Experimental 'Beauty Contests' with Homogeneous and Heterogeneous Players and with Interior and Boundary Equilibria

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Abstract

We study behavior in experimental beauty contests with, first, boundary and interior equilibria, and, second, homogeneous and heterogenous types of players. We find quicker and better convergence to the game-theoretic equilibrium with interior equilibria and homogeneous players.

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1 Introduction

The 'beauty-contest' game - which was likened by Keynes (1936) to professional investment activity - has inspired many experimental studies (see, for instance, Nagel, 1995, Duffy and Nagel, 1997, Ho et al., 1998). All beauty-contest games have in common that n decision makers simultaneously choose a number from a closed interval. In its simple form the winner is the decision maker whose number is closest to $q\emptyset(s)$, where $\emptyset(s)$ is the average of all guesses and q is a real number announced at the beginning of the game. Due to its favorable environment and its simplicity, the beauty-contest game or 'guessing game', as it is sometimes also called[1], has mainly been used to analyze reasoning processes and the extent of boundedly rational behavior. Furthermore, it has gained more and more importance in testing various learning theories (see, for instance, Camerer and Ho, 1999).

One reason for the popularity of the beauty-contest game, apart from its simplicity, is its obvious resemblance with decision making in financial markets. Another reason might be the flexibility in setting game parameters and modifying equilibrium choices to create ideal environments for diverse questions at hand. Some of the obvious modifications of the basic game have already been explored.[2] The latter statement does not only hold true for parameter changes, but also for the effects of different subject pools (e.g. the usual student experiments or readers of newspapers, see Nagel et al., 2000) or of institutional changes like distinguishing single and team players (Kocher and Sutter, 2000). A common feature of all hitherto performed beauty-contest game experiments is that only the winner (i.e., the one closest to the best guess) is rewarded and subject groups are homogeneous (i.e., the best guess is the same for all participants). Nearly all prior studies, furthermore, rely on boundary equilibria.

We modify the standard design with regard to three important features. First, in our version of the beauty-contest game payoffs are continuous. Every participant receives a monetary endowment to pay a fine whose size is determined by how far the chosen number deviates from the best guess. Second, we explore and compare behavior both with boundary and interior equilibria. The latter are generated by adding a constant to the

[1] The third term in use is 'average game', introduced by Moulin (1986).
[2] For an overview see Nagel (1999).

average number in the group. Third, in addition to the common case with homogeneous groups where all players have to guess the same target number in a group of n decision makers we introduce heterogeneous groups, where $\frac{n}{2}$ decision makers have to guess $q_i\emptyset(s)$ and $\frac{n}{2}$ decision makers have to guess $q_{j\neq i}\emptyset(s)$.

The chosen set-up resembles financial decision much more than the basic beauty-contest game. Take, e.g., reasoning processes associated with picking stocks in stock markets. Returns are, obviously, continuous and not dichotomous as in all prior beauty-contest games, boundary equilibria rarely exist and heterogeneous groups are the rule and not the exception (e.g., short versus long positions), which has also been neglected in beauty-contest games, hitherto. Our new modification to the basic game allows to explore several important issues. Typcially, first round guesses in the standard design are quite far away from the boundary equilibrium (see Ho et al., 1998). By comparing guesses in boundary and interior equilibirum environments we want to analyze whether deviations are smaller in an interior equilibrium framework. The explanation for such a difference in the results of the two treamtents might lie in the existence of a desire to choose interior instead of extreme, boundary strategies (see Rubinstein et al., 1997). The introduction of heterogeneous types of players allows to investigate whether a more complex situation induces participants to think harder about other players' behavior, and, thus, promotes convergence to equilibrium due to a better understanding of the game. We deem our payment scheme to be more appropriate if one likens beauty-contest games to financial decision making, but we do not expect it to alter results qualitatively, although deviation from optimality should be slightly smaller.

