

EOF  
Universidad de la Amazonia, Colombia

Diego, Rafael e Jaiderbr

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## 1. DataStructure

### 1.1. indexed set

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;

template<class T> using T_set = tree<T, null_type, less<T>,
    rb_tree_tag, tree_order_statistics_node_update>;
template<class L> using T_multiset = tree<L, null_type,
    less_equal<L>, rb_tree_tag,
    tree_order_statistics_node_update>;

/*
```

```
st.find_by_order(k) - Retorna un iterador al k-esimo
    elemento, >= sz .end()
st.order_of_key(k) - Retorna el numero de elementos
    estrictamente menores que k
st.lower_bound(k) - Retorna un iterador al primer
    elemento >= k, >= sz .end()
st.upper_bound(k) - Retorna un iterador al primer
    elemento > k, >= sz .end()
st.erase(k) - Elimina el elemento k
st.erase_if([](int x) { return x % 2 == 0; }); - Elimina
    todos los elementos que cumplan la condicion

T_set<int> st;
T_multiset<int> mst;
*/

1.2. bint

// big int

struct bint {
    static const int BASE = 1e9;
    vector<int> v;
    bool neg;

    bint() : neg(0) {}
    bint(int val) : bint() { *this = val; }
    bint(long long val) : bint() { *this = val; }

    void trim() {
        while (v.size() and v.back() == 0) v.pop_back();
        if (!v.size()) neg = 0;
    }

    // converter de/para string | cin/cout
    bint(const char* s) : bint() { from_string(string(s)); }
    bint(const string& s) : bint() { from_string(s); }
    void from_string(const string& s) {
        v.clear(), neg = 0;
        int ini = 0;
        while (ini < s.size() and (s[ini] == '-' or s[ini]
            == '+' or s[ini] == '0'))
```

```

        if (s[ini++] == '-') neg = 1;
    for (int i = s.size() - 1; i >= ini; i -= 9) {
        int at = 0;
        for (int j = max(ini, i - 8); j <= i; j++) at =
            10 * at + (s[j] - '0');
        v.push_back(at);
    }
    if (!v.size()) neg = 0;
}

string to_string() const {
    if (!v.size()) return "0";
    string ret;
    if (neg) ret += '-';
    for (int i = v.size() - 1; i >= 0; i--) {
        string at = ::to_string(v[i]);
        int add = 9 - at.size();
        if (i + 1 < v.size()) for (int j = 0; j < add;
            j++) ret += '0';
        ret += at;
    }
    return ret;
}

friend istream& operator>>(istream& in, bint& val) {
    string s; in >> s;
    val = s;
    return in;
}

friend ostream& operator<<(ostream& out, const bint&
    val) {
    string s = val.to_string();
    out << s;
    return out;
}

// operators
friend bint abs(bint val) {
    val.neg = 0;
    return val;
}

friend bint operator-(bint val) {
    if (val != 0) val.neg ^= 1;
    return val;
}

```

```

bint& operator=(const bint& val) { v = val.v, neg =
    val.neg; return *this; }
bint& operator=(long long val) {
    v.clear(), neg = 0;
    if (val < 0) neg = 1, val *= -1;
    for (; val; val /= BASE) v.push_back(val % BASE);
    return *this;
}

int cmp(const bint& r) const { // menor: -1 | igual: 0
    | maior: 1
    if (neg != r.neg) return neg ? -1 : 1;
    if (v.size() != r.v.size()) {
        int ret = v.size() < r.v.size() ? -1 : 1;
        return neg ? -ret : ret;
    }
    for (int i = int(v.size()) - 1; i >= 0; i--) {
        if (v[i] != r.v[i]) {
            int ret = v[i] < r.v[i] ? -1 : 1;
            return neg ? -ret : ret;
        }
    }
    return 0;
}

friend bool operator<(const bint& l, const bint& r) {
    return l.cmp(r) == -1; }
friend bool operator>(const bint& l, const bint& r) {
    return l.cmp(r) == 1; }
friend bool operator<=(const bint& l, const bint& r) {
    return l.cmp(r) <= 0; }
friend bool operator>=(const bint& l, const bint& r) {
    return l.cmp(r) >= 0; }
friend bool operator==(const bint& l, const bint& r) {
    return l.cmp(r) == 0; }
friend bool operator!=(const bint& l, const bint& r) {
    return l.cmp(r) != 0; }

bint& operator+=(const bint& r) {
    if (!r.v.size()) return *this;
    if (neg != r.neg) return *this -= -r;
    for (int i = 0, c = 0; i < r.v.size() or c; i++) {
        if (i == v.size()) v.push_back(0);
        v[i] += c + (i < r.v.size() ? r.v[i] : 0);
        if ((c = v[i] >= BASE)) v[i] -= BASE;
    }
}

```

```

    }
    return *this;
}
friend bint operator+(bint a, const bint& b) { return a
    += b; }
bint& operator --(const bint& r) {
    if (!r.v.size()) return *this;
    if (neg != r.neg) return *this += -r;
    if ((!neg and *this < r) or (neg and r < *this)) {
        *this = r - *this;
        neg ^= 1;
        return *this;
    }
    for (int i = 0, c = 0; i < r.v.size() or c; i++) {
        v[i] -= c + (i < r.v.size() ? r.v[i] : 0);
        if ((c = v[i] < 0)) v[i] += BASE;
    }
    trim();
    return *this;
}
friend bint operator-(bint a, const bint& b) { return a
    -= b; }

// operators de * / %
bint& operator *=(int val) {
    if (val < 0) val *= -1, neg ^= 1;
    for (int i = 0, c = 0; i < v.size() or c; i++) {
        if (i == v.size()) v.push_back(0);
        long long at = (long long)v[i] * val + c;
        v[i] = at % BASE;
        c = at / BASE;
    }
    trim();
    return *this;
}
friend bint operator *(bint a, int b) { return a *= b; }
friend bint operator *(int a, bint b) { return b *= a; }
using cplx = complex<double>;
void fft(vector<cplx>& a, bool f, int N, vector<int>&
    rev) const {
    for (int i = 0; i < N; i++) if (i < rev[i])
        swap(a[i], a[rev[i]]);
    vector<cplx> roots(N);

```

```

    for (int n = 2; n <= N; n *= 2) {
        const static double PI = acos(-1);
        for (int i = 0; i < n / 2; i++) {
            double alpha = (2 * PI * i) / n;
            if (f) alpha = -alpha;
            roots[i] = cplx(cos(alpha), sin(alpha));
        }
        for (int pos = 0; pos < N; pos += n)
            for (int l = pos, r = pos + n / 2, m = 0; m
                < n / 2; l++, r++, m++) {
                auto t = roots[m] * a[r];
                a[r] = a[l] - t;
                a[l] = a[l] + t;
            }
    }
    if (!f) return;
    auto invN = cplx(1) / cplx(N);
    for (int i = 0; i < N; i++) a[i] *= invN;
}
vector<long long> convolution(const vector<int>& a,
    const vector<int>& b) const {
    vector<cplx> l(a.begin(), a.end()), r(b.begin(),
        b.end());
    int ln = l.size(), rn = r.size(), N = ln + rn + 1,
        n = 1, log_n = 0;
    while (n <= N) n <<= 1, log_n++;
    vector<int> rev(n);
    for (int i = 0; i < n; i++) {
        rev[i] = 0;
        for (int j = 0; j < log_n; j++) if (i >> j & 1)
            rev[i] |= 1 << (log_n - 1 - j);
    }
    l.resize(n), r.resize(n);
    fft(l, false, n, rev), fft(r, false, n, rev);
    for (int i = 0; i < n; i++) l[i] *= r[i];
    fft(l, true, n, rev);
    vector<long long> ret;
    for (auto& i : l) ret.push_back(round(i.real()));
    return ret;
}
vector<int> convert_base(const vector<int>& a, int
    from, int to) const {
    static vector<long long> pot(10, 1);

```

```

    if (pot[1] == 1) for (int i = 1; i < 10; i++)
        pot[i] = 10 * pot[i - 1];
    vector<int> ret;
    long long at = 0;
    int digits = 0;
    for (int i : a) {
        at += i * pot[digits];
        digits += from;
        while (digits >= to) {
            ret.push_back(at % pot[to]);
            at /= pot[to];
            digits -= to;
        }
    }
    ret.push_back(at);
    while (ret.size() and ret.back() == 0)
        ret.pop_back();
    return ret;
}

bint operator*(const bint& r) const { // O(n log(n))
    bint ret;
    ret.neg = neg ^ r.neg;
    auto conv = convolution(convert_base(v, 9, 4),
        convert_base(r.v, 9, 4));
    long long c = 0;
    for (auto i : conv) {
        long long at = i + c;
        ret.v.push_back(at % 10000);
        c = at / 10000;
    }
    for (; c; c /= 10000) ret.v.push_back(c % 10000);
    ret.v = convert_base(ret.v, 4, 9);
    if (!ret.v.size()) ret.neg = 0;
    return ret;
}

bint& operator*=(const bint& r) { return *this = *this
    * r; };

bint& operator/=(int val) {
    if (val < 0) neg ^= 1, val *= -1;
    for (int i = int(v.size()) - 1, c = 0; i >= 0; i--)
    {
        long long at = v[i] + c * (long long)BASE;
        v[i] = at / val;

```

```

        c = at % val;
    }
    trim();
    return *this;
}

friend bint operator/(bint a, int b) { return a /= b; }
int operator%=(int val) {
    if (val < 0) val *= -1;
    long long at = 0;
    for (int i = int(v.size()) - 1; i >= 0; i--)
        at = (BASE * at + v[i]) % val;
    if (neg) at *= -1;
    return at;
}

friend int operator%(bint a, int b) { return a %= b; }
friend pair<bint, bint> divmod(const bint& a_, const
    bint& b_) { // O(n^2)
    if (a_ == 0) return { 0, 0 };
    int norm = BASE / (b_.v.back() + 1);
    bint a = abs(a_) * norm;
    bint b = abs(b_) * norm;
    bint q, r;
    for (int i = a.v.size() - 1; i >= 0; i--) {
        r *= BASE, r += a.v[i];
        long long upper = b.v.size() < r.v.size() ?
            r.v[b.v.size()] : 0;
        int lower = b.v.size() - 1 < r.v.size() ?
            r.v[b.v.size() - 1] : 0;
        int d = (upper * BASE + lower) / b.v.back();
        r -= b * d;
        while (r < 0) r += b, d--; // roda O(1) vezes
        q.v.push_back(d);
    }
    reverse(q.v.begin(), q.v.end());
    q.neg = a_.neg ^ b_.neg;
    r.neg = a_.neg;
    q.trim(), r.trim();
    return { q, r / norm };
}

bint operator/(const bint& val) { return divmod(*this,
    val).first; }
bint& operator/=(const bint& val) { return *this =
    *this / val; }

```

```

    bint operator%(const bint& val) { return divmod(*this,
        val).second; }
    bint& operator%=(const bint& val) { return *this =
        *this % val; }
};

```

### 1.3. inversion count

```

// Computa el numero de inversiones para transformar
// l en r (si no es posible, retorna -1)

template<typename T> int inversion_count(vector<T> l,
    vector<T> r = {}) {
    if (!sz(r)) {
        r = l;
        sort(all(r));
    }
    int n = sz(l);
    vector<int> v(n), bit(n);
    vector<pair<T, int>> w;
    forn(i, n) w.pb({ r[i], i + 1 });
    sort(all(w));
    forn(i, n) {
        auto it = lower_bound(w.begin(), w.end(),
            make_pair(l[i], int(0)));
        if (it == w.end() or it->first != l[i]) return -1;
        // no da
        v[i] = it->second;
        it->second = -1;
    }

    int ans = 0;
    for (int i = n - 1; i >= 0; i--) {
        for (int j = v[i] - 1; j; j -= j & -j) ans +=
            bit[j];
        for (int j = v[i]; j < n; j += j & -j) bit[j]++;
    }

    return ans;
}

```

### 1.4. Min queue deque

```

// para max negar...
template<class T> struct Queue {
    deque<pair<T, int>> q;

    void push(T x) {
        int ct = 1;
        while (sz(q) and x < q.front().first)
            ct += q.front().second, q.pop_front();
        q.emplace_front(x, ct);
    }
    void pop() {
        if (q.back().second > 1) q.back().second--;
        else q.pop_back();
    }
    T min() { return q.back().first; }
};

```

### 1.5. Min queue stack

```

// para max negar...
template<class T> struct Stack {
    stack<pair<T, T>> s;

    void push(T x) {
        if (!sz(s)) s.push({ x, x });
        else s.emplace(x, min(s.top().second, x));
    }
    T top() { return s.top().first; }
    T pop() {
        T ans = s.top().first;
        s.pop();
        return ans;
    }
    int size() { return sz(s); }
    T min() { return s.top().second; }
};

```

### 1.6. Dsu



```

struct dsu {
    vector<int> pad, tam;
    int size;

    dsu(int n) : pad(n), tam(n, 1), size(n) {
        iota(all(pad), 0);
    }

    void make() {
        pad.pb(sz(pad));
        tam.pb(1);
        size++;
    }

    int find(int v) {
        if (v == pad[v]) return v;
        return pad[v] = find(pad[v]);
    }

    void unite(int a, int b) {
        a = find(a);
        b = find(b);
        if (a != b) {
            if (tam[a] < tam[b]) swap(a, b);
            pad[b] = a;
            tam[a] += tam[b];
            size--;
        }
    }

    int same(int a, int b) {
        return find(a) == find(b);
    }

    int count(int v) {
        return tam[find(v)];
    }
};

```

## 1.7. Segment Tree ( )

```

struct node { int start, end, maxlen; };

```

```

struct STregularBracket {
    vector<node> seg;
    int size;

    STregularBracket(string S) {
        S = "0" + S;
        size = S.size();
        seg.resize(4 * size);
        build(1, 1, size - 1, S);
    }

    void build(int idx, int s, int e, string& S) {
        if (s == e) {
            if (S[s] == '(') seg[idx] = { 1, 0, 0 };
            else seg[idx] = { 0, 1, 0 };
            return;
        }
        int m = (s + e) / 2;
        build(idx << 1, s, m, S);
        build(idx << 1 | 1, m + 1, e, S);
        pull(idx);
    }

    void pull(int idx) {
        node& L = seg[idx << 1], & R = seg[idx << 1 | 1], &
        P = seg[idx];
        P.start = R.start;
        P.end = L.end;
        P.maxLen = L.maxLen + R.maxLen;
        int pares = min(L.start, R.end);
        P.maxLen += pares * 2;
        int dif = L.start - R.end;
        if (dif > 0) P.start += dif;
        else P.end -= dif;
    }

    node query(int idx, int s, int e, int l, int r) {
        if (l > e || s > r) return { 0, 0, 0 };
        if (s >= l && e <= r) return seg[idx];
        int m = (s + e) / 2;
        node p1 = query(idx << 1, s, m, l, r);
        node p2 = query(idx << 1 | 1, m + 1, e, l, r);
    }
};

```

```

node ans;
ans.start = p2.start;
ans.end = p1.end;
ans.maxLen = p1.maxLen + p2.maxLen;
int pares = min(p1.start, p2.end);
ans.maxLen += pares * 2;
int dif = p1.start - p2.end;
if (dif > 0) ans.start += dif;
else      ans.end -= dif;
return ans;
}

void update(int idx, int s, int e, int pos, char val) {
    if (s == e) {
        if (val == '(') seg[idx] = { 1, 0, 0 };
        else          seg[idx] = { 0, 1, 0 };
        return;
    }
    int m = (s + e) / 2;
    if (pos <= m) update(idx << 1, s, m, pos, val);
    else        update(idx << 1 | 1, m + 1, e, pos,
        val);
    pull(idx);
}

// [1, n]
node query(int l, int r) { return query(1, 1, size - 1,
    l, r); }
void update(int pos, char val) { update(1, 1, size - 1,
    pos, val); }
};

```

## 1.8. STable

```

struct STable {
    int n, K;
    vector<vector<int>> st;

    STable(const vector<int>& a) {
        n = sz(a);
        K = int(log2(n)) + 1;
        st.assign(n + 1, vector<int>(K));
    }
};

```

```

    forn(i, n) st[i][0] = a[i];
    forn(j, K - 1)
        for (int i = 0; i + (1 << (j + 1)) <= n; ++i)
            st[i][j + 1] = oper(st[i][j], st[i + (1 <<
                j)][j]);
}

int oper(int a, int b) { return __gcd(a, b); }

int query(int l, int r) {
    int k = 31 - __builtin_clz(r - l + 1);
    return oper(st[l][k], st[r - (1 << k) + 1][k]);
}
};

```

## 1.9. WaveletTree

```

struct WaveletTree {
    int lo, hi;
    WaveletTree* left = nullptr, * right = nullptr;
    vector<int> freq, pref;

    // Build from [from, to) with values in [x, y] x = min
    // value, y = max value
    WaveletTree(vector<int>::iterator from,
        vector<int>::iterator to, int x, int y) : lo(x),
        hi(y) {
        if (from >= to) return;
        int mid = (lo + hi) >> 1;
        auto f = [mid](int v) { return v <= mid; };

        int sz = to - from;
        freq.reserve(sz + 1);
        freq.push_back(0);
        pref.reserve(sz + 1);
        pref.push_back(0);

        for (auto it = from; it != to; ++it) {
            freq.push_back(freq.back() + f(*it));
            pref.push_back(pref.back() + *it);
        }
    }
};

```

```

    if (lo == hi) return;

    auto pivot = stable_partition(from, to, f);
    left = new WaveletTree(from, pivot, lo, mid);
    right = new WaveletTree(pivot, to, mid + 1, hi);
}

// k-th smallest in [l,r]
int kth(int l, int r, int k) {
    if (l > r) return 0;
    if (lo == hi) return lo;

    int lb = freq[l - 1], rb = freq[r];
    int inLeft = rb - lb;
    if (k <= inLeft) return left->kth(lb + 1, rb, k);
    else return right->kth(l - lb, r - rb, k - inLeft);
}

// number of elements == k in [l,r]
int eq(int l, int r, int k) {
    if (l > r || k < lo || k > hi) return 0;
    if (lo == hi) return r - l + 1;

    int lb = freq[l - 1], rb = freq[r];
    int mid = (lo + hi) >> 1;
    if (k <= mid) return left->eq(lb + 1, rb, k);
    else return right->eq(l - lb, r - rb, k);
}

// number of elements <= k in [l,r]
int le(int l, int r, int k) {
    if (l > r || k < lo) return 0;
    if (hi <= k) return r - l + 1;

    int lb = freq[l - 1], rb = freq[r];
    return left->le(lb + 1, rb, k) + right->le(l - lb,
        r - rb, k);
}

// number of elements < k in [l,r]
int lt(int l, int r, int k) {
    if (l > r || k <= lo) return 0;
    if (hi < k) return r - l + 1;

```

```

    int lb = freq[l - 1], rb = freq[r];
    return left->lt(lb + 1, rb, k) + right->lt(l - lb,
        r - rb, k);
}

// number of elements >= k in [l,r]
int ge(int l, int r, int k) {
    if (l > r || k > hi) return 0;
    if (k <= lo) return r - l + 1;

    int lb = freq[l - 1], rb = freq[r];
    return left->ge(lb + 1, rb, k) + right->ge(l - lb,
        r - rb, k);
}

// number of elements > k in [l,r]
int gt(int l, int r, int k) {
    if (l > r || k >= hi) return 0;
    if (k < lo) return r - l + 1;

    int lb = freq[l - 1], rb = freq[r];
    int mid = (lo + hi) >> 1;
    if (k < mid) return left->gt(lb + 1, rb, k) +
        right->count(l - lb, r - rb);
    else return right->gt(l - lb, r - rb, k);
}

// helper to count total in node
int count(int l, int r) {
    if (l > r) return 0;
    return r - l + 1;
}

// number of elements in [l,r] between [a,b]
int between(int l, int r, int a, int b) {
    return le(l, r, b) - lt(l, r, a);
}

// sum of elements <= k in [l,r]
int sum_le(int l, int r, int k) {
    if (l > r || k < lo) return 0;
    if (hi <= k) return pref[r] - pref[l - 1];

```

```

    int lb = freq[l - 1], rb = freq[r];
    return left->sum_le(lb + 1, rb, k) +
           right->sum_le(l - lb, r - rb, k);
}
};

```

## 1.10. Mint

```

template <typename T, T m>
struct modint {
    T x;
    constexpr static T mod() { return m; }
    constexpr T val() const { return x; }
    constexpr modint() : x(0) {}
    modint(T x_) : x(x_% mod()) { if (x < 0) x += mod(); }
    modint& operator+=(modint b) { if ((x += b.x) >= mod())
        x -= mod(); return *this; }
    modint& operator-=(modint b) { if ((x -= b.x) < 0) x +=
        mod(); return *this; }
    modint& operator*=(modint b) { x = (T)(x)*b.x % mod();
        return *this; }
    modint pow(T e) const {
        modint r = 1, b = *this;
        while (e) {
            if (e & 1) r *= b;
            b *= b, e >>= 1;
        }
        return r;
    }
    modint inv() { return pow(mod() - 2); }
    modint& operator /=(modint b) { return *this *=
        b.pow(mod() - 2); }
    friend modint operator+ (modint a, modint b) { return a
        += b; }
    friend modint operator- (modint a, modint b) { return a
        -= b; }
    friend modint operator/ (modint a, modint b) { return a
        /= b; }
    friend modint operator* (modint a, modint b) { return a
        *= b; }
}

```

```

friend bool operator< (modint a, modint b) { return
    a.x < b.x; }
friend bool operator> (modint a, modint b) { return
    a.x > b.x; }
friend bool operator==(modint a, modint b) { return
    a.x == b.x; }
friend bool operator!=(modint a, modint b) { return
    a.x != b.x; }
friend ostream& operator<<(ostream& os, const modint&
    a) { return os << a.val(); }
};
constexpr int mod = 1000000007;
using mint = modint<int, mod>;

```

## 1.11. Eval

```

template <typename T> struct eval {
    string s;
    int n;
    eval(string s) : s(s), n(sz(s)) {}

    stack<T> nums;
    stack<char> oper;

    int order(char op) {
        if (op < 0) return 3;
        if (op == '+' || op == '-') return 1;
        if (op == '*' || op == '/') return 2;
        return 0;
    }
    bool is_op(char c) { return c == '+' || c == '-' || c
        == '*' || c == '/'; }

    bool is_unary(char c) { return c == '+' || c == '-'; }

    T apply(T a, T b, char op) {
        if (op == '+') return a + b;
        if (op == '-') return a - b;
        if (op == '*') return a * b;
        if (op == '/') return a / b;
        return 0;
    }
}

```

```

T go() {
    int op = oper.top(); oper.pop();
    if (op < 0) {
        T v = nums.top(); nums.pop();
        return apply(0, v, -op);
    }
    T v2 = nums.top(); nums.pop();
    T v1 = nums.top(); nums.pop();
    return apply(v1, v2, op);
}

T get() {
    bool ok = 1;
    forn(i, n) {
        if (s[i] == ' ') continue;
        if (s[i] == '(') oper.push('('), ok = 1;
        else if (s[i] == ')') {
            while (oper.top() != '(') nums.push(go());
            oper.pop(), ok = 0;
        }
        else if (is_op(s[i])) {
            char alt = s[i];
            if (ok && is_unary(alt)) alt = -alt;
            while (sz(oper) && ((alt >= 0 &&
                order(oper.top()) >= order(alt)) || (alt
                < 0 && order(oper.top()) > order(alt))))
                nums.push(go());
            oper.push(alt), ok = 1;
        }

        // else {
        //     int val = 0;
        //     while (i < n && isalnum(s[i])) val = val
        //         * 10 + s[i++] - '0';
        //     --i;
        //     nums.push(val), ok = 0;
        // }

        else {
            T val = 0;
            int dec = -1;
            while (i < n && (isdigit(s[i]) || s[i] ==

```

```

        '.')) {
            if (s[i] == '.') dec = 0;
            else {
                val = val * 10 + (s[i] - '0');
                if (dec >= 0) ++dec;
            }
            ++i;
        }
        if (dec > 0) val /= pow(10, dec);
        --i;
        nums.push(val);
        ok = 0;
    }
}

while (sz(oper)) nums.push(go());

return nums.top();
}

};

```

## 1.12. Fenwick Tree

```

template<typename T>
struct BIT {
    vector<T> ft;
    BIT(int n) : ft(n + 1) {}
    BIT(const vector<T>& a) : ft(sz(a) + 1) {
        forn(i, sz(a)) { upd(i + 1, a[i]); }
    }

    T qry(int i) {
        T ans = 0;
        for (; i; i -= i & -i) ans += ft[i];
        return ans;
    }

    T qry(int l, int r) { return qry(r) - qry(l - 1); }

    void upd(int i, T v) {
        for (; i < sz(ft); i += i & -i) ft[i] += v;
    }
}

```

```

    }
};

```

### 1.13. Segment Tree 2D

```

template<typename T>
struct STree {
    int n, m;
    T neutro = T(0);
    vector<vector<T>> st;

    STree(vector<vector<T>>& a) {
        n = sz(a);
        m = sz(a[0]);
        st = vector<vector<T>>(2 * n, vector<T>(2 * m,
            neutro));
        build(a);
    }

    inline T oper(T a, T b) { return a + b; }

    void build(vector<vector<T>>& a) {
        forn(i, n) forn(j, m) st[i + n][j + m] = a[i][j];
        forn(i, n) {
            for (int j = m - 1; j >= 1; --j) {
                st[i + n][j] = oper(st[i + n][j << 1], st[i
                    + n][j << 1 | 1]);
            }
        }
        for (int i = n - 1; i >= 1; --i) {
            forn(j, 2 * m) {
                st[i][j] = oper(st[i << 1][j], st[i << 1 | 1
                    ][j]);
            }
        }
    }

    T qry(int x1, int y1, int x2, int y2) { // [x1, y1]
        [x2, y2]
        T ans = neutro;
        for (int i0 = x1 + n, i1 = x2 + n + 1; i0 < i1; i0
            >= 1, i1 >= 1) {

```

```

            int t[4], q = 0;
            if (i0 & 1) t[q++] = i0++;
            if (i1 & 1) t[q++] = --i1;
            forn(k, q)
                for (int j0 = y1 + m, j1 = y2 + m + 1; j0 <
                    j1; j0 >= 1, j1 >= 1) {
                    if (j0 & 1) ans = oper(ans,
                        st[t[k]][j0++]);
                    if (j1 & 1) ans = oper(ans,
                        st[t[k]][--j1]);
                }
        }
        return ans;
    }

    void upd(int l, int r, T val) {
        st[l + n][r + m] = val;
        for (int j = r + m; j > 1; j >= 1) {
            st[l + n][j >> 1] = oper(st[l + n][j], st[l +
                n][j ^ 1]);
        }
        for (int i = l + n; i > 1; i >= 1) {
            for (int j = r + m; j > 1; j >= 1) {
                st[i >> 1][j] = oper(st[i][j], st[i ^ 1
                    ][j]);
            }
        }
    }
};

```

### 1.14. Segment Tree Iterative

```

template<typename T>
struct STree {
    vector<T> st;
    int n;
    T neutro = T(0);
    T oper(T a, T b) { return a + b; }
    STree(vector<T>& a) {
        n = sz(a);
        st.resize(n * 2);
        forn(i, n) st[n + i] = a[i];

```

```

        for (int i = n - 1; i >= 1; i -= 1) st[i] =
            oper(st[i << 1], st[i << 1 | 1]);
    }

    void upd(int p, T val) {
        for (st[p += n] = val; p > 1; p >>= 1) st[p >> 1] =
            oper(st[p], st[p ^ 1]);
    }

    T query(int l, int r) { //[l, r)
        T v = neutro;
        for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
            if (l & 1) v = oper(v, st[l++]);
            if (r & 1) v = oper(v, st[--r]);
        }
        return v;
    }
};

```

### 1.15. Segment Tree Lazy

```

template<typename T>
struct STree {
    int n; vector<T> st, lazy;
    T neutro = T(0);

    STree(int m) {
        n = m;
        st.resize(n * 4);
        lazy.resize(n * 4);
    }

    STree(vector<T>& a) {
        n = sz(a);
        st.resize(n * 4);
        lazy.resize(n * 4);
        build(1, 0, n - 1, a);
    }

    T oper(T a, T b) { return a + b; }

    void build(int v, int tl, int tr, vector<T>& a) {

```

```

        if (tl == tr) {
            st[v] = a[tl];
            return;
        }
        int tm = (tl + tr) / 2;
        build(v * 2, tl, tm, a);
        build(v * 2 + 1, tm + 1, tr, a);
        st[v] = oper(st[v * 2], st[v * 2 + 1]);
    }

    void push(int v, int tl, int tr) {
        if (!lazy[v]) return;
        st[v] += (tr - tl + 1) * lazy[v];
        if (tl != tr) {
            lazy[v * 2] += lazy[v];
            lazy[v * 2 + 1] += lazy[v];
        }
        lazy[v] = 0;
    }

    void upd(int v, int tl, int tr, int l, int r, T val) {
        push(v, tl, tr);
        if (tr < l || tl > r) return;
        if (tl >= l && tr <= r) {
            lazy[v] = val;
            push(v, tl, tr);
            return;
        }
        int tm = (tl + tr) / 2;
        upd(v * 2, tl, tm, l, r, val);
        upd(v * 2 + 1, tm + 1, tr, l, r, val);
        st[v] = oper(st[v * 2], st[v * 2 + 1]);
    }

    T query(int v, int tl, int tr, int l, int r) {
        push(v, tl, tr);
        if (tl > r || tr < l) return neutro;
        if (l <= tl && tr <= r) return st[v];
        int tm = (tl + tr) / 2;
        return oper(query(v * 2, tl, tm, l, r), query(v * 2
            + 1, tm + 1, tr, l, r));
    }
}

```

```

void upd(int l, int r, T val) { upd(1, 0, n - 1, l, r,
    val); }
T query(int l, int r) { return query(1, 0, n - 1, l,
    r); }
};

```

## 1.16. Segment Tree

```

template<typename T>
struct STree {
    int n; vector<T> st;
    T neutro = T(0);

    STree(vector<T>& a) {
        n = sz(a);
        st.resize(n * 4);
        build(1, 0, n - 1, a);
    }

    T oper(T a, T b) { return max(a, b); }

    void build(int v, int tl, int tr, vector<T>& a) {
        if (tl == tr) {
            st[v] = a[tl];
            return;
        }
        int tm = (tr + tl) / 2;
        build(v * 2, tl, tm, a);
        build(v * 2 + 1, tm + 1, tr, a);
        st[v] = oper(st[v * 2], st[v * 2 + 1]);
    }

    T query(int v, int tl, int tr, int l, int r) {
        if (tl > r || tr < l) return neutro;
        if (l <= tl && tr <= r) return st[v];
        int tm = (tl + tr) / 2;
        return oper(query(v * 2, tl, tm, l, r), query(v * 2
            + 1, tm + 1, tr, l, r));
    }

    void upd(int v, int tl, int tr, int pos, T val) {
        if (tl == tr) {

```

```

            st[v] = val;
            return;
        }
        int tm = (tr + tl) / 2;
        if (pos <= tm) upd(v * 2, tl, tm, pos, val);
        else upd(v * 2 + 1, tm + 1, tr, pos, val);
        st[v] = oper(st[v * 2], st[v * 2 + 1]);
    }
    void upd(int pos, T val) { upd(1, 0, n - 1, pos, val); }
    T query(int l, int r) { return query(1, 0, n - 1, l,
        r); }
};

```

## 1.17. SqrtBlocks

```

template<typename T>
struct SqrtBlocks {
    int n, blk_sz, blk_n;
    vector<T> st;
    vector<vector<T>> blocks;

    SqrtBlocks(vector<T>& a) {
        n = sz(a), st = a;
        blk_sz = sqrt(n) + 1, blk_n = (n + blk_sz - 1) /
            blk_sz;
        blocks.resize(blk_n);
        forn(i, n) blocks[i / blk_sz].pb(st[i]);
        forn(i, blk_n) sort(all(blocks[i]));
    }

    void update(int pos, int val) {
        int blk = pos / blk_sz;
        auto& b = blocks[blk];
        auto it = lower_bound(all(b), st[pos]);
        b.erase(it);
        b.insert(lower_bound(all(b), val), val);
        st[pos] = val;
    }

