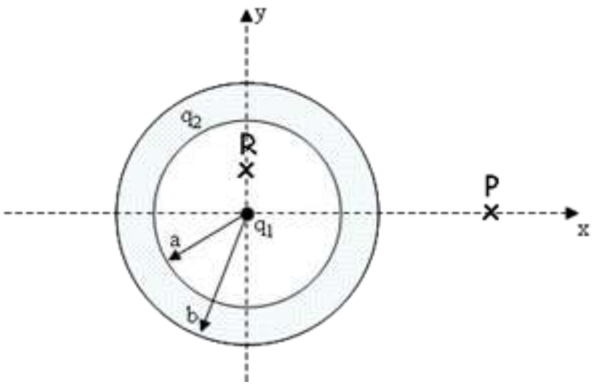
Physics 2220 Homework 4 Solutions

Point Charge and Charged Sphere



A point charge q_1 = -8.7 μC is located at the center of a thick conducting shell of inner radius a = 2.8 cm and outer radius b = 4.6 cm, The conducting shell has a net charge of q_2 = 1.1 μC.

1)

What is $E_x(P)$, the value of the x-component of the electric field at point P, located a distance 7.1 cm along the x-axis from q_1?

-1.36E7 N/C

Since we're dealing with point charges and shells, radial symmetry applies here. This means we also know the E field will only be in the radial direction, which at point P means the X direction. Using Gauss' law we derive:

$$\int \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\boldsymbol{E} = \frac{Q_{enclosed}}{\boldsymbol{4\pi r^2 \varepsilon_0}}$$

At point P both the conducting shell and the inner point charge are enclosed and we add both together to get $Q_{enclosed}$. Then using our derived formula we find **E**.

2)

What is $E_y(P)$, the value of the y-component of the electric field at point P, located a distance 7.1 cm along the x-axis from q_1?

0 N/C

Since Y is perpendicular to the radial axis here, none of the E field will be in the Y direction

3)

What is $E_x(R)$, the value of the x-component of the electric field at point R, located a distance 1.4 cm along the y-axis from q_1?

0 N/C

Here the X axis is perpendicular to the radial direction and so there is no E field component in that direction.

4)

What is $E_y(R)$, the value of the y-component of the electric field at point R, located a distance 1.4 cm along the y-axis from q_1?

-3.99E8 N/C

Exactly the same as in problem 1 using:

$$\boldsymbol{E} = \frac{Q_{enclosed}}{\boldsymbol{4\pi r^2 \varepsilon_0}}$$

The only differences are that the radius is shorter than in (1) and that only the point charge is enclosed now.

5)

What is σ_b, the surface charge density at the outer edge of the shell?

-2.86E-4 C/m^2

The total charge on the surface has to represent all the charges enclosed, so we take

$$\sigma_b = \frac{Q_{enclosed}}{Area} = \frac{Q_{point} + Q_{shell}}{4\pi r^2}$$

6)

What is σ_a, the surface charge density at the inner edge of the shell?

8.83E-4 C/m^2

We know for a conductor the interior has an E field of zero. Since we saw that the E field is proportional to the charge enclosed, to get E=0 we need $Q_{enclosed}=0$. That must mean that $Q_{point}+Q_{innner\ shell}=0$ or in other words the charge on the inner shell must be exactly equal and opposite to the point charge. Then just divide by the area of a sphere to get charge density.

7)

For how many values of x: (4.6 cm < x < infinity) is it true that $E_x = 0$?

(•) none

While E will get really small as we approach infinity, outside of the conducting shell the field never truly goes to zero.

