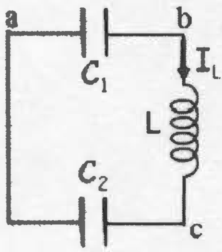
The next six questions pertain to the situation described below.



put your calculator in radians!

A circuit is constructed with two capacitors and an inductor as shown. The values for the capacitors are: $C_1 = 350\ \mu F$ and $C_2 = 77\ \mu F$. The inductance is $L = 275$ mH. At time t =0, the current through the inductor has its maximum value $I_L(0) = 200$ mA and it has the direction shown.

1)What is ω_o, the resonant frequency of this circuit?

✓240.031481603074 radians/s

capacitors in series for this circuit

$$C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = 63.11\ \mu F$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{63 \times 10^{-6} \cdot .275}}$$

$$\underline{\omega_o = 240.\ \text{rad/s}}$$

2)What is $Q_1(t_1)$, the charge on the capacitor C_1 at time $t = t_1 = 15.3$ ms? The sign of Q_1 is defined to be the sameas the sign of the potential difference $V_{ab} = V_a - V_b$ at time $t = t_1$.

✓-4.21861487770325 × 10^{-4} C

I, V_L, V_c, Q_c all oscillate at freq. ω_o, initial conditions determine whether each has $\sin(\omega t)$ or $\cos(\omega t)$. $I = I_{max}\cos(\omega t)$, $Q_c = Q_{max}\sin(\omega t)$

because ⊕ charge is initially flowing clockwise.

★ note $Q_1 = Q_2 = Q$ series capacitors

Note that $I = \frac{dQ}{dt}$ $\quad I(t) = \omega\, Q_{max} \cos(\omega)$

so $Q_{max} = \frac{I_{max}}{\omega_o} = 8.33 \times 10^{-4}$ C

$$Q(t_1) = Q_{max} \sin(\omega_o t_1) = (8.33 \times 10^{-4}) \sin(240. \times .0153)$$

$$\underline{Q_1(t_1) = -4.22 \times 10^{-4}\ C}$$

3)What is $V_{bc}(t_1) = V_b - V_c$, the voltage across the inductor at time $t_1 = 15.3$ ms? Note that this voltage is a signed number.

✓6.68403915688049 V

$$I(t) = 200\ \text{mA} \cos(\omega t)$$

$$|V_{bc}| = |\mathcal{E}| = L\frac{dI}{dt} = L\omega I_{max} \sin(\omega t)$$

$$V_{bc} = -L\omega_o I_{max} \sin(\omega t)$$ initial conditions show (−)

$$V_{bc}(t_1) = -(240.)(.2)(.275) \sin(240. \cdot .0153)$$

$$\underline{= +6.68\ V}$$

4)What is $Q_{1,max}$, the magnitude of the maximum charge on capacitor C_1?

✓ $8.33224036548374 \times 10^{-4}$ C

found in 2) $Q_{max} = \frac{I_{max}}{\omega_0} = \underline{8.33 \times 10^{-4} \text{ C}}$

from $I = \frac{dQ}{dt} = \omega Q_{max} \cos(\omega t)$

5)At time $t = t_2$, the magnitude of the current through the inductor has its maximum value. What are the magnitudes of Q_1, the charge on capacitor C_1, and V_L, the voltage across the inductor at this time?

1. $Q_1 = 0$ and $V_L = V_{max}$
2. $Q_1 = Q_{max}$ and $V_L = V_{max}$
3. ✓ $Q_1 = 0$ and $V_L = 0$
4. $Q_1 = Q_{max}$ and $V_L = 0$

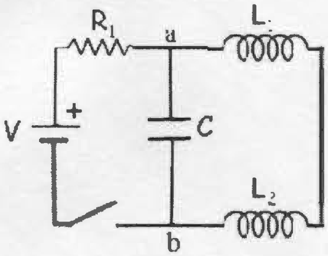
I is maximum so $V_L = 0$

when $V_L = 0$ $V_{c_1} = V_{c_2} = 0$ by Kirchoff's Voltage loop equation

when $V_c = 0$ $Q_c = 0$ since $V = \frac{Q}{C}$

so $Q_1 = Q_2 = 0$

The next seven questions pertain to the situation described below.



