Digital Image Processing

Image Enhancement in the Frequency Domain

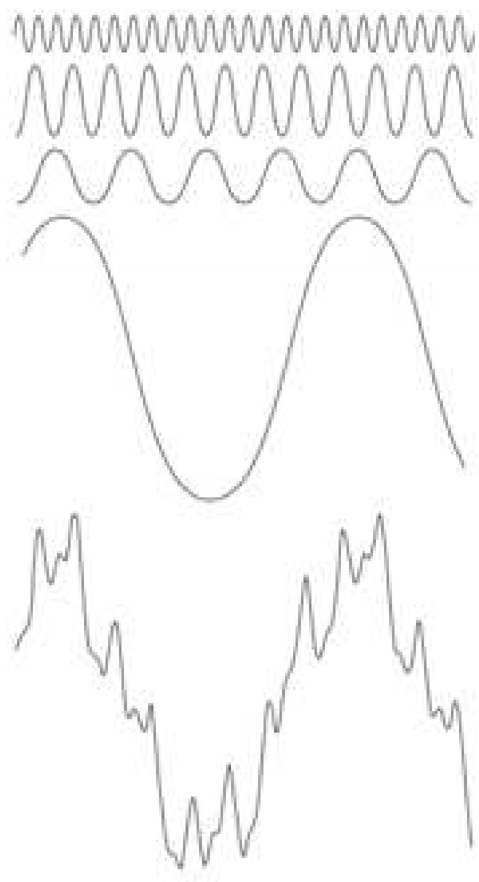
Topics

- Frequency Domain Enhancements
- Fourier Transform
- Convolution
- High Pass Filtering in Frequency Domain
- Low Pass Filtering in Frequency Domain

Frequency Domain

- A frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies.
- A frequency-domain representation can also include information on the phase shift that must be applied to each sinusoid in order to be able to recombine the frequency components to recover the original time signal.
- A given function or signal can be converted between the time and frequency domains with a pair of mathematical operators called a transform

Frequency Domain



Fourier Transform

- Fourier transform is given by

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux}\, dx$$

- Inverse Fourier transform is

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux}\, du.$$

Two Dimensional Fourier Transform

- Forward Fourier Transform is:

$$F(u, v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)e^{-j2\pi(ux+vy)}\, dx\, dy$$

- Inverse Fourier Transform is:

$$f(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u, v)e^{j2\pi(ux+vy)}\, du\, dv.$$

Discrete Fourier Transform

- 1D forward transform

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad \text{for } u = 0, 1, 2, \ldots, M-1.$$

- 1D inverse transform

$$f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad \text{for } x = 0, 1, 2, \ldots, M-1.$$

Fourier Transform Properties

- From Euler's Equation we have:

$$e^{j\theta} = \cos\theta + j\sin\theta.$$

- Hence by replacing we get

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)[\cos 2\pi ux/M - j\sin 2\pi ux/M]$$

Magnitude, Phase, and Power Spectrum of Fourier Transform

- Magnitude of Fourier transform is found by:

$$|F(u)| = \left[R^2(u) + I^2(u)\right]^{1/2}$$

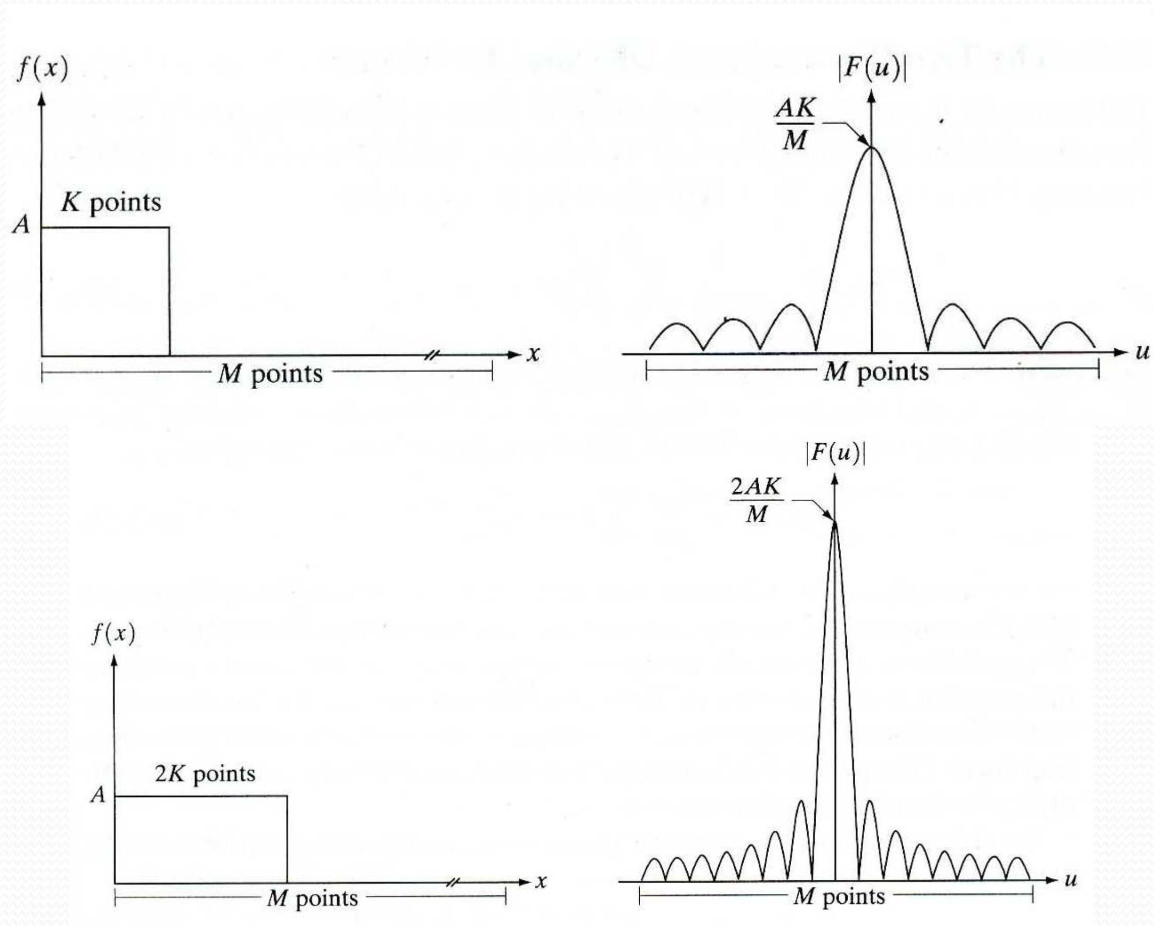
- Phase is given by:

