A Fully Quantum, Modular, and Reprogrammable Arithmetic Logic Unit Using QFT and Quantum Multiplexed Unitaries

Jaikaran Singh Pahwa

BCA Student (Age: 18)
New Delhi, India
jaikaransinghpahwa7513@gmail.com

***Abstract*—We present a novel design for a fully quantum Arithmetic Logic Unit (QALU) constructed entirely from unitary quantum gates. This architecture supports modular, programmable arithmetic and logic operations including quantum Fourier transform (QFT)-based addition and subtraction, XOR via CNOT, and AND via Toffoli gates. Instruction control is implemented using Quantum Multiplexed Unitaries with identity-padded subunitaries to ensure gate-width uniformity. This architecture preserves coherence, supports superposition and entanglement, and scales in qubit space without classical logic. No measurement or classical feedback is used, and operations remain reversible. The design assumes error-corrected hardware and serves as a foundation for scalable quantum processor units (QPUs).**

I. Introduction

Quantum computation is traditionally focused on algorithm-specific circuits. However, as we move toward general-purpose quantum processing, a fundamental unit analogous to the classical Arithmetic Logic Unit (ALU) becomes necessary. This work introduces a fully quantum QALU that handles core logic and arithmetic operations without relying on any classical gates or measurements. Designed at age 18 by a single researcher, this architecture was ideated as a spontaneous insight—proof that curiosity and independent effort can push the boundaries of computation.

II. QFT-Based Arithmetic Operations

The quantum Fourier transform (QFT) maps a computational basis state $|x\rangle$ into a superposition:

$$\mathrm{QFT}\,|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{2\pi i x k/2^n} |k\rangle \tag{1}$$

To add a constant a to $|x\rangle$:

1) Apply QFT to $|x\rangle$
2) Apply a diagonal phase shift gate R_a
3) Apply Inverse QFT

Phase Gate Construction: The diagonal gate R_a applies phase rotations proportional to a:

$$R_a = \mathrm{diag}(1, e^{2\pi i a/N}, \ldots, e^{2\pi i a(N-1)/N}) \tag{2}$$

Thus:

$$|\psi\rangle = \mathrm{IQFT} \cdot R_a \cdot \mathrm{QFT}\,|x\rangle \tag{3}$$

$$= |x + a\rangle \pmod{2^n} \tag{4}$$

Subtraction is implemented by applying the conjugate transpose:

$$R_{-a} = R_a^\dagger = \mathrm{diag}(1, e^{-2\pi i a/N}, \ldots, e^{-2\pi i a(N-1)/N}) \tag{5}$$

III. Logical Gates via Quantum Operations

A. XOR via CNOT

Let $|c\rangle\,|t\rangle$ be two qubits. Applying CNOT yields:

$$\mathrm{CNOT}\,|c\rangle\,|t\rangle = |c\rangle\,|t \oplus c\rangle \tag{6}$$

B. AND via Toffoli

Toffoli gate acts on three qubits $|c_1\rangle, |c_2\rangle, |t\rangle$:

$$\mathrm{Toffoli}(|c_1\rangle\,|c_2\rangle\,|t\rangle) = |c_1\rangle\,|c_2\rangle\,|t \oplus (c_1 \wedge c_2)\rangle \tag{7}$$

IV. Identity Padding for Uniform Multiplexed Gates

To preserve unitarity and apply multiplexed operations across instructions of different width, each unitary U_i is padded as:

$$U_i \mapsto U_i \otimes I^{\otimes(q_{\max} - q_i)} \tag{8}$$

where $q_{\max}$ is the maximum qubit width of all instructions, and q_i is the width of U_i. This ensures every gate in the multiplexing structure has the same dimensionality, a necessity for defining a global unitary.

V. Quantum Multiplexed Unitary Control

The instruction control is encoded using a quantum multiplexer:

$$M = \sum_i |i\rangle\langle i| \otimes U_i \tag{9}$$

Where $|i\rangle$ is the control register and U_i is the quantum operation (QFT-Add, XOR, etc). The QALU thus becomes a reprogrammable core determined by quantum state combinations.

Instruction Table

Index	Operation
00	QFT Adder
01	XOR
10	AND
11	QFT Subtractor

VI. Conclusion

We present a fully quantum arithmetic logic unit constructed entirely from unitary operations, capable of general-purpose computation and coherent logic on superposed or entangled states. Our identity-padded multiplexed structure ensures scalability and uniformity, with no classical logic or measurement required. This architecture is a step toward modular, scalable quantum CPUs.

Acknowledgments

This project was conceptualized and completed independently by the author, without institutional collaboration or team support. What began as a simple shower thought evolved into a fully realized quantum architecture.

References

[1] T. G. Draper, ”Addition on a quantum computer”, arXiv:quant-ph/0008033, 2000.
[2] S. A. Cuccaro, et al., ”A new quantum ripple-carry addition circuit”, arXiv:quant-ph/0410184, 2004.
[3] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*. Cambridge University Press, 2010.
[4] P. Shor, ”Algorithms for quantum computation: Discrete logarithms and factoring”, in Proc. of the 35th Annual FOCS, 1994.