In section 2 we introduce our 2 x 2-design, the equilibria of our beauty contests and our main hypotheses. Details of the computerized experiment are given in section 3. The results are described in section 4 before concluding in section 5.

2 The experimental beauty contests

Let $n\,(>2)$ denote the number of players $i = 1, ..., n$ in the game who all choose a real number

$$s_i \in S_i = [0, 100] \text{ for } i = 1, ..., n.$$

For any strategy vector $s = (s_1, ..., s_n)$ let

$$\emptyset(s) = \frac{1}{n}\sum_{i=1}^{n} s_i$$

denote the average number. The general form of the payoff function $u_i(s)$ is given by

$$u_i(s_i) = C - c\,|s_i - q_i[\emptyset(s) + d]|$$

where $|r|$ denotes the absolute value of the real number r; d is a constant added to the average number $\emptyset(s)$ and $q_i \in (0,1)$ is the quota of $[\emptyset(s) + d]$ which determines player i's best guess. If player i does not guess correctly, he must pay a fine of $c\,(> 0)$ for every unit of deviation which he can pay out of his positive (monetary) endowment C. The 2×2-factorial design of our experiment (Table 1) distinguishes $d = 0$ (left column) and $d = 50$ (right column) as well as $q_i = 1/2$ for all $i = 1, ..., n = 4$ (upper row) and $q_i = 1/3$ for one half and $q_i = 2/3$ for the other half of the 4 players (lower row).

quota q_i	$d = 0$	$d = 50$
$q_i = 1/2\ \forall i$	$s_i^* = 0, \forall i$	$s_i^* = 50, \forall i$
$q_i = 1/3$ for $i = 1, 2$ $q_i = 2/3$ for $i = 3, 4$	$s_i^* = 0$ for $i = 1, ..., 4$	$s_i^* = 100/3$ if $q_i = 1/3$ $s_i^* = 200/3$ if $q_j = 2/3$

Table 1: The 2×2-factorial design and equilibria.

The equilibria s_i^* of our beauty contest games are also displayed in Table 1. In the Appendix, we show how the solutions can either be justified as the unique equilibrium of each game or by repeated elimination of strictly dominated strategies.

Our main hypotheses can be summarized as follows:[3]

1. An interior equilibrium ($d = 50$) is supposed to yield smaller deviations of the guesses from the game-theoretic equilibrium than a boundary equilibrium, since participants often try to avoid extreme choices (see, for instance, Rubinstein et al., 1997).

[3] Our continuous payment scheme should not cause different behavior than under the standard winner-takes-all rule (see, for instance, Bolle and Ockenfels, 1990, Cubitt et al., 1998).

2. Introducing heterogeneity of players should induce subjects to think more thoroughly about the strategies of the other player types in order to make a reasonable guess. In a less complex situation, with homogeneous players, it might not be that obvious for many subjects that they thoroughly consider what others will do. Therefore, from a behavioral point of view, we would expect participants in heterogeneous groups to be closer to equilibrium than participants in homogeneous groups.

3 Experimental Procedure

The experiments were run in December 2000 and January 2001 at the Humboldt-University Berlin. Nearly all participants were students attending an undergraduate course in microeconomics[4], which one usually attends already during the 1st semester. Thus, for most participants it should have been their first experience with experimental economics. The software of the computerized experiment has been developed with the help of z-Tree (Fischbacher, 1998).

In each of our 4 treatment conditions we had 5 sessions involving 8 participants. The average time needed to run a session was 40 minutes (about 15 for reading instructions and asking privately for clarifications and 25 minutes for playing 10 rounds). Participants were divided into two (matching) groups. They were told that they are matched randomly in each of the 10 rounds to form 4 person-player groups. In treatments with heterogeneous groups subjects were told that in each round there would be two subjects of each type in each group. Note that in spite of repeated interaction, the average of a session's matching groups qualifies as an independent observation.