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right]$$

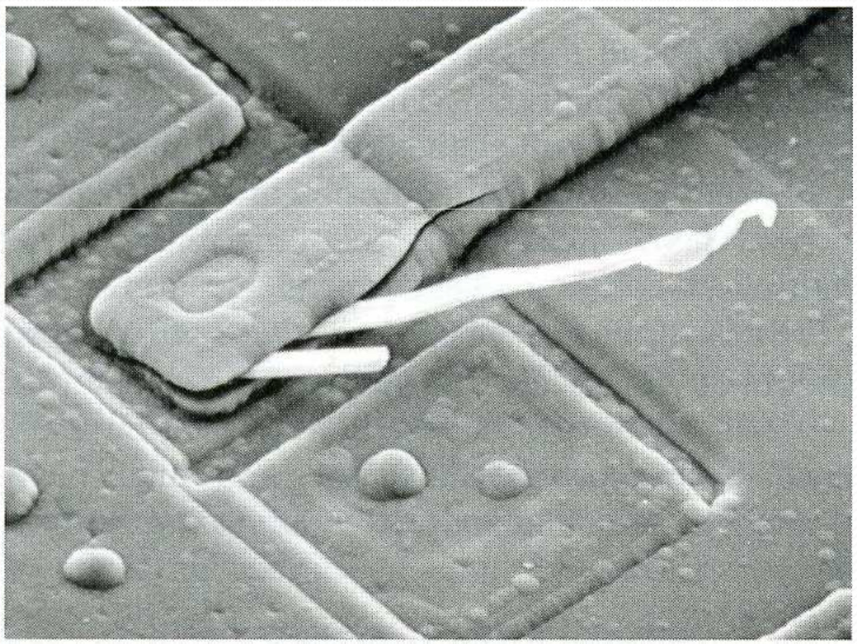
- Power Spectrum is:

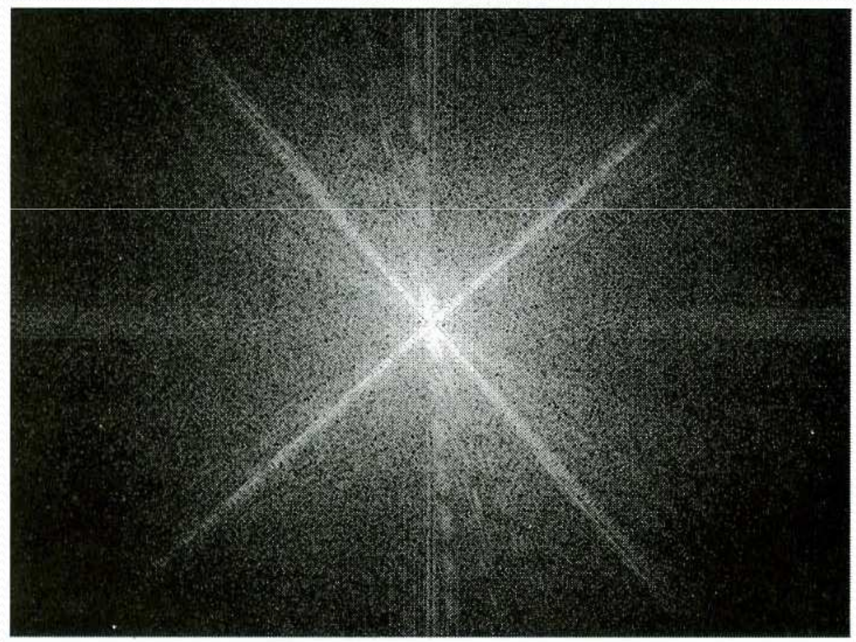
$$\begin{aligned} P(u) &= |F(u)|^2 \\ &= R^2(u) + I^2(u). \end{aligned}$$