    // >
    T query_greater(int l, int r, int val) {
        T res = 0;
        int bl = l / blk_sz, br = r / blk_sz;

```



```

    if (bl == br) {
        forne(i, l, r + 1) res += (st[i] > val);
        return res;
    }
    int end_l = (bl + 1) * blk_sz;
    forne(i, l, end_l) res += (st[i] > val);
    forne(b, bl + 1, br) res += end(blocks[b]) -
        upper_bound(all(blocks[b]), val);
    int start_r = br * blk_sz;
    forne(i, start_r, r + 1) res += (st[i] > val);
    return res;
}

// >=
T query_ge(int l, int r, int val) {
    T res = 0;
    int bl = l / blk_sz, br = r / blk_sz;
    if (bl == br) {
        forne(i, l, r + 1) res += (st[i] >= val);
        return res;
    }
    int end_l = (bl + 1) * blk_sz;
    forne(i, l, end_l) res += (st[i] >= val);
    forne(b, bl + 1, br) res += end(blocks[b]) -
        lower_bound(all(blocks[b]), val);
    int start_r = br * blk_sz;
    forne(i, start_r, r + 1) res += (st[i] >= val);
    return res;
}

// <
T query_less(int l, int r, int val) {
    T res = 0;
    int bl = l / blk_sz, br = r / blk_sz;
    if (bl == br) {
        forne(i, l, r + 1) res += (st[i] < val);
        return res;
    }
    int end_l = (bl + 1) * blk_sz;
    forne(i, l, end_l) res += (st[i] < val);
    forne(b, bl + 1, br) res +=
        lower_bound(all(blocks[b]), val) -
        begin(blocks[b]);

```

```

    int start_r = br * blk_sz;
    forne(i, start_r, r + 1) res += (st[i] < val);
    return res;
}

// <=
T query_le(int l, int r, int val) {
    T res = 0;
    int bl = l / blk_sz, br = r / blk_sz;
    if (bl == br) {
        forne(i, l, r + 1) res += (st[i] <= val);
        return res;
    }
    int end_l = (bl + 1) * blk_sz;
    forne(i, l, end_l) res += (st[i] <= val);
    forne(b, bl + 1, br) res +=
        upper_bound(all(blocks[b]), val) -
        begin(blocks[b]);
    int start_r = br * blk_sz;
    forne(i, start_r, r + 1) res += (st[i] <= val);
    return res;
}

// ==
T query_equal(int l, int r, int val) {
    T res = 0;
    int bl = l / blk_sz, br = r / blk_sz;
    if (bl == br) {
        forne(i, l, r + 1) res += (st[i] == val);
        return res;
    }
    int end_l = (bl + 1) * blk_sz;
    forne(i, l, end_l) res += (st[i] == val);
    forne(b, bl + 1, br) res +=
        upper_bound(all(blocks[b]), val) -
        lower_bound(all(blocks[b]), val);
    int start_r = br * blk_sz;
    forne(i, start_r, r + 1) res += (st[i] == val);
    return res;
}

// between [a,b]
T query_between(int l, int r, int a, int b) {

```

```

T res = 0;
int bl = l / blk_sz, br = r / blk_sz;
if (bl == br) {
    forne(i, l, r + 1) res += (st[i] >= a && st[i]
        <= b);
    return res;
}
int end_l = (bl + 1) * blk_sz;
forne(i, l, end_l) res += (st[i] >= a && st[i] <=
    b);
forne(bk, bl + 1, br) res +=
    upper_bound(all(blocks[bk]), b) -
    lower_bound(all(blocks[bk]), a);
int start_r = br * blk_sz;
forne(i, start_r, r + 1) res += (st[i] >= a &&
    st[i] <= b);
return res;
}
};

```

### 1.18. Coo Compress

```

template<typename T>
struct COO_COMPRESS {
    vector<T> nums;
    bool is_compress = true;

    int size() {
        if (!is_compress) compress();
        return sz(nums);
    }

    void clear() {
        nums.clear();
        is_compress = true;
    }

    void insert(T x) {
        nums.pb(x);
        is_compress = false;
    }
}

```

```

void compress() {
    sort(all(nums));
    nums.resize(unique(all(nums)) - nums.begin());
    is_compress = true;
}

vector<T> compress_offline(vector<T> nums) {
    if (!sz(nums)) return nums;
    vector<pair<T, int>> vvv;
    forn(i, sz(nums)) vvv.pb({ nums[i], i });
    sort(all(vvv));
    int cont = 0;
    T last = vvv[0].first;
    nums[vvv[0].second] = 0;
    forne(i, 1, sz(vvv)) {
        if (vvv[i].first != last) cont++, last =
            vvv[i].first;
        nums[vvv[i].second] = cont;
    }
    return nums;
}

int get(T x) {
    if (!is_compress) compress();
    int pos = lower_bound(all(nums), x) - nums.begin();
    assert(pos != sz(nums) && nums[pos] == x);
    return pos;
}

T iget(int x) {
    if (!is_compress) compress();
    assert(0 <= x && x < sz(nums));
    return nums[x];
}
};

```

### 1.19. Mo's

```

void add(int x) {}
void del(int x) {}
int get_ans() {}

```

```

vector<int> mo(const vector<pair<int, int>>& q) {
    int l = 0, r = -1, blk = 350; // sqrt(n)
    vector<int> inx(sz(q)), ans(sz(q));
    auto K = [&](const pair<int, int>& x) -> pair<int, int> {
        return pair<int, int>(x.f / blk, x.s ^ -(x.f / blk & 1));
    };
    iota(all(inx), 0);
    sort(all(inx), [&](int a, int b) -> bool { return K(q[a]) < K(q[b]); });
    /*
    sort(all(inx), [&](int l, int r) {
        if (q[l].f / blk != q[r].f / blk) return q[l].f < q[r].f;
        if ((q[l].f / blk) % 2) return q[l].s > q[r].s;
        return q[l].s < q[r].s;
    });
    */
    for (int nxt : inx) {
        pair<int, int> it = q[nxt];
        while (r < it.s) add(++r);
        while (l > it.f) add(--l);
        while (r > it.s) del(r--);
        while (l < it.f) del(l++);
        ans[nxt] = get_ans();
    }
    return ans;
}

```

## 2. DP

### 2.1. Knapsack

```

int n, x; cin>>n>>x;
vector<array<int,2>>arr(n);
for(i,n) cin>>arr[i][0];
for(i,n) cin>>arr[i][1];

vector<vector<int>>dp(n+1,vector<int>(x+1,0));
forne(i,1,n+1){

```

```

    forne(j,1,x+1){
        dp[i][j]=dp[i-1][j];
        if(j-arr[i-1][0]>=0){
            int libro=arr[i-1][1];
            int price=arr[i-1][0];
            dp[i][j]=max(dp[i][j],
                libro+dp[i-1][j-price]);
        }
    }
}
cout<<dp[n][x]<<endl;

```

```

const ll inf=1e18+7;

ll Knapsack(ll n, ll cty, vector<ll>& W,vector<ll>& V) {
    ll sum=accumulate(all(V),0LL);
    vector<ll>dp(sum+1,inf);
    dp[0]=0;
    forn(i, n){
        for(int j = sum-V[i]; j >= 0; j--){
            dp[j+V[i]] = min(dp[j+V[i]], dp[j]+W[i]);
        }
    }
    ll ans=0;
    forn(i,sum+1){
        if(dp[i]<= cty) ans=max(ans,ll(i));
    }
    return ans;
}

```

### 2.2. Lis

```

int lis(vector<int>& a) {
    vector<int>dp;
    forn(i, sz(a)) {
        auto it = lower_bound(all(dp), a[i]);
        if (it != dp.end()) *it = a[i];
        else dp.pb(a[i]);
    }
}

```

```

    return sz(dp);
}

constexpr int INF = ((1ULL << 63) - 1) >> 32;
template<typename T> vector<T> lis(vector<T>& v) {
    int n = sz(v), m = -1;
    vector<T> d(n + 1, INF);
    vector<int> l(n);
    d[0] = -INF;

    forn(i, n) {
        // Para non-decreasing use upper_bound()
        int t = lower_bound(all(d), v[i]) - begin(d);
        d[t] = v[i], l[i] = t, m = max(m, t);
    }

    int p = n;
    vector<T> ans;
    while (p-- > 0) if (l[p] == m) {
        ans.pb(v[p]); m--;
    }
    reverse(all(ans));
    return ans;
}

int lis_2(int n, vector<int>& A, vector<int>& B) {

    vector<vector<int>>> C(inf, vector<int>());
    reverse(all(A)); reverse(all(B));
    for (int i = n - 1; i >= 0; i--) C[B[i]].pb(i);

    STree<int> dp(n); // Stree de max, neutro = INT_MIN

    for (int j = 0; j < n; j++) {
        for (auto&& i : C[A[j]]) {
            int mx_pre = dp.query(0, i);
            if (mx_pre != INT_MIN) dp.upd(i, mx_pre + 1);
            else dp.upd(i, 1);
        }
    }
    return dp.query(0, n);
}

```

## 2.3. Divide and Conquer dp

```

/*
Divide and Conquer DP
Particiona o array en k subarrays
minimizando la suma de las queries
*/

ll dp[MAX][2];

void solve(int k, int l, int r, int lk, int rk) {
    if (l > r) return;
    int m = (l + r) / 2, p = -1;
    auto& ans = dp[m][k & 1] = LINF;
    for (int i = max(m, lk); i <= rk; i++) {
        ll at = dp[i + 1][~k & 1] + query(m, i);
        if (at < ans) ans = at, p = i;
    }
    solve(k, l, m - 1, lk, p), solve(k, m + 1, r, p, rk);
}

ll DC(int n, int k) {
    dp[n][0] = dp[n][1] = 0;
    for (int i = 0; i < n; i++) dp[i][0] = LINF;
    for (int i = 1; i <= k; i++) solve(i, 0, n - i, 0, n - i);
    return dp[0][k & 1];
}

```

## 2.4. Edit Distance

```

/*
The edit distance between two strings is the minimum number
of operations required to transform one string into the
other.
*/
{
    string a, b; cin >> a >> b;
    int n = sz(a), m = sz(b);
    vector<vector<int>>> dp(n + 1, vector<int>(m + 1, inf));
    forne(i, 0, n + 1) dp[i][0] = i;
    forne(j, 0, m + 1) dp[0][j] = j;
}

```

```

    forne(i, 1, n + 1) {
        forne(j, 1, m + 1) {
            dp[i][j] = min({ dp[i][j - 1] + 1, dp[i - 1][j -
                1] + (a[i - 1] != b[j - 1]), dp[i - 1][j] + 1
            });
        }
    }
    cout << dp[n][m] << endl;
}

constexpr int INF = (1e18 - 1);

int edit_distance(const string& s, const string& t) {
    int n = sz(s), m = sz(t);
    vector<int> dp(m + 1);
    iota(all(dp), 0);

    forn(i, n) {
        vector<int> ndp(m + 1, INF);
        ndp[0] = i + 1;
        forn(j, m) {
            ndp[j + 1] = min({ ndp[j] + 1, dp[j + 1] + 1,
                dp[j] + (s[i] != t[j]) });
        }
        dp.swap(ndp);
    }

    return dp[m];
}

vector<string> construct_edit_distance(const string& s,
    const string& t) {

    int n = sz(s), m = sz(t);
    vector<vector<int>> dp(n + 1, vector<int>(m + 1, INF));

    forn(i, n + 1) dp[i][0] = i;
    forn(j, m + 1) dp[0][j] = j;

    forn(i, n) {
        forn(j, m) {
            dp[i + 1][j + 1] = min({ dp[i + 1][j] + 1,
                dp[i][j + 1] + 1, dp[i][j] + (s[i] != t[j])
            });
        }
    }
}

```

```

    }
}

vector<string> left = { s }, right = { t };

while (n > 0 || m > 0) {
    if (n > 0 && dp[n][m] == dp[n - 1][m] + 1) {
        n--;
        string str = left.back();
        str.erase(str.begin() + n);
        left.push_back(str);
    }
    else if (m > 0 && dp[n][m] == dp[n][m - 1] + 1) {
        m--;
        string str = right.back();
        str.erase(str.begin() + m);
        right.push_back(str);
    }
    else if (n > 0 && m > 0 && dp[n][m] == dp[n - 1][m
        - 1] + (s[n - 1] != t[m - 1])) {
        n--, m--;
        if (s[n] != t[m]) {
            string str = left.back();
            str[n] = t[m];
            left.push_back(str);
        }
    }
    else {
        assert(false);
    }
}

assert(left.back() == right.back());
right.pop_back();

while (!right.empty()) {
    left.push_back(right.back());
    right.pop_back();
}

return left;
}

```

## 2.5. groups

```
/*
Dado N pesos y un limite Q, se quiere saber el minimo numero
de grupos en los que se pueden dividir los pesos tal que la
suma de los pesos de cada grupo sea menor o igual a Q
n => sz(nums);
q => maximo peso
nums => vector con los pesos
*/
int calculate(int n, int q, vector<int>& nums) {
    pair<int, int> best[1 << n];
    best[0] = { 1, 0 };
    forne(i, 1, 1 << n) {
        best[i] = { n + 1, 0 };
        forn(j, n) {
            if (i & (1 << j)) {
                auto cur = best[i ^ (1 << j)];
                if (cur.s + nums[j] <= q) {
                    cur.s += nums[j];
                }
                else {
                    cur.f++;
                    cur.s = nums[j];
                }
                best[i] = min(best[i], cur);
            }
        }
    }
    return best[(1 << n) - 1].f;
}

/*
Dado N pesos y un limite Q, se quiere saber el numero de
grupos
consecutivos en los que se pueden dividir los pesos tal que
la
suma de los pesos de cada grupo sea menor o igual a Q
n => sz(nums);
q => maximo peso
nums => vector con los pesos
*/
```

```
int get(int n, int q, vector<int>& nums) {
    sort(all(nums));
    int l = 0, r = n - 1, ans = n;
    while (l < r) {
        if (nums[l] + nums[r] <= q) ans--, l++, r--;
        else r--;
    }
    return ans;
}
```

## 2.6. Shortest Hamiltonian Path

```
/*
Shortest Hamiltonian Path
Resuelve problemas del tipo de encontrar el camino mas corto
que recorre todos los nodos de un grafo una sola vez.
*/
vector<vector<pair<int, int>>> ady;
int n, m, target;
const int N = 18;
const int MASK = 1 << N;
const int INF = int(1e7);
int dp[N][MASK];

int solve(int v, int mask) {
    if (mask == target) return 0;
    int& ans = dp[v][mask];
    if (ans != -1) return ans;
    ans = INF;
    for (auto& u : ady[v]) {
        if (!(mask & (1 << u.first))) {
            ans = min(ans, solve(u.first, mask | (1 <<
                u.first)) + u.second);
        }
    }
    return ans;
}

int main() {
    cin >> n >> m;
    target = (1 << n) - 1;
    ady.assign(n, {});
    forn(i, m) {
```

```

    int v, u, w; cin >> v >> u >> w;
    v--, u--;
    ady[v].push_back({ u, w });
    ady[u].push_back({ v, w });
}
memset(dp, -1, sizeof dp);
cout << solve(0, 1) << endl;

cout << flush;
return 0;
}

```

## 2.7. Money Sums

```

// find all money sums you can create using these coins.
int n; cin >> n;
vector<int>nums(n), sums;
for(i, n) cin >> nums[i];
vector<vector<bool>>dp(mxN + 1, vector<bool>(n * mxS + 1
));
dp[0][0] = 1;
forne(i, 1, n + 1) {
    forn(j, mxS * n + 1) {
        dp[i][j] = dp[i - 1][j];
        if (j - nums[i - 1] >= 0 && dp[i - 1][j -
            nums[i - 1]]) dp[i][j] = 1;
    }
}

for(i, mxS * n + 1) {
    if (i && dp[n][i]) sums.pb(i);
}

cout << sz(sums) << endl;
for(i, sz(sums)) cout << sums[i] << " \n"[i + 1 ==
    sz(sums)];
cout << endl;

```

## 2.8. Digit dp

```

// - Descripcion: Cuenta la cantidad de numeros entre [a,
    b] que no tienen digitos iguales seguidos
// - Complejidad: O(NUM_E * NUM_T)

const int MOD = 998244353;
int tam, NUM[55], dp[55][2][2][11];

int solve(int i, bool menor, bool ncero, int last) {
    if (i == tam) return 1;
    int& ans = dp[i][menor][ncero][last];
    if (ans != -1) return ans;
    ans = 0;
    forn(dig, 10) {
        if (dig == last && (ncero || dig)) continue;
        if (menor || dig <= NUM[i]) {
            ans = (ans + solve(i + 1, menor || dig <
                NUM[i], ncero || dig, dig)) % MOD;
        }
    }
    return ans;
}

bool g(string s) {
    forn(i, sz(s) - 1) {
        if (s[i] == s[i + 1]) return false;
    }
    return true;
}

int build(string s) {
    tam = sz(s);
    forn(i, sz(s)) {
        NUM[i] = s[i] - '0';
    }
    memset(dp, -1, sizeof dp);
    return solve(0, false, false, 10);
}

void solve() {
    string l, r;
    while (cin >> l >> r) {
        cout << ((build(r) - build(l) + MOD) % MOD + g(l))
            % MOD << endl;
    }
}

```

```
}
```

## 2.9. LCS

```
constexpr int mxN = 105;
vector<vector<int>> dp(mxN, vector<int>(mxN, -1));
// n=sz(s), m=sz(p)
int cntsub(const string& s, const string& p, int n, int m) {
    if ((n == 0 && m == 0) || m == 0) return 1;
    if (n == 0) return 0;
    int& ans = dp[n][m];
    if (~ans) return ans;
    if (s[n - 1] == p[m - 1]) {
        return ans = cntsub(s, p, n - 1, m - 1) + cntsub(s,
            p, n - 1, m);
    }
    else {
        return ans = cntsub(s, p, n - 1, m);
    }
}

bool issub(const string& str, const string& sub) {
    int idx = 0;
    for (auto&& i : str) {
        if (idx < sz(sub) && i == sub[idx]) {
            idx++;
        }
    }
    return idx == sz(sub);
}

//quadratic_memory
int lcs(const string& s, const string& t) {
    int n = sz(s);
    int m = sz(t);
    vector<vector<int>> dp(n + 1, vector<int>(m + 1, 0));
    forn(i, n) {
        forn(j, m) {
            dp[i + 1][j + 1] = max({ dp[i + 1][j], dp[i][j]
                + 1, dp[i][j] + (s[i] == t[j]) });
        }
    }
}
```

```
return dp[n][m];
}

//best
int lcs(const string& s, const string& t) {
    int n = sz(s);
    int m = sz(t);
    vector<int> dp(m + 1, 0);
    forn(i, n) {
        vector<int> newdp(m + 1, 0);
        forn(j, m) {
            newdp[j + 1] = max({ newdp[j], dp[j + 1], dp[j]
                + (s[i] == t[j]) });
        }
        dp.swap(newdp);
    }
    return dp[m];
}

//construct lcs
string clcs(const string& s, const string& t) {
    int n = sz(s);
    int m = sz(t);
    vector<int> dp(m + 1, 0);
    vector<vector<bool>> pre(n + 1, vector<bool>(m + 1,
        false));
    forn(i, n) {
        vector<int> newdp(m + 1, 0);
        forn(j, m) {
            newdp[j + 1] = max({ newdp[j], dp[j + 1], dp[j]
                + (s[i] == t[j]) });
            pre[i + 1][j + 1] = newdp[j + 1] == newdp[j];
        }
        dp.swap(newdp);
    }
    int a = n, b = m;
    string common;
    while (a > 0 && b > 0) {
        if (s[a - 1] == t[b - 1]) {
            common += s[a - 1];
            a--; b--;
            continue;
        }
    }
```



```

        if (pre[a][b]) b--;
        else a--;
    }
    reverse(all(common));
    return common;
}

//best: construct lcs with Hirschberg Algorithm
string clcsh(const string_view& s, const string_view& t) {
    int n = sz(s), int m = sz(t);
    if (n == 0 || m == 0) return "";
    if (n == 1) return t.find(s[0]) == string::npos ? "" :
        string(1, s[0]);
    int mid = n >> 1;
    vector<int> dp_ff(m + 1, 0);
    vector<int> dp_ss(m + 1, 0);
    vector<int> newdp(m + 1, 0);

    forn(i, mid) {
        forn(j, m) newdp[j + 1] = max({ newdp[j], dp_ff[j + 1], dp_ff[j] + (s[i] == t[j]) });
        dp_ff.swap(newdp);
    }

    newdp.assign(m + 1, 0);
    for (int i = n - 1; i >= mid; i--) {
        for (int j = m - 1; j >= 0; j--) {
            newdp[j] = max({ newdp[j + 1], dp_ss[j], dp_ss[j + 1] + (s[i] == t[j]) });
        }
        dp_ss.swap(newdp);
    }

    int splt = 0;
    forne(j, 1, m + 1) {
        if (dp_ff[j] + dp_ss[j] > dp_ff[splt] + dp_ss[splt]) {
            splt = j;
        }
    }

    dp_ff.clear();
    dp_ss.clear();
    newdp.clear();
    return (clcsh(s.substr(0, mid), t.substr(0, splt)) +

```

```

        clcsh(s.substr(mid), t.substr(splt)));
    }

// lcs con tolerncia de 1% de eliminaciones al inicio
int lcs(const string& s, const string& t) {
    int n = sz(s);
    int poda = (n * 1) / 100 + 1;
    int ans = 0;
    vector<vector<int>> dp(poda + 1, vector<int>(poda + 1, 0));
    forn(i, poda + 1) {
        forn(j, poda + 1) {
            while (i + dp[i][j] < n && j + dp[i][j] < n && s[i + dp[i][j]] == t[j + dp[i][j]])
                dp[i][j]++;
            if (i + 1 <= poda) dp[i + 1][j] = max(dp[i + 1][j], dp[i][j]);
            if (j + 1 <= poda) dp[i][j + 1] = max(dp[i][j + 1], dp[i][j]);
            ans = max(ans, dp[i][j]);
        }
    }
    return ans;
}

```

## 2.10. Subsequences

```

struct mint {
    static constexpr int m = 1e9 + 7;
    //static inline int m = 998244353; //to change mod
    int x;
    mint() : x(0) {}
    mint(long long x_) : x(x_% m) { if (x < 0) x += m; }
    int val() { return x; }
    mint& operator+=(mint b) { if ((x += b.x) >= m) x -= m;
        return *this; }
    mint& operator-=(mint b) { if ((x -= b.x) < 0) x += m;
        return *this; }
    mint& operator*=(mint b) { x = (long long)(x)*b.x % m;
        return *this; }
    mint pow(long long e) const {
        mint r = 1, b = *this;

```

```

        while (e) {
            if (e & 1) r *= b;
            b *= b;
            e >>= 1;
        }
        return r;
    }
    mint inv() { return pow(m - 2); }
    mint& operator/=(mint b) { return *this *= b.pow(m - 2); }
    friend mint operator+(mint a, mint b) { return a += b; }
    friend mint operator-(mint a, mint b) { return a -= b; }
    friend mint operator/(mint a, mint b) { return a /= b; }
    friend mint operator*(mint a, mint b) { return a *= b; }
    friend bool operator<(mint a, mint b) { return a.x < b.x; }
    friend bool operator==(mint a, mint b) { return a.x == b.x; }
    friend bool operator!=(mint a, mint b) { return a.x != b.x; }
};
// Find the number of distinct subsequences of a given string.
// distinct subsequences ending at each of the 26 letters of the alphabet.
template<typename T>int distinctsub(const T& sub) {
    int n = sz(sub);
    vector<mint> dp(n + 1, 0);
    vector<int> last(26, -1);
    // vector<mint> end_count(26, 0);
    dp[0] = 1;
    forn(i, n) {
        dp[i + 1] += 2 * dp[i];
        // end_count[sub[i] - 'a'] += dp[i];
        if (~last[sub[i] - 'a']) {
            dp[i + 1] -= dp[last[sub[i] - 'a']];
            // end_count[sub[i] - 'a'] -= dp[last[sub[i] - 'a']];
        }
        last[sub[i] - 'a'] = i;
    }
    return dp[n].x - 1;
}

```

```

// find the number of distinct subsequences of a given string.
// number of distinct subsequences of each length from 1 to n
// number of distinct subsequences of size i -> dp[n][i]
template<typename T>int distinctsub(const T& sub) {
    int n = sz(sub);
    vector<vector<mint>> dp(n + 1, vector<mint>(n + 1, 0));
    dp[0][0] = 1;
    vector<int> last(26, -1);
    // vector<mint> end_count(26, 0);
    forn(i, n) {
        forn(j, i + 1) {
            dp[i + 1][j + 1] = dp[i][j];
            dp[i + 1][j] += dp[i][j];
            // end_count[sub[i] - 'a'] += dp[i][j].x;
        }
        if (~last[sub[i] - 'a']) {
            forn(j, i + 1) {
                dp[i + 1][j + 1] -= dp[last[sub[i] - 'a']][j];
                // end_count[sub[i] - 'a'] -= dp[last[sub[i] - 'a']][j].x;
            }
        }
        last[sub[i] - 'a'] = i;
    }

    mint ans = 0;
    forne(i, 1, n + 1) ans += dp[n][i];
    return ans.x;
}

```

### 3. Flows

#### 3.1. Dinic

```

constexpr int INF = ((1ULL << 63) - 1) >> 1;

struct Dinic {

```

```

const bool scaling = 0;      // con scaling -> O(nm
    log(MAXCAP)),
int lim;                      // con constante alta
struct edge {
    int to, cap, rev, flow;
    bool res;
    edge(int to_, int cap_, int rev_, bool res_) :
        to(to_), cap(cap_), rev(rev_), flow(0),
        res(res_) {}
};

vector<vector<edge>> g;
vector<int> lev, beg;
int64_t F;
Dinic(int n) : g(n), F(0) {}

void add(int a, int b, int c) {
    g[a].eb(b, c, sz(g[b]), 0);
    g[b].eb(a, 0, sz(g[a]) - 1, 1);
}

bool bfs(int s, int t) {
    lev = vector<int>(sz(g), -1); lev[s] = 0;
    beg = vector<int>(sz(g), 0);
    queue<int> q; q.push(s);
    while (sz(q)) {
        int u = q.front(); q.pop();
        for (auto& i : g[u]) {
            if (lev[i.to] != -1 or (i.flow == i.cap))
                continue;
            if (scaling and i.cap - i.flow < lim)
                continue;
            lev[i.to] = lev[u] + 1;
            q.push(i.to);
        }
    }
    return lev[t] != -1;
}

int dfs(int v, int s, int f = INF) {
    if (!f or v == s) return f;
    for (int& i = beg[v]; i < sz(g[v]); i++) {
        auto& e = g[v][i];
        if (lev[e.to] != lev[v] + 1) continue;
        int foi = dfs(e.to, s, min(f, e.cap - e.flow));

```

```

        if (!foi) continue;
        e.flow += foi, g[e.to][e.rev].flow -= foi;
        return foi;
    }
    return 0;
}

int64_t max_flow(int s, int t) {
    for (lim = scaling ? (1 << 30) : 1; lim; lim /= 2)
        while (bfs(s, t)) while (int ff = dfs(s, t)) F
            += ff;
    return F;
}

// Recupera las aristas del corte s-t
vector<pair<int, int>> get_cut(Dinic& g, int s, int t) {
    g.max_flow(s, t);
    vector<pair<int, int>> cut;
    vector<int> vis(sz(g.g), 0), st = { s };
    vis[s] = 1;
    while (sz(st)) {
        int u = st.back(); st.pop_back();
        for (auto e : g.g[u]) if (!vis[e.to] and e.flow <
            e.cap)
            vis[e.to] = 1, st.pb(e.to);
    }
    for (int i = 0; i < sz(g.g); i++) for (auto e : g.g[i])
        if (vis[i] and !vis[e.to] and !e.res) cut.eb(i,
            e.to);
    return cut;
}

```

### 3.2. Blossom

```

struct Blossom { // O(E * V^2)
    struct struct_edge { int v; struct_edge* n; };
    typedef struct_edge* edge;
    int n;
    struct_edge pool[MAXE]; // 2 * n * n;
    edge top;
    vector<edge> g;

```

```

queue<int> q;
vector<int> f, base, inq, inb, inp, match;
vector<vector<int>> ed;

Blossom(int n) :
    n(n), match(n, -1), g(n), top(pool),
    f(n), base(n), inq(n), inb(n), inp(n),
    ed(n, vector<int>(n)) {}

void add_edge(int u, int v) {
    if (ed[u][v]) return;
    ed[u][v] = 1;
    top->v = v, top->n = g[u], g[u] = top++;
    top->v = u, top->n = g[v], g[v] = top++;
}

int get_lca(int root, int u, int v) {
    fill(all(inp), 0);
    while (1) {
        inp[u = base[u]] = 1;
        if (u == root) break;
        u = f[match[u]];
    }
    while (1) {
        if (inp[v = base[v]]) return v;
        else v = f[match[v]];
    }
}

void mark(int lca, int u) {
    while (base[u] != lca) {
        int v = match[u];
        inb[base[u]] = 1;
        inb[base[v]] = 1;
        u = f[v];
        if (base[u] != lca) f[u] = v;
    }
}

void blossom_contraction(int s, int u, int v) {
    int lca = get_lca(s, u, v);
    fill(all(inb), 0);

```

```

    mark(lca, u); mark(lca, v);
    if (base[u] != lca) f[u] = v;
    if (base[v] != lca) f[v] = u;
    forn(u, n) {
        if (inb[base[u]]) {
            base[u] = lca;
            if (!inq[u]) {
                inq[u] = 1;
                q.push(u);
            }
        }
    }
}

int bfs(int s) {
    fill(all(inq), 0);
    fill(all(f), -1);
    forn(i, n) base[i] = i;
    q = queue<int>();
    q.push(s);
    inq[s] = 1;
    while (sz(q)) {
        int u = q.front(); q.pop();
        for (edge e = g[u]; e; e = e->n) {
            int v = e->v;
            if (base[u] != base[v] && match[u] != v) {
                if ((v == s) || (match[v] != -1 &&
                    f[match[v]] != -1))
                    blossom_contraction(s, u, v);
                else if (f[v] == -1) {
                    f[v] = u;
                    if (match[v] == -1) return v;
                    else if (!inq[match[v]]) {
                        inq[match[v]] = 1;
                        q.push(match[v]);
                    }
                }
            }
        }
    }
    return -1;
}

```

```

int doit(int u) {
    if (u == -1) return 0;
    int v = f[u];
    doit(match[v]);
    match[v] = u; match[u] = v;
    return u != -1;
}

int matching() {
    int ans = 0;
    forn(u, n)
        ans += (match[u] == -1) && doit(bfs(u));
    return ans;
}

// (i < net.match[i]) => means match
vector<pair<int,int>> get_edges() {
    vector<pair<int,int>> ans;
    forn(u, n) if (u < match[u])
        ans.pb({ u, match[u] });
    return ans;
}
};

```

### 3.3. Hopcroft Karp

```

struct mbm { // O(E * sqrt(V))
    int nl, nr, flow = 0;
    vector<vector<int>> g;
    vector<int> dist, mfl, mfr;

    mbm(int nl, int nr) :
        nl(nl), nr(nr), g(nl), mfl(nl, -1),
        mfr(nr, -1), dist(nl) {}

    void add(int u, int v) { g[u].pb(v); }

    void bfs() {
        queue<int> q;
        forn(u, nl)
            if (!mfl[u]) q.push(u), dist[u] = 0;
    }
};

```

```

        else dist[u] = -1;
    while (sz(q)) {
        int u = q.front();
        q.pop();
        for (auto& v : g[u])
            if (!mfr[v] && !dist[mfr[v]]) {
                dist[mfr[v]] = dist[u] + 1;
                q.push(mfr[v]);
            }
    }
}

bool dfs(int u) {
    for (auto& v : g[u])
        if (!mfr[v]) {
            mfl[u] = v, mfr[v] = u;
            return true;
        }
    for (auto& v : g[u])
        if (dist[mfr[v]] == dist[u] + 1 && dfs(mfr[v]))
            {
                mfl[u] = v, mfr[v] = u;
                return true;
            }
    return false;
}

int get_matching() {
    while (true) {
        bfs();
        int agt = 0;
        forn(u, nl)
            if (!mfl[u]) agt += dfs(u);
        if (!agt) break;
        flow += agt;
    }
    return flow;
}

pair<vector<int>, vector<int>> MVC() {
    vector<int> L, R;
    forn(u, nl)
        if (!dist[u]) L.pb(u);
}

```