We set $c = 0.05$ DM and $C = 2$ DM in the payoff function.[5] Table 2 shows average earnings separately for the first and the second five rounds for players with $q_i = 1/2$ (in treatments with homogeneous groups), or $q_i = 1/3$ and $q_i = 2/3$, respectively (in

[4] The topic of this course is general equilibrium theory and did not yet introduce game theoretic concepts.

[5] If a subject was more than 40 units away from her target value, she made a loss. That happened to 5 out of 160 subjects in the first round. These subjects were informed that losses could be balanced and gains accumulated in the rounds to follow.

case of heterogeneous groups). In general, we observe that subjects earned on average always more in the second five rounds of the experiment. Subjects in homogeneous groups with the interior equilibrium ($d = 50$) earned most, namely 18.41 DM in total (out of a maximum of 20 DM). Subjects with $q_i = 2/3$ and the boundary equilibrium ($d = 0$) earned the smallest average amount.

	$d = 0$		$d = 50$	
quotas	$1 - 5$	$6 - 10$	$1 - 5$	$6 - 10$
$q_i = 1/2$	8.16	9.26	8.65	9.76
$q_i = 1/3$,	7.55	9.10	8.10	9.39
$q_i = 2/3$	6.84	8.70	7.34	8.64

Table 2. Average earnings (in DM) per type of player

4 Results

Figures 1 and 2 show the average guesses in each treatment in the course of the experiment as well as the corresponding equilibria.[6] In the heterogenous treatments we split the data for the two types of players with $q_i = 1/3$ and $q_i = 2/3$, respectively. As can be seen, guesses converge steadily towards the equilibrium[7], in particular in our treatments with interior equilibria ($d = 50$).

We can confirm our first hypothesis, stating that interior equilibria trigger more equilibrium-like behavior than boundary equilibria. For testing this hypothesis, we rely on the averages of rounds 1 to 5, rounds 6 to 10, and the overall average. Referring to homogeneous groups we find that groups with an interior equilibrium ($d = 50$) are significantly closer to the equilibrium than groups with a boundary equilibrium ($d = 0$).[8] Heterogeneous groups

[6] A data file with session, respectively type averages is included in the Appendix.

[7] Deviations from equilibrium get significantly smaller from round t to round $t + 1$ in any case (considering both homogeneous and heterogeneous groups), with the exception of round 9 to 10 with $d = 0$, and round 3 to 4 and 9 to 10 with $d = 50$ (Wilcoxon signed ranks test, $p < 0.05$).

[8] $p < 0.1$ for rounds 1-5; $p < 0.05$ for rounds 6-10, and 1-10, respectively (U-test, two-sided).