Sample Functions and their Fourier Transforms



Sample Fourier Analysis





Convolution

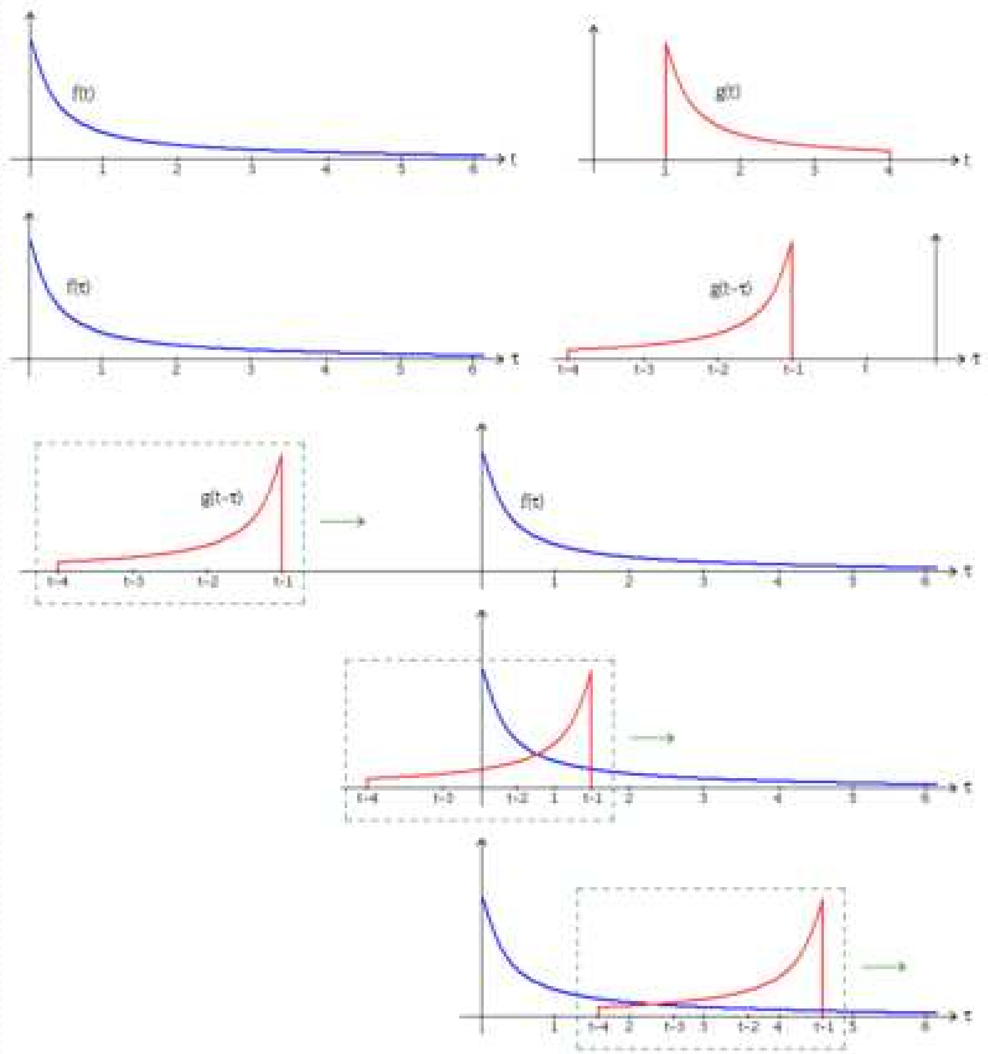
- In 2D continuous space, convolution is defined by:

$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

- In 2D discrete space convolution is given by:

$$(f * g)[n] \stackrel{\text{def}}{=} \sum_{m=-\infty}^{\infty} f[m]\, g[n-m]$$

Convolution



Filtering in Frequency Domain

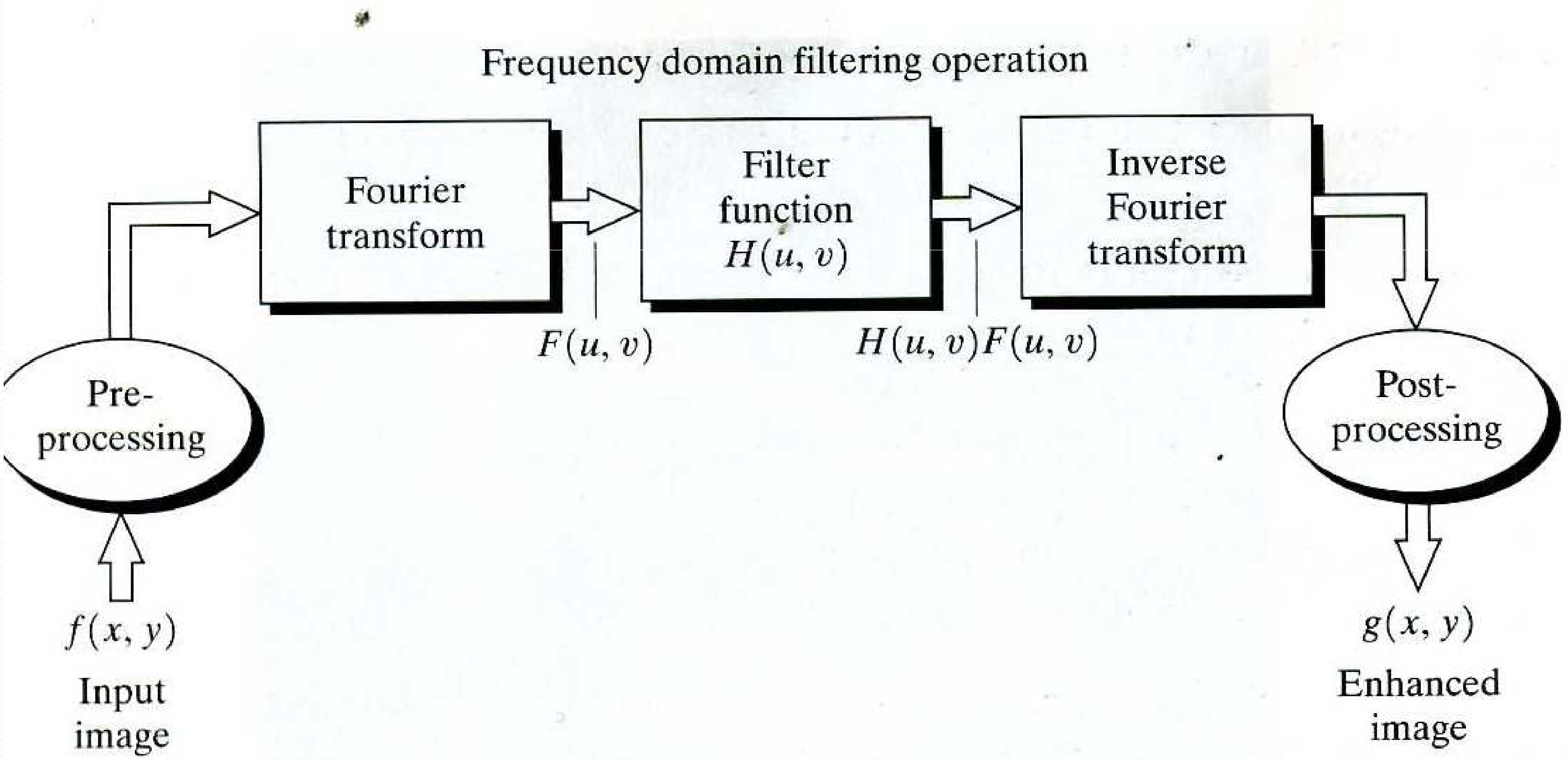
Basic Steps of Filtering in Frequency Domain Are:

