

## HW 6 - Electric Potential

1)

What is  $E_x(P)$ , the x-component of the electric field at point P, located a distance  $d = 34$  cm from the origin along the x-axis as shown?

-1.16E4

N/C

Here we just use our standard electric field equation for spherical symmetry and have:

$$E = k \frac{Q_{\text{enclosed}}}{r^2}$$

Here the only charge to take into account comes from the inner sphere. We can find its value as follows

$$Q = \rho * \text{Volume} = \rho * \frac{4\pi}{3} r^3$$

Since  $\mathbf{X}$  is in the radial direction at point P, the full value of the  $\mathbf{E}$  field goes into  $E_x$ .

2)

What is  $V(b)$ , the electric potential at the inner surface of the conducting shell? Define the potential to be zero at infinity.

V

The potential at the inner surface of the conducting shell will be the same as at the outer surface of the conductor because since the  $\mathbf{E}$  field is zero in the conductor there is no change of potential while traveling across it. At the outer shell it's just

$$-\int_B^\infty \mathbf{E} \cdot d\mathbf{l} = k \frac{Q_{\text{enclosed}}}{r}$$

3)

What is  $V(a)$ , the electric potential at the outer surface of the insulating sphere? Define the potential to be zero at infinity.

V

This will just be the potential from the inner surface of the conducting sphere added to the potential acquired while traveling from there to the surface of the insulating sphere.

$$V_0 - \int_A^B \mathbf{E} \cdot d\mathbf{l} = V_0 + kQ \left( \frac{1}{r_a} - \frac{1}{r_b} \right)$$

4)

What is  $V(c) - V(a)$ , the potential difference between the outer surface of the conductor and the outer surface of the insulator?

V

Since we had to find it in the last question just subtract the answer for (3) from the answer to (2)

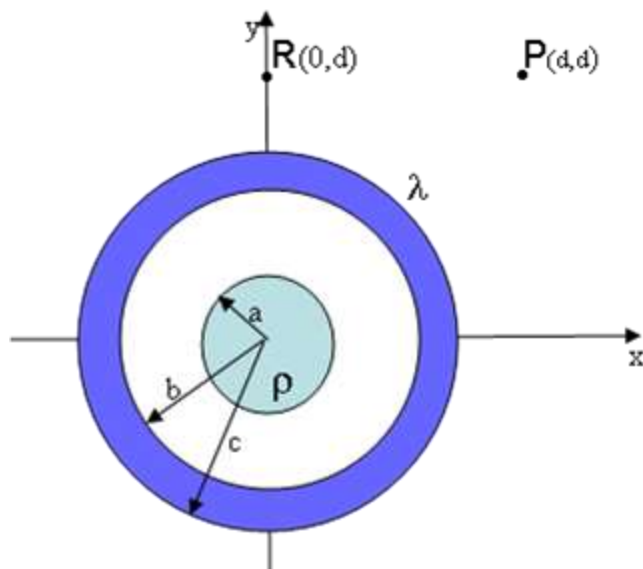
5)

A charge  $Q = 0.0695 \mu\text{C}$  is now added to the conducting shell. What is  $V(a)$ , the electric potential at the outer surface of the insulating sphere, now? Define the potential to be zero at infinity.

V

This will only change the contribution up to the surface of the conducting sphere. Repeat the process of part (2) with the modified charge and add to the unchanged result in (4)

### Potential of Concentric Cylindrical Insulator and Conducting Shell



An infinitely long solid insulating cylinder of radius  $a = 2.2$  cm is positioned with its symmetry axis along the  $z$ -axis as shown. The cylinder is uniformly charged with a charge density  $\rho = 28 \mu\text{C}/\text{m}^3$ . Concentric with the cylinder is a cylindrical conducting shell of inner radius  $b = 15$  cm, and outer radius  $c = 18$  cm. The conducting shell has a linear charge density  $\lambda = -0.36 \mu\text{C}/\text{m}$ .

1)

What is  $E_y(R)$ , the  $y$ -component of the electric field at point R, located a distance  $d = 47$  cm from the origin along the  $y$ -axis as shown?

N/C

Here we just use our standard electric field equation for a cylindrical symmetry:

$$E = 2k \frac{\lambda_{\text{enclosed}}}{r}$$

To find the  $\lambda$  of the inner cylinder we use the fact that:

$$\rho = \frac{\lambda}{A} = \frac{\lambda}{\pi r^2}$$

Then our  $\lambda_{\text{enclosed}}$  will be  $\lambda_{\text{inner}} + \lambda_{\text{outer}}$ .

