Report on

**Analysis of standard elliptic curves for the implementation of ECC Cipher in resource-constrained environments**

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Towards the partial fulfillment for the Award of the Degree of

**BACHELOR OF TECHNOLOGY**

In Computers Science and Engineering

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**1. Introduction:**

Nowadays, many publicly observable Internet protocols

offer cipher suites that contain elliptic curve based

cryptographic algorithms. Today, ECC is increasingly used

to implement public-key cryptography protocols, such as

digital signatures and key agreement protocols. Bitcoin,

secure shell (SSH), Transport Layer Security (TLS), and the

Austrian e-ID card is some of the application protocols

which make use of ECC in the real world [1]. Many E-

commerce applications also use elliptic curve cryptography,

due to its security features.

There are many international standards available for the

selection of elliptic curves. The choice of the curve is

dependent on the security requirement and the required

efficiency of the curve in terms of computation speed. In [2]

the first release of cryptographic standards specifying elliptic

curves for use in practice is given for the different key sizes.

The standards that recommend curves for various security

levels are defined over either prime or binary extension fields.

Currently, across the internet, the elliptic curves deployed are

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elliptic curves defined over prime fields are the subject of

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is absent compared to systems based on the multiplicative

group of a finite field, so it is possible to maintain the same

level of security with an elliptic curve group that is smaller in

size [4]. The advantages of using smaller elliptic groups are

smaller key sizes, bandwidth savings, and faster

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**1.1 Motivation:**

In the current digital world and public-key cryptography segment, the majority of deployments are fulfilled by the RSA based cryptosystems. Cryptosystems based on elliptic curves emerge as an alternative to the RSA cryptosystems. The security of the RSA cryptosystem is based on the integer factorization problem (IFP) whereas the security of ECC is based on the elliptic curve discrete logarithm problem (ECDLP). The significant attraction towards ECC is that the best-known algorithm for solving the ECDLP takes full exponential time while for solving IFP of RSA takes sub exponential time. ECC takes less memory than RSA and is better than RSA, especially on memory-constrained devices. Moreover, ECC provides same level of security as the RSA but with reduced key size.

**2. Literature:**

**Selection of Elliptic Curve** -

All of the different international standards available today

focus on the selection of safe and efficient elliptic curves for

implementing ECC. Each standard of curve strives to

maintain the difficulty of the Elliptic Curve Discrete

Logarithm Problem (ECDLP). For the sake of efficiency,

each standard has a particular form of the elliptic curve along

with a recommendation for the specific values of the prime

fields and the constants in the equation. The security of ECC

is primarily dependent on the selection of curve. The main

idea is to select a curve which is safe against the known

attacks on ECDLP. If a weak curve is selected for any

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**Analysis of Elliptic Curve** –

There are many elliptic curves available for use in ECC

suggested by the different standards. The types of curve are

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There are many elliptic curves available for use in ECC suggested by the different standards. The types of curve are basically classified based on prime field size and the form of the curve. Each selected curve has different field sizes which are nothing but the key sizes of ECC. The selected curves are analyzed by performing the two algorithms used in ECC, namely ECDH and ECDSA. The analysis is carried out using Sage Math which is open-source mathematical software using a system having an Intel Core i3 processor with 4 GB of RAM. The same algorithm is used while performing the analysis for each curve. Only the curve parameters are changed according to the type of curve. For each type of curve, the results are taken regarding computation time needed for different operations during ECDH and ECDSA.

**Example**: Let us consider the elliptic curve y^2 = x^3 + x^2 + x+ 6; p = 11.

Then, set of points of E11(1, 6) are

(2, 4) (2, 7) (3, 5) (3, 6)

(5, 2) (5, 9) (7, 2) (7, 9)

(8, 3) (8, 8) (10, 2) (10, 9)

Let us consider G = (2, 7) and user B’s private key nab = 7, then user B’s public key

PB = nab × G = 7 (2, 7) = (7, 2)

Suppose that user A wishes to send a message to user B that is encoded in the elliptic curve point Pm = (10, 9) (say) and let A select a random number k = 3.

Then cipher text Cm = {kg., Pm + kept}

= {3 (2, 7), (10,9) + 3(7,2)}

= {(8, 3), (10,9) + (3,5)}

= {(8, 3), (10,2)}

Next User B will recover Pm from Cm as follows:

(Pm + kept)– nab (kg.) = (10, 2) – 7 (8,3)

= (10, 2) – (3,5)

= (10, 2) + (3,6)

= (10, 9) = Pm

**Problem Statement** -

Implementation of the Elliptic Curve Diffie-Hellman Algorithm and Elliptic Curve Digital Signature Algorithm.

