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Note

The two-person beauty contest

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Abstract

We introduce a two-person beauty contest game with a unique Nash equilibrium that is identical to the game with many players. However, iterative reasoning is unnecessary in the two-person game as choosing zero is a weakly dominant strategy. Despite this "easier" solution concept, we find that a large majority of players do not choose zero. This is the case even with a sophisticated subject pool.

JEL classification: C7; C9

Keywords: Beauty contest; Guessing game; Bounded rationality; Weak dominance

1. Introduction

In a beauty-contest game (BCG) n participants simultaneously choose a number in the interval [0, 100]. The winner is the person whose number is closest to a given proportion of the average of all chosen numbers. The winner receives a fixed prize. If there is a tie, the prize is split between those who tie. So far, BCGs have been conducted in groups of 3 to several thousands.[1] Here we introduce the *two*-person BCG.[2]

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[1] See Nagel (1995), Stahl (1996), Duffy and Nagel (1997), Ho et al. (1998), Stahl (1998), Bosch-Domenech et al. (2002) and Weber (2003).

[2] Since our first collection of data on the $n = 2$ BCG, Costa-Gomes and Crawford (2006) also conducted two-person BCGs. Their two-person games are, however, quite different, i.e. they are asymmetric, with different p-values for different players in the pair and have continuous payoffs. Their games are dominance solvable in 3–52 iterations. Camerer et al. (2004) also ran two-person BCGs for estimation purposes of their cognitive hierarchy model.

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When n is large the theoretical solution can be found by an infinite process of iterated elimination of weakly dominated strategies or through infinite iterated elimination of best replies.[3] While the theoretical solution is that all choose zero, the experimental evidence is that most players do not choose zero.

The special feature of our two-person BCG is that zero is a weakly dominant strategy and is *always* the winning number regardless of the choice of the other player. Furthermore the two-person BCG is isomorphic to the very simple game "Whoever chooses the smaller number wins."[4] Given the fact that most subjects in related experiments have no problem in applying one level of reasoning, and the ease of computing the equilibrium in the $n = 2$ case, there is good reason to expect that a large fraction of our subjects would choose zero in the two-person BCG.

However, we observe that the overwhelming majority chooses dominated strategies in the two-person BCG. Surprisingly, even fairly sophisticated subjects, i.e. game theorists, get it wrong and seem to apply iterated reasoning to the two-person BCG when first confronted with the task.

The paper is organized as follows. In Section 2 we present the experimental design and our choice of subject pools. Section 3 illustrates and discusses our experimental results. Section 4 concludes.

2. Experimental design

In this study we focus on the BCG where the winning number is defined as the number that is closest to two thirds of the mean of the chosen numbers. We conducted $n = 2$ BCGs as well as $n > 2$ BCGs in order to compare observed choices.[5] We collected data in the controlled environment of the laboratory as well as at various conferences, summer schools and seminar presentations. All winners were compensated. In the remainder of this paper we shall refer to the participants in the laboratory as *students* and to participants at conferences as *professionals*.

2.1. Students

Since the BCG is often used as a teaching tool in undergraduate economics, we carefully selected participants to ensure they had not been exposed to the BCG. We only allowed freshmen to participate and solicited students from different campuses for different sessions. The experimental sessions were conducted during April, September, October 2000, October 2004 and June 2005 using first year undergraduate students majoring in either economics, political science, law, medicine or humanities with no formal training in game theory at the University Pompeu Fabra in Barcelona.[6]

The fixed prize in each round was equal to 100 pesetas (at the time of the experiment roughly equal to \$0.64) in the $n = 2$ BCG.[7] We paid the winner of each round 1000 pesetas in the

[3] See Brandenburger and Keisler (2001) for a theoretical analysis of the finitely iterated elimination of weakly dominated strategies and its requirements.

[4] Two thirds of the average of two positive numbers is always closer to the smaller number in the pair.