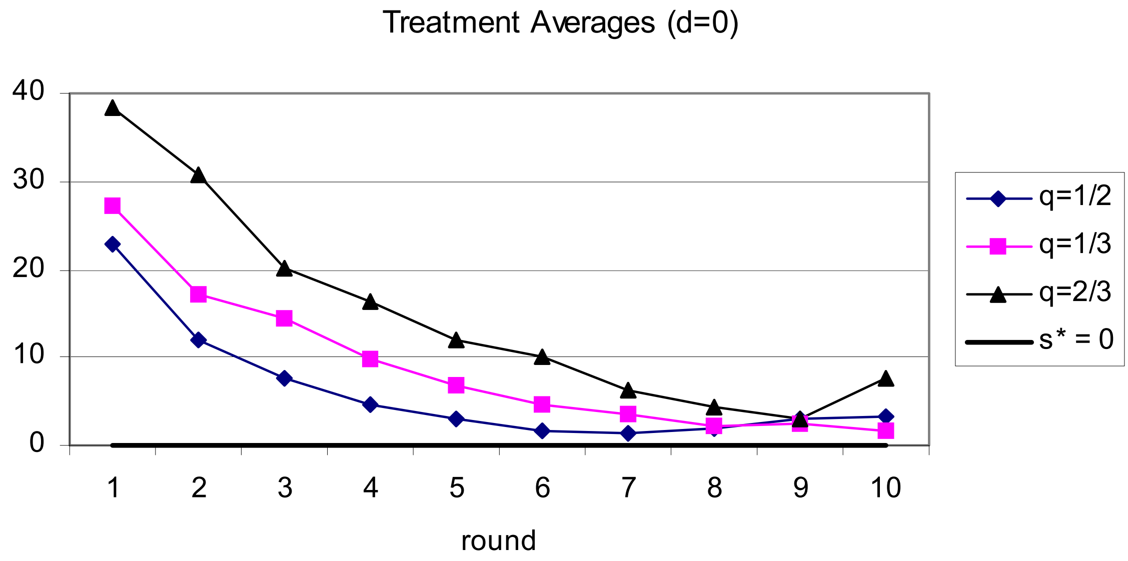

Figure 1:

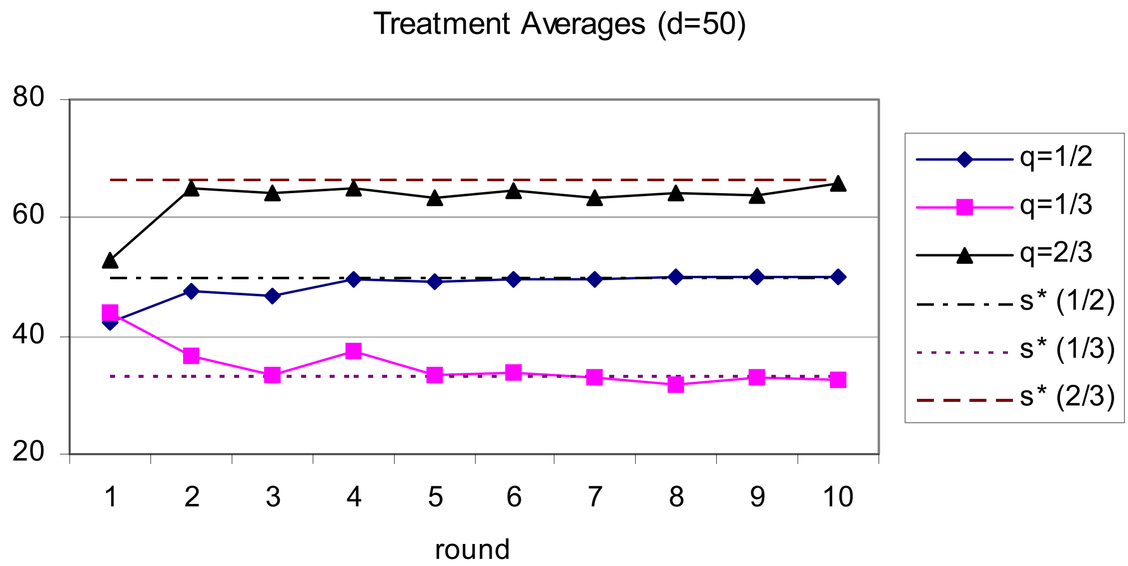

Figure 2:

guess the equilibrium solution more closely when the equilibrium is interior, however, this holds only for the first five rounds.[9]

Furthermore, exact equilibrium guesses are much more frequent in the interior equilibrium treatments with homogeneous groups. 49.25% of all guesses in the treatment with homogeneous groups and an interior equilibrium are exactly at the equlibrium ($s_i^* = 50$), whereas the corresponding figure in case of the boundary equilibrium ($s_i^* = 0$) is 27.75%.[10] The frequency of equilibrium choices is, however, in both treatments remarkably high. Note that in Nagel (1995), who also had homogeneous groups and either $q = 1/2$ or $q = 2/3$, only 3 out of 115 subjects chose exactly zero (the equilibrium choice). We see the continuous payment scheme as the driving force behind the high frequency of equilibrium choices. Looking at groups with heterogeneous players we find equilibrium choices to be much less frequent (7.5% in case of an interior equilibrium and 3.5% with boundary equilibrium, respectively) and no significant difference in equilibrium choices between both types of equilibria. With respect to profits, subjects facing an interior equilibrium earn - ceteris paribus - significantly more than those facing a boundary equilibrium.[11]