```

        else if (~mfl[u]) R.pb(mfl[u]);
    return { L, R };
}

vector<pair<int,int>> get_edges() {
    vector<pair<int,int>> ans;
    forn(u, nl)
        if (mfl[u] != -1)
            ans.pb({ u, mfl[u] });
    return ans;
}
};

```

### 3.4. Matching

```

struct mbm { // O(V * E)
    int l, r;
    vector<int> mat;
    vector<bool> vis;
    vector<vector<int>> g;

    mbm(int l, int r) : l(l), r(r), mat(r), vis(l), g(l) {}

    bool match(int v) {
        if (vis[v]) return false;
        vis[v] = true;
        for (int& u : g[v]) {
            if (mat[u] == -1 || match(mat[u])) {
                mat[u] = v;
                return true;
            }
        }
        return false;
    }

    vector<pair<int,int>> matching() {
        vector<pair<int,int>> ans;
        fill(all(mat), -1);
        forn(i, l) {
            fill(all(vis), false);
            match(i);
        }
    }
};

```

```

        forn(i, r) if (~mat[i]) ans.pb({ mat[i], i });
    return ans;
}
};

```

### 3.5. Maximum flow minimum cost

```

struct mcmf {
    const ll INF = LONG_LONG_MAX;
    struct Edge { int to, rev; ll flo, cap, cost; };
    int n;
    vector<ll> p, dist;
    vector<pair<int, int>> pre;
    vector<vector<Edge>> g;

    mcmf(int m) : n(m), p(n), dist(n), pre(n), g(n) {}

    void add_edge(int v, int u, ll cap, ll cost) {
        g[v].pb({ u, sz(g[u]), 0, cap, cost });
        g[u].pb({ v, sz(g[v]) - 1, 0, 0, -cost });
    }

    bool path(int s, int t) {
        dist.assign(n, INF);
        using T = pair<ll, int>;
        priority_queue<T, vector<T>, greater<T>> todo;
        todo.push({ dist[s] = 0, s });
        while (sz(todo)) {
            T x = todo.top(); todo.pop();
            if (x.f > dist[x.s]) continue;
            for (auto& e : g[x.s]) {
                if (e.flo < e.cap && dist[e.to] > x.f +
                    e.cost + p[x.s] - p[e.to]) {
                    dist[e.to] = x.f + e.cost + p[x.s] -
                        p[e.to];
                    pre[e.to] = { x.s, e.rev };
                    todo.push({ dist[e.to], e.to });
                }
            }
        }
        return dist[t] != INF;
    }
};

```

```

pair<ll, ll> calc(int s, int t) {
    forn(_, n) forn(i, n) for (auto& e : g[i])
        if (e.cap) p[e.to] = min(p[e.to], p[i] +
            e.cost);
    ll totFlow = 0, totCost = 0;
    while (path(s, t)) {
        forn(i, n) p[i] += dist[i];
        ll df = INF;
        for (int x = t; x != s; x = pre[x].f) {
            Edge& e = g[pre[x].f][g[x][pre[x].s].rev];
            df = min(df, e.cap - e.flo);
        }
        totFlow += df; totCost += (p[t] - p[s]) * df;
        for (int x = t; x != s; x = pre[x].f) {
            Edge& e = g[x][pre[x].s]; e.flo -= df;
            g[pre[x].f][e.rev].flo += df;
        }
    }
    return { totFlow, totCost };
}
};

```

### 3.6. Hungarian

```

template<typename T>
struct Hungarian { // O(V^3)
    int n, m;
    const T inf = 1e18;
    vector<T> u, v; vector<int> p, way;
    vector<vector<T>> g;

    Hungarian(int n, int m) :
        n(n), m(m), g(n + 1, vector<T>(m + 1, inf - 1)),
        u(n + 1), v(m + 1), p(m + 1), way(m + 1) {}

    void set(int u, int v, T w) { g[u + 1][v + 1] = w; }

    T assign() {
        forne(i, 1, n + 1) {
            int j0 = 0; p[0] = i;

```

```

        vector<T> minv(m + 1, inf);
        vector<char> used(m + 1, false);
        do {
            used[j0] = true;
            int i0 = p[j0], j1; T delta = inf;
            forne(j, 1, m + 1) if (!used[j]) {
                T cur = g[i0][j] - u[i0] - v[j];
                if (cur < minv[j]) minv[j] = cur,
                    way[j] = j0;
                if (minv[j] < delta) delta = minv[j],
                    j1 = j;
            }
            forn(j, m + 1)
                if (used[j]) u[p[j]] += delta, v[j] -=
                    delta;
            else minv[j] -= delta;
            j0 = j1;
        } while (p[j0]);
        do {
            int j1 = way[j0]; p[j0] = p[j1]; j0 = j1;
        } while (j0);
    }
    return -v[0];
}
};

```

## 4. Geometry

### 4.1. GeometriaInt

```

#define sq(x) ((x)*(x))

struct pt { // punto
    int x, y;
    pt(int x_ = 0, int y_ = 0) : x(x_), y(y_) {}
    bool operator < (const pt p) const {
        if (x != p.x) return x < p.x;
        return y < p.y;
    }
    bool operator == (const pt p) const {

```

```

        return x == p.x and y == p.y;
    }
    pt operator + (const pt p) const { return pt(x + p.x, y
+ p.y); }
    pt operator - (const pt p) const { return pt(x - p.x, y
- p.y); }
    pt operator * (const int c) const { return pt(x * c, y
* c); }
    ll operator * (const pt p) const { return x * (ll)p.x +
y * (ll)p.y; }
    ll operator ^ (const pt p) const { return x * (ll)p.y -
y * (ll)p.x; }
    friend istream& operator >> (istream& in, pt& p) {
        return in >> p.x >> p.y;
    }
};

struct line { // recta
    pt p, q;
    line() {}
    line(pt p_, pt q_) : p(p_), q(q_) {}
    friend istream& operator >> (istream& in, line& r) {
        return in >> r.p >> r.q;
    }
};

// PONTO & VETOR

ll dist2(pt p, pt q) { // cuadrado de la distancia
    return sq(p.x - q.x) + sq(p.y - q.y);
}

ll sarea2(pt p, pt q, pt r) { // 2 * area con signo
    return (q - p) ^ (r - p);
}

bool col(pt p, pt q, pt r) { // si p, q y r son colineales
    return sarea2(p, q, r) == 0;
}

bool ccw(pt p, pt q, pt r) { // si p, q, r estan en sentido
    antihorario
    return sarea2(p, q, r) > 0;
}

```

```

}

int quad(pt p) { // cuadrante de un punto
    return (p.x < 0) ^ 3 * (p.y < 0);
}

bool compare_angle(pt p, pt q) { // retorna si ang(p) <
    ang(q)
    if (quad(p) != quad(q)) return quad(p) < quad(q);
    return ccw(q, pt(0, 0), p);
}

pt rotate90(pt p) { // rota 90 grados
    return pt(-p.y, p.x);
}

// RETA

bool isinseg(pt p, line r) { // si p pertenece al segmento
    de r
    pt a = r.p - p, b = r.q - p;
    return (a ^ b) == 0 and (a * b) <= 0;
}

bool interseg(line r, line s) { // si el segmento de r
    interseca el segmento de s
    if (isinseg(r.p, s) or isinseg(r.q, s)
        or isinseg(s.p, r) or isinseg(s.q, r)) return 1;

    return ccw(r.p, r.q, s.p) != ccw(r.p, r.q, s.q) and
        ccw(s.p, s.q, r.p) != ccw(s.p, s.q, r.q);
}

int segpoints(line r) { // numero de puntos enteros en el
    segmento
    return 1 + __gcd(abs(r.p.x - r.q.x), abs(r.p.y -
        r.q.y));
}

double get_t(pt v, line r) { // retorna t tal que t*v
    pertenece a la recta r
    return (r.p ^ r.q) / (double)((r.p - r.q) ^ v);
}

```



```

// POLIGONO

// cuadrado de la distancia entre los rectangulos a y b
// (lados paralelos a los ejes)
// asume que esta representado (inferior izquierdo,
// superior derecho)
ll dist2_rect(pair<pt, pt> a, pair<pt, pt> b) {
    int hor = 0, vert = 0;
    if (a.second.x < b.first.x) hor = b.first.x -
        a.second.x;
    else if (b.second.x < a.first.x) hor = a.first.x -
        b.second.x;
    if (a.second.y < b.first.y) vert = b.first.y -
        a.second.y;
    else if (b.second.y < a.first.y) vert = a.first.y -
        b.second.y;
    return sq(hor) + sq(vert);
}

ll polarea2(vector<pt> v) { // 2 * area del poligono
    ll ret = 0;
    for (int i = 0; i < v.size(); i++)
        ret += sarea2(pt(0, 0), v[i], v[(i + 1) %
            v.size()]);
    return abs(ret);
}

// si el punto esta dentro del poligono: retorna 0 si esta
// afuera,
// 1 si esta en el interior y 2 si esta en el borde
int inpol(vector<pt>& v, pt p) { // O(n)
    int qt = 0;
    for (int i = 0; i < v.size(); i++) {
        if (p == v[i]) return 2;
        int j = (i + 1) % v.size();
        if (p.y == v[i].y and p.y == v[j].y) {
            if ((v[i] - p) * (v[j] - p) <= 0) return 2;
            continue;
        }
        bool abajo = v[i].y < p.y;
        if (abajo == (v[j].y < p.y)) continue;
        auto t = (p - v[i]) ^ (v[j] - v[i]);

```

```

        if (!t) return 2;
        if (abajo == (t > 0)) qt += abajo ? 1 : -1;
    }
    return qt != 0;
}

vector<pt> convex_hull(vector<pt> v) { // envolvente
    convexa - O(n log(n))
    sort(v.begin(), v.end());
    v.erase(unique(v.begin(), v.end()), v.end());
    if (v.size() <= 1) return v;
    vector<pt> l, u;
    for (int i = 0; i < v.size(); i++) {
        while (l.size() > 1 and !ccw(l.end()[-2],
            l.end()[-1], v[i]))
            l.pop_back();
        l.push_back(v[i]);
    }
    for (int i = v.size() - 1; i >= 0; i--) {
        while (u.size() > 1 and !ccw(u.end()[-2],
            u.end()[-1], v[i]))
            u.pop_back();
        u.push_back(v[i]);
    }
    l.pop_back(); u.pop_back();
    for (pt i : u) l.push_back(i);
    return l;
}

ll interior_points(vector<pt> v) { // puntos enteros dentro
    de un poligono simple
    ll b = 0;
    for (int i = 0; i < v.size(); i++)
        b += segpoints(line(v[i], v[(i + 1) % v.size()])) -
            1;
    return (polarea2(v) - b) / 2 + 1;
}

struct convex_pol {
    vector<pt> pol;

    // no puede tener punto colineal en el convex hull
    convex_pol() {}

```

```

convex_pol(vector<pt> v) : pol(convex_hull(v)) {}

// si el punto esta dentro del hull - O(log(n))
bool is_inside(pt p) {
    if (pol.size() == 0) return false;
    if (pol.size() == 1) return p == pol[0];
    int l = 1, r = pol.size();
    while (l < r) {
        int m = (l + r) / 2;
        if (ccw(p, pol[0], pol[m])) l = m + 1;
        else r = m;
    }
    if (l == 1) return isinseg(p, line(pol[0], pol[1]));
    if (l == pol.size()) return false;
    return !ccw(p, pol[l], pol[l - 1]);
}

// punto extremo en relacion a cmp(p, q) = p mas
// extremo q
int extreme(const function<bool(pt, pt)>& cmp) {
    int n = pol.size();
    auto extr = [&](int i, bool& cur_dir) {
        cur_dir = cmp(pol[(i + 1) % n], pol[i]);
        return !cur_dir and !cmp(pol[(i + n - 1) % n],
            pol[i]);
    };
    bool last_dir, cur_dir;
    if (extr(0, last_dir)) return 0;
    int l = 0, r = n;
    while (l + 1 < r) {
        int m = (l + r) / 2;
        if (extr(m, cur_dir)) return m;
        bool rel_dir = cmp(pol[m], pol[l]);
        if ((!last_dir and cur_dir) or
            (last_dir == cur_dir and rel_dir ==
                cur_dir)) {
            l = m;
            last_dir = cur_dir;
        }
        else r = m;
    }
    return l;
}

int max_dot(pt v) {

```

```

    return extreme([&](pt p, pt q) { return p * v > q *
        v; });
}

pair<int, int> tangents(pt p) {
    auto L = [&](pt q, pt r) { return ccw(p, r, q); };
    auto R = [&](pt q, pt r) { return ccw(p, q, r); };
    return { extreme(L), extreme(R) };
}

bool operator <(const line& a, const line& b) { //
    comparador para recta
    // asume que las rectas tienen p < q
    pt v1 = a.q - a.p, v2 = b.q - b.p;
    bool b1 = compare_angle(v1, v2), b2 = compare_angle(v2,
        v1);
    if (b1 or b2) return b1;
    return ccw(a.p, a.q, b.p); // mismo angulo
}

bool operator ==(const line& a, const line& b) {
    return !(a < b) and !(b < a);
}

// comparador para set para hacer sweep line con segmentos
struct cmp_sweepline {
    bool operator () (const line& a, const line& b) const {
        // asume que los segmentos tienen p < q
        if (a.p == b.p) return ccw(a.p, a.q, b.q);
        if (a.p.x != a.q.x and (b.p.x == b.q.x or a.p.x <
            b.p.x))
            return ccw(a.p, a.q, b.p);
        return ccw(a.p, b.q, b.p);
    }
};

// comparador para set para hacer sweep angle con segmentos
pt dir;
struct cmp_sweepangle {
    bool operator () (const line& a, const line& b) const {
        return get_t(dir, a) < get_t(dir, b);
    }
};

```

## 4.2. sweep line

```
/*
0(nlogn)
Par de puntos cuya distancia es la mas corta
ans = idx de los puntos en el vector de puntos dado
best = la mejor distancia entre dos puntos (la mas corta)
*/

struct P {
    //double para puntos con decimales ej (x , y) ->
    //      (1.234, 2.341)
    double x, y;
    int id;

    //int x, y, id;
};

struct Cx {
    bool operator()(const P& a, const P& b) const {
        return a.x < b.x || (a.x == b.x && a.y < b.y);
    }
};

struct Cy {
    bool operator()(const P& a, const P& b) const {
        return a.y < b.y;
    }
};

int n;
vector<P> a, buf;
double best;
pair<int, int> ans = { -1, -1 };

inline void upd(const P& u, const P& v) {
    double dx = u.x - v.x, dy = u.y - v.y;
    double d = sqrt(dx * dx + dy * dy);
    if (d < best) { best = d; ans = { u.id, v.id }; }
}

void rec(int l, int r) {
    if (r - l <= 3) {
        for (int i = l; i < r; i++)
            for (int j = i + 1; j < r; j++)
```

```
                upd(a[i], a[j]);
        sort(a.begin() + l, a.begin() + r, Cy());
        return;
    }
    int m = (l + r) >> 1;
    //double para puntos con decimales ej (x , y) ->
    //      (1.234, 2.341)
    double midx = a[m].x;
    //int midx = a[m].x;
    rec(l, m);
    rec(m, r);
    merge(a.begin() + l, a.begin() + m, a.begin() + m,
          a.begin() + r, buf.begin(), Cy());
    copy(buf.begin(), buf.begin() + (r - l), a.begin() + l);
    int sz = 0;
    for (int i = l; i < r; i++) {
        if (fabs(double(a[i].x) - midx) < best) {
            for (int j = sz - 1, k = 0; j >= 0 && k < 8;
                --j, ++k) {
                if ((a[i].y - buf[j].y) >= best) break;
                upd(a[i], buf[j]);
            }
            buf[sz++] = a[i];
        }
    }

}

//test con enteros

void test() {
    cin >> n;
    a.resize(n);
    buf.resize(n);
    for (int i = 0; i < n; i++) {
        cin >> a[i].x >> a[i].y;
        a[i].id = i;
    }
    sort(all(a), Cx());
    best = 1e18;
    rec(0, n);
    int i = ans.f, j = ans.s;
    if (i > j) {
        swap(i, j);
    }
}
```

```

    }
    cout << i << ' ' << j << ' ' << fixed <<
        setprecision(6) << best << '\n';
}

//test con decimales

void testd() {
    int n;
    while (cin >> n) {
        if (n == 0) {
            break;
        }
        a.resize(n);
        buf.resize(n);
        vector<pair< string, string >> tempA;
        for (int i = 0; i < n; i++) {
            string sx, sy; cin >> sx >> sy;
            P p;
            p.x = stod(sx);
            p.y = stod(sy);
            p.id = i;
            a[i] = p;
            tempA.pb({ sx, sy });
        }
        buf.assign(n, {});
        sort(all(a), Cx());
        best = 1e300;
        ans = { 0, 0 };
        rec(0, n);
        int idx1 = ans.f, idx2 = ans.s;
        cout << tempA[idx1].f << ' ' << tempA[idx1].s << '
            ' << tempA[idx2].f << ' ' << tempA[idx2].s <<
            endl;
    }
}

```

### 4.3. Polygon

```

// corta poligono con la recta r dejando los puntos p tal
// que
// ccw(r.p, r.q, p)
vector<pt> cut_polygon(vector<pt> v, line r) { // O(n)
    vector<pt> ret;
    for (int j = 0; j < v.size(); j++) {
        if (ccw(r.p, r.q, v[j])) ret.push_back(v[j]);
        if (v.size() == 1) continue;
        line s(v[j], v[(j + 1) % v.size()]);
        pt p = inter(r, s);
        if (isinseg(p, s)) ret.push_back(p);
    }
    ret.erase(unique(ret.begin(), ret.end()), ret.end());
    if (ret.size() > 1 and ret.back() == ret[0])
        ret.pop_back();
    return ret;
}

// distancia entre los rectangulos a y b (lados paralelos a
// los ejes)
// asume que esta representado (inferior izquierdo,
// superior derecho)
ld dist_rect(pair<pt, pt> a, pair<pt, pt> b) {
    ld hor = 0, vert = 0;
    if (a.second.x < b.first.x) hor = b.first.x -
        a.second.x;
    else if (b.second.x < a.first.x) hor = a.first.x -
        b.second.x;
    if (a.second.y < b.first.y) vert = b.first.y -
        a.second.y;
    else if (b.second.y < a.first.y) vert = a.first.y -
        b.second.y;
    return dist(pt(0, 0), pt(hor, vert));
}

ld polarea(vector<pt> v) { // area del poligono
    ld ret = 0;
    for (int i = 0; i < v.size(); i++)
        ret += sarea(pt(0, 0), v[i], v[(i + 1) % v.size()]);
    return abs(ret);
}

// si el punto esta dentro del poligono: retorna 0 si esta

```

```

    afuera,
// 1 si esta en el interior y 2 si esta en el borde
int inpol(vector<pt>& v, pt p) { // O(n)
    int qt = 0;
    for (int i = 0; i < v.size(); i++) {
        if (p == v[i]) return 2;
        int j = (i + 1) % v.size();
        if (eq(p.y, v[i].y) and eq(p.y, v[j].y)) {
            if ((v[i] - p) * (v[j] - p) < eps) return 2;
            continue;
        }
        bool abajo = v[i].y + eps < p.y;
        if (abajo == (v[j].y + eps < p.y)) continue;
        auto t = (p - v[i]) ^ (v[j] - v[i]);
        if (eq(t, 0)) return 2;
        if (abajo == (t > eps)) qt += abajo ? 1 : -1;
    }
    return qt != 0;
}

bool interpol(vector<pt> v1, vector<pt> v2) { // si dos
    poligonos se intersectan - O(n*m)
    int n = v1.size(), m = v2.size();
    for (int i = 0; i < n; i++) if (inpol(v2, v1[i]))
        return 1;
    for (int i = 0; i < n; i++) if (inpol(v1, v2[i]))
        return 1;
    for (int i = 0; i < n; i++) for (int j = 0; j < m; j++)
        if (interseg(line(v1[i], v1[(i + 1) % n]),
            line(v2[j], v2[(j + 1) % m]))) return 1;
    return 0;
}

ld distpol(vector<pt> v1, vector<pt> v2) { // distancia
    entre poligonos
    if (interpol(v1, v2)) return 0;

    ld ret = DINF;

    for (int i = 0; i < v1.size(); i++) for (int j = 0; j <
        v2.size(); j++)
        ret = min(ret, distseg(line(v1[i], v1[(i + 1) %
            v1.size()]),

```

```

            line(v2[j], v2[(j + 1) % v2.size()]));
    return ret;
}

vector<pt> convex_hull(vector<pt> v) { // envolvente
    convexa - O(n log(n))
    sort(v.begin(), v.end());
    v.erase(unique(v.begin(), v.end()), v.end());
    if (v.size() <= 1) return v;
    vector<pt> l, u;
    for (int i = 0; i < v.size(); i++) {
        while (l.size() > 1 and !ccw(l.end()[-2],
            l.end()[-1], v[i]))
            l.pop_back();
        l.push_back(v[i]);
    }
    for (int i = v.size() - 1; i >= 0; i--) {
        while (u.size() > 1 and !ccw(u.end()[-2],
            u.end()[-1], v[i]))
            u.pop_back();
        u.push_back(v[i]);
    }
    l.pop_back(); u.pop_back();
    for (pt i : u) l.push_back(i);
    return l;
}

struct convex_pol {
    vector<pt> pol;

    // no puede tener punto colineal en el convex hull
    convex_pol() {}
    convex_pol(vector<pt> v) : pol(convex_hull(v)) {}

    // si el punto esta dentro del hull - O(log(n))
    bool is_inside(pt p) {
        if (pol.size() == 0) return false;
        if (pol.size() == 1) return p == pol[0];
        int l = 1, r = pol.size();
        while (l < r) {
            int m = (l + r) / 2;
            if (ccw(p, pol[0], pol[m])) l = m + 1;
            else r = m;
        }
    }

```

```

    }
    if (l == 1) return isinseg(p, line(pol[0], pol[1]));
    if (l == pol.size()) return false;
    return !ccw(p, pol[l], pol[l - 1]);
}
// punto extremo en relacion a cmp(p, q) = p mas
// extremo q
int extreme(const function<bool(pt, pt)>& cmp) {
    int n = pol.size();
    auto extr = [&](int i, bool& cur_dir) {
        cur_dir = cmp(pol[(i + 1) % n], pol[i]);
        return !cur_dir and !cmp(pol[(i + n - 1) % n],
            pol[i]);
    };
    bool last_dir, cur_dir;
    if (extr(0, last_dir)) return 0;
    int l = 0, r = n;
    while (l + 1 < r) {
        int m = (l + r) / 2;
        if (extr(m, cur_dir)) return m;
        bool rel_dir = cmp(pol[m], pol[l]);
        if ((!last_dir and cur_dir) or
            (last_dir == cur_dir and rel_dir ==
                cur_dir)) {
            l = m;
            last_dir = cur_dir;
        }
        else r = m;
    }
    return l;
}
int max_dot(pt v) {
    return extreme([&](pt p, pt q) { return p * v > q *
        v; });
}
pair<int, int> tangents(pt p) {
    auto L = [&](pt q, pt r) { return ccw(p, r, q); };
    auto R = [&](pt q, pt r) { return ccw(p, q, r); };
    return { extreme(L), extreme(R) };
}
};

// CIRCUNFERENCIA

```

```

pt getcenter(pt a, pt b, pt c) { // centro de la circunf
    dado 3 puntos
    b = (a + b) / 2;
    c = (a + c) / 2;
    return inter(line(b, b + rotate90(a - b)),
        line(c, c + rotate90(a - c)));
}

vector<pt> circ_line_inter(pt a, pt b, pt c, ld r) { //
    interseccion de la circunf (c, r) y recta ab
    vector<pt> ret;
    b = b - a, a = a - c;
    ld A = b * b;
    ld B = a * b;
    ld C = a * a - r * r;
    ld D = B * B - A * C;
    if (D < -eps) return ret;
    ret.push_back(c + a + b * (-B + sqrt(D + eps)) / A);
    if (D > eps) ret.push_back(c + a + b * (-B - sqrt(D)) /
        A);
    return ret;
}

vector<pt> circ_inter(pt a, pt b, ld r, ld R) { //
    interseccion de la circunf (a, r) y (b, R)
    vector<pt> ret;
    ld d = dist(a, b);
    if (d > r + R or d + min(r, R) < max(r, R)) return ret;
    ld x = (d * d - R * R + r * r) / (2 * d);
    ld y = sqrt(r * r - x * x);
    pt v = (b - a) / d;
    ret.push_back(a + v * x + rotate90(v) * y);
    if (y > 0) ret.push_back(a + v * x - rotate90(v) * y);
    return ret;
}

bool operator <(const line& a, const line& b) { //
    comparador para recta
    // asume que las rectas tienen p < q
    pt v1 = a.q - a.p, v2 = b.q - b.p;
    if (!eq(angle(v1), angle(v2))) return angle(v1) <
        angle(v2);
}

```

```

    return ccw(a.p, a.q, b.p); // mismo angulo
}
bool operator ==(const line& a, const line& b) {
    return !(a < b) and !(b < a);
}

// comparador para set para hacer sweep line con segmentos
struct cmp_sweepline {
    bool operator () (const line& a, const line& b) const {
        // asume que los segmentos tienen p < q
        if (a.p == b.p) return ccw(a.p, a.q, b.q);
        if (!eq(a.p.x, a.q.x) and (eq(b.p.x, b.q.x) or
            a.p.x + eps < b.p.x))
            return ccw(a.p, a.q, b.p);
        return ccw(a.p, b.q, b.p);
    }
};

// comparador para set para hacer sweep angle con segmentos
pt dir;
struct cmp_sweepangle {
    bool operator () (const line& a, const line& b) const {
        return get_t(dir, a) + eps < get_t(dir, b);
    }
};

```

#### 4.4. Circle

```

//Requiere pt y line

pt circumCenter(pt a, pt b, pt c) {
    b = b - a, c = c - a; // consider coordinates relative
    to A
    assert(cross(b,c) != 0); // no circumcircle if A,B,C
    aligned
    return a + perp(b * c.norm() - c * b.norm()) * (1.0 /
        cross(b,c)/2.0);
}

// (x- x0)^2 + (y-y0)^2
// (x0 + r cos(ang), y0 + r sin(ang))

template <typename ld> int64_t sgn(ld x) {

```

```

    return (ld(0) < x) - (x < ld(0));
}

int64_t circleLine(pt o, ld r, line l, pair<pt,pt> &out) {
    ld h2 = r * r - l.sqDist(o);
    if (h2 >= 0) { // the line touches the circle
        pt p = l.proj(o); // point P
        pt h = l.v*sqrt(h2) * (1.0 / abs(l.v)); // vector
            parallel to l, of
            //length h
        out = {p-h, p+h};
    }
    return 1 + sgn(h2);
}

int64_t circleCircle(pt o1, ld r1, pt o2, ld r2,
    pair<pt,pt> &out) {
    pt d=o2-o1; ld d2= d.norm();
    if (d2 == 0) {assert(r1 != r2); return 0;} //
        concentric circles
    ld pd = (d2 + r1 * r1 - r2 * r2)/2; // = |O_1P| * d
    ld h2 = r1 * r1 - pd * pd / d2; // = h^2
    if (h2 >= 0) {
        pt p = o1 + (d * pd)*(1.0 / d2), h = perp(d) *
            sqrt(h2/d2);
        out = {p-h, p+h};
    }
    return 1 + sgn(h2);
}

int64_t tangents(pt o1, ld r1, pt o2, ld r2, bool inner,
    vector<pair<pt,pt>> &out) {
    if (inner) r2 = -r2;
    pt d = o2-o1;
    ld dr = r1 - r2, d2 = d.norm(), h2 = d2 - dr * dr;
    if (d2 == 0 || h2 < 0) {assert(h2 != 0); return 0;}
    for (ld sign : {-1, 1}) {
        pt v = (d * dr + perp(d) * sqrt(h2) * sign) * (1.0
            / d2);
        out.push_back({o1 + v * r1, o2 + v * r2});
    }
    return 1 + (h2 > 0);
}

```

## 4.5. geometria

```
typedef double ld;
const ld DINF = 1e18;
const ld pi = acos(-1.0);
const ld eps = 1e-9;

#define sq(x) ((x)*(x))
bool eq(ld a, ld b) { return abs(a - b) <= eps; }

// Punto: (x, y)
struct pt {
    ld x, y;
    pt(ld x_ = 0, ld y_ = 0) : x(x_), y(y_) {}

    bool operator < (const pt p) const {
        if (!eq(x, p.x)) return x < p.x;
        if (!eq(y, p.y)) return y < p.y;
        return 0;
    }

    bool operator == (const pt p) const { return eq(x, p.x)
        and eq(y, p.y); }

    pt operator + (const pt p) const { return pt(x + p.x, y
        + p.y); }

    pt operator - (const pt p) const { return pt(x - p.x, y
        - p.y); }

    pt operator * (const ld c) const { return pt(x * c, y *
        c); }

    pt operator / (const ld c) const { return pt(x / c, y /
        c); }

    ld operator * (const pt p) const { return x * p.x + y *
        p.y; }

    ld operator ^ (const pt p) const { return x * p.y - y *
        p.x; }

    friend istream& operator >> (istream& in, pt& p) {
        return in >> p.x >> p.y; }

    friend ostream& operator << (ostream& out, const pt& p)
    { return out << p.x << ' ' << p.y; }
};

ld DEG_TO_RAD(ld n) { return n * pi / 180.0; }
ld RAD_TO_DEG(ld n) { return n * 180.0 / pi; }
```

```
// Recta: (p(x,y), q(x,y))
struct line {
    pt p, q;
    line() {}
    line(pt p_, pt q_) : p(p_), q(q_) {}
    friend istream& operator >> (istream& in, line& r) {
        return in >> r.p >> r.q;
    }
};

// distancia
ld dist(pt p, pt q) {
    return hypot(p.y - q.y, p.x - q.x);
}

ld dist2(pt p, pt q) { // cuadrado de la distancia
    return sq(p.x - q.x) + sq(p.y - q.y);
}

ld norm(pt v) { // norma euclidiana del vector
    return dist(pt(0, 0), v);
}

ld angle(pt v) { // angulo del vector con el eje x
    ld ang = atan2(v.y, v.x);
    if (ang < 0) ang += 2 * pi;
    return ang;
}

ld sarea(pt p, pt q, pt r) { // area con signo
    return ((q - p) ^ (r - q)) / 2;
}

bool col(pt p, pt q, pt r) { // si p, q y r son colineales
    return eq(sarea(p, q, r), 0);
}

bool ccw(pt p, pt q, pt r) { // si p, q, r estan en sentido
    antihorario
    return sarea(p, q, r) > eps;
}
```



```

pt rotate(pt p, ld th) { // rota el punto th radianes
    return pt(p.x * cos(th) - p.y * sin(th), p.x * sin(th)
        + p.y * cos(th));
}

pt rotate90(pt p) { // rota 90 grados
    return pt(-p.y, p.x);
}

// RECTA

bool isvert(line r) { // si r es vertical
    return eq(r.p.x, r.q.x);
}

bool isinseg(pt p, line r) { // si p pertenece al segmento
    de r
    pt a = r.p - p, b = r.q - p;
    return eq((a ^ b), 0) and (a * b) < eps;
}

ld get_t(pt v, line r) { // retorna t tal que t*v pertenece
    a la recta r
    return (r.p ^ r.q) / ((r.p - r.q) ^ v);
}

pt proj(pt p, line r) { // proyeccion del punto p en la
    recta r
    if (r.p == r.q) return r.p;
    r.q = r.q - r.p; p = p - r.p;
    pt proj = r.q * ((p * r.q) / (r.q * r.q));
    return proj + r.p;
}

pt inter(line r, line s) { // interseccion de r con s
    if (eq((r.p - r.q) ^ (s.p - s.q), 0)) return pt(DINF,
        DINF);
    r.q = r.q - r.p, s.p = s.p - r.p, s.q = s.q - r.p;
    return r.q * get_t(r.q, s) + r.p;
}

bool interseg(line r, line s) { // si el segmento de r
    intersecta el segmento de s

```

