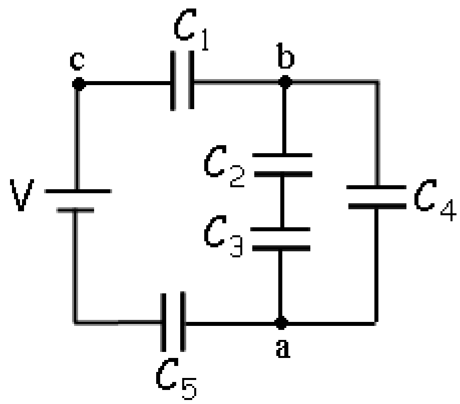
A circuit is constructed with five capacitors and a battery as shown. The values for the capacitors are: $C_1 = C_5 = 5.7\ \mu F$, $C_2 = 1.4\ \mu F$, $C_3 = 6.6\ \mu F$, and $C_4 = 3\ \mu F$. The battery voltage is $V = 12$ V.



1) What is C_{ab}, the equivalent capacitance between points a and b?

$$C_{23} = \frac{C_2 C_3}{C_2 + C_3} \Rightarrow C_{ab} = C_4 + \frac{C_2 C_3}{C_2 + C_3}$$

=(3+(1.4)(6.6)/(1.4+6.6))uF= 4.155 μF

2) What is C_{ac}, the equivalent capacitance between points a and c?

$$C_{ac} = \frac{C_1 C_{ab}}{C_1 + C_{ab}}$$

=((5.7)(4.155)/(5.7+4.155))uF = 2.40 μF

3) What is Q_5, the charge on capacitor C_5?

$$C_{equiv} = \frac{C_5 C_{ac}}{C_5 + C_{ac}} \Rightarrow Q_5 = C_{equiv} V = V \frac{C_5 C_{ac}}{C_5 + C_{ac}}$$

Q5=12 V ((5.7)(2.40)/(5.7+2.40))uF= 20.28 μC

4) What is Q_2, the charge on C_2?

$$V_{ab} = V - 2V_{bc} = V - 2\frac{Q_5}{C_5}$$

$$V_{ab} = \frac{Q_2}{C_2} + \frac{Q_2}{C_3} \Rightarrow Q_2 = V_{ab} \frac{C_2 C_3}{C_2 + C_3}$$

Vab=12 V- 2 (20.28uC)/5.7uF=4.88V

Q2=4.88V((1.4)(6.6)/(1.4+6.6))uF= 5.63 μC

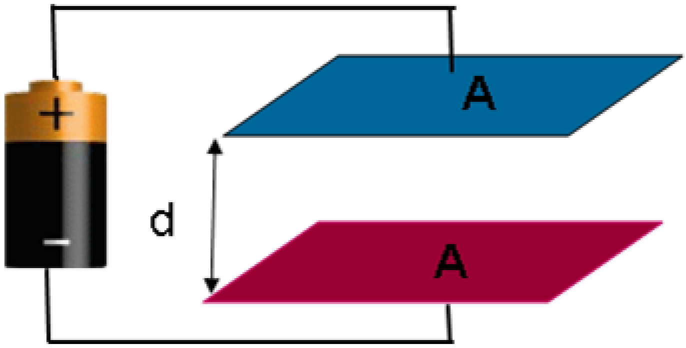
5) What is Q_1, the charge on capacitor C_1?

$$Q_1 = Q_5 \Rightarrow Q_1 = V \frac{C_5 C_{ac}}{C_5 + C_{ac}}$$

Q1=12V((5.7)(2.4)/(5.7+2.4))uF= 20.28 μC

6) What is V_4, the voltage across capacitor C_4?

V4= Vab=12 V- 2 (20.28uC)/5.7uF=4.88V



Two parallel plates, each having area A = 3899cm^2 are connected to the terminals of a battery of voltage V_b = 6 V as shown. The plates are separated by a distance d = 0.34cm. You may assume (contrary to the drawing) that the separation between the plates is small compared to a linear dimension of the plate.

1) What is C, the capacitance of this parallel plate capacitor?

$$C = \varepsilon_0 \frac{A}{d} = \frac{1}{4\pi k}\frac{A}{d}$$

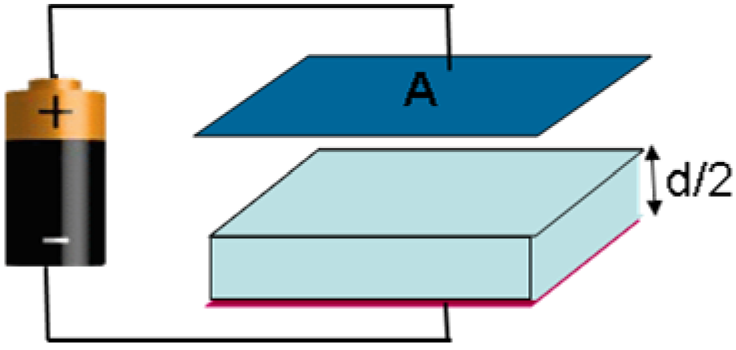
=3899/(4(3.13)(9)(.34) x 10e-9x10e-4x10e2=101x10e-11= .001 µF

2) What is Q, the charge stored on the top plate of the this capacitor?

$$Q = CV_b$$

=(.001 uF)(6V)= .006 µC

3)



A dielectric having dielectric constant κ = 3.2 is now inserted in between the plates of the capacitor as shown. The dielectric has area A = 3899 cm^2 and thickness equal to half of the separation (= 0.17 cm) .