[5] See our working paper Grosskopf and Nagel (2001) for behavior over time in different information conditions and order effects. Choices in the two-person BCG (with fixed pairing) seem to converge to equilibrium predictions only when players are informed about each other's choices. All data reported in this note are first round choices where participants have not been exposed to any other task.

[6] All sessions run in 2005 used only non-Economics majors and were computerized using z-Tree (Fischbacher, 1999).

[7] While 100 pesetas may seem like a small amount in \$-equivalents it would buy a delicious cafe con leché (latté) in the cafeteria of Pompeu Fabra.

$n > 2$ BCG. The $n > 2$ BCGs were conducted in groups of 18 students. In the later experiments, conducted after the introduction of the Euro, we paid a 1 Euro prize (at that time about $1.20) in the $n = 2$ BCG and a 10 Euro prize in the $n > 2$ BCG.[8] Altogether we have gathered data from 168 students. 132 students participated in the $n = 2$ BCG and 36 participated in the $n > 2$ BCG.

2.2. Professionals

We presented preliminary results of this paper at five conferences and in seminars. Four out of the five were economics conferences and one was a psychology-decision making conference.[9] Before each talk, we played the two-person BCG with the audience and collected their choices. At each conference the winner of one randomly selected pair was paid the equivalent of $10 at the end of the presentation. Altogether we have gathered data from 130 professionals participating in the $n = 2$ BCG. We complement these data with data from 59 professionals participating in the $n > 2$ BCG taken from Bosch-Domenech et al. (2002).[10]

3. Experimental results

3.1. Students

Only 9.85% (13 out of 132) of the laboratory participants choose zero. Thus, roughly 90% play dominated strategies, which is probably the highest amount ever observed in an experimental game. 21.21% (17 out of 132) of the choices are above 50, which is similar to results for other $n > 2$ BCGs. It seems that 50 is a strong focal point in the $n = 2$ case, where it is chosen 12.88% (17 out of 132) of the time.

Figure 1 shows the observed cumulative density functions of choices of our student subjects in the $n = 2$ and $n > 2$ treatments. Surprisingly, choices in the BCGs are not stochastically different from one another (Kolmogorov–Smirnov test, $p = 0.091$). If anything, they are marginally higher in the $n = 2$ BCG than in the $n > 2$ BCG.

3.2. Professionals

We find 36.92% (48 out of 130) of professionals choose zero in the $n = 2$ BCG. While the professionals choose fewer dominated strategies, they still choose numbers greater than 50 13.08% (17 out of 130) of the time. More interestingly, their choices are not significantly different from professional choices in the $n > 2$ BCG (Kolmogorov–Smirnov test, $p = 0.166$). Figure 2 shows the observed cumulative density functions of choices of our professional subject.

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8 We chose these higher stakes to show robustness of our results as suggested by a referee.

9 The conferences were the ESA (Economic Science Association) European Meeting held in Amsterdam, The Netherlands in October 2000 (18), the 20th Arne Ryde Symposium held in Lund, Sweden in November 2000 (18), the GEW (Gesellschaft für Experimentelle Wirtschaftsforschung) Meeting held at Burg Warberg, Germany in November 2000 (26), the Far Eastern Meeting of the Econometric Society (FEMES) held in Kobe, Japan in July 2001 (16) and the annual meeting of the Society for Judgement and Decision Making in Orlando, Florida, US (29). The seminars were held at the Economics Department of the University of Arizona in Tucson, Arizona, US in October 2001 (19) and at the Economics Department of the University of Erfurt, Germany in December 2001 (4). The number of participants are given in parentheses after each conference or seminar.

10 Choices in sessions 11 and 12 in Bosch-Domenech et al. (2002) come from data collected at game theory conferences with varying group sizes.

