MATH 4P06
HOMEWORK #2

Question 1: For each of the following curves with parameter σ in $\mathbb{R}^3$:
i) Describe the curve geometrically (make a sketch or plot if possible)
ii) Parameterize $\vec{r}$ in terms of arclength s
iii) Calculate the curvature and torsion functions κ and τ
iv) Find the Frenet frame
v) Find the parallel frame

a) $\vec{r} = \sigma\vec{a}$ where $\vec{a}$ is a constant vector
b) $\vec{r} = (\sin\sigma\cos\phi_0, \sin\sigma\sin\phi_0, \cos\sigma)$ in Cartesian coordinates where ϕ_0 is a fixed angle between 0 and 2π
c) $\vec{r} = (\sin\theta_0\cos\sigma, \sin\theta_0\sin\sigma, \cos\theta_0)$ in Cartesian coordinates where θ_0 is a fixed angle between 0 and π
d) $\vec{r} = \sigma\hat{a} + \cos\sigma\hat{b} + \sin\sigma\hat{a}\times\hat{b}$ where $\hat{a}, \hat{b}$ are a pair of orthonormal vectors

Question 2: Determine whether the following curve flows $\vec{r}(\sigma, t)$ parameterized by σ in $\mathbb{R}^3$ are non-stretching or stretching:

a) $\sigma\vec{a} + t\,\vec{b}$ where $\vec{a}, \vec{b}$ are fixed non-parallel vectors
b) $(\sigma + t)\hat{a} + \cos\sigma\hat{b} + \sin\sigma\hat{a}\times\hat{b}$ where $\hat{a}, \hat{b}$ are a pair of orthonormal vectors

Question 3: For the 4th order NLS equation $u_t = \mathcal{R}^2(i(u_{ss} + \frac{1}{2}|u|^2u))$:

i) Find a tri-Hamiltonian formulation of the equation
ii) Find the curve flow equation expressed in terms of the Frenet frame
iii) Express the curve flow in terms of $\vec{r}$ and its s-derivatives