```

    if (isinseg(r.p, s) or isinseg(r.q, s)
        or isinseg(s.p, r) or isinseg(s.q, r)) return 1;

    return ccw(r.p, r.q, s.p) != ccw(r.p, r.q, s.q) and
        ccw(s.p, s.q, r.p) != ccw(s.p, s.q, r.q);
}

ld disttoline(pt p, line r) { // distancia del punto a la
    recta
    return 2 * abs(sarea(p, r.p, r.q)) / dist(r.p, r.q);
}

ld disttoseg(pt p, line r) { // distancia del punto al
    segmento
    if ((r.q - r.p) * (p - r.p) < 0) return dist(r.p, p);
    if ((r.p - r.q) * (p - r.q) < 0) return dist(r.q, p);
    return disttoline(p, r);
}

ld distseg(line a, line b) { // distancia entre segmentos
    if (interseg(a, b)) return 0;

    ld ret = DINF;
    ret = min(ret, disttoseg(a.p, b));
    ret = min(ret, disttoseg(a.q, b));
    ret = min(ret, disttoseg(b.p, a));
    ret = min(ret, disttoseg(b.q, a));

    return ret;
}

```

#### 4.6. geometria3D

```

typedef double ld;
const ld DINF = 1e18;
const ld eps = 1e-9;

#define sq(x) ((x)*(x))

bool eq(ld a, ld b) { return abs(a - b) <= eps; }

struct pt { // punto

```

```

ld x, y, z;
pt(ld x_ = 0, ld y_ = 0, ld z_ = 0) : x(x_), y(y_),
    z(z_) {}
bool operator < (const pt p) const {
    if (!eq(x, p.x)) return x < p.x;
    if (!eq(y, p.y)) return y < p.y;
    if (!eq(z, p.z)) return z < p.z;
    return 0;
}
bool operator == (const pt p) const {
    return eq(x, p.x) and eq(y, p.y) and eq(z, p.z);
}
pt operator + (const pt p) const { return pt(x + p.x, y
    + p.y, z + p.z); }
pt operator - (const pt p) const { return pt(x - p.x, y
    - p.y, z - p.z); }
pt operator * (const ld c) const { return pt(x * c, y *
    c, z * c); }
pt operator / (const ld c) const { return pt(x / c, y /
    c, z / c); }
ld operator * (const pt p) const { return x * p.x + y *
    p.y + z * p.z; }
pt operator ^ (const pt p) const { return pt(y * p.z -
    z * p.y, z * p.x - x * p.z, x * p.y - y * p.x); }
friend istream& operator >> (istream& in, pt& p) {
    return in >> p.x >> p.y >> p.z;
}
};

ld DEG_TO_RAD(ld n) { return n * acos(-1) / 180.0; }
ld RAD_TO_DEG(ld n) { return n * 180.0 / acos(-1); }

struct line { // recta
    pt p, q;
    line() {}
    line(pt p_, pt q_) : p(p_), q(q_) {}
    friend istream& operator >> (istream& in, line& r) {
        return in >> r.p >> r.q;
    }
};

struct plane { // plano

```

```

    array<pt, 3> p; // puntos que definen el plano
    array<ld, 4> eq; // ecuacion del plano
    plane() {}
    plane(pt p_, pt q_, pt r_) : p({ p_, q_, r_ }) {
        build(); }

    friend istream& operator >> (istream& in, plane& P) {
        return in >> P.p[0] >> P.p[1] >> P.p[2];
        P.build();
    }
    void build() {
        pt dir = (p[1] - p[0]) ^ (p[2] - p[0]);
        eq = { dir.x, dir.y, dir.z, dir * p[0] * (-1) };
    }
};

// convierte de coordenadas polares a cartesianas
// (angulos deben estar en radianes)
// phi es el angulo con el eje z (arriba) theta es el
// angulo de rotacion alrededor de z
pt convert(ld rho, ld th, ld phi) {
    return pt(sin(phi) * cos(th), sin(phi) * sin(th),
        cos(phi)) * rho;
}

// proyeccion del punto p en la recta r
pt proj(pt p, line r) {
    if (r.p == r.q) return r.p;
    r.q = r.q - r.p; p = p - r.p;
    pt proj = r.q * ((p * r.q) / (r.q * r.q));
    return proj + r.p;
}

// proyeccion del punto p en el plano P
pt proj(pt p, plane P) {
    p = p - P.p[0], P.p[1] = P.p[1] - P.p[0], P.p[2] =
        P.p[2] - P.p[0];
    pt norm = P.p[1] ^ P.p[2];
    pt proj = p - (norm * (norm * p) / (norm * norm));
    return proj + P.p[0];
}

// distancia

```

```

ld dist(pt a, pt b) {
    return sqrt(sq(a.x - b.x) + sq(a.y - b.y) + sq(a.z -
        b.z));
}

// distancia punto recta
ld distline(pt p, line r) {
    return dist(p, proj(p, r));
}

// distancia de punto a segmento
ld distseg(pt p, line r) {
    if ((r.q - r.p) * (p - r.p) < 0) return dist(r.p, p);
    if ((r.p - r.q) * (p - r.q) < 0) return dist(r.q, p);
    return distline(p, r);
}

// distancia de punto a plano con signo
ld sdist(pt p, plane P) {
    return P.eq[0] * p.x + P.eq[1] * p.y + P.eq[2] * p.z +
        P.eq[3];
}

// distancia de punto a plano
ld distplane(pt p, plane P) {
    return abs(sdist(p, P));
}

// si punto pertenece a recta
bool isinseg(pt p, line r) {
    return eq(distseg(p, r), 0);
}

// si punto pertenece al triangulo definido por P.p
bool isinpol(pt p, vector<pt> v) {
    assert(v.size() >= 3);
    pt norm = (v[1] - v[0]) ^ (v[2] - v[1]);
    bool inside = true;
    int sign = -1;
    for (int i = 0; i < v.size(); i++) {
        line r(v[(i + 1) % 3], v[i]);
        if (isinseg(p, r)) return true;
    }
}

```

```

    pt ar = v[(i + 1) % 3] - v[i];
    if (sign == -1) sign = ((ar ^ (p - v[i])) * norm > 0
        );
    else if (((ar ^ (p - v[i])) * norm > 0) != sign)
        inside = false;
    }
    return inside;
}

// distancia de punto a poligono
ld distpol(pt p, vector<pt> v) {
    pt p2 = proj(p, plane(v[0], v[1], v[2]));
    if (isinpol(p2, v)) return dist(p, p2);
    ld ret = DINF;
    for (int i = 0; i < v.size(); i++) {
        int j = (i + 1) % v.size();
        ret = min(ret, distseg(p, line(v[i], v[j])));
    }
    return ret;
}

// interseccion de plano y segmento
// BOTH = el segmento esta en el plano
// ONE = uno de los puntos del segmento esta en el plano
// PARAL = segmento paralelo al plano
// CONCOR = segmento concurrente al plano
enum RETCODE { BOTH, ONE, PARAL, CONCOR };
pair<RETCODE, pt> intersect(plane P, line r) {
    ld d1 = sdist(r.p, P);
    ld d2 = sdist(r.q, P);
    if (eq(d1, 0) and eq(d2, 0)) return pair(BOTH, r.p);
    if (eq(d1, 0)) return pair(ONE, r.p);
    if (eq(d2, 0)) return pair(ONE, r.q);
    if ((d1 > 0 and d2 > 0) or (d1 < 0 and d2 < 0)) {
        if (eq(d1 - d2, 0)) return pair(PARAL, pt());
        return pair(CONCOR, pt());
    }
    ld frac = d1 / (d1 - d2);
    pt res = r.p + ((r.q - r.p) * frac);
    return pair(ONE, res);
}

// rota p alrededor del eje u por un angulo a

```

```

pt rotate(pt p, pt u, ld a) {
    u = u / dist(u, pt());
    return u * (u * p) + (u ^ p ^ u) * cos(a) + (u ^ p) *
        sin(a);
}

```

#### 4.7. isfigure

```

typedef ll T;
struct pt {
    T x, y;
    pt() : x(0), y(0) {}
    pt(T _x, T _y) : x(_x), y(_y) {}
    pt operator+(pt p) { return { x + p.x, y + p.y }; }
    pt operator-(pt p) { return { x - p.x, y - p.y }; }
    pt operator*(T d) { return { x * d, y * d }; }
    pt operator/(T d) { return { x / d, y / d }; }
    bool operator==(pt b) { return x == b.x && y == b.y; }
    bool operator!=(pt b) { return x != b.x || y != b.y; }
    bool operator<(pt b) { return x == b.x ? y < b.y : x <
        b.x; }

    void read() {
        cin >> x >> y;
    }
};

const double PI = acos(-1);
double DEG_TO_RAD(double n) { return n * PI / 180.0; }
double RAD_TO_DEG(double n) { return n * 180.0 / PI; }
T sq(pt p) { return p.x * p.x + p.y * p.y; }
T cross(pt v, pt w) { return v.x * w.y - v.y * w.x; }
double abs(pt p) { return sqrt(sq(p)); }
T dot(pt v, pt w) { return v.x * w.x + v.y * w.y; }
T dis(pt a, pt b) { return sq(a - b); }
//Transformaciones
pt translate(pt v, pt p) { return p + v; }
pt scale(pt c, double factor, pt p) { return c + (p - c) *
    factor; }
pt rot(pt p, double ang) { return { p.x * cos(ang) - p.y *
    sin(ang), p.x * sin(ang) + p.y * cos(ang) }; }
pt perp(pt p) { return { -p.y, p.x }; }
T isParall(pt v, pt w) { return cross(v, w) == 0; }

```

```

// A square has four right angles and four sides with equal
lengths.
bool isSquare(pt a, pt b, pt c, pt d) {
    T ab = dis(a, b);
    T bc = dis(b, c);
    T cd = dis(c, d);
    T ad = dis(a, d);
    return isParall(a - b, c - d) && isParall(a - d, b - c)
        && dot(b - a, d - a) == 0 && ab == bc && bc == cd &&
        cd == ad;
}

// A rectangle has four right angles.
bool isRectangle(pt a, pt b, pt c, pt d) {
    return isParall(a - b, c - d) && isParall(a - d, b - c)
        && dot(b - a, d - a) == 0;
}

// A rhombus has four sides with equal lengths.
bool isRhombus(pt a, pt b, pt c, pt d) {
    T ab = dis(a, b);
    T bc = dis(b, c);
    T cd = dis(c, d);
    T ad = dis(a, d);
    return ab == bc && bc == cd && cd == ad;
}

// A parallelogram has two pairs of parallel sides.
bool isParallelogram(pt a, pt b, pt c, pt d) {
    return isParall(a - b, c - d) && isParall(a - d, b - c);
}

// A trapezium has one pair of parallel sides.
bool isTrapezium(pt a, pt b, pt c, pt d) {
    return isParall(a - b, c - d) || isParall(a - d, b - c);
}

// A kite has reflection symmetry across a diagonal.
bool isKite(pt a, pt b, pt c, pt d) {
    T ab = dis(a, b);
    T bc = dis(b, c);

```

```

T cd = dis(c, d);
T ad = dis(a, d);
return (ab == bc && cd == ad) || (ab == ad && bc == cd);
}
int main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);

    pt a, b, c, d;
    a.read(); b.read(); c.read(); d.read();

    if (isSquare(a, b, c, d))          cout << "square"
        << endl;
    else if (isRectangle(a, b, c, d))  cout <<
        "rectangle" << endl;
    else if (isRhombus(a, b, c, d))    cout << "rhombus"
        << endl;
    else if (isParallelogram(a, b, c, d)) cout <<
        "parallelogram" << endl;
    else if (isTrapezium(a, b, c, d))  cout <<
        "trapezium" << endl;
    else if (isKite(a, b, c, d))       cout << "kite" <<
        endl;
    else cout << "none" << endl;

    cout << flush;
    return 0;
}

```

## 5. Graph

### 5.1. hld

```

#define int int64_t
#define MAX 200005
namespace seg {
    int seg[4 * MAX], lazy[4 * MAX];
    int n, * v;

    int build(int p = 1, int l = 0, int r = n - 1) {
        lazy[p] = 0; // lazy[p] = 1;
        if (l == r) return seg[p] = v[l];
        int m = (l + r) / 2;
        return seg[p] = build(2 * p, l, m) + build(2 * p + 1,
            m + 1, r);
    }
    void build(int n2, int* v2) {
        n = n2, v = v2;
        build();
    }
    void prop(int p, int l, int r) {
        seg[p] += lazy[p] * (r - l + 1);
        if (l != r) lazy[2 * p] += lazy[p], lazy[2 * p + 1]
            += lazy[p];
        lazy[p] = 0;
        // seg[p] *= pow(lazy[p], r - l + 1); || seg[p] =
            lazy[p] * seg[p];
        // if (l != r) lazy[2*p] *= lazy[p], lazy[2*p+1] *=
            lazy[p];
        // lazy[p] = 1;
    }
    int query(int a, int b, int p = 1, int l = 0, int r = n
        - 1) {
        prop(p, l, r);
        if (a <= l and r <= b) return seg[p];
        if (b < l or r < a) return 0;
        int m = (l + r) / 2;

        return query(a, b, 2 * p, l, m) + query(a, b, 2 * p
            + 1, m + 1, r);
    }
    int update(int a, int b, int x, int p = 1, int l = 0,
        int r = n - 1) {
        prop(p, l, r);
        if (a <= l and r <= b) {

```

```

        lazy[p] += x; // lazy[p] *= x;
        prop(p, l, r);
        return seg[p];
    }
    if (b < l or r < a) return seg[p];
    int m = (l + r) / 2;
    return seg[p] = update(a, b, x, 2 * p, l, m) +
        update(a, b, x, 2 * p + 1, m + 1, r);
}

};

namespace hld {
    vector<pair<int, int> > g[MAX];
    int pos[MAX], sz[MAX];
    int sobe[MAX], pai[MAX];
    int h[MAX], v[MAX], t;

    void build_hld(int k, int p = -1, int f = 1) {
        v[pos[k] = t++] = sobe[k]; sz[k] = 1;
        for (auto& i : g[k]) if (i.first != p) {
            auto [u, w] = i;
            sobe[u] = w; pai[u] = k;
            h[u] = (i == g[k][0] ? h[k] : u);
            build_hld(u, k, f); sz[k] += sz[u];

            if (sz[u] > sz[g[k][0].first] or g[k][0].first
                == p)
                swap(i, g[k][0]);
        }
        if (p * f == -1) build_hld(h[k] = k, -1, t = 0);
    }

    void build(int root = 0) {
        t = 0;
        build_hld(root);
        seg::build(t, v);
    }

    int query_path(int a, int b) {
        if (a == b) return 0;
        if (pos[a] < pos[b]) swap(a, b);

        if (h[a] == h[b]) return seg::query(pos[b] + 1,
            pos[a]);
        return seg::query(pos[h[a]], pos[a]) +

```

```

        query_path(pai[h[a]], b);
    }
    void update_path(int a, int b, int x) {
        if (a == b) return;
        if (pos[a] < pos[b]) swap(a, b);

        if (h[a] == h[b]) return (void)seg::update(pos[b] +
            1, pos[a], x);
        seg::update(pos[h[a]], pos[a], x);
        update_path(pai[h[a]], b, x);
    }
    int query_subtree(int a) {
        if (sz[a] == 1) return 0;
        return seg::query(pos[a] + 1, pos[a] + sz[a] - 1);
    }
    void update_subtree(int a, int x) {
        if (sz[a] == 1) return;
        seg::update(pos[a] + 1, pos[a] + sz[a] - 1, x);
    }
    int lca(int a, int b) {
        if (pos[a] < pos[b]) swap(a, b);
        return h[a] == h[b] ? b : lca(pai[h[a]], b);
    }
}

```

## 5.2. cycle len

```

// constexpr int mxN = 2500 + 50;
constexpr int inf = 1e9 + 7;
vector<int> adj[mxN];
int n, m;
int cycle_len(int start) {
    int ans = inf;

    vector<int> dist(n, -1);
    queue<int> bfs;

    dist[start] = 0;
    bfs.push(start);

    while (!bfs.empty()) {
        int node = bfs.front();

```

```

    bfs.pop();
    for (int adj_node : adj[node]) {
        if (dist[adj_node] == -1) {
            dist[adj_node] = dist[node] + 1;
            bfs.push(adj_node);
        }
        else if (dist[adj_node] >= dist[node]) {
            ans = min(ans, 1 + dist[adj_node] +
                dist[node]);
        }
    }
}
return ans;
}

```

### 5.3. Topo Sort DFS

```

int n, m; cin >> n >> m;
vector<int> ady[n];
for (i, m) {
    int v, u; cin >> v >> u;
    v--, u--;
    ady[v].pb(u);
}

vector<int> topo;
vector<bool> vis(n);

function<void(int)> dfs = [&](int v) {
    vis[v] = true;
    for (int &u : ady[v]) {
        if (!vis[u]) dfs(u);
    }
    topo.pb(v);
};

for (i, n) if (!vis[i]) dfs(i);

```

### 5.4. Topo Sort BFS

```

int n, m; cin >> n >> m;
vector<int> ady[n];
vector<int> grado(n);
for (i, m) {
    int v, u; cin >> v >> u;
    v--, u--;
    ady[v].pb(u);
    grado[u]++;
}

vector<int> topo;
queue<int> qu;

for (i, n) if (!grado[i]) qu.push(i);

while (sz(qu)) {
    int v = qu.front();
    qu.pop();
    topo.pb(v);
    for (int &u : ady[v]) {
        if (--grado[u] == 0) {
            qu.push(u);
        }
    }
}

```

### 5.5. Kosaraju

```

int n, m; cin >> n >> m;

vector<int> ady[n], rady[n];
vector<int> grado(n);

for (i, m) {
    int a, b; cin >> a >> b;
    a--, b--;
    ady[a].pb(b);
    rady[b].pb(a);
}

vector<int> order;
vector<bool> vis(n);

```

```

vector<vector<int>> comp;

function<void(int)> dfs1 = [&](int v) {
    vis[v] = true;
    for (int& u : ady[v]) {
        if (!vis[u]) {
            dfs1(u);
        }
    }
    order.pb(v);
};

for(i, n) if (!vis[i]) dfs1(i);

vis.assign(n, false);

function<void(int)> dfs2 = [&](int v) {
    vis[v] = true;
    comp.back().pb(v);
    for (int& u : rady[v]) {
        if (!vis[u]) {
            dfs2(u);
        }
    }
};

for (int i = n - 1; i >= 0; --i) {
    if (!vis[order[i]]) {
        comp.pb({});
        dfs2(order[i]);
    }
}

for(i, sz(comp)) {
    cout << "Component #" << i + 1 << ":";
    for (int& j : comp[i]) {
        cout << " " << j + 1;
    }
    cout << endl;
}

```

## 5.6. Floyd Warshall

```

int n; cin >> n;
int ady[n][n];
const int INF = int(1e9);

for(i, n) {
    for(j, n) {
        ady[i][j] = (i == j ? 0 : INF);
    }
}

for(i, n) {
    int v, u, w; cin >> v >> u >> w;
    v--, u--;
    ady[v][u] = ady[u][v] = w;
}

for(k, n) {
    for(i, n) {
        for(j, n) {
            ady[i][j] = min(ady[i][j], ady[i][k] + ady[k][j]);
        }
    }
}

```

## 5.7. ArtiBridges

```

struct ArtiBridges {
    int n, timer;
    vector<bool> vis, is_articulation;
    vector<int> tin, low;
    vector<pair<int, int>> bridges;

    ArtiBridges(int m) :
        n(m), timer(0), vis(n), tin(n, -1),
        low(n, -1), is_articulation(n) {
        for(i, n) if (!vis[i]) dfs(i);
    }

    void dfs(int v, int p = -1) {
        vis[v] = true;

```



```

tin[v] = low[v] = timer++;
int children = 0;
for (int& u : g[v]) {
    if (u == p) continue;
    if (vis[u]) {
        low[v] = min(low[v], tin[u]);
    }
    else {
        dfs(u, v);
        low[v] = min(low[v], low[u]);
        if (low[u] >= tin[v] && p != -1)
            is_articulation[v] = true;
        ++children;

        if (low[u] > tin[v])
            bridges.pb({ v, u });
    }
}
if (p == -1 && children > 1)
    is_articulation[v] = true;
};

```

## 5.8. Biconnected Components

```

struct BiConn {
    int n, timer;
    vector<bool> vis;
    vector<int> tin, low;
    stack<pair<int,int>> stk;
    vector<vector<pair<int,int>>> bcc;

    BiConn(int m) :
        n(m), timer(0), vis(n), tin(n, -1),
        low(n, -1) {
        forn(i, n) if (!vis[i]) dfs(i);
    }

    void dfs(int v, int p = -1) {
        vis[v] = true;
        tin[v] = low[v] = timer++;
        for (int& u : g[v]) {

```

```

            if (u == p) continue;
            if (vis[u]) {
                low[v] = min(low[v], tin[u]);
                if (tin[u] < tin[v]) stk.push({ v, u });
            }
            else {
                stk.push({ v, u });
                dfs(u, v);
                low[v] = min(low[v], low[u]);
                if (low[u] >= tin[v]) {
                    vector<pair<int,int>> comp;
                    pair<int,int> edge;
                    do {
                        edge = stk.top(); stk.pop();
                        comp.pb(edge);
                    } while (edge != make_pair(v, u));
                    bcc.pb(comp);
                }
            }
        }
    }
};

```

## 5.9. Dijkstra

```

struct edge {
    int v; ll w;

    bool operator < (const edge &x) const {
        return x.w < w;
    }
};

vector<ll> dist(n, LONG_LONG_MAX);

auto dijkstra = [&](edge v) {
    priority_queue<edge> pq;
    pq.push(v);
    dist[v.v] = 0;
    while (sz(pq)) {
        v = pq.top();
        pq.pop();

```

```

    if (v.w > dist[v.v]) continue;
    for (edge &u : g[v.v]) {
        if (dist[u.v] > dist[v.v] + u.w) {
            dist[u.v] = dist[v.v] + u.w;
            pq.push({u.v, dist[u.v]});
        }
    }
}
};

```

### 5.10. Kruskal

```

struct edge {
    int v, u, w;

    bool operator < (const edge &x) const {
        return w < x.w;
    }
};

vector<edge> edges;
int n, m; cin >> n >> m;
for (i, m) {
    int v, u, w; cin >> v >> u >> w;
    v--, u--;
    edges.push_back({v, u, w});
}
sort(all(edges));
dsu UF(n);
int nodes = 0, mst = 0;
for (edge &i : edges) {
    if (!UF.same(i.v, i.u)) {
        mst += i.w;
        UF.unite(i.v, i.u);
        nodes++;
    }
    if (nodes == n - 1) break;
}

```

### 5.11. Prim

```

struct edge {
    int v, w;

    bool operator < (const edge &x) const {
        return w > x.w;
    }
};

int n, m; cin >> n >> m;
vector<edge> ady[n];
for (i, m) {
    int v, u, w; cin >> v >> u >> w;
    v--, u--;
    ady[v].pb({u, w});
    ady[u].pb({v, w});
}

priority_queue<edge> pq;
bool vis[n];
memset(vis, false, sizeof vis);

vis[0] = true;
for (edge &i : ady[0]) if (!vis[i.v]) pq.push(i);

int mst = 0;
while (sz(pq)) {
    edge v = pq.top();
    pq.pop();

    if (!vis[v.v]) {
        mst += v.w;
        vis[v.v] = true;
        for (edge &i : ady[v.v]) {
            if (!vis[i.v]) {
                pq.push(i);
            }
        }
    }
}

```

### 5.12. Bellman Ford

```

struct Edge { int v, u; ll w; };

const ll INF = 1e18;
vector<Edge> edges;
vector<ll> d;
vector<int> p;

vector<int> BellmanFord(int n, int src = -1) {
    d.assign(n, ~src ? INF : 0);
    if (~src) d[src] = 0;
    p.assign(n, -1);
    int x = -1;
    forn (i, n) {
        x = -1;
        for (Edge &e : edges)
            if (d[e.v] < INF)
                if (d[e.u] > d[e.v] + e.w) {
                    d[e.u] = max(-INF, d[e.v] + e.w);
                    p[e.u] = e.v;
                    x = e.u;
                }
    }
    if (x == -1) return {};
    forn (i, n) x = p[x];
    vector<int> path;
    for (int cur = x;; cur = p[cur]) {
        path.pb(cur);
        if (cur == x && sz(path) > 1)
            break;
    }
    reverse(all(path));
    return path;
}

vector<int> BellmanFord(int n, int s, int t) {
    d.assign(n, INF);
    d[s] = 0;
    p.assign(n, -1);
    while (1) {
        bool any = false;
        for (Edge &e : edges)
            if (d[e.v] < INF)
                if (d[e.u] > d[e.v] + e.w) {

```

```

                    d[e.u] = d[e.v] + e.w;
                    p[e.u] = e.v;
                    any = true;
                }
            if (!any) break;
    }
    if (d[t] == INF) return {};
    vector<int> path;
    for (int cur = t; cur != -1; cur = p[cur])
        path.pb(cur);
    reverse(all(path));
    return path;
}

```

### 5.13. LCA

```

struct LCA {
    vector<int> height, euler, first, segtree;
    int n;

    LCA(vector<vector<int>> &g, int root = 0) {
        n = sz(g);
        height.resize(n);
        first.resize(n);
        dfs(g, root, root);
        int m = sz(euler);
        segtree.resize(m * 4);
        build(1, 0, m - 1);
    }

    void dfs(vector<vector<int>> &g, int v, int p, int h = 0)
    {
        height[v] = h;
        first[v] = sz(euler);
        euler.pb(v);
        for (int &u : g[v]) {
            if (u == p) continue;
            dfs(g, u, v, h + 1);
            euler.pb(v);
        }
    }
}

```

```

void build(int node, int b, int e) {
    if (b == e) {
        segtree[node] = euler[b];
    } else {
        int mid = (b + e) / 2;
        build(node << 1, b, mid);
        build(node << 1 | 1, mid + 1, e);
        int l = segtree[node << 1], r = segtree[node << 1 | 1];
        segtree[node] = (height[l] < height[r]) ? l : r;
    }
}

int query(int node, int b, int e, int L, int R) {
    if (b > R || e < L) return -1;
    if (b >= L && e <= R) return segtree[node];
    int mid = (b + e) >> 1;
    int left = query(node << 1, b, mid, L, R);
    int right = query(node << 1 | 1, mid + 1, e, L, R);
    if (left == -1) return right;
    if (right == -1) return left;
    return height[left] < height[right] ? left : right;
}

int lca(int u, int v) {
    int left = first[u], right = first[v];
    if (left > right) swap(left, right);
    return query(1, 0, sz(euler) - 1, left, right);
}
};

```

## 5.14. LCA Binary Lifting

```

struct LCA {
    int timer, l, n;
    vector<int> tin, tout;
    vector<vector<int>> up;

    LCA(int n, int root = 0) {
        timer = 0;
        this->n = n;
        tin.resize(n);

```

```

        tout.resize(n);
        l = ceil(log2(n));
        up.assign(n, vector<int>(l + 1));
        dfs(root, root);
    }

    void dfs(int v, int p) {
        tin[v] = ++timer;
        up[v][0] = p;
        for (i, l) up[v][i + 1] = up[up[v][i]][i];
        for (int &u : g[v]) if (u != p) dfs(u, v);
        tout[v] = ++timer;
    }

    bool is_ancestor(int v, int u) {
        return tin[v] <= tin[u] && tout[v] >= tout[u];
    }

    int lca(int v, int u) {
        if (is_ancestor(v, u)) return v;
        if (is_ancestor(u, v)) return u;
        for (i, l)
            if (!is_ancestor(up[u][i], v))
                u = up[u][i];
        return up[u][0];
    }
};

```

## 5.15. Two Sat

```

struct sat {
    int n, tot;
    vector<vector<int>> g;
    vector<int> vis, comp, id, ans;
    stack<int> s;

    sat() {}
    sat(int n_) : n(n_), tot(n), g(2*n) {}

    int dfs(int i, int& t) {
        int lo = id[i] = t++;
        s.push(i), vis[i] = 2;

```

```

    for (int j : g[i]) {
        if (!vis[j]) lo = min(lo, dfs(j, t));
        else if (vis[j] == 2) lo = min(lo, id[j]);
    }
    if (lo == id[i]) while (1) {
        int u = s.top(); s.pop();
        vis[u] = 1, comp[u] = i;
        if ((u>>1) < n and ans[u>>1] == -1) ans[u>>1] =
            ~u&1;
        if (u == i) break;
    }
    return lo;
}

void add_impl(int x, int y) { // x -> y = !x ou y
    x = x >= 0 ? 2*x : -2*x-1;
    y = y >= 0 ? 2*y : -2*y-1;
    g[x].push_back(y);
    g[y^1].push_back(x^1);
}

void add_cl(int x, int y) { // x ou y
    add_impl(~x, y);
}

void add_xor(int x, int y) { // x xor y
    add_cl(x, y), add_cl(~x, ~y);
}

void add_eq(int x, int y) { // x = y
    add_xor(~x, y);
}

void add_true(int x) { // x = T
    add_impl(~x, x);
}

void at_most_one(vector<int> v) { // no max um
    verdadeiro
    g.resize(2*(tot+v.size()));
    for (int i = 0; i < v.size(); i++) {
        add_impl(tot+i, ~v[i]);
        if (i) {
            add_impl(tot+i, tot+i-1);
            add_impl(v[i], tot+i-1);
        }
    }
    tot += v.size();
}

```

```

}

pair<bool, vector<int>> solve() {
    ans = vector<int>(n, -1);
    int t = 0;
    vis = comp = id = vector<int>(2*tot, 0);
    for (int i = 0; i < 2*tot; i++) if (!vis[i]) dfs(i,
        t);
    for (int i = 0; i < tot; i++)
        if (comp[2*i] == comp[2*i+1]) return {false,
            {}};
    return {true, ans};
}
};

```

## 5.16. Tarjan

```

struct Tarjan {
    vector<int> low, num, comp;
    stack<int> st;
    int n, scc, cont;
    const int INF = int(1e9);

    Tarjan(int n) {
        this->n = n;
        low.resize(n);
        num.assign(n, -1);
        comp.resize(n);
        scc = cont = 0;
    }

    void dfs(int v) {
        low[v] = num[v] = cont++;
        st.push(v);
        for (int &u : g[v]) {
            if (num[u] == -1) dfs(u);
            low[v] = min(low[v], low[u]);
        }
        if (low[v] == num[v]) {
            int u;
            do {
                u = st.top(); st.pop();
            }
        }
    }
}

```

```

        low[u] = INF;
        comp[u] = scc;
    } while (u != v);
    scc++;
}
};

void go() {
    forn (i, n)
        if (num[i] == -1) dfs(i);
}
};

```

### 5.17. bipartite Graph

```

template<typename T>
struct Graph {
    int n;
    vector<vector<T>> adj;
    vector<T> side;
    Graph(int size) {
        n = size;
        adj.resize(n);
        side.resize(n, -1);
    }

    void addEdge(int u, int v, int uno) {
        v -= uno; u -= uno;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }

    bool is_bipartite() {
        bool check = true;
        queue<int> q;
        for (int edge = 0; edge < n; ++edge) {
            if (side[edge] == -1) {
                q.push(edge);
                side[edge] = 0;
                while (q.size()) {
                    int curr = q.front();
                    q.pop();

```

```

                        for (auto neig : adj[curr]) {
                            if (side[neig] == -1) {
                                side[neig] = (1 ^ side[curr]);
                                q.push(neig);
                            }
                            else {
                                check &= (side[neig] !=
                                            side[curr]);
                            }
                        }
                    }
                }
            }
        }
        return check;
    }
};

```

### 5.18. nx-ny-8