A circuit is constructed with a resistor, two inductors, one capacitor, one battery and a switch as shown. The value of the resistance is $R_l = 300\ \Omega$. The values for the inductances are: $L_l = 210$ mH and $L_2 = 182$ mH. The capacitance is $C = 98\ \mu$F and the battery voltage is $V = 12$ V. The positive terminal of the battery is indicated with a + sign.

7)The switch has been closed for a long time when at time $t = 0$, the switch is opened. What is $U_{Ll}(0)$, the magnitude of the energy stored in inductor L_l just after the switch is opened?

✓ 1.68×10^{-4} J

before $t=0$ C is uncharged, current flows in outer loop. At $t=0$ current remains same through inductors.

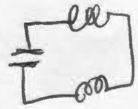
$I = \frac{V}{R_1} = .04$ A

$U_l = \frac{1}{2} L_l I^2 = \frac{1}{2}(.21\text{ H})(.04\text{ A})^2 = 1.68 \times 10^{-4}$ J

8)What is ω_o, the resonant frequency of the circuit just after the switch is opened?

✓ 161.340696947366 radians/s

LC circuit



L in series add $L = L_1 + L_2 = 392$ mH

$\omega_o = \frac{1}{\sqrt{LC}} = 161.$ rad/s

9)What is Q_{max}, the magnitude of the maximum charge on the capacitor after the switch is opened?

✓ 247.922568557201 μC

since $I = \frac{\partial Q}{\partial t}$ and $Q = -Q_{max} \sin(\omega t)$ (set by initial conditions)
$I = I_{max} \cos(\omega t)$

we see that $I_{max} = \omega \cdot Q_{max}$

so $Q_{max} = \frac{I_{max}}{\omega_o} = \frac{(.04\text{ A})}{(161\text{ rad/s})} = 2.48 \times 10^{-4}$ C

$= 248\ \mu$C

10)What is $Q(t_1)$, the charge on the capacitor at time $t = t_1 = 3.22$ ms. $Q(t_1)$ is defined to be positive if $V(a) - V(b)$ is positive.

✓-123.083876783424 µC

$$Q(t) = -Q_{max}\sin(\omega t)$$

$$= -248\,\mu C \sin(161 \cdot .00322)$$

$$= -123.\ \mu C$$

Current is flowing clockwise at $t=0$, so V_a is decreasing while V_b is increasing (and $V_a - V_b = 0$ at $t=0$), which determines $-\sin(\omega t)$ factor

11)What is t_2, the first time after the switch is opened that the energy stored in the capacitor is a maximum?

✓9.73588827692023 ms

V_c, Q_c, and energy U_c all maximum after $1/4$ period

so when $\omega_0 t = \frac{\pi}{2}$

$$t = \frac{\pi}{2\omega_0} = .00974\ s$$

$$= 9.74\ ms$$

12)What is the total energy stored in the inductors plus the capacitor at time $t = t_2$?

✓3.136×10^{-4} J

at t_2 current $= 0$ so no energy stored in inductors.

$$U_{total} = U_c(t_2) = \frac{1}{2}\frac{Q_{max}^2}{C} = 3.136 \times 10^{-4}\ J$$

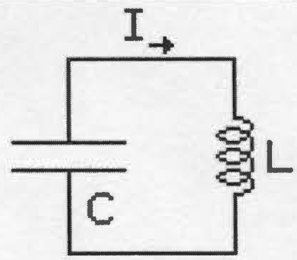
Since there is no resistor connected to the circuit at this point, energy is conserved and is constant, so $U = U_c + U_L$ at all times. So U also $= \frac{1}{2}L(I_{max})^2$

LC Circuit

LC CIRCUIT BASE

Welcome to this IE. You may navigate to any page you've seen already using the IE tab on the right.

