Homework 1

Randomized Experiments and Observartional Studies

Jaiden Atterbury

Problem 1:

(i) First Experiment: "41 Shades of Blue"

(1) Overall Evalutation Criterion (OEC):
The Overall Evalutation Criterion is known as a quantitative measure of an experiments main objective. In this experiment, the OEC was user engagement in the form of click through rate.

(2) The Unit of Randomization:
The unit of randomization is known as the subjects/units that will be randomly assigned to different treatments. In this experiment, the unit of randomization is the users of Google who are making searches.

(3) The Relevant Treatments/Variants:
In this experiment, the relevant treatments/variants are the 41 competing gradations of blue on the Google search results pages.

(4) Null and Alternative Hypotheses:
In this experiment,

The Null Hypothesis (H_0): Different shades of blue that the users experience on the search results page do not change click through rates.

The Alternative Hypothesis (H_A): Different shades of blue that the users experience on the search results page do change click through rates.

(5) Citation:
Holson, Laura M. 2009. "Putting a Bolder Face on Google." NY Times. February 28. https://www.nytimes.com/2009/03/01/business/01marissa.html.

(ii) Second Experiment: "Her Majesty's Revenue and Customs Tax Experiment"

(1) Overall Evalutation Criterion (OEC):
In this experiment, the overall evaluation criterion was the amount of taxes paid back by an individual who has yet to pay their taxes.

(2) The Unit of Randomization:
In this experiment, the units of randomization were the taxpayers who had already declared their income to be taxed, but who had not yet paid their tax liabilities.

(3) The Relevant Treatments/Variants:
In this experiment, the relevant treatments/variants were five randomized messages across around 100,000 individual taxpayers. These messages included three norm-based messages and two public services messages. There also existed a control group who received a standard letter with no persuasive message of this kind.

(4) Null and Alternative Hypotheses:
In this experiment,

The Null Hypothesis (H_0): Different message variants sent to tax payers has no impact on if/how much of their taxes an individual pays off.

The Alternative Hypothesis (H_A): Different message variants sent to tax payers has an impact on if/how much of their taxes an individual pays off.

(5) Citation:
Michael Hallsworth, John A. List, Robert D. Metcalfe, and Ivo Vlaev, "The Behavioralist as Tax Collector: Using Natural Field Experiments to Enhance Tax Compliance," Journal of Public Economics 148 (2017): 14–31.

Problem 2: Randomized Experiment

Useful tables (Count Data):

	$Y=0$	$Y=1$
$X=A$	1080	840
$X=B$	920	1160

(i) Since the individual we are concerned about is in the control and had a good outcome, we know that the potential outcome $Y(X_A)=1$. However, due to the fundamental problem of causal inference, we can't determine the value of the potential outcome $Y(X_B)$. Thus, $Y(X_B)=0$ or $Y(X_B)=1$. With that said we know our possible pairs of outcomes are the pairs $Y(X_A)=1, Y(X_B)=1$ and $Y(X_A)=1, Y(X_B)=0$ which correspond to the types AG and HU.

(ii) Since the individual we are concerned about is in the treatment group and had a bad outcome, we know that the potential outcome $Y(X_B)=0$. However, due to the fundamental problem of causal inference, we can't determine the value of the potential outcome $Y(X_A)$. Thus, $Y(X_A)=0$ or $Y(X_A)=1$. With that said we know our possible pairs of outcomes are the pairs $Y(X_A)=0, Y(X_B)=0$ and $Y(X_A)=1, Y(X_B)=0$ which correspond to the types NG and HU.

(iii)

(a) Find $P(Y=1)$:
$P(Y=1)=\frac{P(Y=1,X=A)+P(Y=1,X=B)}{4000}=\frac{840+1160}{4000}=\frac{1}{2}$.

(b) Find $P(Y=1|X=B)$:
$P(Y=1|X=B)=\frac{P(Y=1,X=B)}{P(X=B)}=\frac{1160}{2080}=\frac{29}{52}$.

(c) Find $P(Y(X_B)=1|X=B)$:
By the Consistency Axiom we know that $P(Y(X_B)=1|X=B)=P(Y=1|X=B)$, thus we know that $P(Y(X_B)=1|X=B)=\frac{29}{52}$.