**Approach** -

There are multiple standard Elliptical Curves that can be used with the ECC scheme

Each standard of curve strives to maintain the difficulty of the Elliptic Curve Discrete Logarithm Problem (ECDLP).

The main idea is to select a curve which is safe against the known attacks on ECDLP

The analysis is carried out using SageMath, which is a free open-source mathematics software system licensed under the GPL, using Python 3.

**Security of ECC** –

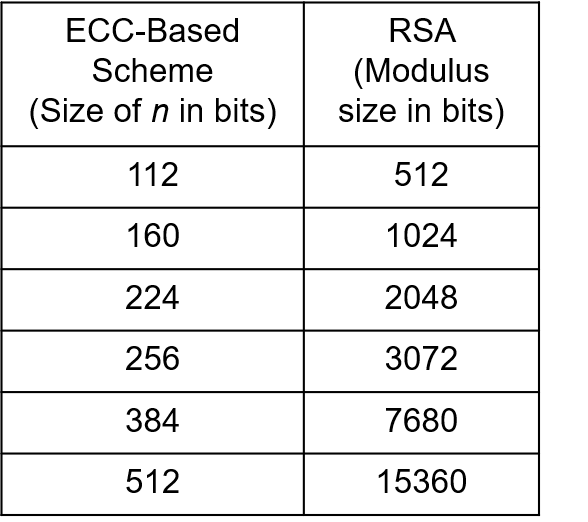
The security of elliptic curve cryptosystems such as the ECDSA is the apparent intractability of the following elliptic curve discrete logarithm problem(ECDLP):

“Given an elliptic curve *E* defined over, a point *P E ()* of order *n*,and a point *Q E ()*, determine the integer *k*, 0 ≤ *k* ≤ *n-*1*,* such that *Q* = *kip,* provided that such an integer exists”.

Over the past many years, the ECDLP has received considerable attention from leading mathematician around the world, and no significant weaknesses have been reported till date. However, in order to achieve the maximum possible security level, *n* should be prime.

Comparison of bit size between RSA and ECC

Table 1.1



**3. Methodology and Framework:**

The analysis will be performed by considering each curve for the implementation of the Elliptic Curve Diffie-Hellman (ECDH) algorithm and the Elliptic Curve Digital Signature Algorithm (ECDSA). The analysis is carried out using Sage Math, which is a free open-source mathematics software system licensed under the GPL, using Python 3.

* SOFTWARE
  + Windows 10
  + Python 3.x
* HARDWARE
  + HDD/SSD with sufficient storage capacity
  + Core i5 Processor
  + At least 4GB RAM

**4. Work Done:**

Homepage-

Figure 1.1



* Understood the mathematical foundations reqd.
* Understood the algorithmic concepts reqd.
* Created a web-app that plots the curves that are pre-defined
* Basic interface ready
* Back-end majorly completed
* Currently gives O/P for predefined parameters

The above bulletins have been implemented using following techniques:

Frontend: Html, CSS, bootstrap

Backend: python3, Flask, SQLAlchemy

Packages: Matplotlib, Seaborn, wtforms

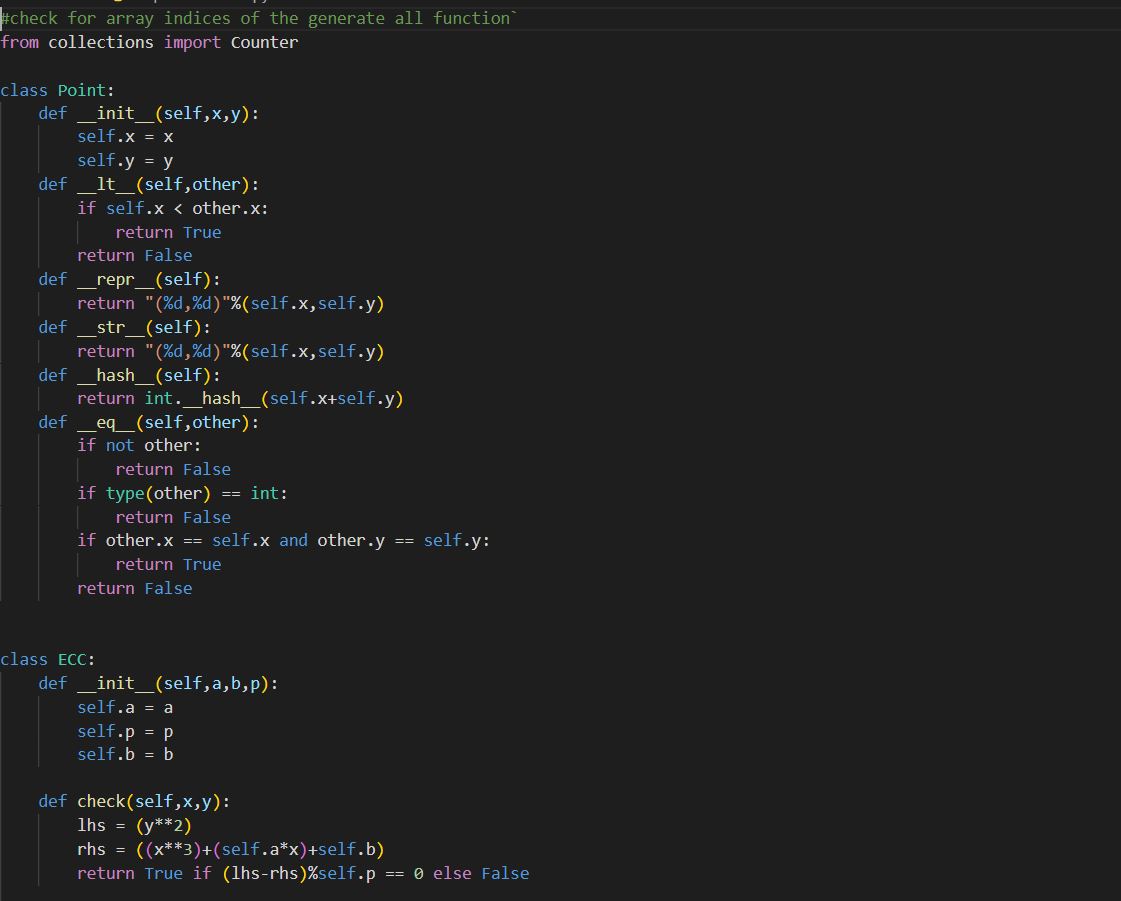
Flask: Flask is a micro web framework written in Python. It is classified as a microframework because it does not require particular tools or libraries. It has no database abstraction layer, form validation, or any other components where pre-existing third-party libraries provide common functions.

SQLAlchemy: SQLAlchemy is an extension for Flask that adds support for SQLAlchemy to your application. It aims to simplify using SQLAlchemy with Flask by providing useful defaults and extra helpers that make it easier to accomplish common tasks.

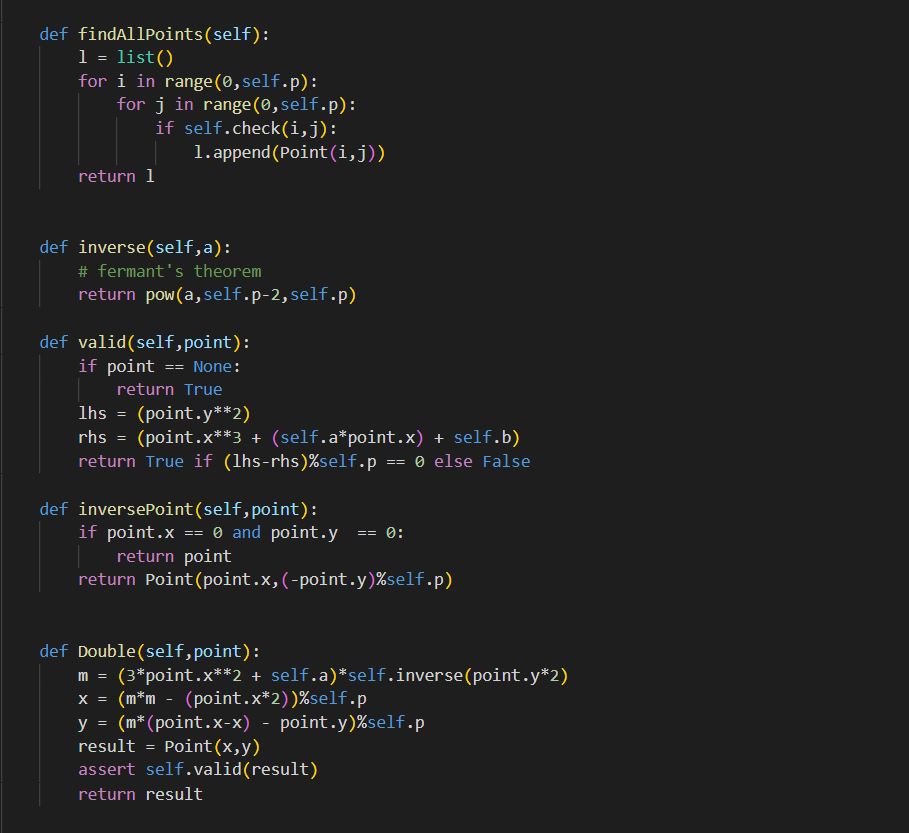
Matplotlib: **Matplotlib is a cross-platform, data visualization and graphical plotting library for Python and its numerical extension NumPy. As such, it offers a viable open-source alternative to MATLAB. Developers can also use matplotlib’s APIs (Application Programming Interfaces) to embed plots in GUI applications.**