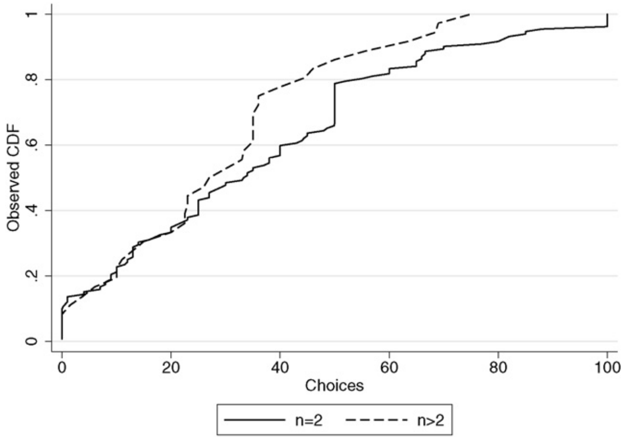

Fig. 1. Cumulative density functions of choices by students.

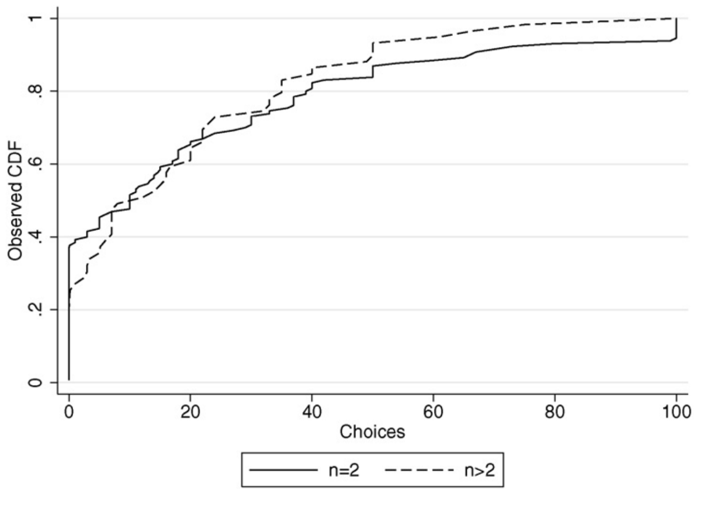

Fig. 2. Cumulative density functions of choices by professionals.

Table 1 shows summary statistics for all treatments. Comparing the proportion of choices of zero for the student population, we find that this proportion is not significantly different in the $n = 2$ BCG and the $n > 2$ BCG (test of equality of proportions, z-value $= 0.8007$, $p = 0.4238$).[11] We do find, however, that professionals choose zero significantly more often in the $n = 2$ BCG than in the $n > 2$ BCG (test of equality of proportions, z-value $= 2.2696$, $p = 0.0232$). Also, for

[11] The specific test statistic is $z = (p_1 - p_2)/S_{p_c}$, where p_i is the proportion of choices of zero in subsample i, and $S_{p_c} = \sqrt{p_c(1 - p_c)(\frac{1}{N_1} + \frac{1}{N_2})}$ is an estimate of the standard error of the difference in proportions, $p_1 - p_2$, p_c is an estimate of the population proportion under the null hypothesis of equal proportions, $p_c = (p_1N_1 + p_2N_2)/(N_1 + N_2)$, where N_i is the total number of choices in subsample i (see Glasnapp and Poggio, 1985).

Table 1
Summary statistics for all treatments

	Choices of zero	Mean	Median
Students $n = 2$	9.85% (13/132)	35.57	33.65
Students $n > 2$	5.56% (2/36)	29.31	28.5
Professionals $n = 2$	36.92% (48/130)	21.73	10
Professionals $n > 2$	20.34% (12/59)	18.98	12.10

each BCG we observe that professionals choose zero significantly more often than students (tests of equality of proportions: $n = 2$ BCG, z-value $= -5.1845$, $p < 0.001$ and $n > 2$, z-value $= -1.9720$, $p = 0.0488$).

