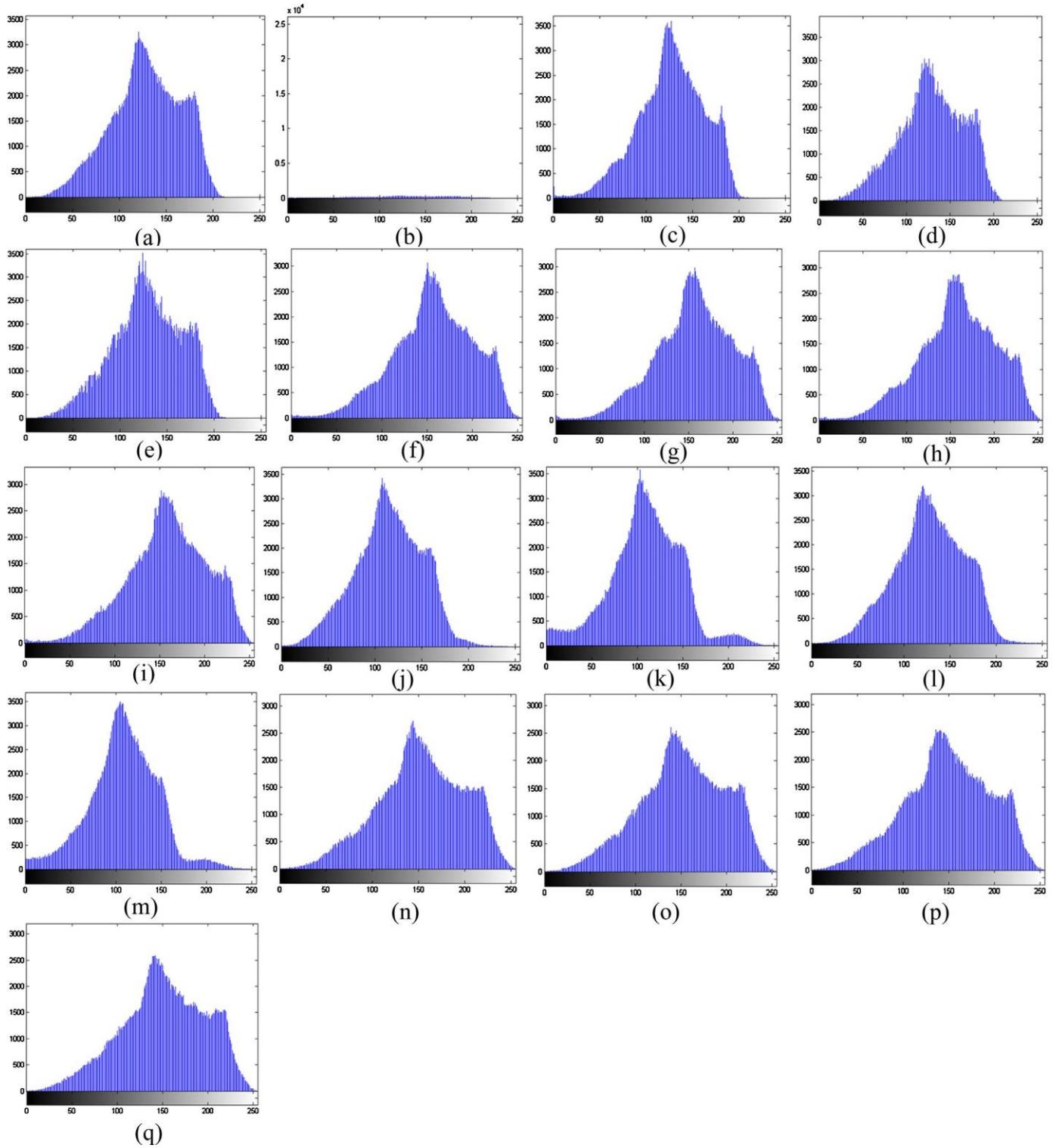
**Fig. 12.** Comparison graph of CORR at various noise levels for different tolerance values and methods. (a) For tolerance = 0, (b) tolerance = 10, (c) tolerance = 20, (d) tolerance = 30, and (e) tolerance = 40.