- Compute F(u,v), the DFT of the input image
- Multiply F(u,v) by a filter function H(u,v)

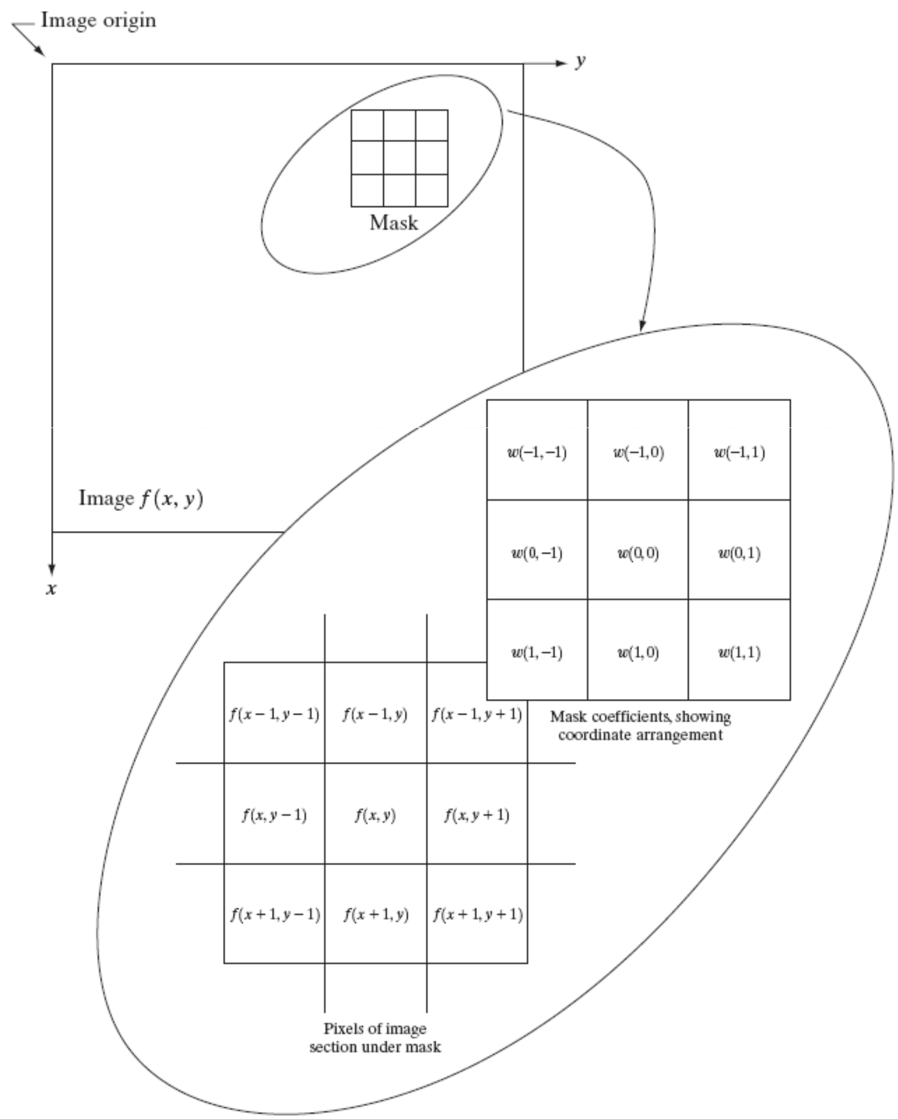
$$G(u, v) = H(u, v)F(u, v).$$

- Compute inverse DFT of the result
- Obtain real part of the inverse DFT

Filtering in Frequency Domain



Spatial Masks in Time Domain



Convolving Mask with Image in Time Domain

- Convolving mask with image is carried out by sliding the mask over the image, multiplying mask values with the pixel values falling beneath them and obtaining the sum.

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

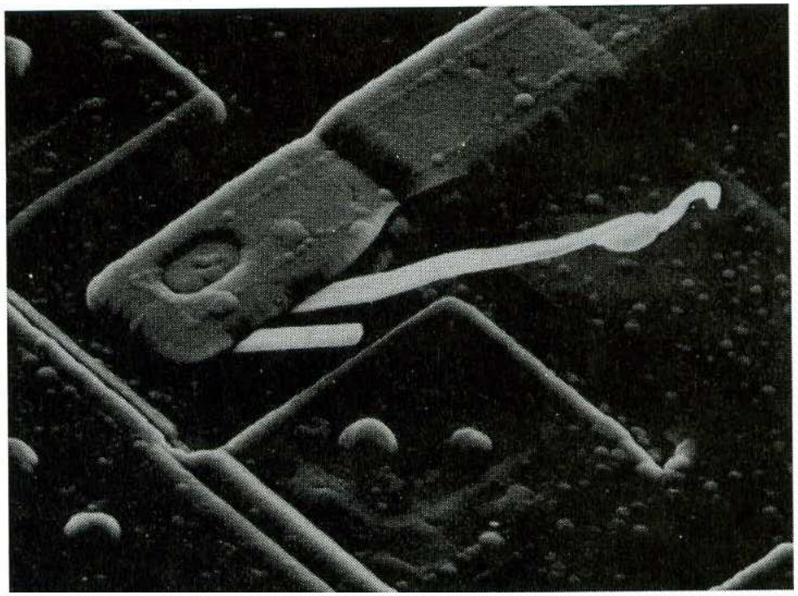
- The convolution is converted to multiplication in frequency domain

