Project 1

Particle trajectory via Verlet method

Consider Earth's magnetic field

$$B_r = -2B_0\left(\frac{R_E}{r}\right)^3\cos(\theta) \tag{1}$$

$$B_\theta = -B_0\left(\frac{R_E}{r}\right)^3\sin(\theta) \tag{2}$$

$$B_\phi = 0 \tag{3}$$

Here the usual spherical co-ordinates (r, θ, ϕ) are used with the magnetic north-south axis along the z direction. $B_0 = 3.12 \times 10^{-5}T$.

An electron with energy 30keV (assume non-relativistic dynamics) starts moving at the magnetic equatorial plane at an altitude of $1R_E$ above Earth's surface. It's velocity is in the north-east direction (45° between north and east). Show the trajectory of this particle in 3 dimensions for different times. Use the magnetic velocity Verlet method to solve for the trajectory.

The magnetic velocity Verlet update is given as follows:

$$\begin{aligned}
\mathbf{d} &\equiv \left(\mathbf{v}(t) + \frac{q\Delta t}{2m}\mathbf{v}(t) \times \mathbf{B}(\mathbf{x}(t))\right), \\
\mathbf{x}(t+\Delta t) &= \mathbf{x}(t) + \mathbf{d}\Delta t, \\
\mathbf{C} &\equiv \mathbf{B}(\mathbf{x}(t+\Delta t)), \\
\mathbf{v}(t+\Delta t) &= \frac{1}{1+\left(\frac{q\Delta t}{2m}\right)^2\mathbf{C}\cdot\mathbf{C}}\left[\mathbf{d} + \frac{q\Delta t}{2m}\mathbf{d}\times\mathbf{C} + \left(\frac{q\Delta t}{2m}\right)^2\mathbf{C}\left(\mathbf{d}\cdot\mathbf{C}\right)\right].
\end{aligned}$$

Check the energy conservation.