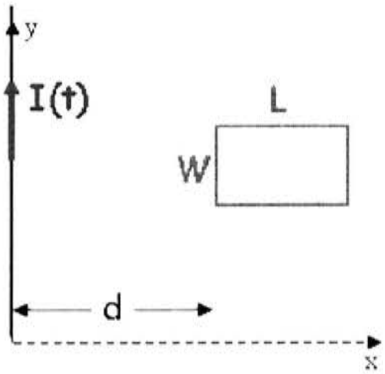
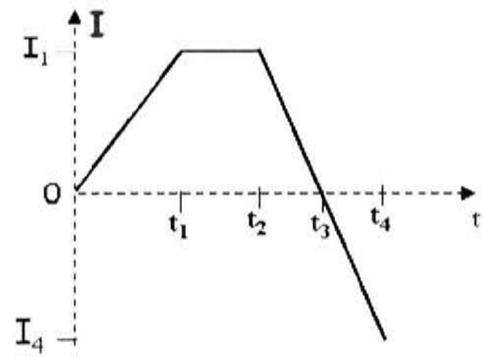
The next six questions pertain to the situation described below.

An infinite straight wire carries a current I that varies with time as shown above. It increases from 0 at t = 0 to a maximum value I_1 = 4.4 A at t = t_1 = 18 s, remains constant at this value until t = t_2 when it decreases linearly to a value I_4 = -4.4 A at t = t_4 = 30 s, passing through zero at t = t_3 = 26.5 s. A conducting loop with sides W = 21 cm and L = 61 cm is fixed in the x-y plane at a distance d = 40 cm from the wire as shown.





$- I = 4.4\ A$

1)What is the magnitude of the magnetic flux Φ through the loop at time t = t_1 = 18 s?

T-m^2

✓ 1.71169348392009 × 10^{-7} T-m^2

B through loop is $B(r) = \frac{\mu_0 I}{2\pi r}$

$\Phi = \int B \cdot da = \int_d^{d+L} B(r) \cdot W\,dr$

$\Phi = \frac{\mu_0 I}{2\pi} W \int_d^{d+L} \frac{1}{r} dr = \frac{(4\pi \times 10^{-7})(4.4A)}{2\pi}(.21) \ln\left(\frac{.61+.4}{.4}\right)$

$\Phi = 1.71 \times 10^{-7}\ T\cdot m^2$

2)What is ε_1, the induced emf in the loop at time t = 9 s? Define the emf to be positive if the induced current in the loop is clockwise and negative if the current is counter-clockwise.

V

✓ -9.5094082440005 × 10^{-9} V

$\varepsilon = -\frac{\partial \Phi}{\partial t} = -\frac{\Delta \Phi}{\Delta t}$

emf is constant for I increasing with constant rate.

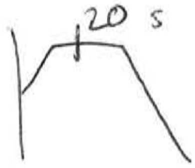
$\varepsilon = -\frac{1.71 \times 10^{-7}\ T\cdot m^2}{18\ s} = -9.51 \times 10^{-9}\ V$ constant from 0-18 seconds

flux is increasing into the page, so current goes counter-clockwise to oppose change in flux (negative)

3)What is ε_2, the induced emf in the loop at time t = 20 s? Define the emf to be positive if the induced current in the loop is clockwise and negative if the current is counter-clockwise.

V

✓0 V

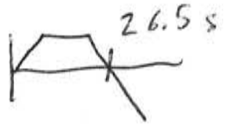


current and flux are constant, so induced emf is 0

since $\mathcal{E} = -\frac{\partial \Phi}{\partial t} = 0$

4)What is the direction of the induced current in the loop at time t = t_3 = 26.5 seconds?

1. There is no induced current at t = t_3
2. ✓Clockwise
3. Counterclockwise



flux is decreasing through the loop

so current flows clockwise to oppose change in flux

5)What is ε_4, the induced emf in the loop at time t = 28.25 s? Define the emf to be positive if the induced current in the loop is clockwise and negative if the current is counter-clockwise.