As to our second hypothesis, namely that heterogeneity of players should trigger more thorough deliberations and, thus, more equilibrium like decisions, we test whether deviations from equilibrium are smaller in heterogeneous than in homogeneous groups, given that either $d = 0$ or $d = 50$. Contrary to our expectations, we find that homogeneous groups are closer to equilibrium than heterogeneous groups. With $d = 0$, session averages with homogeneous groups are significantly smaller (and, thus, closer to equilibrium) than average guesses in heterogeneous groups in each of the first seven rounds, in the averages of the first five rounds, and in the average over all ten rounds.[12] In our $d = 50$ treatments, homogeneous groups are closer to the equilibrium than heterogeneous groups in all rounds but rounds 2 and 4.[13] We, therefore, believe that the complexity generated by two types

[9] $p < 0.05$ (U-test, two-sided).

[10] The difference is statistically significant for the first round and the average of the first five rounds, taking sessions as independent observations (p < 0.05, U-test, two-sided).

[11] $p < 0.05$ (U-test, two-sided).

[12] $p < 0.05$ for rounds 2, 5, and 6, and for the average of rounds 1-5. $p < 0.1$ in all other cases. (U-test, two-sided).

[13] $p < 0.05$ for rounds 5, 7, 8, 9, and 10, and for the average of rounds 6-10. $p < 0.1$ in all other cases. (U-test, two-sided).

of players makes it more difficult to approach equilibrium behavior and that subjects do not reason more thoroughly when there are different types of group members.

Next, we explore whether there are systematic differences between the different types of players in heterogeneous groups. $q_i = 1/3$-players might converge faster to the game-theoretic equilibrium because of the faster elimination of weakly dominated strategies.[14] In Figure 1 ($d = 0$) we find that, on average, guesses of $q_i = 2/3$-players are higher than those of $q_i = 1/3$-players in all rounds. However, taking session averages as the only independent observations we do not find a significant difference between the guesses of both types of players, neither in any single round nor considering the averages of rounds 1 to 5 or 6 to 10. Referring to Figure 2 ($d = 50$) we see that average guesses of $q_i = 2/3$-players are slightly further away from equilibrium than those of $q_i = 1/3$-players. Yet, the difference is driven by a single subject who had difficulties in understanding the experiment and who chose numbers in the range from 0 to 10 in all rounds.[15] Therefore, deviations from equilibrium (33.33 and 66.67, respectively) do also not differ between player types in case of $d = 50$. We may, therefore, conclude that heterogeneity of players does affect group behavior by increasing deviations from equilibrium, compared with groups of homogeneous players. However, heterogeneity affects both types of players and both types do not systematically differ in their deviations from optimality.

5 Conclusion

We have explored behavior in four different types of an experimental beauty-contest. We find that decisions are closer to the game-theoretic equilibrium when the equilibrium is interior. Competition in groups of homogeneous players also promotes convergence to the equilibrium. More complexity through heterogeneous players, however, is detrimental for profits as well as for convergence to the equilibrium.

[14] See the Appendix for details.

[15] This subject even ended up with a loss.

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Appendix

A1. Equilibrium strategies in the beauty contest games

For $d = 0$ the best reply to $s_j^* = 0$ for $j \neq i$ is obviously $s_i^* = 0$ due to $q_i < 1$. This proves that the behavior in the left column of Table 1 defines a strict equilibrium in both cases. To show its uniqueness assume an equilibrium s^* with $\emptyset(s^*) > 0$. From the best reply-condition

$$s_i^* = q_i \emptyset(s^*) \text{ for } i = 1, ..., n$$

it follows that

$$\sum_{i=1}^{n} s_i^* = \emptyset(s^*) \sum_{i=1}^{n} q_i \text{ for } i = 1, ..., n$$

or

$$n = \sum_{i=1}^{n} q_i \text{ due to } \sum_{i=1}^{n} s_i^* = n\emptyset(s^*) .$$

This contradicts $q_i < 1$ for $i = 1, ..., n$. Thus, for $d = 0$ the behavior in Table 1 is the only equilibrium.