$$f(x, y) * h(x, y) \Leftrightarrow F(u, v) H(u, v).$$

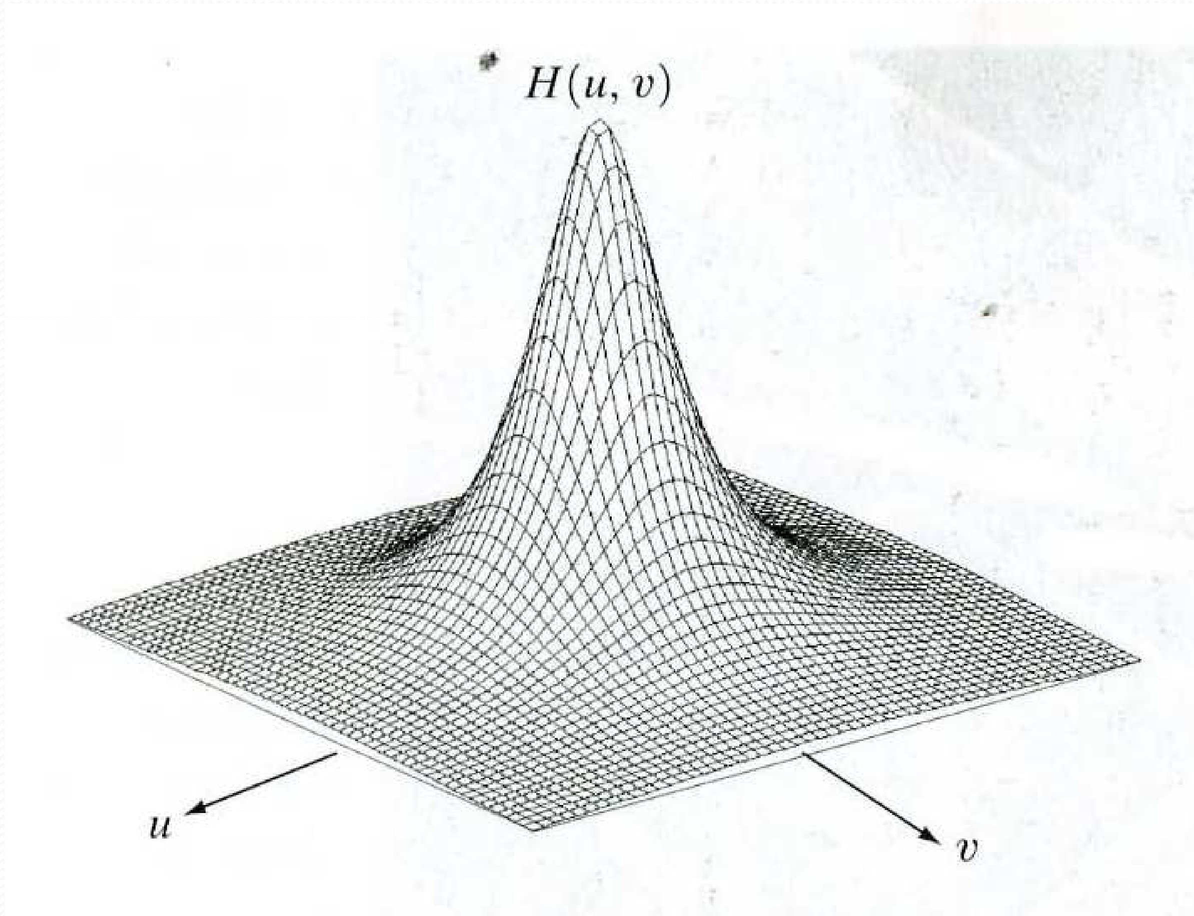
Sample Frequency Domain Filters

- Notch Filter: Changes the average value of an image to zero

$$H(u, v) = \begin{cases} 0 & \text{if} \quad (u, v) = (M/2, N/2) \\ 1 & \text{otherwise.} \end{cases}$$

