Homework 1

Graded

Student

Jaiden Rain Atterbury

Total Points

36 / 40 pts

Question 1

Question 1

9 / 10 pts

+ 0 pts Note: each part (a)-(e) out of 2 points, with possible scores of 0, 1, or 2 for each part.

(a) 2 points

✔ **+ 1 pt** Correctly calculated the eigenvalues and eigenvectors of A^3, and showed work.

✔ **+ 1 pt** Correctly calculated the eigenvalues and eigenvectors of $A + 10I$, and showed work.

+ 1 pt *Correct answers but not sufficiently justified

+ 1 pt *Partial credit, see comments

(b) 2 points

✔ **+ 1 pt** Correctly identified the eigenvalues and eigenvectors of A^T

+ 1 pt Explained how the eigenvalues of A and A^T are related in general, **including** full reasoning why. If the fact $\det(A) = \det(A^T)$ is used, then it must be explained.

(c) 2 points

✔ **+ 1 pt** Correctly found the diagonalization of A.

✔ **+ 1 pt** Correctly computed A^3.

+ 1 pt *Diagonalization and/or A^3 incorrect, but the general idea is good

+ 0 pts *(c) missing

(d) 2 points

✔ **+ 2 pts** Explained *why* A is not invertible. (One way to do this is to note that $\lambda = 0$ is an eigenvalue of A.)

+ 1 pt *Said A is not invertible, but didn't give a reason why.

+ 0 pts *Question not answered

(e) 2 points

✔ **+ 2 pts** Explained how the eigenvalues of A can tell you if A is invertible or not. Citing a theorem from Math 208 is acceptable.

+ 1 pt *Needs more explanation, see comments

+ 0 pts Question not answered

2 explain why taking the transpose does not change the determinant. this is not a fact that is in the 208 notes, so it needs to be justified here

Question 2

Question 2

10 / 10 pts

- ✔ **+ 2 pts** (a) invertible + diagonalizable
- ✔ **+ 2 pts** (b) invertible + not diagonalizable
- ✔ **+ 2 pts** (c) singular + diagonalizable
- ✔ **+ 2 pts** (d) singular + not diagonalizable
- ✔ **+ 2 pts** conclusion

− 1 pt (b) or (d) justification

+ 0 pts no submission

Question 3

Question 3

10 / 10 pts

- ✔ **+ 3 pts** (a) not always similar
- ✔ **+ 2 pts** (b) same eigenvalues + similar
- ✔ **+ 2 pts** (c) same eigenvalues + not similar
- ✔ **+ 3 pts** (d) same distinct eigenvalues implies similar

− 1 pt (a) minor error/omission in reasoning

− 2 pts (d) error in reasoning

− 1 pt (d) minor error/omission in reasoning

+ 0 pts no progress

1 You are using the fact that similarity of matrices is transitive. Make sure that you know how to prove this!

Question 4

Question 4

7 / 10 pts

✔ **+ 1 pt** (a) check eigenvectors

+ 1 pt (a) justify that it is a basis

✔ **+ 1 pt** (b) diagonalization

+ 2 pts (c) draw grid + points with respect to the standard basis in $\mathbb{R}^2$

− 1 pt (c) minor error

✔ **+ 1 pt** (d) $y = (1, 1)$

✔ **+ 1 pt** (e) $z = (0, 5)$

✔ **+ 1 pt** (f) check $z = \Lambda y$

✔ **+ 2 pts** (g) explanation

− 1 pt (g) insufficient

7) Jaiden Atterbury 03/29/2022 Homework 1

(1) Finding Eigenspaces

$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ $\det(A-\lambda I)=0 \to \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & 1 & 1-\lambda \end{bmatrix}$ $(1-\lambda)\det\begin{vmatrix} 2-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} + \det\begin{vmatrix} -1 & -1 \\ 0 & 1-\lambda \end{vmatrix}$

$(1-\lambda)^2(2-\lambda)+(-1+\lambda)+(-1+\lambda) = -2+2\lambda + (1-\lambda^2)(2-\lambda)$

$= -2+2\lambda + 2-5\lambda+4\lambda^2-\lambda^3$

$= -\lambda(3-4\lambda+\lambda^2) \to -\lambda(\lambda-3)(\lambda-1)$

$\hookrightarrow \boxed{\lambda = 0, 3, 1}$

$\text{null}(A) = E_0(A) = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_3 = s$
$x_2 = s \to \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\} \to \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector for $\lambda = 0$
$x_1 = s$

$\text{null}(A-1I) = E_1(A) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_3 = s$
$x_2 = 0 \to \text{span}\left\{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\} \to \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ is an eigenvector for $\lambda = 1$
$x_1 = -s$

$\text{null}(A-3I) = E_3(A) = \begin{bmatrix} -2 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{bmatrix} \sim \begin{bmatrix} -1 & -1 & -1 \\ -2 & -1 & 0 \\ 0 & -1 & -2 \end{bmatrix} \sim \begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \sim \begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$x_3 = s$
$x_2 = -2s \to$ ~~$\text{span}\left\{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right\} \to \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is an eigenvector for $\lambda = 3$~~
$x_1 = s$