LC Circuit



The LC circuit shown above has a capacitance C = 0.05 μF and inductance L = 420 mH. Suppose that at time t = 0, the stored electric and magnetic energies are equal to one another and the instantaneous current is 75 mA. What is Q_{max}, the maximum charge that is stored on the capacitor in this situation? Q_{max} =

[ ] C Submit Graph

Help

Right Answer: 1.53704261489395E-05 C ✓

magnetic: $U_L = \frac{1}{2} L I^2$

electric: $U_C = \frac{1}{2}\frac{Q^2}{C}$

> given that $U_L = U_C$ at $t=0$ and $I = 75\text{ mA}$ at $t=0$

total energy is conserved since no resistor is present, so

$U_T = U_L + U_C$ = constant

at $t=0$ $U_T = 2 \times U_L = L I(0)^2 = .00236\text{ J}$

when $Q_C = Q_{max}$, $I = 0$ so $U_L = 0$ and $U_C = U_T = .00236\text{J}$

$U_C = \frac{1}{2}\frac{Q_{max}^2}{C}$ when $Q = Q_{max}$

so $\frac{1}{2}\frac{Q_{max}^2}{C} = U_T$

$Q_{max} = \sqrt{2 C U_T} = \sqrt{2 C L I(0)^2}$

$= 1.54 \times 10^{-5}\text{ C}$

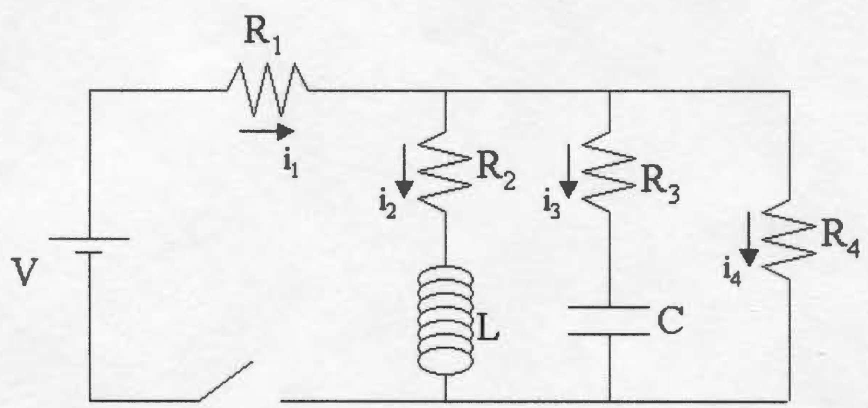
LCR Energy

LCR ENERGY BASE

Welcome to this IE. You may navigate to any page you've seen already using the IE tab on the right.

Stored Energy in LCR Circuit

Four resistors (R_1 = 60 Ohms, R_2 = 220 Ohms, R_3 = 330 Ohms, and R_4 = 480 Ohms), an ideal inductor (L = 8 mH), and a capacitor (C = 250 microF) are connected to a battery (V = 9 V) through a switch as shown in the figure below.



The switch has been open for a long time before it is closed at t = 0. What is U_{stored}, the total stored energy in the circuit elements (not including the battery) a long time after the switch is closed? U_{stored} =

.00519 J Submit Graph

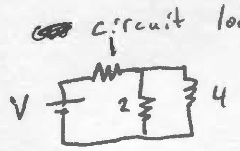
Help

Right Answer: 0.005186056 J ✓

Only L and C store energy. $U_L = \frac{1}{2}LI^2$ (magnetic field) $U_C = \frac{1}{2}CV^2$ (electric field)

At t = ∞ capacitor is fully charged and prevents current through it.
inductor allows current to flow like a wire and has no Voltage $V_L = 0$

so circuit looks like:



we need to find: $I_L = I_2$
$V_C = V_{R2} = V_{R4}$ because it is in a parallel branch.

solving this circuit gives $I_2 = .0293\ A$
$V_{R2} = V_{R4} = 6.44\ V$

so $U = \frac{1}{2}LI_L^2 + \frac{1}{2}CV_C^2$
$= \frac{1}{2}\left[(.008)(.0293)^2 + (250 \times 10^{-6})(6.44)^2\right]$

$\boxed{U = .00519\ J}$