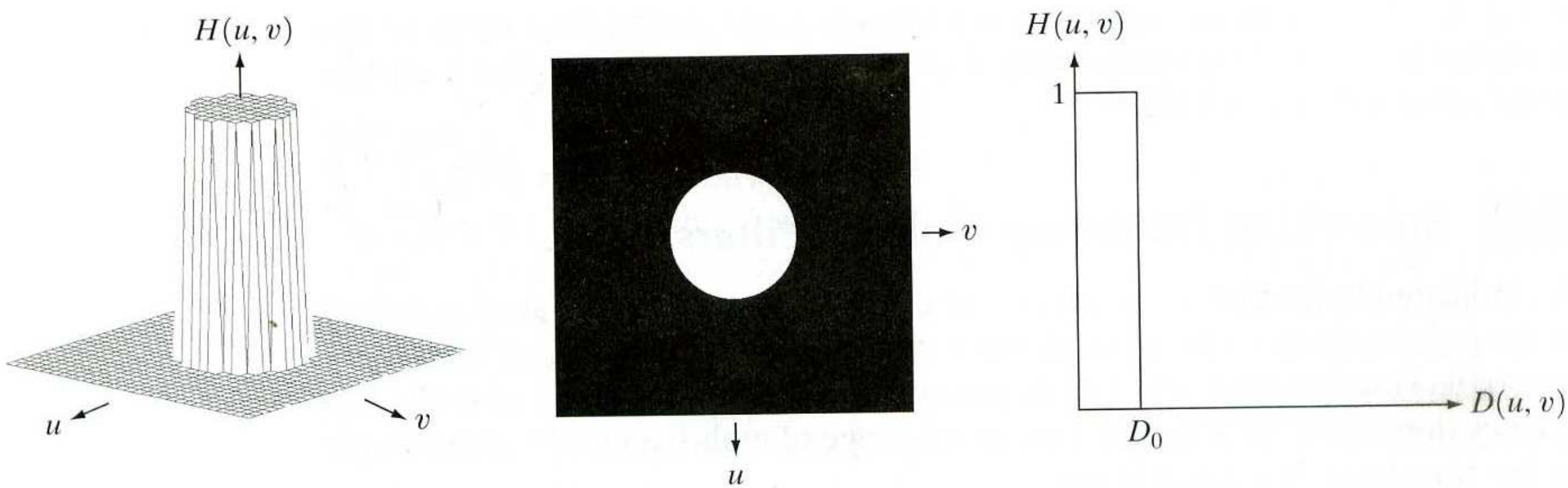


Low Pass Filters



Ideal Low Pass Filter

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$



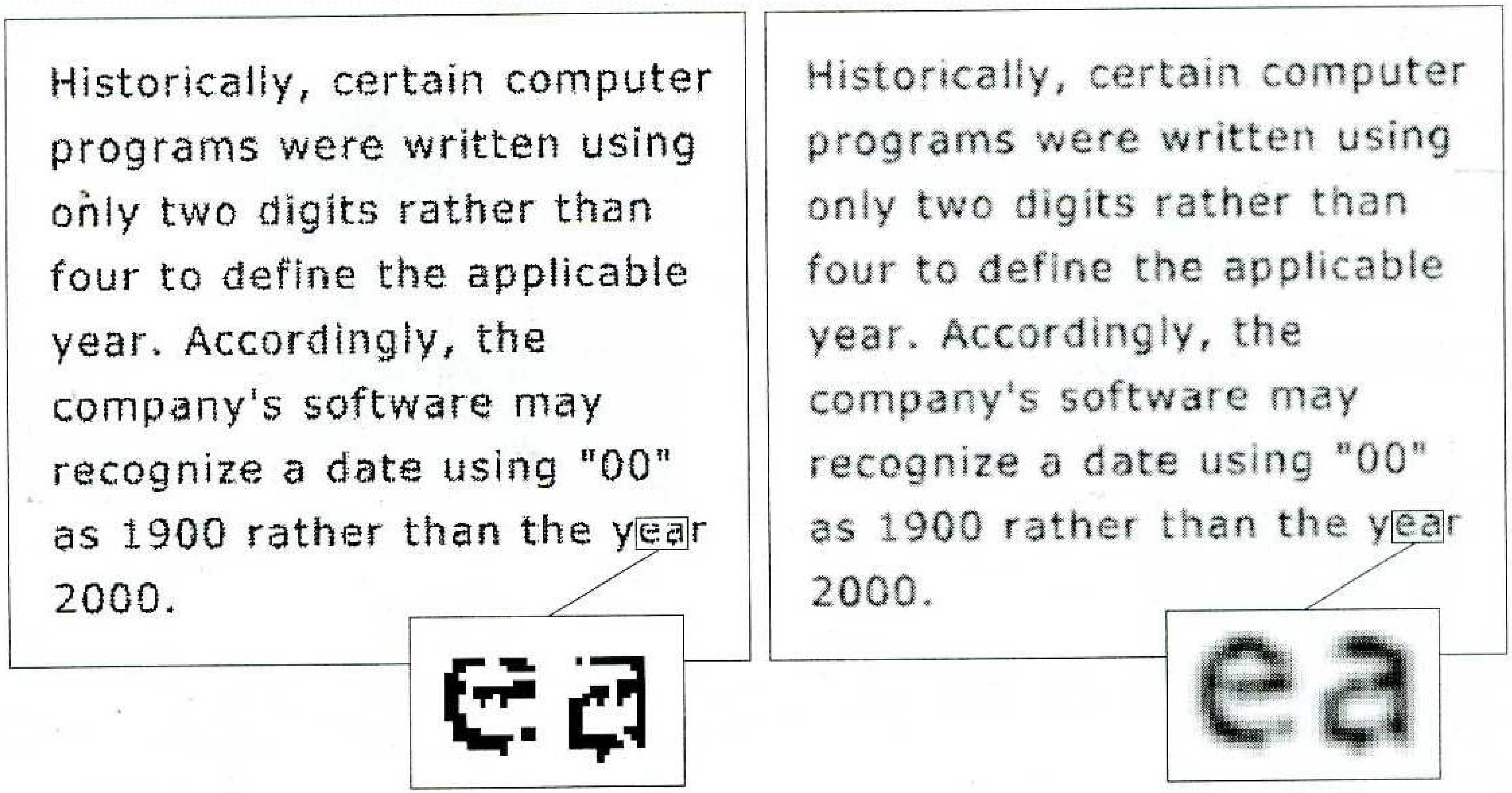
Some Low Pass Filters

- Gaussian low pass filter (GLPF)

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

D(u,v) is the distance from the origin of the Fourier transform.