**Fig. 13.** Histogram of restored Lena for various methods. (a) Histogram of original Lena, (b) histogram of 90% salt and Pepper noise added Lena, histogram of restored Lena using, (c) TSAMFT, (d) ITSAMFT, (e) Level-2 ITSAMFT, (f) TSAMFT + Level-1 Wavelet + Soft thresholding, (g) TSAMFT + Level-1 Wavelet + Hard thresholding, (h) TSAMFT + Level-2 Wavelet + Soft thresholding, (i) TSAMFT + Level-2 Wavelet + Hard thresholding, (j) ITSAMFT + Level-1 Wavelet + Soft thresholding, (k) ITSAMFT + Level-1 Wavelet + Hard thresholding, (l) ITSAMFT + Level-2 Wavelet + Soft thresholding, (m) ITSAMFT + Level-2 Wavelet + Hard thresholding, (n) Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding, (o) Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding, (p) Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding, and (q) Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding.

range is an indicator of contrast and of how good the image is. In general, an image with a histogram of small spread has low contrast and one with a wide spread has high contrast. Figs. 13 and 14 shows the histograms of the filtered images using various algorithms for the Lena and Baboon images corrupted by

90% impulse noise. The histograms in Figs. 13n-q and 14n-q have widespread than the histograms in Figs. 13c-m and 14c-m. This infers good visual quality and high contrast on denoised images attained by Level-2 ITSAMFT combined with wavelet thresholding methods.



**Fig. 14.** Histogram of restored Baboon for various methods. (a) Histogram of original Baboon, (b) histogram of 90% salt and pepper noise added Baboon, histogram of restored Baboon using (c) TSAMFT, (d) ITSAMFT, (e) Level-2 ITSAMFT, (f) TSAMFT + Level-1 Wavelet + Soft thresholding, (g) TSAMFT + Level-1 Wavelet + Hard thresholding, (h) TSAMFT + Level-2 Wavelet + Soft thresholding, (i) TSAMFT + Level-2 Wavelet + Hard thresholding, (j) ITSAMFT + Level-1 Wavelet + Soft thresholding, (k) ITSAMFT + Level-1 Wavelet + Hard thresholding, (l) ITSAMFT + Level-2 Wavelet + Soft thresholding, (m) ITSAMFT + Level-2 Wavelet + Hard thresholding, (n) Level-2 ITSAMFT + Level-1 Wavelet + Soft thresholding, (o) Level-2 ITSAMFT + Level-1 Wavelet + Hard thresholding, (p) Level-2 ITSAMFT + Level-2 Wavelet + Soft thresholding, and (q) Level-2 ITSAMFT + Level-2 Wavelet + Hard thresholding.

## 5. Conclusion

In this paper, an effective improved tolerance based selective arithmetic mean filtering technique (ITSAMFT) combined with

wavelet thresholding is proposed to remove the high density salt and pepper noise. The proposed algorithm performs well in retaining the edge detail information present in the image for all levels (low, medium and high) of noise densities. Thus, the proposed

method exhibits best visual quality by avoiding blurring effect even at higher noise densities. This, in turn, leads to high peak signal to noise ratio, SSIM and correlation values in addition with better visual quality for images heavily corrupted by impulse noise. The simulation results reveal the fact that the proposed algorithm is much better than the existing denoising algorithms such as SMF, DBA, DBUTM, IDFRWRIM, AMWMF-CI, TSAMFT, ITSAMFT and Level-2 ITSAMFT in both quantitative and qualitative measures. Finally it is suggested that for images corrupted with wide range noise densities (that is, from lesser noise densities to higher noise densities) Level-1/Level-2 ITSAMFT + Level-1/Level-2 Wavelet decomposition + Soft/Hard thresholding method is adequate.

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