```

vector<vector<char>>board;
vector<vector<bool>>vis;
int n, m;
// U,UR, R,RD,D,LD,L, UL
int dx[8] = { -1, -1, 0, 1, 1, 1, 0, -1 };
int dy[8] = { 0, 1, 1, 1, 0, -1, -1, -1 };

void init() {
    board.resize(n + 1, vector<char>(m + 1));
    vis.resize(n + 1, vector<bool>(m + 1, 0));
}

void back(int x, int y) {
    vis[x][y] = 1;
    forn(i, 8) {
        int nx = x + dx[i], ny = y + dy[i];
        if (nx >= 0 && nx < n && ny >= 0 && ny < m &&
            board[nx][ny] != '1' && !vis[nx][ny]) back(nx,
            ny);
    }
}

```

## 5.19. nx-ny-4

```
vector<vector<char>>>board;
vector<vector<bool>>>vis;
int n, m;
// R D L U
int dx[] = { 0, 1, 0, -1 };
int dy[] = { 1, 0, -1, 0 };
void init() {
    board.resize(n + 1, vector<char>(m + 1));
    vis.resize(n + 1, vector<bool>(m + 1, 0));
}

void back(int x, int y) {
    vis[x][y] = 1;
    forn(i, 4) {
        int nx = x + dx[i], ny = y + dy[i];
        if (nx >= 0 && nx < n && ny >= 0 && ny < m &&
            board[nx][ny] != '1' && !vis[nx][ny]) back(nx,
            ny);
    }
}
```

## 6. Math

### 6.1. TernarySearch

```
double f(double x) {
    return x;
}
// ternary_search(0.0, posibleMaximo)
double ternary_search(double l, double r) {
    double eps = 1e-9;
    while (r - l > eps) {
        double m1 = l + (r - l) / 3;
        double m2 = r - (r - l) / 3;
        double f1 = f(m1);
        double f2 = f(m2);
        // if (f1 > c) f1 = f2 minimizar;
        if (f1 < f2) l = m1;
```

```
        else r = m2;
    }

    // return l;
    return f(l);
}
```

### 6.2. Discrete root

```
//find all x -> x^k = a mod n
vector<int> discrete_root(int k, int a, int n) {
    int g = primitive_root(n);
    int gk = bnpow(g, k, n);
    int y = discrete_log(gk, a, n);
    int x = bnpow(g, y, n); //first solution
    int phin = phi(n);
    int delta = phin / __gcd(k, phin);

    vector<int> v;
    for (int i = 0; i < n - 1; i += delta) {
        x = bnpow(g, y + i, n);
        v.pb(x);
    }
    return v;
}
```

### 6.3. squares in a circle

```
ll get(ll mid) {
    ll ans = 0;
    for (ll i = 1; i * i < mid; i++) {
        ans += 4 * floor(sqrt(mid - i * i));
    }

    return ans;
}

const int inf = 1e9 + 7;
```

```

int main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    int test = 1;
#ifdef LOCAL
    freopen("in.txt", "r", stdin);
    freopen("out.txt", "w", stdout);
    test = 2;
#endif

    while (test--) {
        ll n; cin >> n;
        ll r = 4 * n, l = 1;
        double pre = 1.0 * inf;
        while (l <= r) {
            ll mid = (l + r) >> 1;
            if (get(mid) > n) {
                r = mid - 1;
                pre = min(pre, sqrt(mid));
            }
            else l = mid + 1;
        }
        cout << fixed << setprecision(30) << pre << endl;

    }

    cout << flush;
    return 0;
}

```

## 6.4. Pow

```

#define int int64_t
int binpow(int a, int b, int m) {
    a %= m;
    int res = 1;
    while (b > 0) {
        if (b & 1) res = res * a % m;
        a = a * a % m;
        b >>= 1;
    }
    return res;
}

```

```

}

#define int int64_t
int binpow(int a, int b) {
    int res = 1;
    while (b > 0) {
        if (b & 1) res = res * a;
        a = a * a;
        b >>= 1;
    }
    return res;
}

// usar esta pow cuando el modulo puede llegar hasta 1e18
// (segura contra overflow)

int mul(int a, int b, int m) {
    int ret = a * b - int((long double)1 / m * a * b + 0.5)
        * m;
    return ret < 0 ? ret + m : ret;
}

int pow(int x, int y, int m) {
    if (!y) return 1;
    int ans = pow(mul(x, x, m), y / 2, m);
    return y % 2 ? mul(x, ans, m) : ans;
}

```

## 6.5. Pollards Rho

```

#define int int64_t
int mul(int a, int b, int m) {
    int ret = a * b - int((long double)1 / m * a * b + 0.5)
        * m;
    return ret < 0 ? ret + m : ret;
}

int pow(int x, int y, int m) {
    if (!y) return 1;
    int ans = pow(mul(x, x, m), y / 2, m);
    return y % 2 ? mul(x, ans, m) : ans;
}

```



```

}

bool prime(int n) {
    if (n < 2) return 0;
    if (n <= 3) return 1;
    if (n % 2 == 0) return 0;
    int r = __builtin_ctzll(n - 1), d = n >> r;
    for (int a : {2, 325, 9375, 28178, 450775, 9780504, 1795
        265022}) {
        int x = pow(a, d, n);
        if (x == 1 or x == n - 1 or a % n == 0) continue;
        for (int j = 0; j < r - 1; j++) {
            x = mul(x, x, n);
            if (x == n - 1) break;
        }
        if (x != n - 1) return 0;
    }
    return 1;
}

int rho(int n) {
    if (n == 1 or prime(n)) return n;
    auto f = [n](int x) {return mul(x, x, n) + 1;};

    int x = 0, y = 0, t = 30, prd = 2, x0 = 1, q;
    while (t % 40 != 0 or gcd(prd, n) == 1) {
        if (x == y) x = ++x0, y = f(x);
        q = mul(prd, abs(x - y), n);
        if (q != 0) prd = q;
        x = f(x), y = f(f(y)), t++;
    }
    return gcd(prd, n);
}

vector<int> fact(int n) {
    if (n == 1) return {};
    if (prime(n)) return { n };
    int d = rho(n);
    vector<int> l = fact(d), r = fact(n / d);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}

```

## 6.6. Factorization Sieve

```

#define int int64_t
constexpr int MAX = 1e6;
int primediv[MAX]; //10^6
vector<int> primes;

void sieve() {
    forn(i, MAX) primediv[i] = i;
    int root = sqrt(MAX) + 1;
    forne(i, 2, MAX) {
        if (primediv[i] != i) continue;
        primes.pb(i);
        if (i > root) continue;
        for (int j = i * i; j < MAX; j += i) primediv[j] =
            i;
    }
}

map<int, int> factorize(int n) { //n <= 10^12
    map<int, int> factors;
    for (int i = 0; i < primes.size() && n >= MAX; ++i) {
        while (n % primes[i] == 0) {
            factors[primes[i]]++;
            n /= primes[i];
        }
    }
    if (n >= MAX) {
        factors[n]++;
        return factors;
    }
    while (n > 1) {
        factors[primediv[n]]++;
        n /= primediv[n];
    }
    return factors;
}

```

## 6.7. karatsuba

```

/*
Multiplica dos polinomios A y B

```

```

Devuelve un vector C, donde C[i] = coeficiente de xi en A*B
good: n ~ 2e5
*/

//#pragma GCC optimize("Ofast")
//#pragma GCC target ("avx,avx2")
template<typename T> void kar(T* a, T* b, int n, T* r, T*
tmp) {
    if (n <= 64) {
        forn(i, n) forn(j, n) r[i + j] += a[i] * b[j];
        return;
    }
    int mid = n / 2;
    T* atmp = tmp, * btmp = tmp + mid, * E = tmp + n;
    memset(E, 0, sizeof(E[0]) * n);
    forn(i, mid) {
        atmp[i] = a[i] + a[i + mid];
        btmp[i] = b[i] + b[i + mid];
    }
    kar(atmp, btmp, mid, E, tmp + 2 * n);
    kar(a, b, mid, r, tmp + 2 * n);
    kar(a + mid, b + mid, mid, r + n, tmp + 2 * n);
    forn(i, mid) {
        T temp = r[i + mid];
        r[i + mid] += E[i] - r[i] - r[i + 2 * mid];
        r[i + 2 * mid] += E[i + mid] - temp - r[i + 3 *
mid];
    }
}

template<typename T> vector<T> karatsuba(vector<T> a,
vector<T> b) {
    int n = max(sz(a), sz(b));
    while (n & (n - 1)) n++;
    a.resize(n), b.resize(n);
    vector<T> ret(2 * n), tmp(4 * n);
    kar(&a[0], &b[0], n, &ret[0], &tmp[0]);
    return ret;
}

```

## 6.8. Segmented Sieve

```

/*
Math/Sieve.cpp
*/

#define int int64_t
vector<int> segmented_criba(int l, int r) {
    l = max<int>(l, 211);
    vector<bool> vis(r - l + 1);
    for (int& pp : prime) {
        if ((int)pp * pp > r) break;
        int mn = (l + pp - 1) / pp;
        if (mn == 111) mn++;
        mn *= pp;
        for (int i = mn; i <= r; i += pp) {
            vis[i - l] = true;
        }
    }
    vector<int> ans;
    forn(i, sz(vis)) if (!vis[i]) ans.pb(l + i);
    return ans;
}

```

## 6.9. Primitive root

```

/*
Math/Pow.cpp
Math/Phi.cpp
Math/Factorization Sieve.cpp
Math/Pollards Rho.cpp ...?
*/

//find g -> (gk == a mod m) for all a -> gcd(a, m)=1
int primitive_root(int m) {
    int phin = phi(m);
    map<int, int> factors = factorize(phin);
    /*
    phollards rho
    phin = m-1? -> m is prime
    vector<int> ffactors = fact(phin);
    sort(all(f)); f.erase(unique(all(f)), f.end());
    */
}

```

```

forne(i, 1, m + 1) {
    bool ok = true;
    for (auto it : factors) {
        ok = ok && pow(i, phin / it.f, m) != 1;
        if (!ok) break;
    }
    if (ok) return i;
}
return -1;
}

```

## 6.10. Berlekamp massey

```

/*
"Se le da una secuencia y se puede encontrar la k-esima"
Math/Propiedades del modulo.cpp
*/

// #define int int64_t
struct Berlekamp_massey {

    int m; //length of recurrence
    //a: first terms
    //h: relation
    vector<int> a, h, t_, s, t;

    Berlekamp_massey(vector<int>& x) {
        vector<int> v = BM(x);
        m = sz(v);
        h.resize(m), a.resize(m), s.resize(m), t.resize(m),
            t_.resize(2 * m);
        forn(i, m) h[i] = v[i], a[i] = x[i];
    }

    vector<int> BM(vector<int>& x) {
        vector<int> ls, cur;
        int lf, ld;
        forn(i, sz(x)) {
            int t = 0;
            forn(j, sz(cur)) t = (t + x[i - j - 1] *
                (int)cur[j]) % MOD;
            if ((t - x[i]) % MOD == 0) continue;

```

```

            if (!sz(cur)) {
                cur.resize(i + 1);
                lf = i;
                ld = (t - x[i]) % MOD;
                continue;
            }
            int k = -(x[i] - t) * inv_mod(ld);
            vector<int> c(i - lf - 1); c.pb(k);
            forn(j, sz(ls)) c.pb((-ls[j] * k % MOD));
            if (sz(c) < sz(cur)) c.resize(sz(cur));
            forn(j, sz(cur)) c[j] = (c[j] + cur[j]) % MOD;
            if (i - lf + sz(ls) >= sz(cur)) ls = cur, lf =
                i, ld = (t - x[i]) % MOD;
            cur = c;
        }
        forn(i, sz(cur)) cur[i] = (cur[i] % MOD + MOD) %
            MOD;
        return cur;
    }

    //calculate p*q MOD f
    inline vector<int> mul(vector<int>& p, vector<int>& q) {
        forn(i, 2 * m) t_[i] = 0;
        forn(i, m) if (p[i]) {
            forn(j, m) t_[i + j] = (t_[i + j] + p[i] *
                q[j]) % MOD;
        }
        for (int i = 2 * m - 1; i >= m; --i) if (t_[i]) {
            forn(j, m) t_[i - j - 1] = (t_[i - j - 1] +
                t_[i] * h[j]) % MOD;
        }
        forn(i, m) p[i] = t_[i];
        return p;
    }

    inline int magic(int k) {
        if (k < sz(a)) return a[k];
        forn(i, m) s[i] = t[i] = 0;
        s[0] = 1;
        if (m != 1) t[1] = 1;
        else t[0] = h[0];

        while (k) {
            if (k & 1LL) s = mul(s, t);

```

```

        t = mul(t, t); k /= 2;
    }
    int su = 0;
    forn(i, m) su = (su + s[i] * a[i]) % MOD;
    return (su % MOD + MOD) % MOD;
}

};

```

## 6.11. FFT

```
// #include <complex> ...?
```

```

/*
usage complex<double> como tipo de dato
vector<complex<double>> a(k + 1), b(k + 1);
vector<complex<double>> c = convolution(a, b);
(int)(c[i].real() + 0.5) para redondear
*/

```

```

void get_roots(bool f, int n, vector<complex<double>>&
roots) {
    const static double PI = acos(-1);
    forn(i, n / 2) {
        double alpha = i * ((2 * PI) / n);
        if (f) alpha = -alpha;
        roots[i] = { cos(alpha), sin(alpha) };
    }
}

template<typename T> void fft(vector<T>& a, bool f, int N,
vector<int>& rev) {
    forn(i, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
    int l, r, m;
    vector<T> roots(N);
    for (int n = 2; n <= N; n *= 2) {
        get_roots(f, n, roots);
        for (int pos = 0; pos < N; pos += n) {
            l = pos + 0, r = pos + n / 2, m = 0;
            while (m < n / 2) {
                auto t = roots[m] * a[r];
                a[r] = a[l] - t;

```

```

                a[l] = a[l] + t;
                l++, r++, m++;
            }
        }
    }
    if (f) {
        auto invN = T(1) / T(N);
        forn(i, N) a[i] = a[i] * invN;
    }
}

template<typename T> vector<T> convolution(vector<T>& a,
vector<T>& b) {
    vector<T> l(all(a)), r(all(b));
    int N = sz(l) + sz(r) - 1;
    int n = 1, log_n = 0;
    while (n <= N) n *= 2, log_n++;
    vector<int> rev(n);
    forn(i, n) {
        rev[i] = 0;
        forn(j, log_n) if (i >> j & 1) {
            rev[i] |= 1 << (log_n - 1 - j);
        }
    }
    assert(N <= n);
    l.resize(n);
    r.resize(n);
    fft(l, false, n, rev);
    fft(r, false, n, rev);
    forn(i, n) l[i] *= r[i];
    fft(l, true, n, rev);
    l.resize(N);
    return l;
}

```

## 6.12. Discrete Log

```

// Returns minimum x for which a ^ x % m = b % m, a and m
are coprime.
#define int int64_t
constexpr int INF = 1e18;
int discrete_log(int b, int a, int m) {

```

```

if (a == 0) return b ? -1 : 1; // base case?

a %= m, b %= m;
int k = 1, shift = 0;
while (1) {
    int g = __gcd(a, m);
    if (g == 1) break;
    if (b == k) return shift;
    if (b % g) return -1;
    b /= g, m /= g, shift++;
    k = (int)k * a / g % m;
}

int sq = sqrt(m) + 1, giant = 1;
for(i, sq) giant = (int)giant * a % m;

vector<pair<int, int>> baby;
for (int i = 0, cur = b; i <= sq; i++) {
    baby.emplace_back(cur, i);
    cur = (int)cur * a % m;
}
sort(all(baby));

for (int j = 1, cur = k; j <= sq; j++) {
    cur = (int)cur * giant % m;
    auto it = lower_bound(all(baby), make_pair(cur,
        INF));
    if (it != baby.begin() and (--it)->f == cur) return
        sq * j - it->s + shift;
}

return -1;
}

```

### 6.13. Chinese Remainder Theorem

```

//ext_gcd(a,b): return {g, x, y} tal que g = gcd(a,b) y a *
    x + b * y = g.

template<typename T> tuple<T, T, T> ext_gcd(T a, T b) {
    if (!a) return { b, 0, 1 };
    auto [g, x, y] = ext_gcd(b % a, a);

```

```

        return { g, y - b / a * x, x };
    }

    /*
    Dadas dos congruencias
    x == a (mod n)
    x == b (mod m)
    con gcd(n, m) = 1, existe una solución única módulo K = n *
        m.
    a = residuo en [0, lcm)
    m = lcm de los módulos.
    crt<int> c1(a, n), c2(b, m), sol = c1 * c2;
    */

template<typename T = int64_t> struct crt {
    T a, m;
    crt() : a(0), m(1) {}
    crt(T a_, T m_) : a(a_), m(m_) {}
    crt operator * (crt C) {
        auto [g, x, y] = ext_gcd(m, C.m);
        if ((a - C.a) % g) a = -1;
        if (a == -1 or C.a == -1) return crt(-1, 0);
        T lcm = m / g * C.m;
        T ans = a + (x * (C.a - a) / g % (C.m / g)) * m;
        return crt((ans % lcm + lcm) % lcm, lcm);
    }
};

```

### 6.14. polynomial

```

//zx^n+...+cx^2+bx+a
typedef int tp; // type of polynomial
template<class T = tp>
struct poly { // poly<> : 1 variable, poly<poly<>>: 2
    variables, etc.
    vector<T> c;
    T& operator[](int k) { return c[k]; }
    poly(vector<T>& c) : c(c) {}
    poly(initializer_list<T> c) : c(c) {}
    poly(int k) : c(k) {}
    poly() {}
    poly operator+(poly<T> o) {

```

```

    int m = c.size(), n = o.c.size();
    poly res(max(m, n));
    forn(i, m) res[i] = res[i] + c[i];
    forn(i, n) res[i] = res[i] + o.c[i];
    return res;
}
poly operator*(tp k) {
    poly res(c.size());
    forn(i, c.size()) res[i] = c[i] * k;
    return res;
}
poly operator*(poly o) {
    int m = c.size(), n = o.c.size();
    poly res(m + n - 1);
    forn(i, m) forn(j, n) res[i + j] = res[i + j] + c[i]
        * o.c[j];
    return res;
}
poly operator-(poly<T> o) { return *this + (o * -1); }
T operator()(tp v) {
    T sum(0);
    for (int i = c.size() - 1; i >= 0; --i) sum = sum * v
        + c[i];
    return sum;
}
};
// example: p(x,y)=2*x^2+3*x*y-y+4
// poly<poly<>> p={{4,-1},{0,3},{2}}
// printf("%d\n",p(2)(3)) // 27 (p(2,3))
set<tp> roots(poly<> p) { // only for integer polynomials
    set<tp> r;
    while (!p.c.empty() && !p.c.back()) p.c.pop_back();
    if (!p(0)) r.insert(0);
    if (p.c.empty()) return r;
    tp a0 = 0, an = abs(p[p.c.size() - 1]);
    for (int k = 0; !a0; a0 = abs(p[k++]));
    vector<tp> ps, qs;
    forne(i, 1, sqrt(a0) + 1) if (a0 % i == 0) ps.pb(i),
        ps.pb(a0 / i);
    forne(i, 1, sqrt(an) + 1) if (an % i == 0) qs.pb(i),
        qs.pb(an / i);
    for (auto pt : ps) for (auto qt : qs) if (pt % qt == 0) {
        tp x = pt / qt;

```

```

        if (!p(x)) r.insert(x);
        if (!p(-x)) r.insert(-x);
    }
    return r;
}
pair<poly<>, tp> ruffini(poly<> p, tp r) { // returns pair
    (result, rem)
    int n = p.c.size() - 1;
    vector<tp> b(n);
    b[n - 1] = p[n];
    for (int k = n - 2; k >= 0; --k) b[k] = p[k + 1] + r * b[k]
        + 1];
    return { poly<>(b), p[0] + r * b[0] };
}
// only for double polynomials
pair<poly<>, poly<> > polydiv(poly<> p, poly<> q) { //
    returns pair (result, rem)
    int n = p.c.size() - q.c.size() + 1;
    vector<tp> b(n);
    for (int k = n - 1; k >= 0; --k) {
        b[k] = p.c.back() / q.c.back();
        forn(i, q.c.size()) p[i + k] -= b[k] * q[i];
        p.c.pop_back();
    }
    while (!p.c.empty() && abs(p.c.back()) <
        EPS) p.c.pop_back();
    return { poly<>(b), p };
}
// only for double polynomials
poly<> interpolate(vector<tp> x, vector<tp> y) { //TODO TEST
    poly<> q = { 1 }, S = { 0 };
    for (tp a : x) q = poly<>({ -a, 1 }) * q;
    forn(i, x.size()) {
        poly<> Li = ruffini(q, x[i]).ff;
        Li = Li * (1.0 / Li(x[i])); // change for int
            polynomials
        S = S + Li * y[i];
    }
    return S;
}
vector<ll> coef(vector<ll> roots, bool first = true) {
    int l = roots.size() + 1;
    vector<ll> c(10002, 0), m(10002, 0);

```

```

c[0] = 1;
for(k, roots.size()) {
    forne(i, 1, 1) m[i] = c[i] + c[i - 1] * roots[k];
    forne(i, 1, 1) c[i] = m[i];
}
ll sign = first ? 1 : -1;
for(i, roots.size()) {
    sign *= -1LL;
    m[i + 1] *= sign;
}
return m;
}

inline ll modn(ll x) { x = x % mod; if (x < 0ll) x += mod;
return x; }

//interpolate for consecutive values X and evaluate at K;
ll interpolateAndEvaluate(ll k, int inix, vector<ll>& y) {
    ll den = 1, num = 1;
    int len = inix + sz(y) - 1;

    forne(i, inix, len) {
        num = (num * (k - (i + 1))) % mod;
        den = (den * modn(-1ll * i)) % mod;
    }

    ll res = (y[0] * divmod(num, den)) % mod;
    forne(i, inix, len) {
        num = divmod(num, k - (i + 1));
        num = (num * (k - i)) % mod;
        den = divmod(den, modn(-1ll * (sz(y) - i)));
        den = (den * i) % mod;
        res = (res + (y[i] * divmod(num, den)) % mod) % mod;
    }
    return res;
}

```

### 6.15. LinearDiophantineEquations

```

bool find_any_solution(int a, int b, int c, int& x, int& y,
int& g) {
    g = extEuclid(a, b, x, y);
    if (c % g)
        return false;
}

```

```

x *= (c / g);
y *= (c / g);
return true;
}

void find_all_solutions(int a, int b, int c) {
    int x, y, g, x0, y0;
    if (!find_any_solution(a, b, c, x, y, g))
        return;
    forne(i, -10, 10) {
        x0 = x + i * (b / g);
        y0 = y - i * (a / g);
        printf("%d*%d + %d*%d = %d\n", a, x0, b, y0, a * x0
+ b * y0);
    }
}

```

### 6.16. is prime

```

bool is_prime(int64_t n) {
    if (n < 2) return 0;
    for (int64_t p = 2; p * p <= n; p += p % 2 + 1) if (n %
p == 0) return 0;
    return 1;
}

```

### 6.17. Miller Rabin

```

bool probably_prime(ll n, ll a, ll d, int s){
    ll x = binpow(a, d, n);
    if(x == 1 || x+1 == n) return true;
    forn(r, s){
        x = mulmod(x,x,n);
        if(x == 1) return false;
        if(x+1 == n) return true;
    }
    return false;
}

bool miller_rabin(ll n){//check (n is prime)?
    if(n < 2) return false;
}

```

```

const int a[] = {2,3,5,7,11,13,17,19,23};
int s = -1;
ll d = n-1;
while(!d&1) d >>= 1, s++;

forn(i, 9){
    if(n == a[i]) return true;
    if(!probably_prime(n, a[i], d, s))
        return false;
}
return true;
}

```

## 6.18. Sieve

```

const int MAX = int(1e6);
bitset<MAX + 5> bs;
vector<int64_t> prime;

void sieve() {
    bs.set();
    bs[0] = bs[1] = 0;
    for (int i = 2; i <= MAX; i++) {
        if (bs[i]) {
            prime.pb(i);
            for (int j = i * i; j <= MAX; j += i) {
                bs[j] = 0;
            }
        }
    }
}

```

## 6.19. Propiedades del modulo

```

constexpr int MOD = 1e9 + 7;

int64_t binpow(int64_t a, int64_t b, int64_t m) {
    a %= m;
    int64_t res = 1;
    while (b > 0) {
        if (b & 1) res = res * a % m;

```

```

        a = a * a % m;
        b >>= 1;
    }
    return res;
}

inline int inv_mod(int a) { return binpow(a, MOD - 2, MOD); }
inline int add(int a, int b) { return ((a % MOD) + (b % MOD)) % MOD; }
inline int res(int a, int b) { return ((a % MOD) - (b % MOD) + MOD) % MOD; }
inline int mul(int a, int b) { return ((a % MOD) * (b % MOD)) % MOD; }
inline int divm(int a, int b) { return mul(a, inv_mod(b)); }
}

```

## 6.20. ExtendedEuclid

```

int extEuclid(int a, int b, int &x, int &y){
    if(b == 0){
        x = 1;
        y = 0;
        return a;
    }
    int xi, yi;
    int g = extEuclid(b, a%b, xi, yi);
    x = yi;
    y = xi-yi*(a/b);
    return g;
}

```

## 6.21. Modular Inverse

```

int inv[MAXN];

void modular_inverse_range(int m) {
    inv[0] = 0; inv[1] = 1;
    forne(i, 2, MAXN)
        inv[i] = (-(m / i) * inv[m % i] + m) % m;
}

```



```

int modular_inverse_binpow(int a, int m) {
    return binpow(a, phi(m) - 1, m);
}

int modular_inverse_extEuclid(int a, int m) {
    int x, y;
    int g = extEuclid(a, m, x, y);
    if (g != 1)
        return -1;
    x = (x % m + m) % m;
    return x;
}

vector<int> inversos(vector<int> a, int m) {
    vector<int> inv;
    int v = 1;
    forn(i, sz(a)) {
        inv.pb(v);
        v = (v * a[i]) % m;
    }

    int x, y;
    extEuclid(v, m, x, y);
    x = (x % m + m) % m;
    for (int i = sz(a) - 1; i >= 0; i--) {
        inv[i] = inv[i] * x;
        x = (x * a[i]) % m;
    }
    return inv;
}

```

## 6.22. Phi

```

int phi(int n) {
    int result = n;
    for (int i = 2; i * i <= n; ++i) {
        if (n % i) continue;
        while (n % i == 0)
            n /= i;
        result -= result / i;
    }
}

```

```

if (n > 1)
    result -= result / n;
return result;
}

vector<int> phi_1_to_n(int n) {
    vector<int> phi;
    forn(i, n + 1) phi.pb(i);
    for (int i = 2; i <= n; ++i) {
        if (phi[i] != i) continue;
        for (int j = i; j <= n; j += i) phi[j] -= phi[j] / i;
    }
    return phi;
}

vector<int> phi_1_to_n2(int n) {
    vector<int> phi;
    forn(i, n + 1) phi.pb(i - 1);
    phi[1] = 1;
    for (int i = 2; i <= n; ++i) {
        for (int j = i * 2; j <= n; j += i) phi[j] -= phi[i];
    }

    return phi;
}

```

## 6.23. floordiv ceildiv

```

int64_t floor_div(int64_t a, int64_t b) {
    return a / b - ((a ^ b) < 0 && a % b != 0);
}

int64_t ceil_div(int64_t a, int64_t b) {
    return a / b + ((a ^ b) > 0 && a % b != 0);
}

```

## 6.24. sqrt

```

11 int_sqrt (11 x) {
    11 ans = 0;
    for (11 k = 1LL << 30; k != 0; k /= 2)
        if ((ans + k) * (ans + k) <= x)
            ans += k;
    return ans;
}

```

## 6.25. dec and bin

```

string dec_to_bin(int64_t n) {
    bitset<32> bs(n);
    string s = bs.to_string();
    return s;
}
int64_t bin_to_dec(string s) {
    bitset<32> bs(s);
    int64_t n = bs.to_ullong();
    return n;
}

string dec_to_bin(int64_t n) {
    string s;
    while (n) {
        if (n & 1) s.pb('1');
        else s.pb('0');
        n >>= 1;
    }
    reverse(all(s));
    return s;
}
int64_t bin_to_dec(string s) {
    int64_t res = 0;
    for (auto&& i : s) {
        res <<= 1;
        res += i - '0';
    }
    return res;
}

```

## 6.26. fraction

```

struct fraction {
    int num, den;

    fraction(int num, int den) : num(num), den(den) {
        check_den();
        simplify();
    }
    void check_den() {
        if (den < 0) {
            num = -num;
            den = -den;
        }
    }
    void simplify() {
        int mcd = __gcd(abs(num), abs(den));
        num /= mcd;
        den /= mcd;
    }
    pair<int, int> x() { return { num, den }; }

    fraction operator + (const fraction& x) const {
        return fraction(num * x.den + den * x.num, den *
            x.den);
    }
    fraction operator - (const fraction& x) const {
        return fraction(num * x.den - den * x.num, den *
            x.den);
    }
    fraction operator * (const fraction& x) const {
        return fraction(num * x.num, den * x.den);
    }
    fraction operator / (const fraction& x) const {
        return fraction(num * x.den, den * x.num);
    }
    friend ostream& operator << (ostream& os, const
        fraction& x) {
        return os << x.num << " / " << x.den;
    }
};

```

## 6.27. Matrix

```
struct matrix {
    int n, m;
    vector<vector<int>> v;

    matrix(int n, int m, bool ones = false) : n(n), m(m),
        v(n, vector<int>(m)) {
        if (ones) forn(i, n) v[i][i] = 1;
    }

    matrix operator * (const matrix& o) {
        matrix ans(n, o.m);
        forn(i, n)
            forn(k, m) if (v[i][k])
                forn(j, o.m)
                    ans[i][j] = (1ll * v[i][k] * o.v[k][j] + ans[i][j]) %
                        MOD;
        return ans;
    }

    vector<int>& operator [] (int i) {
        return v[i];
    }
};

matrix binpow(matrix b, ll e) {
    matrix ans(b.n, b.m, true);
    while (e) {
        if (e & 1) ans = ans * b;
        b = b * b;
        e >>= 1;
    }
    return ans;
}
```

## 6.28. nCK

```
struct mint {
    static constexpr int m = 1e9 + 7;
    //static inline int m = 998244353; //to change mod
    int x;
```

```
    mint() : x(0) {}
    mint(long long x_) : x(x_% m) { if (x < 0) x += m; }
    int val() { return x; }
    mint& operator+=(mint b) { if ((x += b.x) >= m) x -= m;
        return *this; }
    mint& operator-=(mint b) { if ((x -= b.x) < 0) x += m;
        return *this; }
    mint& operator*=(mint b) { x = (long long)(x)*b.x % m;
        return *this; }
    mint pow(long long e) const {
        mint r = 1, b = *this;
        while (e) {
            if (e & 1) r *= b;
            b *= b;
            e >>= 1;
        }
        return r;
    }
    mint inv() { return pow(m - 2); }
    mint& operator/=(mint b) { return *this *= b.pow(m - 2); }

    friend mint operator+(mint a, mint b) { return a += b; }
    friend mint operator-(mint a, mint b) { return a -= b; }
    friend mint operator/(mint a, mint b) { return a /= b; }
    friend mint operator*(mint a, mint b) { return a *= b; }
    friend bool operator<(mint a, mint b) { return a.x <
        b.x; }
    friend bool operator==(mint a, mint b) { return a.x ==
        b.x; }
    friend bool operator!=(mint a, mint b) { return a.x !=
        b.x; }
};

const int mxN = 1e6 + 7; // max value of N,K
mint fact[mxN];
mint inv_fact[mxN];
void init() {
    fact[0] = 1;
    forne(i, 1, mxN) fact[i] = fact[i - 1] * i;
    forn(i, mxN) inv_fact[i] = fact[i].inv();
}
```

```

mint nCk(ll n, ll k) {
    return (fact[n] * inv_fact[k]) * inv_fact[n - k];
}

```

## 6.29. Binomial

```

template<typename T>
struct Binomial {
    vector<T> Facto, inv_Facto;
    void extend(int m = -1) {
        int n = sz(Facto);
        if (m == -1) m = n * 2;
        if (n >= m) return;
        Facto.resize(m);
        inv_Facto.resize(m);
        for (int i = n; i < m; i++) Facto[i] = Facto[i - 1]
            * T(i);
        inv_Facto[m - 1] = T(1) / Facto[m - 1];
        for (int i = m - 1; i > n; i--) inv_Facto[i - 1] =
            inv_Facto[i] * T(i);
    }
    Binomial(int MAX = 0) {
        Facto.resize(1, T(1));
        inv_Facto.resize(1, T(1));
        extend(MAX + 1);
    }

