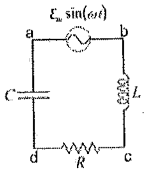
The next six questions pertain to the situation described below.



A circuit is constructed with an AC generator, a resistor, capacitor and inductor as shown. The generator voltage varies in time as $\varepsilon = V_a - V_b = \varepsilon_m \sin\omega t$, where $\varepsilon_m = 120$ V and $\omega = 215$ radians/second. The inductance L = 250 mH. The values for the capacitance C and the resistance R are unkown. What is known is that the current in the circuit leads the voltage across the generator by $\varphi = 38$ degrees and the average power delivered to the circuit by the generator is $P_{avg} = 132$ W.

Known	unknown
$\mathcal{E}_m$	C
ω	R
L	
$\phi = -38°$	
P_{avg}	

Useful equations

$$Z = \sqrt{R^2 + \left(\omega L + \frac{1}{\omega C}\right)^2}$$

$$I_m = \mathcal{E}_m / Z$$

$$I(t) = I_m \sin(\omega t - \phi)$$

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}$$

$$P_{avg} = \frac{1}{2}(I_m)^2 R$$

1) What is I_{max}, the amplitude of the current oscillations in the circuit?

✓2.79183885123368 A

Note that $Z = \sqrt{R^2 + (R\tan\phi)^2}$ which can be further rearranged to give $Z = \frac{R}{\cos\phi}$

So $I_{max} = \frac{\mathcal{E}_m \cos\phi}{R}$ solve for R then substitute here to get $P_{avg} = \frac{1}{2} I_m^2 R$

$\rightarrow P_{avg} = \frac{1}{2}\mathcal{E}_m I_m \cos\phi$ (actually just from here)

$$I_m = \frac{2P_{avg}}{\mathcal{E}_m \cos\phi} = \frac{2(132)}{(120)\cos(38°)}$$

$$I_m = 2.79 \text{ A}$$

2)What is R, the value of the resistance of the circuit?

✓33.8706268015442 Ω

Now use $P_{avg} = \frac{1}{2} I_m^2 R$

$$R = \frac{2P_{avg}}{I_m^2} = \frac{2(132)}{(2.79)^2} = 33.9\ \Omega$$

3)What is C, the value of the capacitance of the circuit?

✓57.9854361775539 μF

Use $\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$ (or $I_m = \mathcal{E}_m / Z$)

$\rightarrow R\tan\phi = \omega L - \frac{1}{\omega C}$

$\frac{1}{\omega C} = \omega L - R\tan\phi$

$\frac{1}{C} = \omega^2 L - \omega R\tan\phi = (215)^2(.250) - (215)(33.9)\tan(38°) = 17{,}246$

$C = \frac{1}{17{,}246} = 58.0 \times 10^{-6}$ F

4)The value of ω is now changed, keeping all other circuit parameters constant, until resonance is reached. How was ω changed?

1. ✓ ω was increased
2. ω was decreased

since current leads voltage, ϕ is negative and

$X_C > X_L$ ~~the~~ to reach resonance ($X_L = X_C$)

$\frac{1}{\omega C} > \omega L$ ω must increase.

5)What is the average power delivered to the circuit when it is in resonance?

✓212.57356830703 W

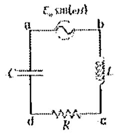
at resonance $Z = R$ (R does not change)

$I_m = \frac{\mathcal{E}_m}{R}$ so $P_{avg} = \frac{1}{2} I_m^2 R = \frac{1}{2}\frac{\mathcal{E}_m^2}{R}$

$$P_{avg} = \frac{1}{2}\frac{\mathcal{E}_m^2}{R} = \frac{1}{2}\frac{(120)^2}{(33.9)}$$

$$\underline{P_{avg} = 212.6 \text{ W}}$$

The next seven questions pertain to the situation described below.



A circuit is constructed with an AC generator, a resistor, capacitor and inductor as shown. The generator voltage varies in time as $\varepsilon = V_a - V_b = \varepsilon_m \sin\omega t$, where $\varepsilon_m = 24$ V and $\omega = 150$ radians/second. At this frequency,the circuit is in resonance with the maximum value of the current $I_{max} = 0.63$ A. The capacitance C = 182μF. The values for the resistance R and the inductance L are unknown.

Known	unknown
ε_m	R
ω	L
I_m	
C	

$\phi = 0$ (resonance)

useful equations (at resonance)

$$I_m = \frac{\varepsilon_m}{R}$$

$$X_L = X_C \rightarrow \frac{1}{\omega C} = \omega L$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$P_{avg} = \frac{1}{2} I_m^2 R$$

$$P_{avg} = \frac{1}{2} \varepsilon_m I_m$$

$$U_{max} = \frac{1}{2} L I_m^2 = \frac{1}{2} \frac{Q_{max}^2}{C}$$

$$\Delta U = P_{avg} \cdot T$$

T = 1 period (in seconds)

$$T = \frac{2\pi}{\omega}$$

$$Q = \frac{U_{max}}{\Delta U}(2\pi)$$

7)What is L, the value of the inductance of the circuit?