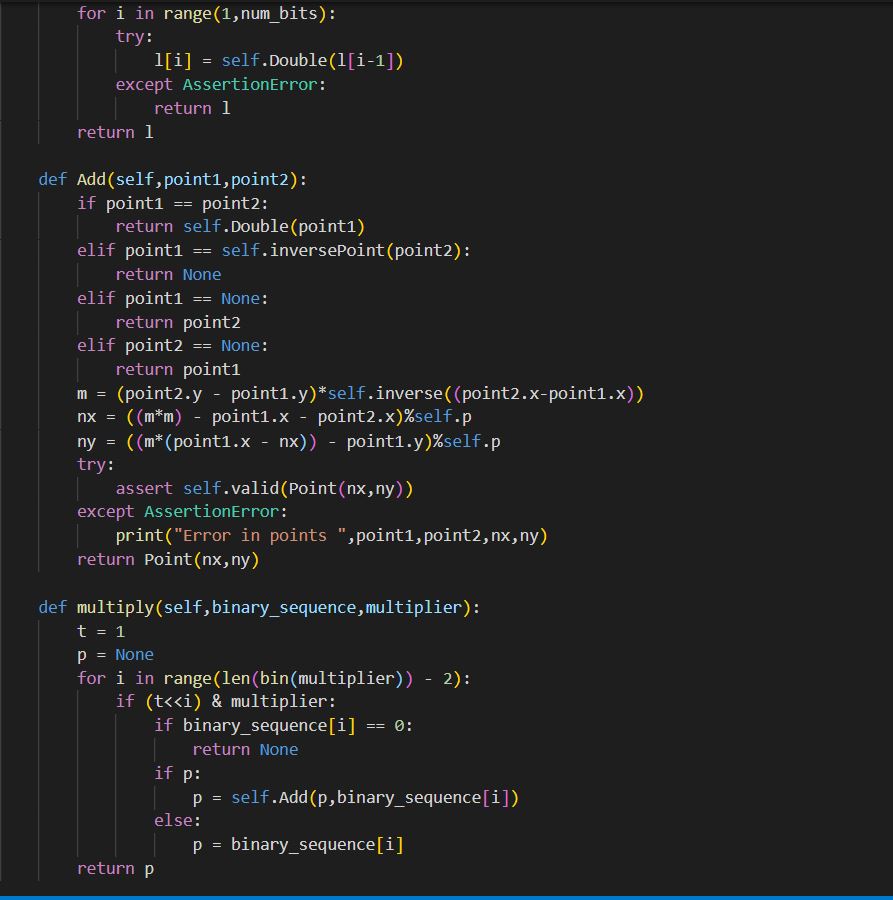
WTForms: Flask-WTForms is a great tool to help with form validation (e.g., avoidance of  CSRF. Flask-WTForms can help create and use web forms with simple Python models, turning tedious and boring form validation into a breeze

**Code of implementation of Algorithm:**



The below function is continuation of the above function i.e., part of the same class ECC.





The entire code has been uploaded on GitHub:

<https://github.com/Jaidev810/ECC-Cipher.git>

\*If you want to run the code:

Step1: install requirement.txt file using code **pip install -r requirement.txt**

Step2: Go in webview and run application.py using command **python application.py**

**5. Conclusion and Future Plan:**

Till now ECC algorithm has been implemented in python and its implementation has been displayed using GUI. A web app has been created which shows the curves for predefined values. The App has been implemented using various technologies and the visualization has been done using matplotlib.

* We plan to learn sage math and implement in our code.
* Selection of 5 best points from the curve.
* Calculation of average time taken by a point.
* Calculation of average deviation of a point.
* Improve the UI of the app.
* Take user input so that the user can plot his/her curve.
* Fixing bugs on the backend.

**References:**

* Cryptography and Network Security, 4th Edition, William Stallings
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* Flask Documentation: <https://flask.palletsprojects.com/en/1.1.x/>
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* Sqlalchemy: [https://docs.sqlalchemy.org/en/14/](%20https:/docs.sqlalchemy.org/en/14/%20)