    T fact(int i) {
        if (i < 0) return 0;
        while (int(sz(Facto)) <= i) extend();
        return Facto[i];
    }

    T invfact(int i) {
        if (i < 0) return 0;
        while (int(sz(inv_Facto)) <= i) extend();
        return inv_Facto[i];
    }

    T C(int a, int b) {
        if (a < b || b < 0) return 0;
        return fact(a) * invfact(b) * invfact(a - b);
    }

    T invC(int a, int b) {

```

```

        if (a < b || b < 0) return 0;
        return fact(b) * fact(a - b) * invfact(a);
    }

    T P(int a, int b) {
        if (a < b || b < 0) return 0;
        return fact(a) * invfact(a - b);
    }

    T inv(int a) {
        if (a < 0) return inv(-a) * T(-1);
        if (a == 0) return 1;
        return fact(a - 1) * invfact(a);
    }

    T Catalan(int n) {
        if (n < 0) return 0;
        return fact(2 * n) * invfact(n + 1) * invfact(n);
    }

    T narayana(int n, int k) {
        if (n <= 0 || n < k || k < 1) return 0;
        return C(n, k) * C(n, k - 1) * inv(n);
    }

    T Catalan_pow(int n, int d) {
        if (n < 0 || d < 0) return 0;
        if (d == 0) {
            if (n == 0) return 1;
            return 0;
        }
        return T(d) * inv(d + n) * C(2 * n + d - 1, n);
    }

    // return [x^a] 1/(1-x)^b
    T ruiseki(int a, int b) {
        if (a < 0 || b < 0) return 0;
        if (a == 0) {
            return 1;
        }
        return C(a + b - 1, b - 1);
    }

    // (a, b) -> (c, d)
    // always x + e >= y
    T mirror(int a, int b, int c, int d, int e = 0) {
        if (a + e < b || c + e < d) return 0;
        if (a > c || b > d) return 0;
        a += e;
        c += e;

```

```

        return C(c + d - a - b, c - a) - C(c + d - a - b, c
            - b + 1);
    }
    // return sum_{i = 0, ... , a} sum_{j = 0, ... , b} C(i
        + j, i)
    // return C(a + b + 2, a + 1) - 1;
    T gird_sum(int a, int b) {
        if (a < 0 || b < 0) return 0;
        return C(a + b + 2, a + 1) - 1;
    }
    // return sum_{i = a, ..., b - 1} sum_{j = c, ... , d -
        1} C(i + j, i)
    // AGC 018 E
    T gird_sum_2(int a, int b, int c, int d) {
        if (a >= b || c >= d) return 0;
        a--, b--, c--, d--;
        return gird_sum(a, c) - gird_sum(a, d) -
            gird_sum(b, c) + gird_sum(b, d);
    }

    // the number of diagonal dissections of a convex n-gon
    // into k+1 regions.
    // OEIS A033282
    // AGC065D
    T diagonal(int n, int k) {
        if (n <= 2 || n - 3 < k || k < 0) return 0;
        return C(n - 3, k) * C(n + k - 1, k) * inv(k + 1);
    }
};

Binomial<mint> bin;

```

### 6.30. divisores

```

vector<int> div(int n) {
    vector<int> ans;
    for (int i = 1; i * i <= n; i++) {
        if (n % i == 0) {
            ans.pb(i);
            if (i != n / i) ans.pb(n / i);
        }
    }
}

```

```

        return ans;
    }

```

## 7. Problems

### 7.1. dp+nck

```

static constexpr int mod = 1e9 + 7;
struct mint {
    static constexpr int m = 1e9 + 7;
    int x;
    mint() : x(0) {}
    mint(long long x_) : x(x_% m) { if (x < 0) x += m; }
    int val() { return x; }
    mint& operator+=(mint b) { if ((x += b.x) >= m) x -= m;
        return *this; }
    mint& operator-=(mint b) { if ((x -= b.x) < 0) x += m;
        return *this; }
    mint& operator*=(mint b) { x = (long long)(x)*b.x % m;
        return *this; }
    mint pow(long long e) const {
        mint r = 1, b = *this;
        while (e) {
            if (e & 1) r *= b;
            b *= b;
            e >>= 1;
        }
        return r;
    }
    mint inv() { return pow(m - 2); }
    mint& operator/=(mint b) { return *this *= b.pow(m - 2
        ); }
    friend mint operator+(mint a, mint b) { return a += b; }
    friend mint operator-(mint a, mint b) { return a -= b; }
    friend mint operator/(mint a, mint b) { return a /= b; }
    friend mint operator*(mint a, mint b) { return a *= b; }
    friend bool operator==(mint a, mint b) { return a.x ==
        b.x; }
    friend bool operator!=(mint a, mint b) { return a.x !=
        b.x; }
}

```

```

};

const int mxN = 105;
mint dp[21][mxN][mxN][mxN];
mint factorial[mxN];
mint inverse_factorial[mxN];

void init() {
    factorial[0] = 1;
    forne(i, 1, mxN) factorial[i] = factorial[i - 1] * i;
    forn(i, mxN) inverse_factorial[i] = factorial[i].inv();
}

mint binomial_coefficient(ll n, ll k) {
    return (factorial[n] * inverse_factorial[k]) *
           inverse_factorial[n - k];
}

mint back(ll n, ll r, ll g, ll b) {
    if (r < 0 || g < 0 || b < 0) return 0;
    if (n == 0) return 1;
    if (dp[n][r][g][b] != -1) return dp[n][r][g][b];
    mint form_1 = 0, form_2 = 0, form_3 = 0;
    form_1 = back(n - 1, r - n, g, b) + back(n - 1, r, g -
        n, b) + back(n - 1, r, g, b - n);
    if (n % 2 == 0) {
        /*
        R G B (R G) - (R B) - (G B)
        */
        form_2 = binomial_coefficient(n, n / 2) * ((back(n
            - 1, r - n / 2, g - n / 2, b) + back(n - 1, r -
            n / 2, g, b - n / 2) + back(n - 1, r, g - n / 2,
            b - n / 2)));
    }

    if (n % 3 == 0) {
        /*
        n=3 bc_1=3 bc_2=2 = 6
        R G B - R B G - G B R -B G R - G R B - B R G
        */
        mint bc_1 = binomial_coefficient(n, n / 3);
        mint bc_2 = binomial_coefficient(2 * n / 3, n / 3);
        form_3 = (bc_1 * bc_2) * back(n - 1, r - n / 3, g -
            n / 3, b - n / 3);
    }
}

```

```

}
return dp[n][r][g][b] = form_1 + form_2 + form_3;
}

int main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    init();
    ll n, r, g, b; cin >> n >> r >> g >> b;
    forn(i, n + 1) forn(j, r + 1) forn(k, g + 1) forn(l, b
        + 1) dp[i][j][k][l] = -1;
    // memset(dp, -1, sizeof dp);
    cout << back(n, r, g, b).val() << endl;

    cout << flush;
    return 0;
}

```

## 7.2. matrixexp

```

/*
The Trinacci sequence is defined as follows:
t(0)=1
t(1)=2
t(2)=3

t(n)=3t(n-1)+2t(n-2)+t(n-3)+3 for n>=3
*/

const int MOD = 1e9 + 7;
struct matrix {
    int n, m;
    vector<vector<int>>>v;

    matrix(int n, int m, bool ones = false) : n(n), m(m),
        v(n, vector<int>(m)) {
        if (ones) forn(i, n) v[i][i] = 1;
    }

    matrix operator * (const matrix& o) {
        matrix ans(n, o.m);

```

```

        forn(i, n)
            forn(k, m) if (v[i][k])
                forn(j, o.m)
                    ans[i][j] = (v[i][k] * o.v[k][j] + ans[i][j]) %
                        MOD;
            return ans;
    }

    vector<int>& operator [] (int i) {
        return v[i];
    }
};

matrix bincpow(matrix b, int e) {
    matrix ans(b.n, b.m, true);
    while (e) {
        if (e & 1) ans = ans * b;
        b = b * b;
        e >>= 1;
    }
    return ans;
}

void solve() {

    matrix a(4, 4), b(4, 4);
    a[0][0] = 4;
    a[0][1] = (-1 + MOD) % MOD;
    a[0][2] = (-1 + MOD) % MOD;
    a[0][3] = (-1 + MOD) % MOD;
    forn(i, 3) a[i + 1][i] = 1;

    b[0][0] = 17;
    b[1][0] = 3;
    b[2][0] = 2;
    b[3][0] = 1;

    int n; cin >> n;
    if (n < 4) {
        cout << b[3 - n][0] << endl;
    }
    else {

```

```

        matrix ans = bincpow(a, n - 3) * b;
        cout << ans[0][0] << endl;
    }

}

main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    int testcase = 1;
#ifdef LOCAL
    freopen("in.txt", "r", stdin);
    freopen("out.txt", "w", stdout);
    testcase = 4;
#endif

    //cin >> testcase;
    while (testcase--) solve();

    cout << flush;
    return 0;
}

```

### 7.3. Conotruncado

```

#define int int64_t

const double PI = acos(-1);
const double eps = 1e-7;

// se requiere llenar 50% del volumen de un cono truncado

```

```

// con radio menor r, radio mayor R y altura h
// se requiere encontrar la altura a la que se debe llenar
// el cono truncado

void solve() {
    double r, R, h; cin >> r >> R >> h;

    auto get = [&](double r1, double r2, double h) {
        return (PI * h / 3.0) * (r1 * r1 + r1 * r2 + r2 *
            r2); //volumen cono truncado
    };

    double vol = get(r, R, h);
    double l = 0, hi = h;

    forn(_, 200) {
        double mid = (l + hi) / 2.0;
        double newmid = r + (R - r) * (mid / h); // Radio
            en la altura mid
        double volmid = get(r, newmid, mid);
        if (volmid < vol / 2.0) l = mid;
        else hi = mid;
    }

    cout << fixed << setprecision(9) << (l + hi) / 2.0 <<
        endl;
}

main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);

    int testcase; cin >> testcase; while (testcase--)
        solve();
}

```

```

    cout << flush;
    return 0;
}

```

## 7.4. Hard-Fibonacci

```

#define int int64_t

constexpr int MOD = 998244353;
constexpr int NMOD = 998244353 + 998244353 + 2;

struct matrix {
    int n, m;
    vector<vector<int>> v;

    matrix(int n, int m, bool ones = false) : n(n), m(m),
        v(n, vector<int>(m)) {
        if (ones) forn(i, n) v[i][i] = 1;
    }

    matrix operator * (const matrix& o) {
        matrix ans(n, o.m);
        forn(i, n)
            forn(k, m) if (v[i][k])
                forn(j, o.m)
                    ans[i][j] = (1ll * v[i][k] * o.v[k][j] +
                        ans[i][j]) % MOD;
        return ans;
    }

    vector<int>& operator [] (int i) {
        return v[i];
    }
};

matrix binpow(matrix b, ll e) {
    matrix ans(b.n, b.m, true);
    while (e) {
        if (e & 1) ans = ans * b;
        b = b * b;
        e >>= 1;
    }
}

```



```

    }
    return ans;
}

void solve() {

    string s; cin >> s;

    int n = 0;
    forn(i, sz(s)) n = ((n * 10) % NMOD + (s[i] - '0')) %
        NMOD;

    matrix fib(2, 2);
    fib[0][0] = fib[0][1] = fib[1][0] = 1; fib[1][1] = 0;

    matrix res = binpow(fib, n);

    cout << res[0][1] << endl;

}

main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);

    int testcase; cin >> testcase; while (testcase--)
        solve();

    cout << flush;
}

```

```

    return 0;
}

```

## 7.5. Eval fractio

```

# 1/2+1/3 -> 5/6
import math
import sys
from fractions import Fraction
import re
input = sys.stdin.readline
write = sys.stdout.write
def print(x): write(str(x) + '\n')

def main():
    s = input().strip()
    while s != "":
        s_eval = re.sub(r'(\d+)/(\d+)', r'Fraction(\1,\2)',
            s)
        ans = eval(s_eval, {'Fraction': Fraction})
        x = ans
        y = f"{x.numerator}/{x.denominator}"
        print(y)
        s = input().strip()

if __name__ == '__main__':
    main()

```

## 7.6. censor

```

/*
Dado un string s y n strings t, se pide eliminar todas las
ocurrencias de los strings t en s.

begintheescapeexecutionatthebreakofdawn
2
escape
execution
-----
beginthatthebreakofdawn
*/

```

```

/*
Usage:
    Good values c = 137, modbest=998244353, mod = 10^9
        + 7, mod = 1e18 + 9.
    If necessary to check too many pairs of hashes, use
        two
    different hashes.
    If hashing something other than english characters:
        - Don't have elements with value 0.
        - Use c > max element value.
*/

#define int int64_t
constexpr int mxN = 1e6 + 7;
vector<int> p(mxN);
void pre(int c, int mod) {
    p[0] = 1;
    for (int i = 0; i < mxN - 1; i++) {
        p[i + 1] = (c * p[i]) % mod;
    }
}

struct Hash {
    #warning llamar pre;
    ll c, mod;
    vector<int> h;
    Hash(const string s, const int c, const int mod) :
        c(c), mod(mod), h(sz(s) + 1) {
        h[0] = 0;
        for (int i = 0; i < sz(s); i++) {
            h[i + 1] = (c * h[i] + s[i]) % mod;
        }
    }
    // Returns hash of interval s[a ... b] (where 0 <= a <=
        b < sz(s))
    ll get(int a, int b) {
        return (h[b + 1] - ((h[a] * p[b - a + 1]) % mod) +
            mod) % mod;
    }
};

```

```

bool same(Hash& Ha, Hash& Hb, int l, int r) {
    int qa = Ha.get(l, r);
    int qb = Hb.get(sz(Hb.h) - 2 - r, sz(Hb.h) - 2 - l);
    return qa == qb;
}

const int mod = (1e9 + 7);
main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);

    pre(137, mod);
    #ifndef LOCAL
        freopen("censor.in", "r", stdin);
        freopen("censor.out", "w", stdout);
    #endif

    string s, t; cin >> s;

    int mm = (1ULL << 63) - 1, n;
    int mx = -((1ULL << 63) - 1);
    cin >> n;
    set<int> mp, tam;

    forn(i, n) {
        cin >> t;
        mm = min(mm, sz(t));
        mx = max(mx, sz(t));
        tam.ins(sz(t));
        Hash ht(t, 137, mod);
        mp.ins(ht.get(0, sz(t) - 1));
    }

    int c = 137;
    vector<int> h(sz(s) + 10);
    h[0] = 0;

    auto get = [&](int a, int b) -> int {
        return (h[b + 1] - ((h[a] * p[b - a + 1]) % mod) +
            mod) % mod;
    };
    int i = 0;
    vector<char> ans(sz(s) + 1);
    each(j, s) {

```

```

    ans[i] = j;
    h[i + 1] = (c * h[i] + j) % mod;
    if (i >= mm - 1) {
        each(l, tam) {
            if (i >= l - 1) {
                int cur = get(i - l + 1, i);
                if (i >= l - 1 && mp.count(cur)) {
                    i = i - l;
                }
            }
            else break;
        }
    }
    i++;
}

for(j, i) cout << ans[j];

cout << flush;
return 0;
}

```

## 7.7. F Less Than G

```

/*
Given two arrays a and b of n non-negative integers, count
the number of good pairs
l,r (1<=l<=r<=n), satisfying F(l,r)<G(l,r)
Where F(l,r) is the sum of the square of numbers in the
range [l,r]
And G(l,r) is the square of the bitwise OR of the range
[l,r]
*/

main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);

    // brute();
    int n; cin >> n;
    vector<int> a(n), b(n), prefix(n + 6);
    set<int> pro[25];
    for(i, n) cin >> a[i];

```

```

    for(i, n) cin >> b[i];
    for(init, 22) pro[init].insert(n);
    for(i, n) prefix[i + 1] = prefix[i] + a[i] * a[i];
    prefix.erase(prefix.begin());
    for(i, n) for(j, 22) if (b[i] & (1 << j))
        pro[j].insert(i);

    int ans = 0;
    for(i, n) {
        int last = i, Or = b[i];
        while (last < n) {
            int best = n;
            for(j, 22) {
                if (!(Or & (1 << j))) best = min(best,
                    *pro[j].upper_bound(last));
            }
            int l = last, r = best - 1, x = -1, need = Or *
                Or;
            while (l <= r) {
                int mid = (l + r) / 2;
                if (prefix[mid] - (i > 0 ? prefix[i - 1] : 0)
                    >= need) r = mid - 1;
                else l = mid + 1, x = mid;
            }
            if (~x) ans += x - last + 1;
            swap(last, best);
            if (last < n) Or |= b[last];
        }
    }
    cout << ans << endl;

    cout << flush;
    return 0;
}

```

## 7.8. graycode

```

/*
Genera una permutacion de 0 a 2^n-1, de modo que
dos posiciones adyacentes difieren en exactamente 1 bit
*/

```

```
vector<string> gray_code(int n) {
    vector<string> ret(1 << n);
    for (int i = 0; i < (1 << n); i++) {
        ret[i] = bitset<32>(i ^ (i >> 1)).to_string();
    }
    return ret;
}
```

## 7.9. Nested Circles

/\*  
You are given n circles numbered from 1 to n.  
Each circle is defined by an integer center (xi,yi) and an  
integer radius ri.

Then we will ask you q questions. In each question,  
we will give you an integer point (xi,yi),  
and you have to find the number of circles that cover this  
point.

\*/

```
void solve() {

    map<pair<int, int>, vector<array<int, 3>>> mp;
    map<pair<int, int>, int> mpans;

    int n, q, x, y, r; cin >> n >> q;

    forn(i, n) {
        cin >> x >> y >> r;
        mp[{int(x / 10), int(y / 10)}].pb({ x,y,r });
    }

    while (q--) {
        cin >> x >> y;
        int gX = int(x / 10), gY = int(y / 10);
        int ans = 0;
        if (!mpans.count({ x,y })) {
            forne(dx, -1, 2) {
                forne(dy, -1, 2) {
```

```
                    auto it = mp.find({ gX + dx, gY + dy
                        });
                    if (it != mp.end()) {
                        each(i, it->s) {
                            int xc = i[0], yc = i[1], rc =
                                i[2];
                            if ((x - xc) * (x - xc) + (y -
                                yc) * (y - yc) <= rc * rc)
                                ans++;
                        }
                    }
                }
            }
            cout << ans << endl;
            mpans[{x, y}] = ans;
        }
        else cout << mpans[{x, y}] << endl;
    }
}
```

## 7.10. Restoring the Expression

```
/*
12345168 = 123+45=168
199100 = 1+99=100
*/

#define int int64_t

constexpr int mxN = 1e6 + 7, mod = 998244353;
vector<int> p(mxN);
void pre(int c, int mod) {
    p[0] = 1;
    for (int i = 0; i < mxN - 1; i++) {
        p[i + 1] = (c * p[i]) % mod;
    }
}

struct Hash {
    #warning llamar pre;
    ll c, mod;
```

```

vector<int> h;
Hash(const string& s, const int c, const int mod) :
    c(c), mod(mod), h(sz(s) + 1) {
    h[0] = 0;
    for (int i = 0; i < sz(s); i++) {
        h[i + 1] = (c * h[i] + s[i] - '0') % mod;
    }
}
// Returns hash of interval s[a ... b] (where 0 <= a <=
// b < sz(s))
ll get(int a, int b) {
    return (h[b + 1] - ((h[a] * p[b - a + 1]) % mod) +
        mod) % mod;
}

};

bool same(Hash& Ha, Hash& Hb, int l, int r) {
    int qa = Ha.get(l, r);
    int qb = Hb.get(sz(Hb.h) - 2 - r, sz(Hb.h) - 2 - l);
    return qa == qb;
}

main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);

    pre(10, mod);

    string s; cin >> s;
    Hash ha(s, 10, mod);
    int n = sz(s);

    auto ok = [&](int i, int j) {
        i--, j--;
        if (i - 1 < 0) return;
        int a = ha.get(0, i - 1), b = ha.get(i, j - 1), c =
            ha.get(j, n - 1);
        if (((a + b) % mod) == c && ((s[i] != '0' ? 1 : i
            == j - 1)) && ((s[j] != '0' ? 1 : j == n - 1))) {
            cout << string(begin(s), begin(s) + i) << "+"
                << string(begin(s) + i, begin(s) + j) << "="
                << string(begin(s) + j, end(s)) << endl;
            exit(0);
        }
    }
}

```

```

};

forne(i, n / 3, ((n / 2) + 1)) {
    ok(i, n - i + 1);
    ok(i + 1, n - i + 1);
    ok(n - i * 2 + 2, n - i + 1);
    ok(n - i * 2 + 1, n - i + 1);
}

cout << flush;
return 0;
}

```

## 7.11. Word Search

```

/*
input
1 2
ab
6 4
abba
baab
abba
baab
abba
baab

output
ab..
..ab
ab..
..ab
ab..
..ab
*/

```

```

int n, m; cin >> n >> m;
vector<string> a(n);

forne(i, n) cin >> a[i];
Hash2D ha(a);

int Ha_full = ha.get(0, 0, n - 1, m - 1);

int nn, mm; cin >> nn >> mm;
vector<string> b(nn);
forne(i, nn) cin >> b[i];
Hash2D hb(b);

auto check = [&](int x, int y) {
    int x1 = x;
    int y1 = y;
    int x2 = x + n - 1;
    int y2 = y + m - 1;

    return hb.get(x1, y1, x2, y2) == Ha_full;
};

vector<string> ans(nn, string(mm, '.'));
vector<vector<int>> vis(nn + 2, vector<int>(mm + 2, 0));
auto paint = [&](int x, int y) {
    int X = x + 1;
    int Y = y + 1;
    vis[X][Y] += 1;
    vis[X + n][Y] -= 1;
    vis[X][Y + m] -= 1;
    vis[X + n][Y + m] += 1;
};

forne(i, nn - n + 1) {
    forne(j, mm - m + 1) {
        if (a[0][0] != b[i][j]) continue;
        if (check(i, j)) {
            paint(i, j);
        }
    }
}

```

```

    }
}

forne(i, 1, nn + 1) {
    forne(j, 1, mm + 1) {
        vis[i][j] += vis[i][j - 1];
    }
}

forne(i, 1, nn + 1) {
    forne(j, 1, mm + 1) {
        vis[i][j] += vis[i - 1][j];
    }
}

forne(i, nn) {
    forne(j, mm) {
        if (vis[i + 1][j + 1]) cout << b[i][j];
        else cout << '.';
    }
    cout << endl;
}

```

## 7.12. non overlapping substrings

```

/*
add:aho corasick.cpp
You want to select as many non-overlapping substrings of S
as possible such that each selected substring is exactly
equal to one of the M strings.

3
7 2
arwhwar
ar
w
7 1
arwhwar
ar
4 1
rrrr

```

```

rr

-----
4
2
2

*/

void solve() {

    int n, m; cin >> n >> m;
    string s; cin >> s;
    vector<string> st(m);
    forn(i, m) cin >> st[i];

    aho_corasick aho(st);
    vector<pair<int, int>> intervals;
    int cur = 0;

    forn(i, n) {
        cur = aho.get_suffix_link(cur, s[i]);
        int dict_node = aho.nodes[cur].word_index < 0 ?
            aho.nodes[cur].dict : cur;

        while (dict_node >= 0) {
            int widx = aho.nodes[dict_node].word_index;
            int len = st[widx].size();
            intervals.pb({ i - len + 1, i });
            dict_node = aho.nodes[dict_node].dict;
        }
    }

    sort(all(intervals), [](auto& a, auto& b) {
        if (a.s == b.s) return a.f < b.f;
        return a.s < b.s;
    });

    int ans = 0, last_end = -1;
    each(it, intervals) {
        if (it.f > last_end) {
            ans++;

```

```

            last_end = it.s;
        }
    }

    cout << ans << endl;
}

```

### 7.13. Maximum Product

```

// Find the number from the range [a,b] which has the
// maximum product of the digits.
pair<int, string> dp[20][2][2][2];
bool vis[20][2][2][2];

pair<int, string> back(string l, string r, int pos, int ta,
    int tb, int st) {
    if (pos == sz(l)) return { 1, "" };
    if (vis[pos][ta][tb][st]) return dp[pos][ta][tb][st];
    int sta = ta ? l[pos] - '0' : 0, end = tb ? r[pos] -
        '0' : 9, ans = -1;
    string s = "";
    forne(i, sta, end + 1) {
        int val = i;
        if (st == 0 && i == 0) val = 1;
        pair<int, string> alt = back(l, r, pos + 1, ta & (i
            == sta), tb & (i == end), st | i > 0);
        if (alt.f * val > ans) {
            ans = alt.f * val;
            if (i == 0 && st == 0) s = alt.s;
            else s = alt.s, s.pb('0' + i);
        }
    }

    vis[pos][ta][tb][st] = 1;
    return dp[pos][ta][tb][st] = { ans, s };
}

pair<int, string> solve(int a, int b) {
    string L = to_string(a), R = to_string(b);
    if (sz(L) < sz(R)) {
        reverse(all(L));

```

```

        L += string(sz(R) - sz(L), '0');
        reverse(all(L));
    }
    memset(vis, 0, sizeof(vis));
    pair<int, string> ans = back(L, R, 0, 1, 1, 0);
    reverse(all(ans.s));
    return { ans.f, ans.s };
}

```

## 7.14. circles touching radius

```

// Problema: hallar el radio maximo de los circulos
// externos que rodean
// un circulo interno de radio R, tocandolo y tocandose
// entre ellos

ld n, R; cin >> n >> R; // n = numero de circulos externos,
// R = radio circulo interno

ld l = 0, r = 1e9, eps = 1e-8; // inicializa binaria: l
// minimo, r grande, eps precision

while (r - l > eps) { // mientras el rango sea mayor que la
// precision

    ld mid = l + (r - l) / 2; // mid = radio candidato
    // circulos externos

    ld sinv = sin(pi / n); // sin(pi/n) = factor
    // trigonometrico para distancia entre centros

    ld d = 2 * (R + mid) * sinv; // d = distancia entre
    // centros de dos circulos externos

    if (d >= 2 * mid) l = mid; // si se tocan o sobra
    // espacio, mid valido, busca mayor
    else r = mid; // si no se tocan, radio muy grande,
    // busca menor
}

cout << fixed << setprecision(7) << l << endl;

```

## 7.15. Dueling Digits

```

const int mxN = 800 + 7, mxS = 14400 + 7, mod = 1e9 + 7;
int dp[mxN][mxS << 1];

int back(int pos, int addA) {
    if (pos == 0) return addA == mxS;
    int& ans = dp[pos][addA];
    if (~ans) return ans;
    ans = 0;
    forn(i, 10) {
        forn(j, 10) {
            if (i == j) continue;
            if ((i == 0 || j == 0) && pos == 1) continue;
            ans = (ans + back(pos - 1, addA + i - j)) % mod;
        }
    }
    return ans;
}

void solve() {

    int n; cin >> n;
    cout << back(n, mxS) << endl;
}

```

## 8. String

### 8.1. SThash

```

/*
query = or.query(l, r)  ror.query(n - 1 - r, n - 1 - l)
udp    = or.upd(pos, a)  ror.upd(n - 1 - pos, a);

LLAMAR PRECAL!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

*/
const int MAX = 2e5 + 6;
const int MOD = 1e9 + 7;

```



```

const int BASE = 137;

int BP[MAX];
void precal() {
    BP[0] = 1;
    BP[1] = 1;
    for (int i = 1; i < MAX; i++) {
        BP[i] = (BP[i - 1] * BASE) % MOD;
    }
}

template<typename T>
struct SThash {
    struct node {
        int tam;
        int h;
        node() {}
    };
    int n;
    vector<node> tree;
    SThash(string& s) {
        n = sz(s);
        vector<T> a(n);
        tree.resize(n * 4);
        for (int i = 0; i < n; i++) a[i] = s[i];
        build(1, 0, n - 1, a);
    }

    node Merge(node a, node b) {
        node ret;
        ret.h = ((a.h * BP[b.tam]) + b.h) % MOD;
        ret.tam = a.tam + b.tam;
        return ret;
    }

    void build(int v, int tl, int tr, vector<T>& a) {
        if (tl == tr) {
            tree[v].h = a[tl];
            tree[v].tam = 1;
            return;
        }
        int mid = (tl + tr) >> 1;
        build(v * 2, tl, mid, a);
        build(v * 2 + 1, mid + 1, tr, a);
    }
};

```

```

        tree[v] = Merge(tree[v * 2], tree[v * 2 + 1]);
    }

    void upd(int v, int tl, int tr, int id, int val) {
        if (tl > id or tr < id) return;
        if (tl == tr and tr == id) {
            tree[v].h = val;
            return;
        }
        int mid = (tl + tr) >> 1;
        upd(v * 2, tl, mid, id, val);
        upd(v * 2 + 1, mid + 1, tr, id, val);
        tree[v] = Merge(tree[v * 2], tree[v * 2 + 1]);
    }

    node query(int v, int tl, int tr, int l, int r) {
        if (tl >= l and tr <= r) return tree[v];
        int mid = (tl + tr) / 2;
        if (mid < l) return query(v + v + 1, mid + 1, tr, l, r);
        else if (mid >= r) return query(v + v, tl, mid, l, r);
        else return Merge(query(v + v, tl, mid, l, r),
            query(v + v + 1, mid + 1, tr, l, r));
    }

    void upd(int pos, int val) { upd(1, 0, n - 1, pos, val); }
    int query(int l, int r) { return query(1, 0, n - 1, l, r).h; }
};

```

## 8.2. HuffmanCoding

```

struct Node {
    char data;
    int freq;
    Node* L, * R;
    Node(char data, int freq) : data(data), freq(freq),
        L(nullptr), R(nullptr) {}
};

```

```

struct Huffman {
    unordered_map<char, int> freqMap;
    unordered_map<char, string> hfCodes;
    string str;
    Node* root;
    Huffman(string& str) : str(str) {
        for (auto&& i : str) freqMap[i]++;
        root = build();
        createHF(root, "");
    }

    struct oper {
        bool operator()(const Node* L, const Node* R) const
        {
            return L->freq > R->freq;
        }
    };

    Node* build() {
        priority_queue<Node*, vector<Node*>, oper> pq;
        each(i, freqMap) {
            pq.push(new Node(i.f, i.s));
        }
        if (sz(pq) == 1) {
            Node* L = pq.top();
            pq.pop();
            Node* parent = new Node('\0', L->freq);
            parent->L = L;
            pq.push(parent);
        }
        while (sz(pq) > 1) {
            Node* L = pq.top();
            pq.pop();
            Node* R = pq.top();
            pq.pop();
            Node* parent = new Node('\0', L->freq +
                                    R->freq);
            parent->L = L;
            parent->R = R;
            pq.push(parent);
        }
        return pq.top();
    }
}

```

```

void createHF(Node* root, string code) {
    if (root == nullptr) return;
    if (!root->L && !root->R) {
        hfCodes[root->data] = code;
    }
    createHF(root->L, code + "0");
    createHF(root->R, code + "1");
}

int LengthBinary() {
    int cnt = 0;
    for (auto&& i : str) cnt += sz(hfCodes[i]);
    return cnt;
}

void _print() {
    each(i, hfCodes) cout << i.f << ' ' << i.s << endl;
}

};

int main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);

    string s; cin >> s;
    Huffman hf(s);
    cout << hf.LengthBinary() << endl;

    // hf._print();

    cout << flush;
    return 0;
}

```

### 8.3. palindrome range

```
vector<vector<int>>>dp(mxN, vector<int>(mxN));
vector<vector<bool>>>pal(mxN, vector<bool>(mxN));
string s; cin >> s;
int n = sz(s), q, l, r;

for (int i = n - 1; i >= 0; i--) {
    dp[i][i] = pal[i][i] = 1;
    for (int j = i + 1; j < n; j++) {
        pal[i][j] = (pal[i + 1][j - 1] || j - i == 1) &
            (s[i] == s[j]);
        dp[i][j] = dp[i + 1][j] + dp[i][j - 1] - dp[i + 1][j - 1] + pal[i][j];
    }
}

cin >> q;
while (q--) {
    cin >> l >> r;
    cout << dp[l - 1][r - 1] << endl;
}
```

### 8.4. hashing-2D-64