(a) $A^3 \to A \cdot A \cdot A X = AA\lambda X = A\lambda^2 X = \lambda^3 X$

Eigenvalues of A^3: $\lambda = 0, 1, 27$

Eigenvectors of A^3: (same as A) $\to \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ for 0, $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, for 1, $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ for 27

$A + 10I \to (A+10I)\vec{x} = A\vec{x} + 10I\vec{x} = \lambda\vec{x} + 10\vec{x} = (\lambda+10)\vec{x}$

Eigenvalues of $A + 10I$: $\lambda = 10, 11, 13$

Eigenvectors of $A+10I$: (same as A) $\to \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ for 10, $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ for 11, $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ for 13

(b) $A^T = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} = A$

$\therefore$ Eigenvalues of $A^T = 0, 1, 3$

Eigenvectors of $A^T = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ for 0, $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ for 1, $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ for 3

- In general A^T and A share eigenvalues since they share the same characteristic polynomial.

  ↳ λ eigenvalue of A if $\det(A - \lambda I) = 0$

  since $\det(A) = \det(A^T) \Rightarrow \det((A - \lambda I)^T) = 0$

  $\Rightarrow \det(A^T - \lambda I) = 0$

  $\therefore$ λ is an eigenvalue of A^T as well.

(c) A is diagonalizable since A is a 3x3 matrix with 3 distinct real eigenvalues, it is also important to note that A is diagonalizable since the dimension of each eigenspace is equal to the multiplicity of each corresponding eigenvalue.

$A = PDP^{-1}$

$A^K = PD^KP^{-1}$

$P = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 3 & & \\ & 1 & \\ & & 0 \end{bmatrix}$

$$P^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\substack{2R_1 + R_2 \to R_2 \\ -R_1 + R_3 \to R_3}}{\sim} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 3 & 2 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 \end{array}\right] \underset{-\frac{1}{2}R_2 \to R_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3/2 & -1 & -1/2 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 \end{array}\right]$$

$$\underset{\substack{R_2 + R_1 \to R_1 \\ -2R_2 + R_3 \to R_3}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & -1/2 & 0 & -1/2 & 0 \\ 0 & 1 & -3/2 & -1 & -1/2 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right] \underset{\substack{\frac{1}{3}R_3 \to R_3 \\ \frac{1}{2}R_3 + R_1 \to R_1}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & -1/3 & 1/6 \\ 0 & 1 & 0 & -1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right]$$

$P^{-1} = \begin{bmatrix} 1/6 & -1/3 & 1/6 \\ -1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$ $\quad D^3 = \begin{bmatrix} 27 & & \\ & 1 & \\ & & 0 \end{bmatrix}$

Question assigned to the following page: 1

(c) continued

$$A^3 = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 27 & & \\ & 1 & \\ & & 0 \end{bmatrix} \begin{bmatrix} 1/6 & -1/3 & 1/6 \\ -1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 27 & -1 & 0 \\ -54 & 0 & 0 \\ 27 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1/6 & -1/3 & 1/6 \\ -1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 5 & -9 & 4 \\ -9 & 18 & -9 \\ 4 & -9 & 5 \end{bmatrix}$$

(d) A is NOT invertible since 0 is an eigenvalue of A.
in general $Av = \lambda v \Rightarrow A^{-1}Av = \lambda A^{-1}v$, so $\frac{1}{\lambda}$ is an eigenvalue.

(e) Yes, you can tell ~~definitey~~ definitely from the eigenvalues of A whether it's invertible or not. You can do this through the unifying theorem which states if 0 is an eigenvalue of A then A^{-1} does not exist.

Question assigned to the following page: 2

(2) Invertibility vs. Diagonalizability

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

- invertible since $\det(A) = 2 \neq 0$

$\lambda = 1, 2$ (eigenvalues)

- diagonalizable since A is a 2x2 matrix with 2 distinct eigenvalues

$null(A - 1I) = E_1(A) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{matrix} x_1 = s \\ x_2 = 0 \end{matrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$null(A - 2I) = E_2(A) = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} x_2 = s \\ x_1 = 0 \end{matrix} \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$A = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{P^{-1}} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A$ ✓

(b) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

invertible since $\det(A) = 1 \neq 0$

$\det(A - \lambda I) = (\lambda - 1)^2 = 0 \rightarrow \lambda = 1$ with multiplicity 2

$null(A - 1I) = E_1(A) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{matrix} x_1 = s \\ x_2 = 0 \end{matrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

↳ No basis of eigenvectors / multiplicity $\neq$ dimension of eigenspace

$\therefore$ NOT diagonalizable.

(c) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

singular since $\det(A) = 0$ / 0 is an eigenvalue of A

$\lambda = 0, 1$ (eigenvalues).

diagonalizable since A is a 2x2 matrix with 2 distinct eigenvalues.