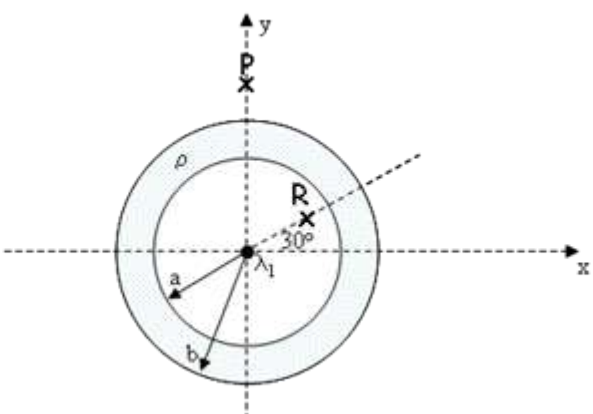
8)

Define E_2 to be equal to the magnitude of the electric field at r = 1.4 cm when the charge on the outer shell (q_2) is equal to 1.1 μC. Define E_o to be equal to the magnitude of the electric field at r = 1.4 cm if the charge on the outer shell (q_2) were changed to 0. Compare E_2 and E_o.

(•) $E_2 = E_o$

Since the E field only depends on the charge *enclosed* then changing the amount of charge past our Gaussian sphere has no effect.

Line Charge and Charged Cylindrical Shell



An infinite line of charge with linear density λ_1 = 6.8 μC/m is positioned along the axis of a thick insulating shell of inner radius a = 2 cm and outer radius b = 4.8 cm. The insulating shell is uniformly charged with a volume density of ρ = -688 μC/m^3.

1)

What is λ_2, the linear charge density of the insulating shell?

-4.11 μC/m

Just think of this as taking a small slice of the cylinder and finding the charge in that slice. We want to go from volume density to linear density so we need to multiply by cross-sectional area.

$$Area = \pi b^2 - \pi a^2$$
$$\lambda_2 = \rho A = \rho\pi(b^2 - a^2)$$

2)

What is $E_x(P)$, the value of the x-component of the electric field at point P, located a distance 6.4 cm along the y-axis from the line of charge?

0 N/C

With a cylindrical symmetry, once again the E fields will only point along the radial direction. At point P the X axis is perpendicular to the radial so there's no X component of the field.

3)

What is $E_y(P)$, the value of the y-component of the electric field at point P, located a distance 6.4 cm along the y-axis from the line of charge?

7.57E5 N/C

Apply Gauss's law again for the cylindrical symmetry:

$$\int \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\boldsymbol{E} = \frac{\lambda_{enclosed}}{2\pi r \varepsilon_0}$$

Since P is outside the insulating shell we need to add the total linear charge densities of both the line of charge and the shell to get our $\lambda_{enclosed}$. Then just plug into our formula for E.

4)

What is $E_x(R)$, the value of the x-component of the electric field at point R, located a distance 1 cm along a line that makes an angle of 30° with the x-axis?

1.06E7 N/C

First we need to find the E field from:

$$\boldsymbol{E} = \frac{\lambda_{enclosed}}{2\pi r \varepsilon_0}$$

Keeping in mind that we only worry about the charge enclosed, i.e. the line of charge. Then use trigonometry to see that:

$$\boldsymbol{E}_x = \boldsymbol{E} Cos\theta$$

5)

What is $E_y(R)$, the value of the y-component of the electric field at point R, located a distance 1 cm along a line that makes an angle of 30° with the x-axis?

6.12E6 N/C

The same principle applies:

$$\boldsymbol{E}_y = \boldsymbol{E}Sin\theta$$

6)

For how many values of r: (2 cm < r < 4.8 cm) is the magnitude of the electric field equal to 0?

none

The field will only go to zero if the charge enclosed is zero but since the linear charge density of the line of charge is of great magnitude than that of the entire shell, the charge enclosed will never cancel out and the field will never become zero.

7)

If we were to double λ_1 (λ_1 = 13.6 μC/m), how would E, the magnitude of the electric field at point P, change?

E would increase by more than a factor of two

Quoting from smart physics:

Feedback:

Your answer is correct! Doubling λ_1 will double the contribution to the field from the line of charge ($2E_{line}$), but the contribution from the shell does not change (E_{shell}). Therefore, the new field will be equal to $2E_{line} - E_{shell}$, which is *greater than* twice the original field ($2(E_{line} - E_{shell})$).