```
#define int int64_t
const int MOD = (1ll << 61) - 1;

int mulmod(int a, int b) {
    const static int L = (1ll << 30) - 1, _31 = (1ll << 31) - 1;
    int l1 = a & L, h1 = a >> 30, l2 = b & L, h2 = b >> 30;
    int m = l1 * h2 + l2 * h1, h = h1 * h2;
    int ans = l1 * l2 + (h >> 1) + ((h & 1) << 60) + (m >> 31) + ((m & _31) << 30) + 1;
    ans = (ans & MOD) + (ans >> 61), ans = (ans & MOD) + (ans >> 61);
    return ans - 1;
}

int rnd(int l, int r) {
    static std::mt19937
        rng(std::chrono::steady_clock::now().time_since_epoch().count());
```

```
    return std::uniform_int_distribution<int>(l, r)(rng);
}

struct Hash2D {
    static int P, Q;
    vector<vector<int>>> H;
    vector<int> powP, powQ, invP, invQ;
    static int modexp(int a, int e) {
        int r = 1;
        while (e > 0) {
            if (e & 1) r = mulmod(r, a);
            a = mulmod(a, a);
            e >>= 1;
        }
        return r;
    }

    Hash2D(vector<string>& g) {
        int n = sz(g), m = sz(g[0]);
        H.assign(n + 1, vector<int>(m + 1, 0));
        powP.assign(n + 1, 1), powQ.assign(m + 1, 1),
            invP.assign(n + 1, 1), invQ.assign(m + 1, 1);

        int invBaseP = modexp(P, MOD - 2);
        int invBaseQ = modexp(Q, MOD - 2);
        forne(i, 1, n + 1) {
            powP[i] = mulmod(powP[i - 1], P);
            invP[i] = mulmod(invP[i - 1], invBaseP);
        }
        forne(j, 1, m + 1) {
            powQ[j] = mulmod(powQ[j - 1], Q);
            invQ[j] = mulmod(invQ[j - 1], invBaseQ);
        }

        forne(i, 1, n + 1) {
            forne(j, 1, m + 1) {
                int val = g[i - 1][j - 1];
                H[i][j] = (H[i - 1][j] + H[i][j - 1] - H[i - 1][j - 1] + mulmod(val,
                    mulmod(powP[i], powQ[j]))) % MOD;
                if (H[i][j] < 0) H[i][j] += MOD;
            }
        }
    }
}
```

```

}

int raw(int x1, int y1, int x2, int y2) {
    int res = H[x2 + 1][y2 + 1];
    res = (res - H[x1][y2 + 1] - H[x2 + 1][y1] +
           H[x1][y1]) % MOD;
    if (res < 0) res += MOD;
    return res;
}

int get(int x1, int y1, int x2, int y2) {
    int res = raw(x1, y1, x2, y2);
    res = mulmod(res, invP[x1]);
    res = mulmod(res, invQ[y1]);
    return res;
}

/*
(x1 = 0, y1 = 0, x2 = n-1, y2 = m-1)
get(i, j, i + n - 1, j + m - 1)
x1,y1  -----  x1,y2
|               |
|   submatriz   |
|               |
x2,y1  -----  x2,y2
*/
};

```

## 8.5. hashing-64

```

#define int int64_t
const int MOD = (1ll << 61) - 1;

int mulmod(int a, int b) {
    const static int L = (1ll << 30) - 1, _31 = (1ll << 31)
        - 1;
    int l1 = a & L, h1 = a >> 30, l2 = b & L, h2 = b >> 30;
    int m = l1 * h2 + l2 * h1, h = h1 * h2;
    int ans = l1 * l2 + (h >> 1) + ((h & 1) << 60) + (m >> 3
        1) + ((m & _31) << 30) + 1;
    ans = (ans & MOD) + (ans >> 61), ans = (ans & MOD) +
        (ans >> 61);
}

```

```

    return ans - 1;
}

int rnd(int l, int r) {
    static std::mt19937
        rng(std::chrono::steady_clock::now().time_since_epoch());
    return std::uniform_int_distribution<int>(l, r)(rng);
}

struct Hash {
    static int P;
    vector<int> h, p;
    Hash(string& s) : h(sz(s)), p(sz(s)) {
        p[0] = 1, h[0] = s[0];
        for (int i = 1; i < sz(s); i++) p[i] = mulmod(p[i - 1], P), h[i]
            = (mulmod(h[i - 1], P) + s[i]) % MOD;
    }
    // Returns hash of interval s[a ... b] (where 0 <= a <=
        b < sz(s))
    int get(int l, int r) {
        int hash = h[r] - (l ? mulmod(h[l - 1], p[r - l + 1
            ]) : 0);
        return hash < 0 ? hash + MOD : hash;
    }
};

int Hash::P = rnd(256, MOD - 1);

```

## 8.6. Z

```

/*
Dado una string s, devuelve un vector Z donde Z[i]
    representa el prefijo
de mayor longitud de s, que tambien es prefijo del sufijo
de s que inicia
en i.
01234567
aabzaaba "aab" es un prefijo de s y "aaba" es un sufijo de
s, Z[4] = 3.

Otra definicion: Dado un string s retorna un vector z donde
z[i] es igual al mayor

```

numero de caracteres desde s[i] que coinciden con los caracteres desde s[0]

Complejidad:  $O(|n|)$

```
*/
vector<int> z_function(string& s) {
    int n = s.size();
    vector<int> z(n);
    for (int i = 1, x = 0, y = 0; i < n; i++) {
        z[i] = max(0ll, min(z[i - x], y - i + 1));
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            x = i, y = i + z[i], z[i]++;
        }
    }
    return z;
}
```

## 8.7. paltree

```
/*
size() number of different palindrome substr
propagate() number of palindrome substr
lps longest palindrome substr {star, len}
*/

struct paltree {
    vector<vector<int>> t;
    int n, last, sz;
    vector<int> s, len, link, qt;
    pair<int, int> lps{ 0, 0 };

    paltree(int N) {
        t.assign(N + 2, vector<int>(26, int()));
        s = len = link = qt = vector<int>(N + 2);
        s[0] = -1, link[0] = 1, len[0] = 0, link[1] = 1,
            len[1] = -1;
        sz = 2, last = 0, n = 1;
    }

    void add(char c) {
        s[n++] = c -= 'a';
    }
}
```

```
while (s[n - len[last] - 2] != c) last = link[last];
if (!t[last][c]) {
    int prev = link[last];
    while (s[n - len[prev] - 2] != c) prev =
        link[prev];
    link[sz] = t[prev][c];
    len[sz] = len[last] + 2;
    t[last][c] = sz++;
    if (len[sz - 1] > lps.s) {
        lps = { n - len[sz - 1] - 1, len[sz - 1] };
    }
}
qt[last = t[last][c]]++;
}

int size() {
    return sz - 2;
}

ll propagate() {
    ll cnt = 0;
    for (int i = n; i > 1; i--) {
        qt[link[i]] += qt[i];
        cnt += qt[i];
    }
    return cnt;
}

};
```

## 8.8. hashing

```
/*
Usage:
    Good values c = 137, modbest=998244353, mod = 10^9
        + 7, mod = 1e18 + 9.
    If necessary to check too many pairs of hashes, use
        two
    different hashes.
    If hashing something other than english characters:
        - Don't have elements with value 0.
        - Use c > max element value.
*/
```

```

//#define int int64_t

constexpr int mxN = 1e6 + 7;
vector<int> p(mxN);
void pre(const int c = 137, const int mod = 998244353) {
    p[0] = 1;
    forn(i, mxN - 1) p[i + 1] = (c * p[i]) % mod;
}

struct Hash {
    #warning llamar pre;
    int c, mod;
    vector<int> h;
    Hash(const string& s, const int c = 137, const int mod = 998244353) : c(c), mod(mod), h(sz(s) + 1) {
        h[0] = 0;
        forn(i, sz(s)) h[i + 1] = (c * h[i] + s[i]) % mod;
    }
    // Returns hash of interval s[a ... b] (where 0 <= a <= b < sz(s))
    int get(int a, int b) {
        return (h[b + 1] - ((h[a] * p[b - a + 1]) % mod) + mod) % mod;
    }
};

bool same(Hash& Ha, Hash& Hb, int l, int r) {
    int qa = Ha.get(l, r);
    int qb = Hb.get(sz(Hb.h) - 2 - r, sz(Hb.h) - 2 - l);
    return qa == qb;
}

```

## 8.9. hash table

```

/*
hash_table
sirve para contar cuantas veces aparece un patron en un
string
en un rango [l,r] en O(1) con O(n) de preprocesamiento
ejemplo:
string s;

```

```

a b a c a b a d a b a c a b a
string p;
b a
hash_table<int> h(s,p);
0 0 1 1 1 1 2 2 2 2 3 3 3 3 4

hash_table(string s, int m)
sirve para contar cuantas veces aparece un patron de
longitud m en un string
modificar bulid() segun condicion

*/

template<typename T>
struct hash_table
{
    string s, p;
    int n, m;
    vector<T> prefix;
    hash_table(string s, string p) {
        this->s = s;
        this->p = p;
        this->n = sz(s);
        this->m = sz(p);
        prefix.resize(n + 5, 0);
        build();
    }
    hash_table(string s, int m) {
        this->s = s;
        this->n = sz(s);
        this->m = m;
        prefix.resize(n + 5, 0);
        build();
    }

    void build() {
        forn(i, n - m + 1) {
            int ok = 1;
            forn(j, m) {
                if (s[i + j] != p[j]) {
                    ok = 0;
                    break;
                }
            }
        }
    }
}

```

```

    }
    }
    prefix[i + 1] = prefix[i] + ok;
}
}
int query(int l, int r) {
    if (r - l + 1 < m) return 0;
    return prefix[r - m + 1] - prefix[l - 1];
}
};

```

## 8.10. ahobit

```

/*
ahobit: used to search for a pattern in a string
- query(l,r): searches for how many times the pattern
  is repeated in the range [l,r]
- numoc: number of occurrences of the pattern in the
  string
- a: vector with the positions of the occurrences of
  the pattern
- szp: size of the pattern
- bs: bitset of the characters in the string
- oc: bitset of the occurrences of the pattern
- N: maximum size of the string
*/
struct ahobit {
    static constexpr int N = 1e5 + 9;
    bitset<N> bs[26], oc, _all;
    int szp;
    ahobit(const string& s) {
        for (int i = 0; i < sz(s); i++) bs[s[i] - 'a'][i] =
            1, _all[i] = 1;
    }
    void add(const string& p) {
        // oc.set();
        oc = _all; szp = sz(p);
        for (int i = 0; i < sz(p); i++) oc &= (bs[p[i] -
            'a'] >> i);
    }
    int num_occu() {
        return oc.count();
    }
};

```

```

}
vector<int> pos_occu() {
    vector<int> a;
    int pos = oc._Find_first();
    a.clear(); a.pb(pos);
    pos = oc._Find_next(pos);
    while (pos < N) {
        a.pb(pos);
        pos = oc._Find_next(pos);
    }
    return a;
}

int query(int l, int r) {
    //1-indexed
    if (szp > r - l + 1) return 0;
    return (oc >> (l - 1)).count() - (oc >> (r - szp + 1
        )).count();
}
};

```

## 8.11. suffixAutomaton

```

// codebreaker suffix automaton
struct suffixAutomaton {
    struct node {
        int len, link; bool end;
        map<char, int> next;
        int cnt; ll in, out, cntSubstrs;
    };

    vector<node> sa;
    //ocurrencias de estados, usar encontrar kth pequena
    lexico all strings
    vector<ll> cntState;
    int last; ll substrs = 0;

    suffixAutomaton() {}
    suffixAutomaton(string& s) {
        sa.reserve(sz(s) * 2);
        // cntState.reserve(sz(s)*2);
        last = add_node();
    }
};

```

```

    sa[0].link = -1;
    sa[0].in = 1;
    for (char& c : s) add_char(c);
    for (int p = last; p; p = sa[p].link) sa[p].end = 1;
}

int add_node() { sa.pb({}); return sa.size() - 1; }

void add_char(char c) {
    int u = add_node(), p = last;
    // cntState[u] = 1;
    sa[u].len = sa[last].len + 1;
    while (p != -1 && !sa[p].next.count(c)) {
        sa[p].next[c] = u;
        sa[u].in += sa[p].in;
        subtrs += sa[p].in;
        p = sa[p].link;
    }
    if (p != -1) {
        int q = sa[p].next[c];
        if (sa[p].len + 1 != sa[q].len) {
            int clone = add_node();
            // cntState[clone] = 0;
            sa[clone] = sa[q];
            sa[clone].len = sa[p].len + 1;
            sa[clone].in = 0;
            sa[q].link = sa[u].link = clone;
            while (p != -1 && sa[p].next[c] == q) {
                sa[p].next[c] = clone;
                sa[q].in -= sa[p].in;
                sa[clone].in += sa[p].in;
                p = sa[p].link;
            }
        }
        else sa[u].link = q;
    }
    last = u;
}

//Cuenta la cantidad de ocurrencias de una cadena s
int match_str(string& s) {
    int u = 0, n = sz(s);
    for (int i = 0; i < n; ++i) {
        if (!sa[u].next.count(s[i])) return 0;

```

```

        u = sa[u].next[s[i]];
    }
    return count_occ(u);
}

int count_occ(int u) {
    if (sa[u].cnt != 0) return sa[u].cnt;
    sa[u].cnt = sa[u].end;
    for (auto& v : sa[u].next)
        sa[u].cnt += count_occ(v.ss);
    return sa[u].cnt;
}

//Calcular la cantidad de caminos que pertenecen al
//estado ti, desde ti hasta tn
ll count_paths(int u) {
    //Out cuenta la cantidad de caminos (cantidad de
    //cadenas distintas)
    if (sa[u].out != 0) return sa[u].out;
    //sa[u].cntSubtrs != 0 return sa[u].cntSubtrs
    for (auto& v : sa[u].next)
        sa[u].out += count_paths(v.ss);
    //sa[u].cntSubtrs += count_paths(v.ss)
    return ++sa[u].out; //sa[u].cntSubtrs +=
    cntState[u];
}

//kth subcadena mas pequena en orden lexicografico
//out para cadenas distintas, cntSubtrs para todas las
//cadenas llamar antes pre
string kth;
void dfs_kth(int u, ll& k) { //Antes llamar a count
    if (k == 0) return; // k < cntState[u] para todas
    //las cadenas
    k--; // k -= cntState[u];
    for (auto& v : sa[u].next) {
        if (k < sa[v.ss].out) { //k <
            sa[v.ss].cntSubtrs
            kth += v.ff;
            return dfs_kth(v.ss, k);
        }
        k -= sa[v.ss].out; //k -= sa[v.ss].cntSubtrs
    }
}

```



```

}
//calcula la cantidad de ocurrencias de los estados
void pre() {
    vector<ii> v(sz(sa));
    forn(i, sz(sa)) v[i] = { sa[i].len, i };
    sort(all(v), greater<ii>());
    for (auto& it : v) {
        int u = it.ss;
        if (sa[u].link != -1)
            cntState[sa[u].link] += cntState[u];
    }
    cntState[0] = 1;
}
//longest common substring
int lcs(string& t) {
    int n = sz(t);
    int u = 0, l = 0, best = 0, bestPosition = 0;
    forn(i, n) {
        while (u && !sa[u].next.count(t[i])) {
            u = sa[u].link;
            l = sa[u].len;
        }
        if (sa[u].next.count(t[i])) u =
            sa[u].next[t[i]], l++;
        if (best < l) best = l, bestPosition = i;
    }
    return best;
}
vector<int> LCS, match;
void lcsMatch(string& t) {
    match.assign(sz(sa), 0); //usar pivote si toca
    resetear mucho
    int u = 0, l = 0;
    for (int i = 0; i < sz(t); ++i) {
        while (u && !sa[u].next.count(t[i])) {
            u = sa[u].link;
            l = sa[u].len;
        }
        if (sa[u].next.count(t[i])) u =
            sa[u].next[t[i]], l++;
        match[u] = max(match[u], l);
    }
    for (int i = sz(sa) - 1; i > 0; --i)

```

```

        match[i] = max(match[i], match[sa[i].link]);
    for (int i = 0; i < sz(sa); ++i)
        LCS[i] = min(LCS[i], match[i]);
}

//longest common substring de n cadenas
int lcs_n(vector<string>& t) {
    const int INF = 1e7;
    LCS.assign(sz(sa), INF);
    forn(i, sz(t)) lcsMatch(t[i]);
    return *max_element(all(LCS));
}

//longitud desde 1 hasta N, return v donde v[i] = num
//distintas substr de i longitud
vector<int> substringDistribution(int lenCadena) {
    vector<int> st(lenCadena + 5);
    forn(i, sz(sa)) {
        int l = sa[sa[i].link].len + 1; // l minlen
        //subcadena que pertenece al conjunto sa[i]
        int r = sa[i].len; // r maxlen subcadena que
        //pertenece al conjunto s[i]
        if (l > 0) st[l]++, st[r + 1]--;
    }
    forn(i, lenCadena + 1) st[i + 1] += st[i];
    return st;
}

//Devuelve V, V[i] = max ocurrencias para una subcadena
//de S de longitud i.
void maxOcurrenciasLengths(int n) { //Llamar antes
    count_occ
    vector<int> ans(n + 1);
    forn(i, sz(sa)) ans[sa[i].len] =
        max(ans[sa[i].len], sa[i].cnt);
    forn(i, n) cout << ans[i + 1] << endl;
}
node& operator[](int i) { return sa[i]; }
};

```

## 8.12. rabin karp

```
// Dado un patron S y un texto T, se desea conocer los
// indices de las ocurrencias del patron S en el texto T.

vector<int> rabin_karp(string const& s, string const& t) {
    const int p = 31;
    const int m = 1e9 + 9;
    int S = sz(s), T = sz(t);

    vector<int64_t> p_pow(max(S, T)), h(T + 1, 0);
    p_pow[0] = 1;
    int64_t h_s = 0;
    forne(i, 1, sz(p_pow)) p_pow[i] = (p_pow[i - 1] * p) %
        m;
    forn(i, T) h[i + 1] = (h[i] + (t[i] - 'a' + 1) *
        p_pow[i]) % m;
    forn(i, S) h_s = (h_s + (s[i] - 'a' + 1) * p_pow[i]) %
        m;

    vector<int> occ;
    for (int i = 0; i + S - 1 < T; i++) {
        int64_t cur_h = (h[i + S] + m - h[i]) % m;
        if (cur_h == h_s * p_pow[i] % m) occ.pb(i);
    }
    return occ;
}
```

### 8.13. aho corasick

```
// Esta version de aho_corasick usa un bitmask de tamano
// ALPHABET, por lo que debe ser modificado para ALPHABET >
// 26.
// suff = el indice del nodo del sufijo estricto mas largo
// del nodo actual que tambien esta en el arbol.
// dict = el indice del nodo del sufijo estricto mas largo
// del nodo actual que esta en la lista de palabras.
// depth = profundidad normal del trie (la raiz es 0). Se
// puede eliminar para ahorrar memoria.
// word_index = el indice de la *primera* palabra que
// termina en este nodo. -1 si no hay ninguna.
// word_count = el numero total de palabras que terminan en
// este nodo. Usado en count_total_matches().
```

```
// first_child = el primer hijo de este nodo (los hijos son
// secuenciales debido al orden BFS), -1 si no hay ninguno.
// child_mask = la mascara de bits de las claves de los
// hijos disponibles desde este nodo. Si ALPHABET > 26,
// cambie el tipo.

const int INF = int(1e9) + 5;
template<char MIN_CHAR = 'a', typename mask_t = uint32_t>
struct aho_corasick {
    struct node {
        int suff = -1, dict = -1, depth = 0, word_index =
            -1, word_count = 0, first_child = -1;
        mask_t child_mask = 0;
        int get_child(char c) const {
            int bit = c - MIN_CHAR;
            if ((child_mask >> bit & 1) == 0) return -1;
            assert(first_child >= 0);
            return first_child +
                __builtin_popcount(child_mask & ((mask_t(1)
                    << bit) - 1));
        }
    };

    vector<node> nodes;
    int W = 0;
    vector<int> word_location, word_indices_by_depth, defer;

    aho_corasick(const vector<string>& words = {}) {
        build(words); }

    // construir la adj list basada en los suffix parents.
    // A menudo queremos realizar DP y/o consultas en este
    // arbol.
    vector<vector<int>> build_suffix_adj() const {
        vector<vector<int>> adj(sz(nodes));
        forne(i, 1, sz(nodes))
            adj[nodes[i].suff].push_back(i);
        return adj;
    }

    int get_or_add_child(int current, char c) {
        int bit = c - MIN_CHAR;
        if (nodes[current].child_mask >> bit & 1) return
```

```

        nodes[current].get_child(c);
assert(nodes[current].child_mask >> bit == 0);
int index = sz(nodes);
nodes[current].child_mask |= mask_t(1) << bit;
if (nodes[current].first_child < 0)
    nodes[current].first_child = index;
nodes.emplace_back();
nodes.back().depth = nodes[current].depth + 1;
return index;
}

// return donde en el trie deberiamos terminar despues
// de comenzar en "location" y agregar el char "C".
// corre en el peor de los casos O(depth) pero se
// amortiza a O(1) en la mayoria de las situaciones.
int get_suffix_link(int location, char c) const {
    int child;
    while (location >= 0 && (child =
        nodes[location].get_child(c)) < 0) location =
        nodes[location].suff;
    return location < 0 ? 0 : child;
}

void build(const vector<string>& words) {
    nodes = { node() };
    W = sz(words);
    vector<int> indices(W);
    iota(all(indices), 0);
    stable_sort(all(indices), [&](int a, int b) -> bool
        { return words[a] < words[b]; });
    word_location.assign(W, 0);
    vector<int> remaining = indices;
    int rem = W;

    for (int depth = 0; rem > 0; depth++) {
        int nrem = 0;
        for (i, rem) {
            int word = remaining[i];
            int& location = word_location[word];
            if (depth >= int(words[word].size())) {
                if (nodes[location].word_index < 0)
                    nodes[location].word_index = word;
                nodes[location].word_count++;
            }
        }
    }
}

```

```

    }
    else {
        location = get_or_add_child(location,
            words[word][depth]);
        remaining[nrem++] = word;
    }
}
rem = nrem;
}

int max_depth = 0;
defer.resize(W);

for (i, W) {
    max_depth = max(max_depth,
        int(words[i].size()));
    defer[i] = nodes[word_location[i]].word_index;
}

// crear una lista de indices de palabras en orden
// decreciente de profundidad, en tiempo lineal a
// traves de counting sort.
word_indices_by_depth.resize(W);
vector<int> depth_freq(max_depth + 1, 0);
for (i, W) depth_freq[words[i].size()]++;
for (int i = max_depth - 1; i >= 0; i--)
    depth_freq[i] += depth_freq[i + 1];
for (int i = W - 1; i >= 0; i--)
    word_indices_by_depth[--depth_freq[words[i].size()]]
    = i;

// Solve suffix parents by traversing in order of
// depth (BFS order).

for (i, sz(nodes)) {
    mask_t child_mask = nodes[i].child_mask;
    while (child_mask != 0) {
        int bit = __builtin_ctzll(child_mask);
        char c = char(MIN_CHAR + bit);
        int index = nodes[i].get_child(c);
        child_mask ^= mask_t(1) << bit;
        // buscamos el suffix parent de index, que
        // es el suffix parent de i que tiene un
        // hijo c.
    }
}

```

```

        int suffix_parent =
            get_suffix_link(nodes[i].suff, c);
        nodes[index].suff = suffix_parent;
        nodes[index].word_count +=
            nodes[suffix_parent].word_count;
        nodes[index].dict =
            nodes[suffix_parent].word_index < 0 ?
            nodes[suffix_parent].dict :
            suffix_parent;
    }
}

// Counts the number of matches of each word in O(text
// length + num words).
vector<int> count_matches(const string& text) const {
    vector<int> matches(W, 0);
    int current = 0;
    for (char c : text) {
        current = get_suffix_link(current, c);
        int dict_node = nodes[current].word_index < 0 ?
            nodes[current].dict : current;
        if (dict_node >= 0)
            matches[nodes[dict_node].word_index]++;
    }

    // Iterate in decreasing order of depth.
    for (int word_index : word_indices_by_depth) {
        int location = word_location[word_index];
        int dict_node = nodes[location].dict;
        if (dict_node >= 0)
            matches[nodes[dict_node].word_index] +=
                matches[word_index];
    }
    forn(i, W) matches[i] = matches[defer[i]];
    return matches;
}

// Finds the last index of the first occurrence of each
// word (INF if not present) in O(text length + num
// words).
vector<int> find_first_occurrence(const string& text)
    const {

```

```

        vector<int> first_occurrence(W, INF);
        int current = 0;
        forn(i, sz(text)) {
            char c = text[i];
            current = get_suffix_link(current, c);
            int dict_node = nodes[current].word_index < 0 ?
                nodes[current].dict : current;
            if (dict_node >= 0) {
                int word = nodes[dict_node].word_index;
                first_occurrence[word] =
                    min(first_occurrence[word], i);
            }
        }

        // Iterate in decreasing order of depth.
        for (int word_index : word_indices_by_depth) {
            int location = word_location[word_index];
            int dict_node = nodes[location].dict;

            if (dict_node >= 0) {
                int word_parent =
                    nodes[dict_node].word_index;
                first_occurrence[word_parent] =
                    min(first_occurrence[word_parent],
                        first_occurrence[word_index]);
            }
        }

        forn(i, W) first_occurrence[i] =
            first_occurrence[defer[i]];
        return first_occurrence;
    }

vector<int> find_last_occurrence(const string& text)
    const {
        vector<int> first_occurrence(W, -INF);
        int current = 0;
        forn(i, sz(text)) {
            char c = text[i];
            current = get_suffix_link(current, c);
            int dict_node = nodes[current].word_index < 0 ?
                nodes[current].dict : current;

```

```

        if (dict_node >= 0) {
            int word = nodes[dict_node].word_index;
            first_occurrence[word] =
                max(first_occurrence[word], i);
        }
    }

    // Iterate in decreasing order of depth.
    for (int word_index : word_indices_by_depth) {
        int location = word_location[word_index];
        int dict_node = nodes[location].dict;

        if (dict_node >= 0) {
            int word_parent =
                nodes[dict_node].word_index;
            first_occurrence[word_parent] =
                max(first_occurrence[word_parent],
                    first_occurrence[word_index]);
        }
    }
    forn(i, W) first_occurrence[i] =
        first_occurrence[defer[i]];
    return first_occurrence;
}

// Counts the number of matches over all words at each
// ending position in "text" in O(text length).
vector<int> count_matches_by_position(const string&
    text) const {
    vector<int> matches(sz(text));
    int current = 0;
    forn(i, sz(text)) {
        current = get_suffix_link(current, text[i]);
        matches[i] = nodes[current].word_count;
    }

    return matches;
}

// Counts the total number of matches of all words
// within "text" in O(text length).
int64_t count_total_matches(const string& text) const {
    int64_t matches = 0;

```

```

        int current = 0;
        for (char c : text) {
            current = get_suffix_link(current, c);
            matches += nodes[current].word_count;
        }
        return matches;
    }
};

```

## 8.14. Manacher

```

// manacher receives a vector of T and returns the vector
// with the size of the palindromes
// ret[2*i] = size of the largest palindrome centered at i
// ret[2*i+1] = size of the largest palindrome centered at
// i and i+1
//
// Complexities:
// manacher - O(n)
// palindrome - <O(n), O(1)>
// pal_end - O(n)

template<typename T> vector<int> manacher(const T& s) {
    int l = 0, r = -1, n = s.size();
    vector<int> d1(n), d2(n);
    for (int i = 0; i < n; i++) {
        int k = i > r ? 1 : min(d1[l + r - i], r - i);
        while (i + k < n && i - k >= 0 && s[i + k] == s[i -
            k]) k++;
        d1[i] = k--;
        if (i + k > r) l = i - k, r = i + k;
    }
    l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1
            ); k++;
        while (i + k <= n && i - k >= 0 && s[i + k - 1] ==
            s[i - k]) k++;
        d2[i] = --k;
        if (i + k - 1 > r) l = i - k, r = i + k - 1;
    }
    vector<int> ret(2 * n - 1);

```

```

    for (int i = 0; i < n; i++) ret[2 * i] = 2 * d1[i] - 1;
    for (int i = 0; i < n - 1; i++) ret[2 * i + 1] = 2 *
        d2[i + 1];
    return ret;
}

// checks if string s[i..j] is palindrome
template<typename T> struct palindrome {
    vector<int> man;

    palindrome(const T& s) : man(manacher(s)) {}
    bool query(int i, int j) {
        return man[i + j] >= j - i + 1;
    }
};

// size of the largest palindrome ending in each position
template<typename T> vector<int> pal_end(const T& s) {
    vector<int> ret(s.size());
    palindrome<T> p(s);
    ret[0] = 1;
    for (int i = 1; i < s.size(); i++) {
        ret[i] = min(ret[i - 1] + 2, i + 1);
        while (!p.query(i - ret[i] + 1, i)) ret[i]--;
    }
    return ret;
}

void print_pals(const string s) {
    vector<int> man = manacher(s);
    int n = sz(s);
    forn(i, n) {
        for (int len = 1; len <= man[2 * i]; len += 2) {
            int start = i - (len - 1) / 2;
            cout << s.substr(start, len) << endl;
        }
        if (i < n - 1) {
            for (int len = 2; len <= man[2 * i + 1]; len +=
                2) {
                int start = i - (len - 2) / 2;
                cout << s.substr(start, len) << endl;
            }
        }
    }
}

```

```

    }
}

//expansion
int odd(int d, int i, int n) {
    // d=(manacher[2 * i], i)
    int l = i - (d - 1) / 2;
    int r = i + (d - 1) / 2;

    while (l >= 0 && r < n) {
        //process
        l -= 1; r += 1;
    }
    return ((r - 1) - (l + 1) + 2) / 2;
}

int even(int d, int i, int n) {
    // d=(manacher[2 * i+1], i)
    if (i == n - 1) return 0;
    if (d == 0) d = 2;
    int l = i - d / 2 + 1;
    int r = i + d / 2;

    while (l >= 0 && r < n) {
        //process
        l -= 1; r += 1;
    }
    return ((r - 1) - (l + 1) + 2) / 2;
}

// largest palindrome
string manacher(const string& s) {
    if (sz(s) == 0) return "";

    string curr = "";
    for (auto&& i : s) {
        curr += i;
        curr += "#";
    }

    curr = "@#" + curr + "&";
    vector<ll> pali(sz(curr), 0);
    ll center = 0;
}

```

```

11 R = 0;
for (11 i = 1; i < sz(curr) - 1; i++) {
    if (i < R) pali[i] = min(pali[2 * center - i], R - i);
    while (curr[i + (pali[i] + 1)] == curr[i - (pali[i] + 1)]) pali[i]++;
    if (i + pali[i] > R) {
        center = i;
        R = i + pali[i];
    }
}
11 HC = 0, CI = 0;
for (11 i = 1; i < sz(curr) - 1; i++) {
    if (pali[i] > HC) {
        HC = pali[i];
        CI = i;
    }
}
string ans = "";
if (HC <= 0) return string(1, s[0]);
for (11 i = CI - HC + 1; i <= CI + HC - 1; i += 2) ans
    += curr[i];
return ans;
}

```

## 8.15. trie

```

// T.count pref(s) number of strings that have a as a prefix
struct trie {
    vector<vector<int>> to;
    vector<int> end, pref;
    int sigma; char norm;
    int lcpsum = 0;
    trie(int sigma_ = 26, char norm_ = 'a') :
        sigma(sigma_), norm(norm_) {
        to = { vector<int>(sigma) };
        end = { 0 }, pref = { 0 };
    }
    void insert(string s) {
        int x = 0;
        for (auto c : s) {
            int& nxt = to[x][c - norm];

```

```

            if (!nxt) {
                nxt = to.size();
                to.pb(vector<int>(sigma));
                end.pb(0), pref.pb(0);
            }
            // else lcpsum += pref[nxt];
            x = nxt, pref[x]++;
        }
        end[x]++, pref[0]++;
    }
    void erase(string s) {
        int x = 0;
        for (char c : s) {
            int& nxt = to[x][c - norm];
            x = nxt, pref[x]--;
            if (!pref[x]) nxt = 0;
        }
        end[x]--, pref[0]--;
    }
    int find(string s) {
        int x = 0;
        for (auto c : s) {
            x = to[x][c - norm];
            if (!x) return -1;
        }
        return x;
    }
    int count_pref(string s) {
        int id = find(s);
        return id >= 0 ? pref[id] : 0;
    }
}

string kth_word(int k, int x = 0, string s = "") {
    if (k <= end[x]) return s;
    k -= end[x];
    for (int i = 0; i < sigma; i++) {
        int nxt = to[x][i];
        if (!nxt) continue;
        if (k <= pref[nxt]) return kth_word(k, nxt, s + char(i + norm));
        k -= pref[nxt];
    }
}

```

```

        return "-1";
    }
};

```

## 8.16. Kmp

//Cuenta las ocurrencias del string p en el string s.