$null(A) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} x_2 = s \\ x_1 = 0 \end{matrix} \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$null(A - I) = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{matrix} x_1 = s \\ x_2 = 0 \end{matrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$A = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{P^{-1}} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = A$ ✓

Question assigned to the following page: 2

(d) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Singular since $\det(A) = 0$ / 0 is an eigenvalue of A.

$\lambda = 0$ with a multiplicity of 2

$null(A) = E_0(A) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{matrix} x_1 = s \\ x_2 = 0 \end{matrix} \rightarrow [\,_0]$

↳ Since $\dim(E_0(A)) < 2$, A is not diagonalizable

- Since there are many matricies that satisfy all 4 cases, there is no direct connection between invertibility and diagonalizability in terms of one implying the other.

Question assigned to the following page: 3

(3) Similar Matricies

(a) A and Λ aren't always similar, if A isn't diagonalizable then the corresponding matrix of eigenvalues Λ won't be similar. For example:

if $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and the corresponding matrix $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A and Λ are not similar.

Since A isn't diagonalizable, because the eigenspace of $\lambda = 1$ has a dimension $\dim(E_1(A)) = 1$ that isn't equal to its algebraic multiplicity of 2, there isn't a matrix of eigenvectors P and its inverse P^{-1} that satisfy the equation $A = P\Lambda P^{-1}$, and therefore A and Λ are not similar. This comes from the fact that a matrix A is diagonalizable if and only if it is similar to a diagonal matrix.

(b) $A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ $\quad C = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

$\lambda = 2, 4$ $\quad \lambda = 2, 4$

1. Eigenvalues are the same for both matricies
2. Geometric multiplicity is the same for corresponding eigenspaces.
3. Both matricies are diagonalizable.
4. There exists a matrix P and its inverse P^{-1} such that $A = PCP^{-1}$.

$\therefore$ They are similar.

(c) $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ $\quad C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\lambda = 2, 2$ $\quad \lambda = 2, 2$

BUT their eigenspaces don't share geometric multiplicity.

$\therefore$ They are NOT similar.

(d) If the eigenvalues of A are distinct, and C shares these same eigenvalues, these matricies will ALWAYS be similar.

Question assigned to the following page: 3

This is because both C and A will be diagonalizable and will share the same diagonal matrix Λ. Since $A \sim \Lambda$ and $C \sim \Lambda$ because in order for a matrix to be diagonalizable it must be similar to a diagonal matrix Λ. Since $A \sim \Lambda$ and $C \sim \Lambda$, therefore C must be similar to A.

Question assigned to the following page: 4

(4) Change of bases and diagonalization

$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(a) $\det(A - \lambda I) = \begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix} = 0$

$4 - 5\lambda + \lambda^2 - 4 = 0 \rightarrow \lambda(-5+\lambda) = 0$

$\lambda = 0, 5$

$\text{null}(A) = E_0(A) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{matrix} x_2 = s \\ x_1 = -2s \end{matrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \cdot -1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \checkmark \rightarrow u_1 = (2, -1)^T$

$\text{null}(A - 5I) = E_5(A) = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \sim \begin{bmatrix} -4 & 2 \\ 0 & 0 \end{bmatrix} \begin{matrix} x_2 = s \\ x_1 = 1/2 s \end{matrix} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \cdot 2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \checkmark \rightarrow u_2 = (1, 2)^T$

$\mathcal{U} = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$

(b) $A = U \Lambda U^{-1}$ $\qquad U = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

PDP^{-1} $\qquad \Lambda = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$

$U^{-1} = \frac{1}{5} \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix}$

(c) $[x_1]_{\mathcal{U}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\qquad x = U[x]_B$

$[x_2]_{\mathcal{U}} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

$x_1 = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$x_2 = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

Picture of grid, vectors, and points on graph paper attatched.

(d) $v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$y = U^{-1} v$

$y = \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(e) $Av = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

$z = \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

(f) $z = \Lambda y$ $\qquad \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \checkmark$

Question assigned to the following page: 4

(4) Cha

(0,5) in u

(g) The transformation A takes a point in S multiplies the point/vector by the inverse change of matrix to get the point in terms of u. Then that new vector will be multiplied by $[T]_u$, which is the linear transformation with respect to the new basis. Then this transformed vector will be multiplied by the change of basis matrix in order to get it back in terms of the standard basis.

$$\begin{tikzcd} S \arrow[d, "A/T"'] & u \arrow[l, "U"'] \arrow[d, "\Lambda/[T]_u"] \\ S \arrow[r, "U^{-1}"'] & u \end{tikzcd}$$

$\leftarrow$ picture example of what A does to a point in u

$$A = U\Lambda U^{-1}\vec{x}$$

With this said, A has the same action as Λ if we change to the eigenbasis of u

Proof: $U^{-1}AV = z = \Lambda y$

furthermore: $U^{-1}AV = U^{-1}U\Lambda U^{-1}V = \Lambda y$

↑ This comes from the fact in 1.2.1 that a diagonalizable matrix behaves like a diagonal matrix if we are willing to change bases.

Question assigned to the following page: 4

(4) Change of bases and diagonalization