Solving the case $d = 0$ by repeated elimination of strictly dominated strategies proceeds by the following algorithm:

- Set $m_j = 100$ for $j = 1, ..., n$.
- For all $i = 1, ..., n$ all strategies $s_i > \frac{q_i}{n} \sum_{j=1}^{n} m_j$ are eliminated.
- Denote for $i = 1, ..., n$ by m_i' the maximal s_i remaining after the previous step. Substitute for $i = 1, ..., n$ the previous m_i by m_i' and repeat the second step.

In the first elimination step this excludes all strategies $s_i > q_i \cdot 100$ for $i = 1, ..., n$. Thus the sequence of the m_j-values are monotonically decreasing until they finally reach $m_j = 0$ for $j = 1, ..., n$. Notice, however, that in the lower row of Table 1 for $d = 0$ this implies a much faster elimination for $q_i = 1/3$-players than for $q_i = 2/3$-players: A $q_i = 1/3$-player would immediately eliminate all $s_i > 100/3$ whereas a $q_i = 2/3$-player starts by eliminating only $s_i > 200/3$. More details can be found in Table A1 illustrating in detail the implications of the first four elimination steps.

number of elimination step	eliminated strategies					
	$q_i = 1/2, d = 0$ for $i = 1, ..., 4$	$q_i = 1/2, d = 50$ for $i = 1, ..., 4$	$d = 0$		$d = 50$	
			$q_i = 1/3$	$q_i = 2/3$	$q_i = 1/3$	$q_i = 2/3$
1	$s_i > 50$	$s_i < 25, s_i > 75$	$s_i > \frac{100}{3}$	$s_i > \frac{200}{3}$	$s_i < \frac{50}{3}$ $s_i > 50$	$s_i < \frac{100}{3}$
2	$s_i > 25$	$s_i < 37.5, s_i > 62.5$	$s_i > \frac{50}{3}$	$s_i > \frac{100}{3}$	$s_i < 25$ $s_i > \frac{125}{3}$	$s_i < 50$ $s_i > \frac{125}{3}$
3	$s_i > 12.5$	$s_i < 43.75, s_i < 56.25$	$s_i > \frac{25}{3}$	$s_i > \frac{50}{3}$	$s_i < \frac{350}{12}$ $s_i > \frac{450}{12}$	$s_i < \frac{350}{6}$ $s_i > \frac{450}{6}$
4	$s_i > 6.25$	$s_i < 46.875, s_i > 53.125$	$s_i > \frac{25}{6}$	$s_i > \frac{25}{3}$	$s_i < \frac{550}{12}$ $s_i > \frac{1700}{48}$	$s_i < \frac{550}{6}$ $s_i > \frac{1700}{24}$
$\vdots$						
∞	$s_i > 0$	$s_i \neq 50$	$s_i > 0$	$s_i > 0$	$s_i \neq \frac{100}{3}$	$s_i \neq \frac{200}{3}$

Table A1: Repeated elimination of strictly dominated strategies for our 2×2-factorial design

For $d = 50$ one proves in the same way that the behavior in the right hand-column of Table 1 is the unique (strict) equilibrium. Elimination of strictly dominated strategies proceeds now from below and above. For $q_i = 1/2$ for $i = 1, ..., 4$, for instance, not only the strategies $s_i > 75$ but also those with $s_i < 25$ have to be eliminated in the first step. For the asymmetric case players i with $q_i = 1/3$ eliminate correspondingly $s_i > 50$ and $s_i < 50/3$ in the first elimination step whereas for $q_i = 2/3$ only the strategies $s_i < 100/3$ can be eliminated in the first step. This again illustrates how the asymmetric case forces participants to engage in quite different considerations when anticipating others with a different quota and when anticipating another with the same quota whose deliberations should be similar to one's own.