```

vector<int> prefix_function(string& s) {
    int n = s.size();
    vector<int> pf(n);
    pf[0] = 0;
    for (int i = 1, j = 0; i < n; i++) {
        while (j && s[i] != s[j]) j = pf[j - 1];
        if (s[i] == s[j]) j++;
        pf[i] = j;
    }
    return pf;
}

int kmp(string& s, string& p) {
    int n = s.size(), m = p.size(), cnt = 0;
    vector<int> pf = prefix_function(p);
    for (int i = 0, j = 0; i < n; i++) {
        while (j && s[i] != p[j]) j = pf[j - 1];
        if (s[i] == p[j]) j++;
        if (j == m) {
            cnt++;
            j = pf[j - 1];
        }
    }
    return cnt;
}

```

## 8.17. Min-Max-SuffixCyclic

//Dado un string s devuelve el indice donde comienza la rotacion lexicograficamente menor de s.

```
int minimum_expression(string s) { //Factorizacion de lyndon
```

```

    s = s+s; // si no se concatena devuelve el indice del
              sufijo menor
    int len = s.size(), i = 0, j = 1, k = 0;
    while (i+k < len && j+k < len) {
        if (s[i+k] == s[j+k]) k++;
        else if (s[i+k] > s[j+k]) i = i+k+1, k = 0; //
            cambiar por < para maximum
        else j = j+k+1, k = 0;
        if (i == j) j++;
    }
    return min(i, j);
}

/*
max_suffix: retorna el inicio del sufijo lexicograficamente
mayor
min_suffix: retorna el inicio del sufijo lexicograficamente
menor
max_cyclic_shift: retorna el inicio del shift ciclico
lexicograficamente mayor
min_cyclic_shift: retorna el inicio del shift ciclico
lexicograficamente menor
*/
template<typename T> int max_suffix(T s, bool mi = false) {
    s.push_back(*min_element(s.begin(), s.end()) - 1);
    int ans = 0;
    for (int i = 1; i < s.size(); i++) {
        int j = 0;
        while (ans + j < i && s[i + j] == s[ans + j]) j++;
        if (s[i + j] > s[ans + j]) {
            if (!mi or i != s.size() - 2) ans = i;
        }
        else if (j) i += j - 1;
    }
    return ans;
}

template<typename T> int min_suffix(T s) {
    for (auto& i : s) i *= -1;
    s.push_back(*max_element(s.begin(), s.end()) + 1);
    return max_suffix(s, true);
}

```



```

template<typename T> int max_cyclic_shift(T s) {
    int n = s.size();
    for (int i = 0; i < n; i++) s.push_back(s[i]);
    return max_suffix(s);
}

template<typename T> int min_cyclic_shift(T s) {
    for (auto& i : s) i *= -1;
    return max_cyclic_shift(s);
}

```

## 8.18. dynamic aho corasick

```

const int MX = 3 * (1e5), SIG = 26, LMX = 20;

struct aho_corasick {
    struct Node {
        Node* sig[SIG], * fail;
        int finish, cnt;
        Node() : fail(this), finish(0), cnt(0) {
            for (int i = 0; i < SIG; i++) sig[i] = this;
        }
        Node(Node* root) : fail(root), finish(0), cnt(0) {
            for (int i = 0; i < SIG; i++) sig[i] = root;
        }
    };

    Node* root;
    aho_corasick() { reset(); }
    void reset() { root = new Node; }

    void insert(string& s, int ind) {
        Node* u = root;

        for (char c : s) {
            c -= 'a';
            if (u->sig[c] == root) {
                u->sig[c] = new Node(root);
                u->sig[c]->finish = -1;
            }
            u = u->sig[c];
        }
    }
}

```

```

    }

    u->finish = ind;
    u->cnt++;
}

Node* getFail(Node* u, int c) {
    while (u != root && u->sig[c] == root) u = u->fail;
    return u->sig[c];
}

void build() {
    queue<Node*> q;

    for (int i = 0; i < SIG; i++) {
        if (root->sig[i] != root) q.push(root->sig[i]);
    }

    while (q.size()) {
        Node* u = q.front();
        q.pop();

        for (int i = 0; i < SIG; i++) {
            Node* v = u->sig[i];
            if (v != root) {
                v->fail = getFail(u->fail, i);
                v->cnt += v->fail->cnt;
                q.push(v);
            }
        }
    }
}

int match(string& t) {
    Node* u = root;
    int res = 0;
    for (int i = 0; i < t.size(); i++) {
        char c = t[i] - 'a';
        if (u->sig[c] != root) u = u->sig[c];
        else u = getFail(u->fail, c);
        res += u->cnt;
    }
    return res;
}

```

```

    }
};

typedef vector<string*> vs;

struct dynamic_aho_corasick {
    aho_corasick ac[LMX];
    vs s[LMX];
    int exi;

    dynamic_aho_corasick() : exi(0) {}

    void insert(string& str) {
        int j = 0;
        while (exi & (1 << j)) j++;
        s[j].push_back(new string(str));

        for (int i = 0; i < j; i++) {
            for (string* t : s[i]) s[j].push_back(t);
            s[i].clear();
            ac[i].reset();
        }

        for (string* t : s[j]) ac[j].insert(*t, 1);
        ac[j].build();
        exi++;
    }

    int match(string& t) {
        int res = 0;

        for (int i = 0; i < LMX; i++)
            if (exi & (1 << i))
                res += ac[i].match(t);

        return res;
    }
};

```

## 8.19. suffixAutomaton1

```
constexpr int MAX = 1e5 + 7;
```

```

namespace sam {
    struct node {
        int len, link, cnt, fpos;
        bool acc;
        map<char, int> next;
    };

    int cur, sz;
    vector<node> sa(MAX * 2);

    void add(char c) {
        int at = cur;
        sa[cur].fpos = sa[sz].len = sa[cur].len + 1;
        sa[cur].fpos -= 1, cur = sz++;
        while (at != -1 && !sa[at].next.count(c))
            sa[at].next[c] = cur, at = sa[at].link;
        if (at == -1) { sa[cur].link = 0; return; }
        int q = sa[at].next[c];
        if (sa[q].len == sa[at].len + 1) { sa[cur].link = q; return; }
        int qq = sz++;
        sa[qq].len = sa[at].len + 1, sa[qq].next = sa[q].next, sa[qq].link = sa[q].link;
        sa[qq].fpos = sa[q].fpos;
        while (at != -1 && sa[at].next[c] == q)
            sa[at].next[c] = qq, at = sa[at].link;
        sa[q].link = sa[cur].link = qq;
    }

    void build(string& s) {
        #warning "clear????";
        sa.assign(MAX * 2, node());
        cur = 0, sz = 0, sa[0].len = 0, sa[0].link = -1, sz++;
        for (auto& i : s) add(i);
        int at = cur;
        while (at) sa[at].acc = 1, at = sa[at].link;
    }

    int64_t distinct_substrings() {
        int ans = 0;
        for (int i = 1; i < sz; i++) ans += sa[i].len - sa[sa[i].link].len;
        return ans;
    }
}

```

```

}
int longest_common_substring(string& S, string& T) {
    build(S);
    int at = 0, l = 0, ans = 0, pos = -1;
    for (int i = 0; i < sz(T); i++) {
        while (at && !sa[at].next.count(T[i])) at =
            sa[at].link, l = sa[at].len;
        if (sa[at].next.count(T[i])) at =
            sa[at].next[T[i]], l++;
        else at = 0, l = 0;
        if (l > ans) ans = l, pos = i;
    }
    return ans;
    // return T.substr(pos - ans + 1, ans);
}

vector<int> LCS, match;
void lcsMatch(string& t) {
    match.assign(MAX, 0);
    int u = 0, l = 0;
    for (int i = 0; i < sz(t); ++i) {
        while (u && !sa[u].next.count(t[i])) u =
            sa[u].link, l = sa[u].len;
        if (sa[u].next.count(t[i])) u =
            sa[u].next[t[i]], l++;
        match[u] = max(match[u], l);
    }
    for (int i = MAX - 1; i > 0; --i) match[i] =
        max(match[i], match[sa[i].link]);
    for (int i = 0; i < MAX; ++i) LCS[i] = min(LCS[i],
        match[i]);
}

int lcs_n(vector<string>& t) {
    const int INF = 1e7;
    LCS.assign(MAX, INF);
    for (i, sz(t)) lcsMatch(t[i]);
    return *max_element(all(LCS));
}

int isSubstr(string& s) {
    int at = 0;
    for (auto& i : s) {
        if (!sa[at].next.count(i)) return 0;

```

```

        at = sa[at].next[i];
    }
    return at;
}

int count_occ(int u) {
    if (sa[u].cnt != 0) return sa[u].cnt;
    sa[u].cnt = sa[u].acc;
    for (auto& v : sa[u].next) sa[u].cnt +=
        count_occ(v.s);
    return sa[u].cnt;
}

int pos_occ(string& s) {
    int x = sam::isSubstr(s);
    return x ? (abs(sam::sa[x].fpos - sz(s)) + 1) : -1;
}

int dp[2 * MAX];
int paths(int i) {
    auto& x = dp[i];
    if (x) return x;
    x = 1;
    for (char j = 'a'; j <= 'z'; j++) {
        if (sa[i].next.count(j)) x +=
            paths(sa[i].next[j]);
    }
    return x;
}

void kth_substring(int k, int at = 0) { // k=1 : menor
    substring lexicog.
    for (int i = 0; i < 26; i++) if (k &&
        sa[at].next.count(i + 'a')) {
        if (paths(sa[at].next[i + 'a']) >= k) {
            cout << char(i + 'a');
            kth_substring(k - 1, sa[at].next[i + 'a']);
            return;
        }
        k -= paths(sa[at].next[i + 'a']);
    }
}

};

```

## 8.20. suffixAutomaton popback

```

struct suffixAutomata {
    struct node {
        int len, link, cnt;
        int next[26];
    };

    vector<node> sa;
    vector<int> last, p1, p2, q1, qlink;
    int ans = 0; //number of distinct strings that occur at
        least twice as substrings of S
    string s;

    suffixAutomata(int mx_len) {
        sa.reserve(mx_len * 2);
        last.pb(add_node());
        sa[0].link = -1;
    }

    int add_node() { sa.pb({}); return sz(sa) - 1; }
    void add_char(char ch) {
        s.pb(ch);
        int c = ch - 'A';
        int u = add_node(), p = last.back();
        sa[u].len = sa[p].len + 1;
        while (p != -1 && !sa[p].next[c]) {
            sa[p].next[c] = u;
            p = sa[p].link;
        }
        p1.pb(p);
        if (p != -1) {
            int q = sa[p].next[c];
            q1.pb(q);
            if (sa[p].len + 1 != sa[q].len) {
                int clone = add_node();
                sa[clone] = sa[q];
                sa[clone].len = sa[p].len + 1;
                qlink.pb(sa[q].link);
                sa[q].link = sa[u].link = clone;
                while (p != -1 && sa[p].next[c] == q) {
                    sa[p].next[c] = clone;
                    p = sa[p].link;
                }
            }
        }
    }
}

```

```

        p2.pb(p);
    }
    else sa[u].link = q;
    int v = sa[u].link;
    if (!sa[v].cnt) ans += sa[v].len -
        sa[sa[v].link].len;
    sa[v].cnt++;
}
last.pb(u);
}

void pop_back() {
    int c = s.back() - 'A'; s.pop_back();
    int u = last.back(); last.pop_back();
    int p = last.back();
    while (p != p1.back()) {
        sa[p].next[c] = 0;
        p = sa[p].link;
    }
    p1.pop_back();
    if (p != -1) {
        int v = sa[u].link;
        sa[v].cnt--;
        if (!sa[v].cnt) ans -= sa[v].len -
            sa[sa[v].link].len;
        int q = q1.back(); q1.pop_back();
        if (sa[p].len + 1 != sa[q].len) {
            sa[q].link = qlink.back(); qlink.pop_back();
            while (p != p2.back()) {
                sa[p].next[c] = q;
                p = sa[p].link;
            }
            p2.pop_back();
            sa.pop_back();
        }
    }
    sa.pop_back();
}

node& operator[](int i) { return sa[i]; }
};

```

## 8.21. Splitear

```
template<typename T>
T split(string& in) {
    T result;
    regex pattern("[^a-zA-Z]|paraagregar mas | |");
    in = regex_replace(in, pattern, " ");
    transform(all(in), in.begin(), ::toupper);
    istringstream iss(in);
    string token;
    if constexpr (is_same<T, vector<string>>::value) while
        (iss >> token) result.pb(token);
    else if constexpr (is_same<T, string>::value) {
        result = ""; while (iss >> token) result += token;
    }
    return result;
}
```

## 8.22. Suffix Array 1

```
vector<int> suffix_array(string s) {
    s += "$";
    int MAX = 260, n = sz(s), N = max(n, MAX);
    vector<int> sa(n), ra(n);
    for (int i = 0; i < n; i++) sa[i] = i, ra[i] = s[i];
    for (int k = 0; k < n; k ? k *= 2 : k++) {
        vector<int> nsa(sa), nra(n), cnt(N);
        for (int i = 0; i < n; i++) nsa[i] = (nsa[i] - k +
            n) % n, cnt[ra[i]]++;
        for (int i = 1; i < N; i++) cnt[i] += cnt[i - 1];
        for (int i = n - 1; i + 1; i--)
            sa[--cnt[ra[nsa[i]]]] = nsa[i];
        for (int i = 1, r = 0; i < n; i++) nra[sa[i]] = r
            += ra[sa[i]] != ra[sa[i - 1]] || ra[(sa[i] + k)
                % n] != ra[(sa[i - 1] + k) % n];
        ra = nra;
        if (ra[sa[n - 1]] == n - 1) break;
    }
    return vector<int>(sa.begin() + 1, sa.end());
}

vector<int> kasai(string s, vector<int> sa) {
```

```
    int n = sz(s), k = 0;
    vector<int> ra(n + 1), lcp(n);
    for (int i = 0; i < n; i++) ra[sa[i]] = i;
    for (int i = 0; i < n; i++, k -= !!k) {
        if (ra[i] == n - 1) { k = 0; continue; }
        int j = sa[ra[i] + 1];
        while (i + k < n && j + k < n && s[i + k] == s[j +
            k]) k++;
        lcp[ra[i]] = k;
    }
    return lcp;
}

/*
find the number of occurrences of the string t in the
string s
*/
int find_str(string& s, string& t, vector<int>& sa) {
    int n = sz(s);
    if (sz(t) > n) return 0;
    int L = 0, R = n - 1;
    int nL, nR;
    for (int i = 0; i < sz(t); i++) {
        int l = L, r = R + 1;
        while (l < r) {
            int m = (l + r) / 2;
            if (i + sa[m] >= n || s[i + sa[m]] < t[i]) l =
                m + 1;
            else r = m;
        }
        if (l == R + 1 || s[i + sa[l]] > t[i]) return 0;
        nL = l, l = L, r = R + 1;

        while (l < r) {
            int m = (l + r) / 2;
            if (i + sa[m] >= n || s[i + sa[m]] <= t[i]) l =
                m + 1;
            else r = m;
        }
        l--;
        nR = l, L = nL, R = nR;
    }
    return (nL <= nR ? nR - nL + 1 : 0);
}
```

```

/*
find the longest common substring what
appear in the string s at least twice
*/
string lcs(vector<int>& sa, vector<int>& ka, string& s) {
    int idx = max_element(all(ka)) - begin(ka);
    return (ka[idx] > 0 ? s.substr(sa[idx], ka[idx]) :
        "-1");
}
/*
Find the longest common substring of two given strings s
and t
create a new string s + '#' + t
compute the suffix array of the new string
compute the LCP array of the new string
pos_t = (i_s ? sa[i + 1] - (n + 1) : sa[i] - (n + 1));
*/
string find_lcs(string& s, string& t, vector<int>& sa,
    vector<int>& lcp) {
    int best = 0, n = sz(s), pos = INT_MAX;
    for (int i = 0; i < sz(lcp) - 1; i++) {
        bool i_s = (0 <= sa[i] && sa[i] <= n - 1);
        bool j_s = (0 <= sa[i + 1] && sa[i + 1] <= n - 1);
        if (i_s != j_s && best < lcp[i]) {
            best = lcp[i];
            pos = min(sa[i], sa[i + 1]);
        }
    }
    return pos == INT_MAX ? "" : s.substr(pos, best);
}
vector<int> substr_begin_by_letter(const string& s, const
    vector<int>& sa, const vector<int>& lcp) {
    vector<int> abc(26);
    int n = sz(s);
    forn(i, n) abc[s[sa[i]] - 'a'] += n - sa[i] - lcp[i];
    return abc;
}
int dis_substr(const string& s, const vector<int>& sa,
    const vector<int>& lcp) {
    int n = sz(s), ans = 0;
    forn(i, n) ans += n - sa[i] - lcp[i];
    return ans;
}

```

## 9. Utilities

### 9.1. cmd

```

"C:\w64devkit\bin\gdb.exe" !.exe
"C:\w64devkit\bin\g++.exe" -g !.cpp -o !.exe

```

```

g++ -o A A.cpp
./A

```

```

A < in.txt
A < in.txt > op.txt

```

### 9.2. Custom Hash

```

#define PI acos(-1)
struct chash {
    // any random-ish large odd number will do
    const uint64_t C = uint64_t(4e18 * PI) + 71;
    const uint32_t RANDOM =
        chrono::steady_clock::now().time_since_epoch().count();
    size_t operator()(uint64_t x) const { return
        __builtin_bswap64((x ^ RANDOM) * C); }
};

template <class K, class V> using u_map = unordered_map<K,
    V, chash>;
template <class K> using u_set = unordered_set<K, chash>;

```

### 9.3. include

```

#include <algorithm>
#include <iostream>
#include <iterator>
#include <sstream>
#include <fstream>
#include <cassert>
#include <climits>
#include <cstdlib>
#include <cstring>

```

```

#include <string>
#include <cstdio>
#include <vector>
#include <cmath>
#include <queue>
#include <deque>
#include <stack>
#include <list>
#include <map>
#include <set>
#include <bitset>
#include <iomanip>
#include <unordered_map>
#include <tuple>
#include <random>
#include <chrono>

```

## 9.4. template

```

#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define f first
#define s second
#define ins insert
#define pb push_back
#define eb emplace_back
#define sz(x) int((x).size())
#define all(x) begin(x), end(x)
typedef long long ll;
typedef unsigned long long ull;
#define forn(i, n) for (int i = 0; i < n; ++i)
#define each(i, x) for (auto &i : x)
#define forne(i, x, n) for (int i = x; i < n; ++i)
#define show(x) for (auto &i : x) {cerr << i << ' ';}
    cerr<< endl;

void dbg_out() { cerr << ']' << endl; }
template<typename Head, typename... Tail>
void dbg_out(Head H, Tail... T) { cerr << H; if
    (sizeof...(T)) cerr << ', ' << ' '; dbg_out(T...); }

```

```

#ifdef LOCAL
#define dbg(...) cerr << '|' << __LINE__ << '|' << '{' <<
    #__VA_ARGS__ << '}' << ':' << ' ' << '[' << dbg_out(__VA_ARGS__)
#else
#define dbg(...)
#endif
#define int int64_t

signed main() {
    ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
#ifdef LOCAL
    freopen("in", "r", stdin);
    freopen("out", "w", stdout);
    freopen("err", "w", stderr);
#endif

    cout << flush;
    return 0;
}

```

## 9.5. Pragma

```

#pragma GCC
    optimize("Ofast,unroll-loops,no-stack-protector,fast-math,in
#pragma GCC
    target("sse,sse2,sse3,ssse3,sse4,popcnt,lzcnt,mmx,abm,avx,av

```

## 9.6. nodes STree

```

/*
segment whit the maximum sum
add to segment tree the node struct
T neutro = T();
T oper(T a, T b) {node::get(a, b);}
Check the base ans
*/

```

```

constexpr int inf = (1e18);
struct node {
    int lt, rt, sum, ans;
    node() : lt(-inf), rt(-inf), sum(0), ans(-inf) {}
    node(int x) : sum(x) {
        lt = rt = ans = (x);
    }
    static node get(node& a, node& b) {
        node res;
        res.sum = a.sum + b.sum;
        res.lt = max(a.lt, a.sum + b.lt);
        res.rt = max(b.rt, b.sum + a.rt);
        res.ans = max({ a.ans, b.ans, a.rt + b.lt });
        return res;
    }
};

/*
    max(al,al+1,...,ar)-min(al,al+1,...,ar)-(r-l),
*/
constexpr int inf = (1e18);
struct node {
    int len, mxl, mxr, mnl, mnr, ans;
    node() : len(0), mxl(-inf), mxr(-inf), mnl(inf),
        mnr(inf), ans(0) {}
    node(int pos, int val) : len(1), mxl(val + pos),
        mxr(val - pos), mnl(val - pos), mnr(val + pos),
        ans(0) {}

    static node get(node& a, node& b) {
        node res;
        res.len = a.len + b.len;
        res.mxl = max(a.mxl, b.mxl);
        res.mxr = max(a.mxr, b.mxr);
        res.mnl = min(a.mnl, b.mnl);
        res.mnr = min(a.mnr, b.mnr);
        res.ans = max({ a.ans, b.ans, a.mxl - b.mnr, b.mxr
            - a.mnl });
        return res;
    }
};

```

## 9.7. util

```

//__builtin_popcount(x); -> Cuenta el numero de bits '1'
    en la representacion binaria de x.
//__builtin_parity(x); -> Devuelve 1 si el numero de
    bits '1' en la representacion binaria de x es impar, 0
    si es par.
//__builtin_clz(x); -> Cuenta el numero de bits en
    '0' a la izquierda, desde el bit mas significativo hasta
    el primer '1'.
//__builtin_ctz(x); -> Cuenta el numero de bits en
    '0' a la derecha, desde el bit menos significativo hasta
    el primer '1'.
//__builtin_ffs(x); -> Encuentra la posicion del
    primer bit en '1' (contando desde 1, desde el bit menos
    significativo).
//__lg(x); -> Devuelve el logaritmo en base 2
//__builtin_bswap32(x); -> Intercambia los bytes de un
    entero de 32 bits.
//__builtin_bswap64(x); -> Intercambia los bytes de un
    entero de 64 bits.
// x ^ (1 << (x & -x)); -> Invierte el bit menos
    significativo en '1' de x.
//n & ~(1 << (x - 1)); -> Apaga el m-esimo bit de n (bit
    1 si m=1 es el menos significativo), Si m=1, apaga el
    bit menos significativo.
//~n; -> Suma 1 a n.
//~- n; -> Resta 1 a n.
//x && (!(x & (x - 1))); -> Comprueba si x es una potencia
    de 2.
// x & (1<<i) -> Verifica si el i-esimo bit
    esta encendido
// x = x | (1<<i) -> Enciende el i-esimo bit
// x = x & ~(1<<i) -> Apaga el i-esimo bit
// x = x ^ (1<<i) -> Invierte el i-esimo bit
// x = ~x -> Invierte todos los bits
// x & -x -> Devuelve el bit encendido mas
    a la derecha (potencia de 2, no el indice)
// ~x & (x+1) -> Devuelve el bit apagado mas a
    la derecha (potencia de 2, no el indice)
// x = x | (x+1) -> Enciende el bit apagado mas a
    la derecha
// x = x & (x-1) -> Apaga el bit encendido mas a
    la derecha

```



```

// x = x & ~y          -> Apaga en x los bits encendidos
// de y

// elementos unicos
sort(all(nums));
nums.resize(unique(all(nums)) - nums.begin());

//Rotar una matriz 90 grados
vector<vector<int>> rotar(vector<vector<int>>& a) {
    int n = sz(a), m = sz(a[0]);
    vector<vector<int>> v(m, vector<int>(n));
    forn(i, n) {
        forn(j, m) {
            v[j][n - 1 - i] = a[i][j];
        }
    }
    return v;
}

//1234567891011121314151617... what is the digit at
// position n?
char digit_at_pos(int n) {
    n--; // 0 index
    int len = 9, mm = 1;
    forne(i, 1, 32) { // change 32 to 64 if needed
        if (n < len) {
            int num = n / i + mm, pos = n % i;
            return to_string(num)[pos];
        }
        n -= len, mm *= 10, len = 9 * mm * (i + 1);
    }
}

// sum of even or odd numbers from l to r
auto eve = [&](int l, int r) { return ((r / 2) * ((r / 2) + 1)) - (((l - 1) / 2) * (((l - 1) / 2) + 1)); };
auto odd = [&](int l, int r) { return (r * (r + 1) / 2) - (((l - 1) * ((l - 1) + 1) / 2) - eve(l, r)); };

// > need
auto upper_bound = [&](int need) ->int {
    int l = -1, r = n;
    while (r - l > 1) {

```

```

        int mid = l + (r - l) / 2;
        if (nums[mid] <= need) l = mid;
        else r = mid;
    }
    return l;
};

// >= need
auto lower_bound = [&](int need) ->int {
    int l = -1, r = n;
    while (r - l > 1) {
        int mid = l + (r - l) / 2;
        if (nums[mid] < need) l = mid;
        else r = mid;
    }
    return r;
};

// == need
auto search = [&](int need) ->bool {
    int l = -1, r = n;
    while (r - l > 1) {
        int mid = l + (r - l) / 2;
        if (nums[mid] < need) l = mid;
        else r = mid;
    }
    return nums[r] == need;
};

// xor sum from 0 to x
int xorsum(int x) {
    if (x % 4 == 0) return x;
    else if (x % 4 == 2) return x + 1;
    else if (x % 4 == 1) return 1;
    else return 0;
};

/* resultado de & en el rango [l, r] */
int rangeAND(int l, int r) {
    int ans = 0;
    for (int i = 63 - 1; i >= 0; i--) {
        if ((l & (1ll << i)) != (r & (1ll << i))) break;
        ans |= (l & (1ll << i));
    }
    return ans;
}

```

```
}
```

## 9.8. Plantillap

```
import math
import sys
input = sys.stdin.readline
write = sys.stdout.write
def fast_print(x): write(str(x) + '\n')
```

```
def main():
```

```
    fast_print("Hello!!")
```

```
if __name__ == '__main__':
    main()
```

## 9.9. Stres

```
import subprocess
```

```
def run_command(command, input_data=None):
    process = subprocess.Popen(command,
                                stdin=subprocess.PIPE, stdout=subprocess.PIPE,
                                text=True)
    stdout, _ = process.communicate(input_data)
    return stdout.strip()
```

```
def compile_cpp(source_file, output_file):
    compile_command = ["g++", source_file, "-o",
                        output_file, "-O2", "-std=c++11"]
    result = subprocess.run(compile_command)
    return result.returncode == 0
```

```
compile_cpp("brute.cpp", "brute")
compile_cpp("main.cpp", "main")
compile_cpp("gen.cpp", "gen")
```

```
for i in range(100000):
    testcase = run_command(["./gen"])
    brute_output = run_command(["./brute"], testcase)
```

```
main_output = run_command(["./main"], testcase)
if brute_output != main_output:
    print("Testcase:\n", testcase)
    print("Output brute:\n", brute_output)
    print("Output sol:\n", main_output)
    break
else :
    print("Testcase", i, "OK")
```

## 9.10. coisaspypytho

```
import sys
from math import gcd, lcm, comb, factorial, sqrt, ceil,
    floor, log2, log10, sin, cos, tan, pi, perm
from itertools import permutations, combinations, product
from collections import deque, defaultdict, Counter
import heapq
```

```
# Entrada y salida rapidas
input = sys.stdin.readline
print = sys.stdout.write
```

```
# Limite de recursion
sys.setrecursionlimit(10**6)
```

```
# Constantes
```

```
MOD = 10**9 + 7
```

```
INF = 10**18
```

```
dirs4 = [(1,0), (-1,0), (0,1), (0,-1)]
```

```
dirs8 = [(1,0), (-1,0), (0,1), (0,-1), (1,1), (1,-1),
          (-1,1), (-1,-1)]
```

```
def modinv(a): return pow(a, -1, MOD)
```

```
# n, m = map(int, input().split())
# arr = list(map(int, input().split()))
```

```
#estructuras
```

```
n, m = 5, 5
```

```
lista_1d = [0] * n
```

```

lista_2d = [[0] * m for _ in range(n)]
# Aplanar listas 2D en 1D
lista_flat = [0] * (n * m)

# Enumerar
for i, val in enumerate(lista_1d):
    pass

# Pilas, colas y heaps
stack = []
stack.append(1)
stack.pop()

q = deque()
q.append(1)
x = q.popleft()

pq = []
heapq.heappush(pq, 3)
heapq.heappush(pq, 1)
heapq.heappush(pq, 2)
min_val = heapq.heappop(pq)
max_val = -heapq.heappop(pq) # Usar valores negativos
                             (pq,-3) para max-heap

#set map

s = set()
s.add(5)
if 5 in s:
    s.remove(5)

d = {}
d[1] = 'a'
for k, v in d.items():
    pass

# defaultdict y Counter
graph = defaultdict(list)
freq = Counter([1, 2, 2, 3])

# Obtener valor ASCII
ord('A') # 65
chr(65)  # 'A'

```

```

x = 36
y = 48
g = gcd(x, y)
l = lcm(x, y)

# combinatoria
n, k = 5, 2
c = comb(n, k)
f = factorial(n)

# raices, redondeos y logaritmos
r = sqrt(16)
up = ceil(3.1)
down = floor(3.9)
lg2 = log2(8)
lg10 = log10(1000)

# trigonometria
ang = pi / 4
seno = sin(ang)
coseno = cos(ang)
tangente = tan(ang)

# combinaciones y permutaciones

nums = [1, 2, 3]
for p in permutations(nums):
    pass

for c in combinations(nums, 2):
    pass

for p in product([0,1], repeat=3): #todas combinaciones con
    repeticion de los elementos [0, 1], tomando 3 posiciones.
    pass #(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0),
        (1,0,1), (1,1,0), (1,1,1)

#Salida rapida de muchas lineas
outputs = []
for i in range(5):
    outputs.append(str(i))

```

```

print('\n'.join(outputs))
}

# -----
# sys.stdin = open("input.in", "r")
# sys.stdout = open("output.out", "w")

```

### 9.11. random

```

int rnd(int l, int r) {
    static std::mt19937
        rng(std::chrono::steady_clock::now().time_since_epoch().count());
    return std::uniform_int_distribution<int>(l, r)(rng);
}

```

### 9.12. int128

```

using lint = __int128;
static inline lint abs128(lint x) { return x < 0 ? -x : x; }
static inline lint gcd128(lint a, lint b) {
    if (a < 0) a = -a;
    if (b < 0) b = -b;
    while (b) { lint t = a % b; a = b; b = t; }
    return a;
}

istream& operator>>(istream& in, lint& x) {
    string s; in >> s;
    x = 0; bool neg = 0; int i = 0;
    if (s[0] == '-') neg = 1, i = 1;
    for (; i < sz(s); i++) x = x * 10 + (s[i] - '0');
    if (neg) x *= -1;
    return in;
}

ostream& operator<<(ostream& out, lint x) {
    if (x == 0) return out << "0";
    if (x < 0) out << '-', x = -x;
    string s;
    while (x) s += '0' + x % 10, x /= 10;
    reverse(all(s));
    return out << s;
}

```