V

✓4.89055281120027 × 10^{-8} V

$\mathcal{E} = -\frac{\Delta \Phi}{\partial t}$

change from $t_3 \to t_4$

$\Delta \Phi$ is same as part 1) 0 A → −4.4 A

$\Delta \Phi = -1.71 \times 10^{-7}$ A

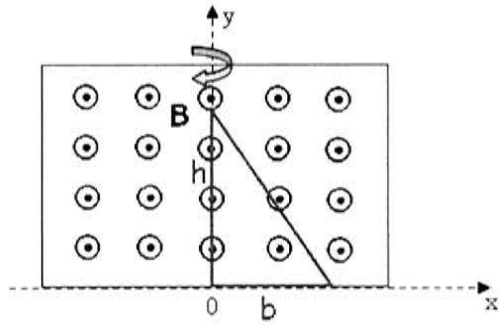
$\Delta t = t_4 - t_3 = 3.5$ s

$\mathcal{E} = -\frac{\Delta \Phi}{\Delta t} = \frac{1.71 \times 10^{-7}}{3.5}$

$\mathcal{E} = 4.89 \times 10^{-8}$ V

clockwise, same as 4)

The next seven questions pertain to the situation described below.



A conducting wire formed in the shape of a right triangle with base b = 30 cm and height h = 52 cm and having resistance R = 1.7 Ω, rotates uniformly around the y-axis in the direction indicated by the arrow (clockwise as viewed from above (loooking down in the negative y-direction)). The triangle makes one complete rotaion in time t = T = 1.6 seconds. A constant magnetic field B = 1.2 T pointing in the positive z-direction (out of the screen) exists in the region where the wire is rotating.

7)What is ω, the angular frequency of rotation?

radians/second

✓3.9269875 radians/second

$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.6\ s}$ period

$\omega = 3.927$ rad/s

8)What is I_{max}, the magnitude of the maximum induced current in the loop?

A

✓0.216215311764706 A

$I = V/R$ so we need to find the emf induced in the loop.

$\mathcal{E} = -\frac{d\Phi}{dt} = \omega BA \sin \omega t$

Area $= \frac{1}{2}bh$

$= \frac{1}{2}(.3)(.52)$

$= .078$

So maximum $\mathcal{E} = \omega BA$

and $I_{max} = \frac{\omega BA}{R}$

$I_{max} = \frac{(3.93)(1.2)(.078)}{1.7}$

$I_{max} = .216$ A

9)At time $t = 0$, the wire is positioned as shown. What is the magnitude of the magnetic flux Φ_1 at time $t = t_1 = 0.6$ s?

T-m^2

✓0.0661851947190608 T-m^2

$t=0$ view from above

$\theta = \omega t = 0$

maximum flux

$t = .6$ s $\quad \theta = \omega(.6\text{ s}) = 2.36$ radians $\approx 135°$

loop top view

$|\Phi| = |B \cdot A \cdot \cos 135°|$

$= (1.2)(.078)\cos 135°$

$= .0662\ T \cdot m^2$

10)What is I_1, the induced current in the loop at time $t = 0.6$ s? I_1 is defined to be positive if it flows in the negative y-direction in the segment of length h.

A

✓0.152887313145187 A

$\mathcal{E} = \omega B A \sin(\omega t)$

$I = \frac{\mathcal{E}}{R} = \frac{(3.93)(1.2)(.078)\sin(135°)}{1.7}$

front view

$I = .153$ A

flux is increasing through loop out of page, so current flows clockwise as shown

11)Which of the folowing statements about Φ_o, the magnitude of the flux through the loop at time $t = t_o = 0.4$ s, and I_o, the magnitude of the current through the loop at time $t = t_o = 0.4$ s, is true? Φ_{max} and I_{max} are defined to be the maximum values these quantites achieve during the complete rotation.

1. $\Phi_o = 0$ and $I_o = 0$
2. $\Phi_o = \Phi_{max}$ and $I_o = 0$
3. $\Phi_o = \Phi_{max}$ and $I_o = I_{max}$
4. ✓ $\Phi_o = 0$ and $I_o = I_{max}$

$t = .4$s top view

loop at 90°

flux is 0 (minimum)

$\frac{\partial \Phi}{\partial t}$ maximum, so emf and current maximum

12)Suppose the frequency of rotation is now doubled. How do Φ_{max}, the maximum value of the flux through the loop, and I_{max}, the maximum value of the induced current in the loop change?

1. Φ_{max} doubles and I_{max} remains the same
2. Both Φ_{max} and I_{max} remain the same
3. Φ_{max} and I_{max} both double
4. ✓ Φ_{max} remains the same and I_{max} doubles

$\Phi_{max} = BA$ neither B or A changes, so same

~~$\Phi = \omega BA$~~ $I_{max} = \frac{\omega BA}{R}$ so if ω doubles then I doubles