What is the charge on the top plate of this capacitor?

$$C_{equiv} = \frac{2C\kappa(2C)}{2C+\kappa(2C)} = C\frac{2\kappa}{1+\kappa} = \varepsilon_o \frac{A}{d}\frac{2\kappa}{1+\kappa}$$

$$Q_{new} = C_{equiv}V_b = \varepsilon_o \frac{A}{d}\frac{2\kappa}{1+\kappa}V_b = Q\frac{2\kappa}{1+\kappa}$$

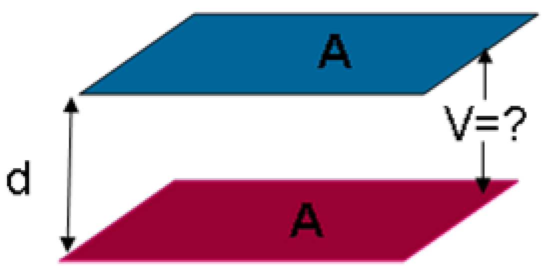
= .009 μC

4) What is U, the energy stored in this capacitor?

$$U_{new} = \frac{1}{2}Q_{new}V_b$$

=.009uC(6V)/2= 2.7e-8 J

5)

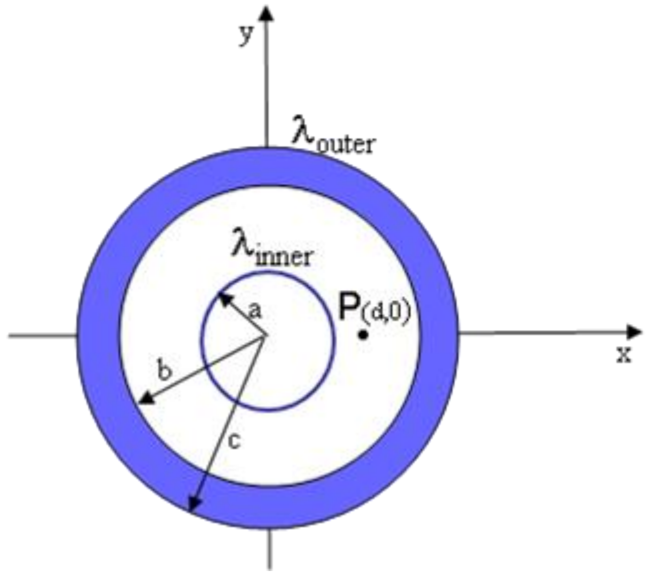


The battery is now disconnected from the capacitor and then the dielectric is withdrawn.
What is V, the voltage across the capacitor?

$$V = \frac{Q_{new}}{C}:$$

=(.009uC/.001uF)= 9.1 V - this required putting in extra significant digits to get smartphysics to recognize the answer

An infiinitely long solid conducting cylindrical shell of radius a = 3 cm and negligible thickness is positioned with its symmetry axis along the z-axis as shown. The shell is charged, having a linear charge density λ_{inner} = -0.44 μC/m. Concentric with the shell is another cylindrical conducting shell of inner radius b = 17.5 cm, and outer radius c = 19.5 cm. This conducting shell has a linear charge density λ_{outer} = 0.44μC/m.



1)

What is $E_x(P)$, the x-component of the electric field at point P, located a distance d = 8.8 cm from the origin along the x-axis as shown?

$$E_x(P) = 2k\frac{\lambda_{inner}}{d}$$

=2(9)(-.44C/m)/.088m x 10e9x10e-6= -9e3 N/C

2)

What is V(c) – V(a), the potential difference between the the two cylindrical shells?

$$V(c) - V(a) = -\int_a^b \frac{2k\lambda_{inner}}{r}dr = -2k\lambda_{inner}\ln\frac{b}{a}$$

=-2(9)(-.44)ln(17.5cm/3cm) x 10e9x10e-6= 13967 V

3)

What is C, the capacitance of a one meter length of this system of conductors?

$$C \equiv \frac{Q}{V} = \frac{\lambda L}{2k\lambda \ln \frac{b}{a}} = \frac{1}{2k \ln \frac{b}{a}}$$

=1/2(9)(ln17.5/3) x 10e-9= 3.15 e-5 μF/m

4)

The magnitudes of the charge densities on the inner and outer shells are now changed (keeping λ_{inner} = -λ_{outer}) so that the resulting potential difference doubles ($V_{ca,new}$ = 2$V_{ca,initial}$). How does C_{new}, the capacitance of a one meter length of the system of conductors when the charge density is changed, compare to C, the initial capacitance of a one meter length of the system of conductors?

Right Answer:
2

Feedback:
Your answer is correct! The capacitance is totally determined by the geometry of the conductors! The electric field between the conductors is proportional to the charge on the conductors. Therefore, the potential difference between the conductors is also proportional to the charge on the conductors. Therefore the capacitance (= Q/V) is independent of the magnitude of the charge on the capacitor.

5) What is $\lambda_{outer,new}$?

$$V_{ca,new} = 2k\lambda_{outer,new} \ln \frac{b}{a}$$

$$V_{ca} = 2k\lambda_{outer} \ln \frac{b}{a}$$

$$\lambda_{outer,new} = \lambda_{outer} \frac{V_{ca,new}}{V_{ca}}$$

=(.44uC/m)/(13967V) x (2(9)(.44uC/m)ln (17.5/3)x10e9= .88 μC/m