2)

What is  $V(P) - V(R)$ , the potential difference between points P and R? Point P is located at  $(x, y) = (47 \text{ cm}, 47 \text{ cm})$ .

V

The potential relates to the electric field by:

$$\Delta V = - \int_P^R \mathbf{E} \cdot d\mathbf{l}$$

This is a dot product and since in our case our path goes in the  $x$  direction, we only concern ourselves with that component of the  $E$  field.

$$\Delta V = - \int_P^R 2k \frac{\lambda_{\text{enclosed}}}{r} \sin\theta dx = \int_P^R 2k \frac{\lambda_{\text{enclosed}}}{r} \frac{x}{r} dx = \int_P^R 2k \frac{\lambda_{\text{enclosed}} x}{x^2 + y^2} dx$$

$$\Delta V = k \lambda_{\text{enclosed}} \ln\left(\frac{R}{P}\right)$$

3)

What is  $V(c) - V(a)$ , the potential difference between the outer surface of the conductor and the outer surface of the insulator?

V

Moving through the conductor will create no change in potential because the E field is zero. We then take the integral of the E field moving from the conductor to the insulator and get.

$$\Delta V = -2k\lambda_{\text{enclosed}} \ln\left(\frac{A}{B}\right)$$

4)

Defining the zero of potential to be along the z-axis ( $x = y = 0$ ), what is the sign of the potential at the surface of the insulator?

☒  $V(a) < 0$

If we move outwards from the center then we're moving with the E field which decreases the potential.

5)

The charge density of the insulating cylinder is now changed to a new value,  $\rho'$  and it is found that the electric field at point P is now zero. What is the value of  $\rho'$ ?

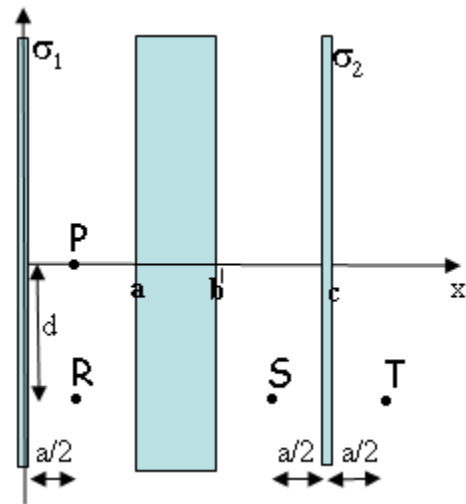
$\mu\text{C}/\text{m}^3$

We know that in order to have an electric field of zero, we need to cancel the effect of the conductor and get a  $\lambda$  of the same value but opposite sign from the inner cylinder. Using the fact that:

$$\rho = \frac{\lambda}{A} = \frac{\lambda}{\pi r^2}$$

And using the radius of the insulating cylinder but the  $\lambda$  of the conductor, we can find the necessary charge density.

### Potential of Infinite Sheets of Charge and Conducting Slab



An infinite sheet of charge is located in the y-z plane at  $x = 0$  and has uniform charge density  $\sigma_1 = 0.5 \mu\text{C}/\text{m}^2$ . Another infinite sheet of charge with uniform charge density  $\sigma_2 = -0.54 \mu\text{C}/\text{m}^2$  is located at  $x = c = 21 \text{ cm}$ . An uncharged infinite conducting slab is placed halfway in between these sheets ( i.e., between  $x = 8.5 \text{ cm}$  and  $x = 12.5 \text{ cm}$ ).

1)

What is  $E_x(P)$ , the x-component of the electric field at point P, located at  $(x,y) = (4.25 \text{ cm}, 0)$ ?

5.88E4

N/C

As we've seen before the electric field of slabs is independent of distance and gives:

$$E = \frac{\sigma}{2\epsilon_0}$$

For this particular case, due to their opposing charges, we can add the contributing E fields from the two sheets

2)

What is  $\sigma_a$ , the charge density on the surface of the conducting slab at  $x = 8.5 \text{ cm}$ ?

-0.52

$\mu\text{C}/\text{m}^2$

Make a Gaussian surface penetrating the slab. Gauss' law gives:

$$-E \cdot A + 0 = \frac{\sigma_a}{\epsilon_0}$$

And putting in the value of E we get

$$\sigma_a = -\frac{\sigma_1 + \sigma_2}{2}$$

3)

What is  $V(R) - V(P)$ , the potential difference between point P and point R, located at  $(x,y) = (4.25 \text{ cm}, -12.5 \text{ cm})$ ?

0

V

Since the E field is only in the X direction and the path is all in the Y, integrating along it gives a potential change of zero.