Result of Filtering with GLPF


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

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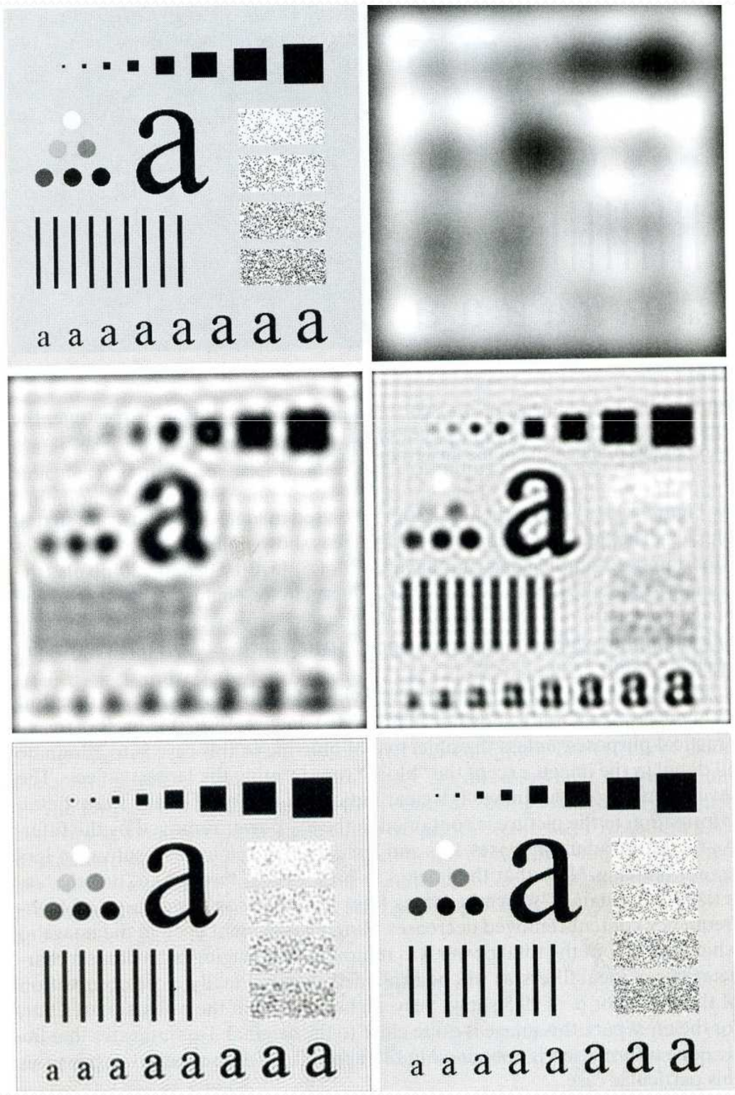
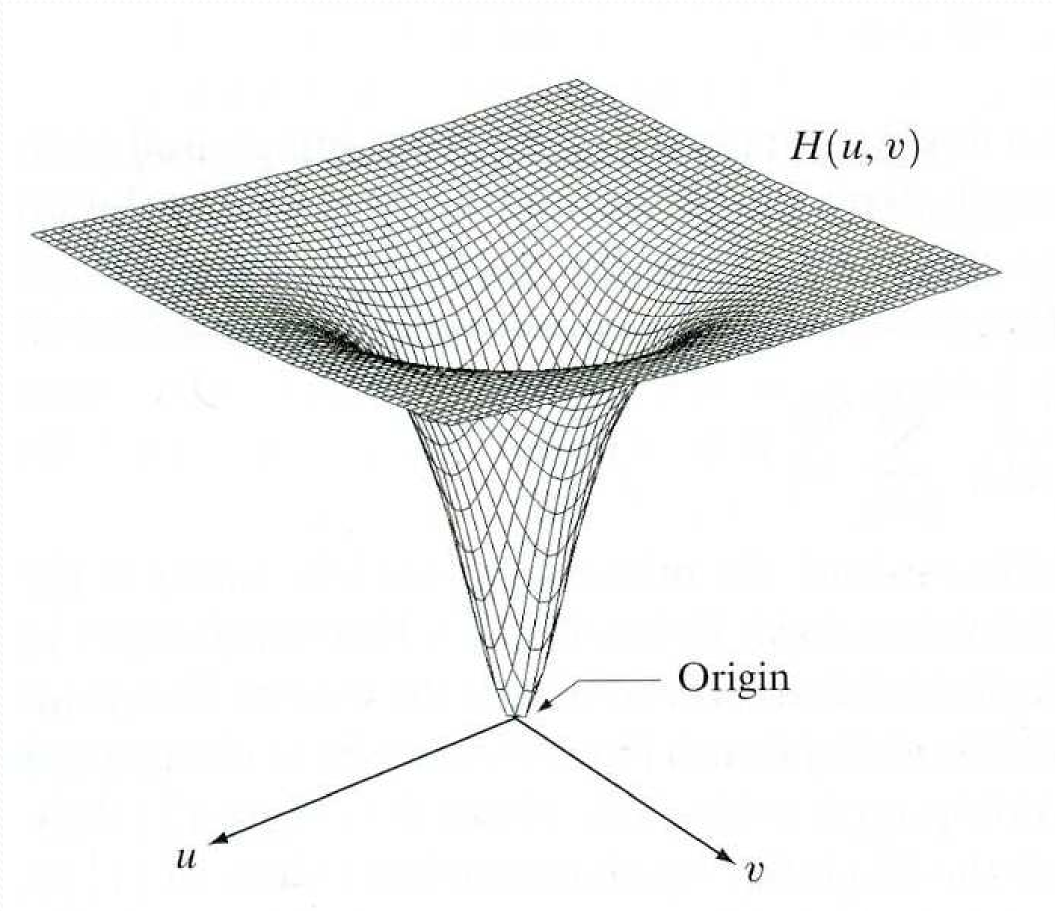
Low Pass Filter Example



High Pass Filter



High Pass Filters

- Simplest high pass filter is the complement of low pass filter

$$H_{\mathrm{hp}}(u, v) = 1 - H_{\mathrm{lp}}(u, v)$$

- Ideal high pass filter is given by:

$$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$

High Pass Filters

- Gaussian high pass filter is given by:

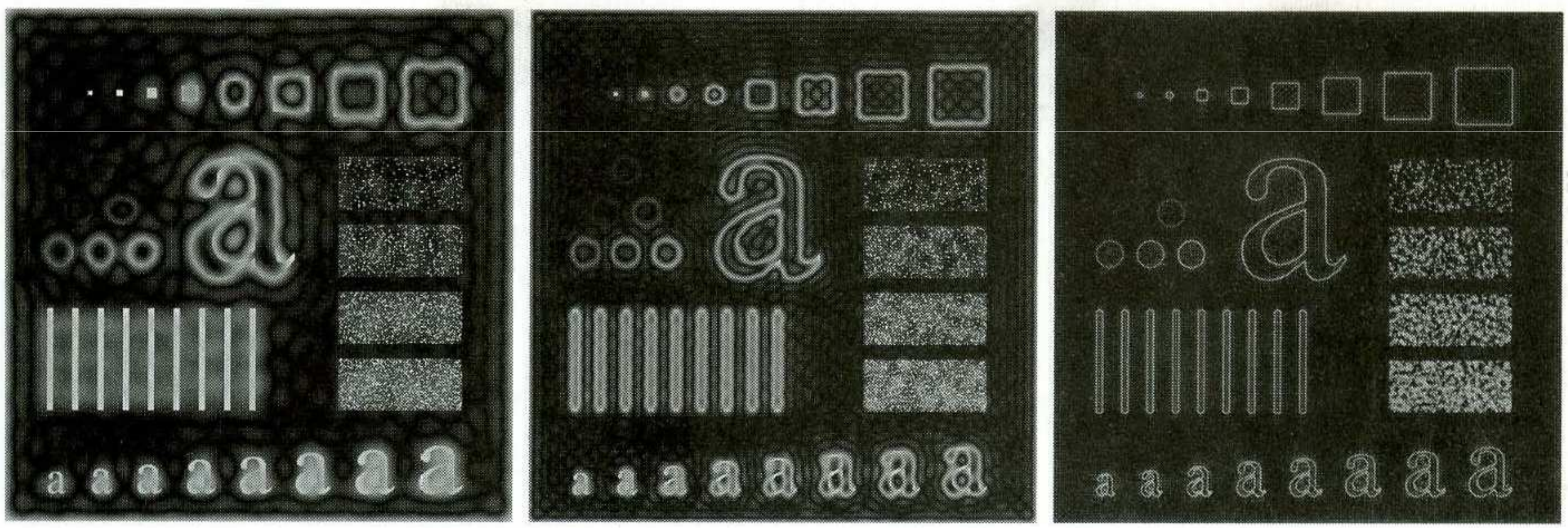
$$H(u, v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