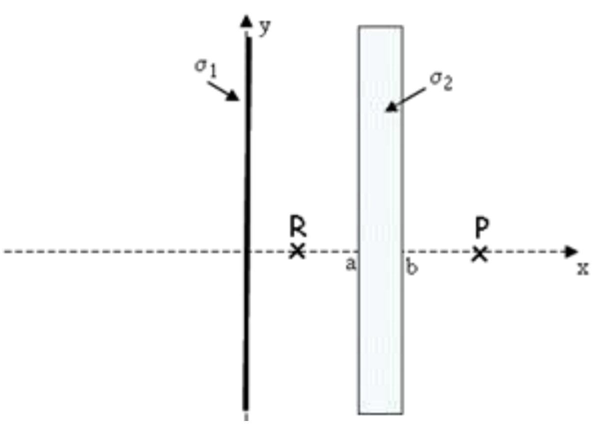
8)

In order to produce an electric field of zero at some point r > 4.8 cm, how would λ_1 have to change?

Keep its sign the same and decrease its magnitude

We would need it to be small enough that $\lambda_1 + \lambda_2 = 0$

Infinite Charged Sheet and Infinite Conducting Slab



An infinite sheet of charge, oriented perpendicular to the x-axis, passes through x = 0. It has a surface charge density σ_1 = -4.2 μC/m^2. A thick, infinite conducting slab, also oriented perpendicular to the x-axis occupies the region between a = 2 cm and b = 4.8 cm. The conducting slab has a net charge per unit area of σ_2 = 71 μC/m^2.

1)

What is $E_x(P)$, the value of the x-component of the electric field at point P, located a distance 6.4 cm from the infinite sheet of charge?

3.77E6 N/C

Infinite sheets of charge are my favorite because they have no dependence on distance and are easy to work with. Using Gauss's law we get:

$$\int \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\boldsymbol{E} = \frac{\boldsymbol{\sigma}}{2\varepsilon_0}$$

And then we just use superposition, adding the E fields from each plane of charge:

$$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2} = -\frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}$$

Because σ_1 is a negative charge and attractive while σ_2 is positive and repulsive, the two fields will work against each other at point P. In the formula I've taken into account the fact that σ_1 is negative

2)

What is $E_y(P)$, the value of the y-component of the electric field at point P, located a distance 6.4 cm from the infinite sheet of charge?

| 0 | N/C

The E field from an infinite plane is always perpendicular to the plane and thus none will be in the y direction here.

3)

What is $E_x(R)$, the value of the x-component of the electric field at point R, located a distance 1 cm from the infinite sheet of charge?

| -4.25E6 | N/C

We follow the same process here as in (1) but with the important difference that the fields work together and thus:

$$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2} = -\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$$

4)

What is $E_y(R)$, the value of the y-component of the electric field at point R, located a distance 1 cm from the infinite sheet of charge?

| 0 | N/C

See the explanation of (2)

5)

What is σ_b, the charge per unit area on the surface of the slab located at x = 4.8 cm?

| 33.4 | $\mu C/m^2$

This is the only tricky part to these problems. Imagine taking a Gaussian surface, a box, which goes partway into the slab, and the other face is to the right of the slab. The part inside the slab, which is conducting, will have a field of zero.

Using Gauss's law I have

$$\int \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E \cdot A + E \cdot 0 = \frac{\sigma_b A}{\varepsilon_0}$$

$$E = \frac{\sigma_b}{\varepsilon_0}$$

But since we already know the E field:

$$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2} = -\frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}$$

Then combining our two formulas we get:

$$\sigma_b = \frac{-\sigma_1 + \sigma_2}{2}$$

6)

What is E_x, the value of the x-component of the electric field at a point on the x-axis located at x = 3.12 cm ?

0 N/C

Since this is insider the conductor the E field will be zero.

7)

What is σ_a, the charge per unit area on the surface of the slab located at x = 2 cm?

37.6 $\mu C/m^2$

Same as in (5) or we can just use the fact that :

$$\sigma_2 = \sigma_a + \sigma_b$$

Which gives:

$$\sigma_a = \frac{\sigma_1 + \sigma_2}{2}$$

8)

Where along the x-axis is the magnitude of the electric field equal to zero?

(•) none of these regions

Since the E field doesn't change with increasing radius the only way the E field can be zero is if the charges cancel out and the only place that happens is in the conductor region which was not one of the choices