A2. Instructions

The written instructions differ only in the description of the quotas (either 1/2 for all 4 players or two players with 1/3 and two with 2/3) as well as of the constant (which was only mentioned in case of $d = 50$). We provide the translated instructions for $d = 0$ and heterogeneous groups. All other instructions are available upon request.

Rules of the experiment (originally in German)

Welcome to our experiment and thank you for participating!

You are member of a group of 4 persons. Each person of this group has to choose a number x_i between zero (0) and one hundred (100). Zero and one hundred are also possible. It is not necessary to choose an integer, however, numbers with more than two decimals are excluded.

Your payoff in the experiment depends on how close your number is to a target number. The closer your number to a target number, the higher your payoff.

Groups consist of 2 participants of type A and 2 participants of type B. Target numbers of type A and type B participants are different. If you are a type A your target number is one-third (factor f_A) of the average of all four numbers chosen in your group. If you are a type B your target number is two-thirds (factor f_B) of the average of all four numbers chosen in your group.

The payoff of each round depends on the difference between your chosen number and the target number. If your chosen number and the target number are identical, you receive 2.00 DM. Each (rounded up) unit difference leads to a deduction of 0.05 DM.

Formally, your per round payoff is calculated as follows:

$$\text{payoff per round} = 2.00 - 0.05 \mid x_i - f_{A,B} \frac{\sum x_j}{4} \mid .$$

The experiment lasts for 10 rounds. Groups are rematched in every round. Note, though, that each group always consists of 2 type A- and 2 type B-participants. You keep your type (either A or B) in all rounds.

After each round you receive information on the group average, your target number and your payoff. Payoffs are accumulated over all rounds and paid in cash and privately at the end of the experiment.

A3. Data file

Session averages															
							Guesses								
	Session						Rounds								
	▼	q	1	2	3	4	5	6	7	8	9	10	1 -- 5	6 -- 10	1 -- 10
d=0	1	0,5	18,56	8,15	4,31	3,13	1,98	0,66	0,36	0,24	0,20	6,44	7,22	1,58	4,40
	2	0,5	20,57	6,98	2,32	0,54	0,06	0,01	0,00	6,25	1,02	5,36	6,09	2,53	4,31
	3	0,5	23,44	12,30	6,52	2,88	1,31	0,99	0,77	0,42	0,31	0,32	9,29	0,56	4,92
	4	0,5	21,19	12,84	7,76	4,60	2,34	1,04	1,05	0,40	12,58	3,73	9,75	3,76	6,75
	5	0,5	30,18	19,71	16,80	12,52	9,45	6,06	4,25	2,46	1,46	0,55	17,73	2,96	10,34
d=50	1	0,5	35,63	42,19	46,01	53,65	50,02	49,83	49,72	49,58	49,77	49,84	45,50	49,75	47,62
	2	0,5	35,94	39,25	42,84	45,50	48,15	49,08	49,96	50,34	50,27	50,34	42,34	50,00	46,17
	3	0,5	47,38	55,69	48,12	49,38	48,68	49,36	49,36	49,72	49,97	49,88	49,85	49,66	49,75
	4	0,5	42,63	52,44	47,70	48,63	48,93	49,03	49,51	49,74	49,78	49,77	48,06	49,56	48,81
	5	0,5	49,06	48,94	49,45	49,87	49,87	49,90	50,00	50,00	50,00	50,00	49,44	49,98	49,71
d=0	1	*	29,98	25,86	24,77	20,61	12,22	6,38	3,40	1,86	0,91	0,41	22,69	2,59	12,64
	2	*	26,50	12,91	7,11	5,06	2,89	2,24	1,54	1,14	1,07	0,67	10,89	1,33	6,11
	3	*	22,14	24,98	16,75	12,39	10,20	7,61	6,07	3,07	2,47	14,97	17,29	6,84	12,06
	4	*	44,49	31,91	25,77	19,72	16,42	17,80	11,67	9,59	8,28	6,85	27,66	10,84	19,25
	5	*	41,12	24,39	11,52	7,31	5,19	2,54	1,43	0,70	0,32	0,07	17,91	1,01	9,46
d=50	1	*	36,63	39,96	34,90	38,12	36,84	37,02	37,59	38,37	40,25	41,14	37,29	38,87	38,08
	2	*	57,00	54,34	54,36	52,72	53,96	53,36	50,81	51,23	50,16	51,27	54,48	51,37	52,92
	3	*	49,63	50,22	50,72	51,61	51,82	51,46	51,06	50,44	50,14	50,03	50,80	50,63	50,71
	4	*	46,81	52,70	48,24	50,39	48,44	50,44	50,41	46,60	49,44	50,10	49,32	49,40	49,36
	5	*	52,56	56,56	55,49	62,36	50,94	53,38	51,72	52,39	51,78	52,81	55,58	52,41	54,00