D_o is the distance from the center to cut-off frequency

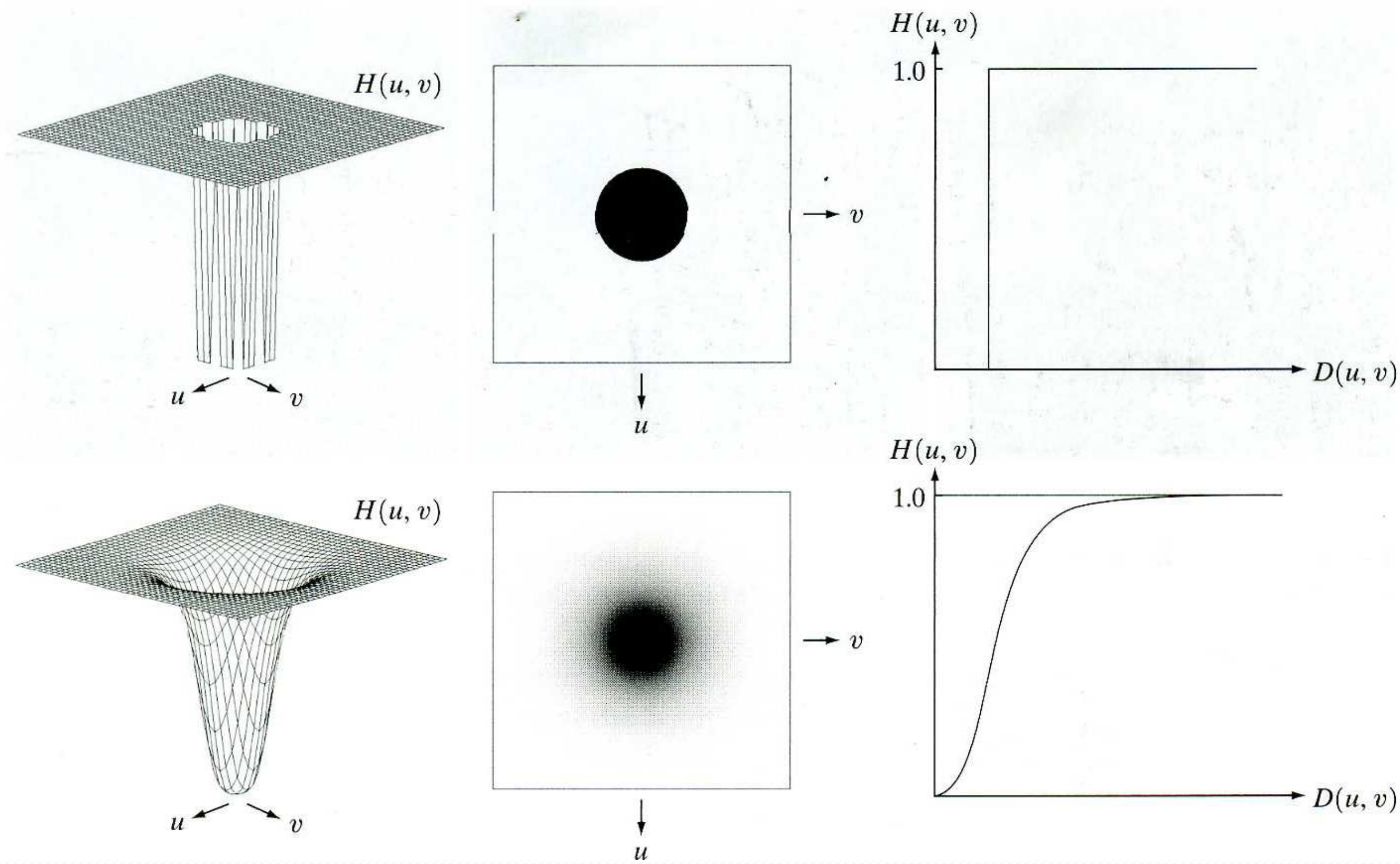
$D(u,v)$ is the distance from any point (u,v) to the center given by

$$D(u, v) = \left[(u - M/2)^2 + (v - N/2)^2\right]^{1/2}.$$

Example Results with D_0 = 15, 30 and 80



Ideal and Gaussian High Pass Filters



MATLAB Implementations

- Use fft(.) function for 1D Fourier transform and fft2(.) for 2D transform.
- Use ifft and ifft2 for inverse Fourier transforms
- Example:
  - I = imread(‘test.jpg’);
  - FI = fft2(I);
  - FI2 = log(abs(FI));
  - imshow(FI2, [-1 5] , 'InitialMagnification','fit')

Fast Convolution

- Read the image
- Create the mask (or read it)
- Find the Fourier transforms of the Image and the mirror of the mask (rotate by 180 degrees)
- Multiply the transformed image and the transformed mask
- Find the inverse Fourier transform of the result

Fast Convolution

- I = imread('test.bmp');
- Mask = [1 1 1

  0 0 0

  -1 -1 -1];
- FI = fft2(I);
- FMask = fft2(Mask, size(I,1), size(I,2));
- RF = FI .* FMask;
- R = ifft2(RF);

Example: Detecting a Letter

- To find a sample pattern in an image convolution can be used
- Assume Image is given at variable I and the pattern at P
- C = real(ifft2(fft2(I) .* fft2(rot90(P,2)));
- MaxC = max(C);
- imshow(C > MaxC – 5)

Questions?

Assignment

- Will be posted on http://ceng503.cankaya.edu.tr