(iv) As mentioned in the slides, $ACE(X\rightarrow Y)$ is the same as the percent recovery if everybody was treated versus if nobody was treated. Thus we can find the ACE as p(Helped) − p(Hurt) which in this case would be $ACE(X\rightarrow Y)=\frac{1160}{2080}-\frac{840}{1920}=\frac{25}{208}$.

(v) Using the Frechet bounds as described in class we know that AG $=t$, where $t\in[max\{0,(c_0+c_1)-1\}, min\{c_0,c_1\}]$, $c_0=P(Y=1|X=A)$, and $c_1=P(Y=1|X=B)$. In our case $c_0=\frac{P(Y=1,X=A)}{P(X=A)}=\frac{7}{16}$ and $c_1=\frac{P(Y=1,X=B)}{P(X=B)}=\frac{29}{52}$ as calculated in part (iii). Thus $t\in[0,\frac{7}{16}]$, and it follows that $AG_{min}=0$. The corresponding values for the other types are $HU=c_0-0=\frac{7}{16}$, $HE=c_1-0=\frac{29}{52}$, and $NG=1-c_1-c_0+0=\frac{1}{208}$. Thus the corresponding two way table is:

	$P(Y=0\|X=0)$	$P(Y=1\|X=0)$
$P(Y=0\|X=1)$	$\frac{1}{208}$	$\frac{7}{16}$
$P(Y=1\|X=1)$	$\frac{29}{52}$	0

(vi) Since $t \in [0, \frac{7}{16}]$, it follows that $AG_{max} = \frac{7}{16}$. The corresponding values for the other types are $HU = c_0 - \frac{7}{16} = 0$, $HE = c_1 - \frac{7}{16} = \frac{25}{208}$, and $NG = 1 - c_1 - c_0 + 0 = \frac{23}{52}$. Thus the corresponding two way table is:

	$P(Y=0\|X=0)$	$P(Y=1\|X=0)$
$P(Y=0\|X=1)$	$\frac{23}{52}$	0
$P(Y=1\|X=1)$	$\frac{25}{208}$	$\frac{7}{16}$

(vii) The main features of the two above tables that I found were in common is that when $\%AG$ is minimized, $\%HE$ and $\%HU$ take their maximum values. Similarly, when $\%AG$ is maximized, $\%HE$ and $\%HU$ take their minimum values. This checks out, since $\%HE$ and $\%HU$ increase and decrease together.

(viii) The maximum and minimum proportions of type helped are, $\%HE \in [\frac{25}{208}, \frac{29}{52}]$, and the maximum and minimum proportions of type hurt are, $\%HU \in [0, \frac{7}{16}]$.

Problem 3: Observational Study

Useful tables (Count Data and Types Breakdown):

	$Y=0$	$Y=1$
$X=A$	HE *or* NG	HU *or* AG
$X=B$	HU *or* NG	HE *or* AG

	$Y=0$	$Y=1$
$X=A$	1080	840
$X=B$	920	1160

(i) If we want the population with the maximum value for the proportion of people who are of type helped and the minimum people of type hurt we must take all of the people in row 1 column 1 and row 2 column 2 to be of type helped and none of the people in row 1 column 2 and row 2 column 1 to be of type hurt. Once we do this we see that the population turns out to be $HE = 2240, HU = 0, NG = 920, AG = 840$. In terms of proportions this equates to $HE = \frac{14}{25}, HU = 0, NG = \frac{23}{100}, AG = \frac{21}{100}$.

(ii) If we want the population with the minimum value for the proportion of people who are of type helped and the maximum people of type hurt we must take none of the people in row 1 column 1 and row 2 column 2 to be of type helped and all of the people in row 1 column 2 and row 2 column 1 to be of type hurt. Once we do this we see that the population turns out to be $HE = 0, HU = 1760, NG = 1080, AG = 1160$. In terms of proportions this equates to $HE = 0, HU = \frac{11}{25}, NG = \frac{27}{100}, AG = \frac{29}{100}$.

(iii) Based on my calculations from parts (ii) and (iii) we can see that the maximum value that ACE can take on is $ACE_{max} = \frac{2240}{4000} - 0 = 0.56$ and the minimum value ACE can take on is $ACE_{min} = 0 - \frac{1760}{4000} = -0.44$. These calculations check out since the distance between the maximum and the minimum is 1 and the bound includes zero.