4)

What is  $V(S) - V(P)$ , the potential difference between point P and point S, located at  $(x,y) = (16.75 \text{ cm}, -12.5 \text{ cm})$ ?

-4.998E3

V

Break the path into two parts: P to R and R to S. As we saw in (3) the first part has no effect. The second will just be

$$-\int_R^S \mathbf{E} \cdot d\mathbf{l} = -El$$

5)

What is  $E_x(T)$ , the x-component of the electric field at point T, located at  $(x,y) = (25.25 \text{ cm}, -12.5 \text{ cm})$ ?

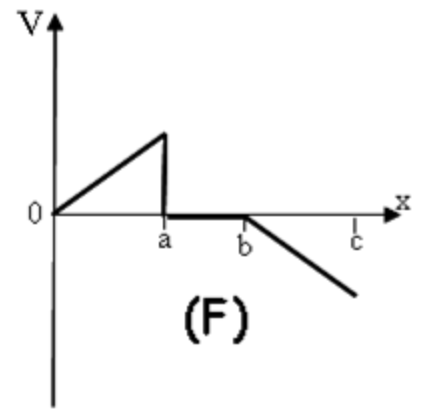
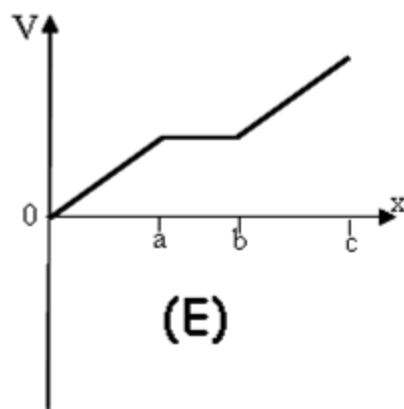
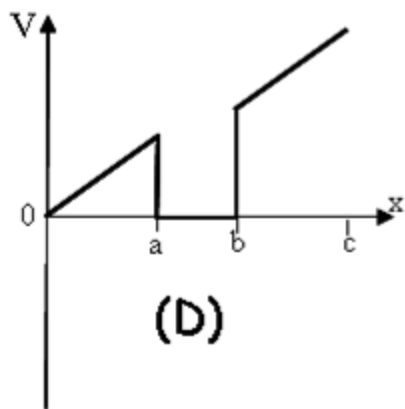
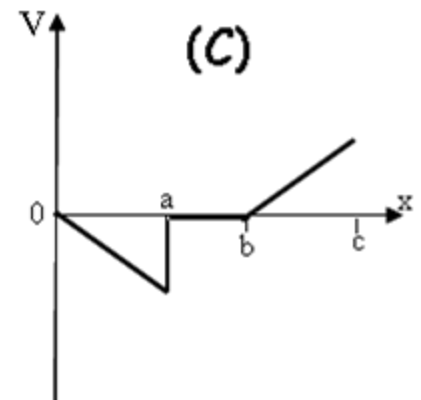
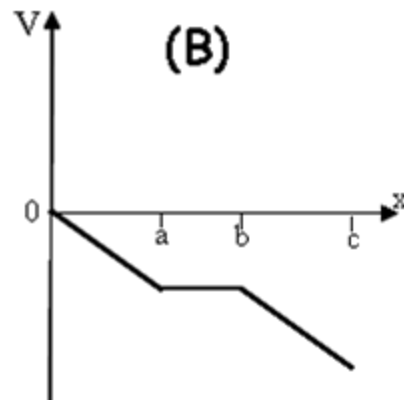
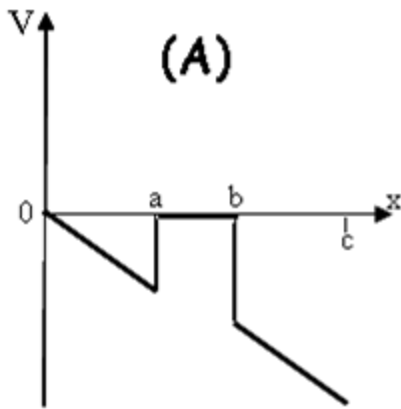
-2.26E3

N/C

The same as in (1) except instead of working together the two field contributions work against each other.

6)

Which of the following plots gives the correct x-dependence for the potential function between  $x = 0$  and  $x = 21 \text{ cm}$ ?



• B

Going from the first sheet to the slab is going with  $E$  field so the potential will decrease. Going through the slab will not change the potential so we'll see a flat line continuing from our sloped line. After the slab the potential drops again until the next sheet. Only (b) fits these requirements.