Why do neither students nor game theorists choose zero more often in the $n = 2$ case? There are at least three different explanations:

(1) Players seem to ignore the important feature that for $n = 2$ their own influence on the mean is very large. Instead, players seem to approximate the mean by the choice of the other, which makes them want to choose a number that is close to $2/3$ of the number chosen by the other. The following reported thought-process of a game theorist makes this point clearer: "*When the other chooses 50, I choose 33.33, but then the other could choose 100 and I should chose 67. If the other chooses 22, I choose 14. What should I choose?*" This comment points out that it is not obvious that zero is the (weakly) dominant answer, even though the game theorist seems to realize that the lower number always wins. As in the $n > 2$ game, players see a conflict between being too far from the behavior of the other player and choosing the lower number.

(2) For our student population, we find a desire to find a kind of "fixed-point." We see students trying to solve the following equation $x = \frac{2}{3}\frac{x+y}{2}$, with x being the own choice and y being the other person's choice. Such reasoning leads to a desire to choose half of the other person's choice.

(3) Players might fall prey to negative transfer (e.g. Luchins and Luchins, 1970 and Chen and Daehler, 1989). In particular, colleagues who were familiar with the $n > 2$ BCG seem to incorrectly transfer their experience without thinking about the changed situation. The following comment from an experimenter who often had used the guessing game as a demonstration experiment illustrates this argument: "*Well, I would just choose 18, which is a good guess in these games.*"

4. Discussion

In this note we have introduced a new version of the beauty contest game. With this version we extend the range of BCGs with respect to the influence of a single players' choice on the final outcome of the game. We can distinguish different strengths of individual impact in the BCGs studied so far. In the large group mean BCG's (see Bosch-Domenech et al., 2002) or when the order statistic used is the median instead of the mean (see Duffy and Nagel, 1997), an individual player has practically no influence on the final result. In the $1/2$ maximum game (see Duffy and Nagel, 1997) and in the small group mean BCG (see Ho et al., 1998) a single player has a large influence on the result.[12] Finally, in our two-person BCG a single person has the greatest influence. An individual can even win the game for sure.

Our experiment is probably the first experimental game where the overwhelming majority of initially observed choices are in dominated strategies, especially if we only consider constant sum games. Even academic professionals (game theorists) are prone to get it wrong initially. Our data suggest that subjects do not use the rational prescriptive reasoning process of the two-person BCG, which essentially means recognizing that the lower number *always* wins and thus zero is the right answer. Instead, players largely ignore their own influence on the order statistic. This approach is correct for a many-person BCG or a $2/3$ maximum game, however, it is wrong in the two-person $2/3$ mean BCG. We observe, surprisingly, that choices in the two-person BCG are not any different from choices in the many-person BCG.

In the economics and psychology literature there is a large discussion on overestimation of one's own influence on outcomes (e.g. illusion of control) or the belief of similarity to other players' behavior (false consensus; e.g. Dawes, 1990). Here we show the opposite, individuals ignore their own influence. A similar situation would be estimating one's influence on congestion. If one decides to drive on a crowded road, it will be more crowded, and slower (see Schelling, 1978). Another example is in auctions: one's own bid might cause others to reevaluate what the item is worth and induce them to raise their bids. People who correctly estimate the impact of their bids "snipe" in the eBay data analyzed by Roth and Ockenfels (2002).

So far, behavior in our two-person game could not be captured with existing descriptive theories. The cognitive hierarchy model by Camerer et al. (2004) predicts a low level of reasoning (either random play or best response to random play), and the fit of the model is rather low in comparison to the fit of the same model for behavior in $n > 2$ person games.

While we have found choices to be mostly in dominated strategies when people are confronted with this new BCG for the first time, it remains to be investigated whether and how people can learn to choose zero in the $n = 2$ BCG.

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[12] Note that in the $1/2$ maximum game, different to the $p > 1/2$ case where there are only mixed equilibria, zero is a "weak" equilibrium.

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