STAT 4352 - Mathematical Statistics Notes

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1 Chapter 11 - Interval Estimation

Point Estimators
θ is a unknown parameter (feature of a population)

- Ex: population mean μ
- **Fixed.**

$\hat{\theta}$ is a point estimator of θ (it is a numerical value)

- Ex: sample mean $\bar{x}$
- **Varies from sample to sample.**
- No guarantee of accuracy
- Must be *supplemented by* Var(θ)
  Standard Error SE($\hat{\theta}$) measures how much $\hat{\theta}$ varies from sample to sample.
  small SE $\implies$ low variance thus a more reliable estimate of θ

Interval Estimators

Interval Estimate
Provides a range of values that best describe the population.
Let L = L(x) be the Lower Limit
U = U(x) be the Upper Limit
Both L,U are Random Variables because they are functions of sample data.

Confidence Level / Confidence Coefficient
Is the probability that the **interval estimate** will include population parameter θ.

- Sample means will follow the normal probability distribution for large sample sizes (n $\geq$ 30)
- For small sample forces us to use the t-distribution probability distribution(n $<$ 30)
- A confidence level of 95% implies that **95% of all samples would give an interval that includes θ, and only 5%of all samples would yield an erroneous interval.**
- The most frequently used confidence levels are 90%, 95%, and 99% with corresponding Z-scores 1.645, 1.96, 2.576.
- The higher the confidence level, the more strongly we believe that the value of the parameter lies within the interval.

Confidence Interval

Gives plausible values for the parameter θ being estimated where degree of plausibility specified by a confidence level.

To construct an interval estimator of unknown parameter θ. We must find two statistics **L** and **U** such that:

$$P\{\mathbf{L} \leq \theta \leq \mathbf{U}\} = 1 - \alpha$$

- $P\{\mathbf{L} \leq \theta \leq \mathbf{U}\}$ **Coverage Probability**, in repeated sampling, what percent of samples or Confident Intervals capture true θ.
- 100(1- α) **Confidence Interval** - for unknown fixed parameter θ.
- L,U - **Lower and Upper Bounds** - RVs because they are functions of sample data. Vary from sample to sample.
- 1-α **Confidence Level** Probability that estimate will include population parameter

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