✓244.200244200244 mH

from $X_L = X_C$

$$L = \frac{1}{\omega^2 C} = \frac{1}{(150^2)(182\times10^{-6})}$$

$$L = .244 \text{ H} = 244 \text{ mH}$$

8)What is $U_{max,C}$, the value of the maximum energy stored in the capacitor during one cycle?

✓0.0484615384615384 J

Energy oscillates between capacitor and inductor, max energies in each are equal

$$U_{max} = \frac{1}{2} L I_m^2 = \frac{1}{2}(244)(.63)^2$$

$$U_{max} = .0485 \text{ J}$$

9)What is ΔU, the total energy dissipated in the circuit in one cycle?

✓0.316672272 J

$$P_{avg} = \frac{1}{2} \varepsilon_m I_m \qquad T = \frac{2\pi}{\omega}$$

$$\Delta U = P_{avg}\left(\frac{2\pi}{\omega}\right) = \frac{\pi \varepsilon_m I_m}{\omega}$$

$$\Delta U = \frac{\pi(24)(.63)}{(150)} = .317 \text{ J}$$

10)What is Q, the quality factor of this ciruit?
✓0.96153846153846

$$Q = \frac{U_{max}}{\Delta U} (\times 2\pi)$$

$$Q = 2\pi \frac{(.0485)}{(.317)} = \underline{.962}$$

11)What is R, the value of the resistance of the circuit?
✓38.0952380952381 Ω

$$R = \frac{\mathcal{E}_m}{I_m} = \frac{24}{.63} = \underline{38.1\,\Omega}$$

12)Suppose now the value of the capacitance in the circuit is doubled (C' = 2C) and the inductance is changed appropriately to keep the circuit in resonance at angular frequency ω = 150 radians/s while the generator voltage and resistance are kept constant. How does Q, the quality factor of the circuit, change, if at all?

1. Q increases
2. Q stays the same
3. ✓ Q decreases

At resonance, $X_L = X_C$ $\omega L = \frac{1}{\omega C}$

So if C increases, L must decrease to maintain resonance.

Since R does not change, I_{max} will not change, so P_{avg} also will not change.

So $\underline{U_{max}} = \frac{1}{2} L I_{max}^2$ will $\underline{\text{decrease}}$.

$\underline{\Delta U = P_{avg} \cdot \frac{2\pi}{\omega}}$ will $\underline{\text{not change}}$.

thus $\underline{Q = 2\pi \frac{U_{max}}{\Delta U} \text{ must decrease}}$ as result.

AC circuit 2

AC CIRCUIT 2 BASE

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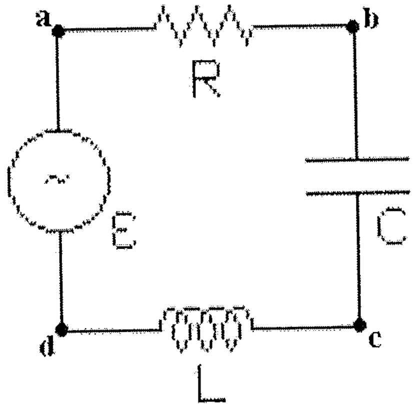
AC Circuit 2



A series RLC circuit (L = 340 ×10^{-3} H, C = 25 ×10^{-6} F, and R = 280 ohms) has an AC generator with amplitude ε_{max} = 120 V and unknown frequency f.

At time t = 0, the following voltages are measured:

$V_{ad} = V_a - V_d$ = + 120 V,

$V_{bc} = V_b - V_c$ = -10.1 V, and

$V_{cd} = V_c - V_d$ = + 60.6 V.

What is t_{imax}, the first time after t = 0 that the current in the circuit attains its maximum value? t_{imax} =

seconds Submit Graph

$\sin(-x) = -\sin x$

Help

Right Answer: 0.0008417383 seconds

Since $\Delta V_{ad} = 120V$ at $t=0$

generator $\mathcal{E} = \mathcal{E}_m \cos(\omega t)$

and

$I(t) = I_m \cos(\omega t - \phi)$

thus we need to find ω and ϕ

We know also $V_c = +X_c I_m \sin(\omega t - \phi)$ (lags current 90°)

$V_L = X_L I_m \sin(\omega t - \phi)$ (leads current 90°)

Note that $\frac{|V_c|}{|V_L|} = \frac{X_c}{X_L}$ is constant

so $\frac{X_c}{X_L} = \frac{1}{\omega^2 LC} = \frac{|\Delta V_{bc}|}{|\Delta V_{cd}|}$

so $\omega = \sqrt{\frac{V_{cd}}{V_{bc}(LC)}} = 840$ rad/s

remember to use both in radians (or degrees)

now can calculate $\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} \rightarrow \phi = 40.3°$

$t_{Imax} = \frac{\phi}{\omega} = .000839$ s