* Averages of both types in heterogeneous groups

Type averages in heterogeneous groups															
	Session														
	▼	q	1	2	3	4	5	6	7	8	9	10	1 -- 5	6 -- 10	1 -- 10
d=0	1	0,33	23,46	14,58	18,47	10,49	6,95	3,80	1,42	0,73	0,44	0,12	14,79	1,30	8,04
	1	0,67	36,50	37,14	31,08	30,74	17,49	8,95	5,38	2,99	1,39	0,69	30,59	3,88	17,23
	2	0,33	31,25	11,56	8,48	6,38	3,25	2,53	2,25	1,63	1,38	0,91	12,18	1,74	6,96
	2	0,67	21,75	14,25	5,75	3,75	2,53	1,95	0,83	0,65	0,76	0,44	9,61	0,92	5,26
	3	0,33	11,52	11,70	14,75	11,28	9,14	6,23	4,23	1,60	1,56	0,76	11,68	2,87	7,28
	3	0,67	32,75	38,25	18,75	13,50	11,25	9,00	7,90	4,54	3,39	29,18	22,90	10,80	16,85
	4	0,33	33,75	23,39	22,16	16,05	11,09	8,10	8,65	6,96	8,19	6,39	21,29	7,66	14,47
	4	0,67	55,23	40,43	29,38	23,39	21,75	27,50	14,69	12,21	8,38	7,30	34,03	14,02	24,02
	5	0,33	36,38	24,90	7,65	4,50	3,38	1,88	0,99	0,51	0,22	0,07	15,36	0,73	8,05
	5	0,67	45,86	23,89	15,39	10,12	7,00	3,20	1,87	0,88	0,43	0,08	20,45	1,29	10,87
d=50	1	0,33	45,25	36,66	23,80	29,43	28,36	29,03	29,69	28,72	29,25	28,72	32,70	29,08	30,89
	1	0,67	28,00	43,25	46,00	46,81	45,31	45,01	45,50	48,01	51,25	53,56	41,87	48,67	45,27
	2	0,33	49,50	38,69	37,47	35,27	36,17	35,56	34,01	33,91	33,90	33,83	39,42	34,24	36,83
	2	0,67	64,50	70,00	71,25	70,17	71,75	71,17	67,62	68,54	66,42	68,72	69,53	68,49	69,01
	3	0,33	45,00	35,96	35,72	36,47	36,22	35,54	34,60	33,85	33,32	32,72	37,87	34,00	35,94
	3	0,67	54,25	64,48	65,72	66,75	67,42	67,39	67,52	67,04	66,96	67,33	63,72	67,25	65,49
	4	0,33	40,13	36,33	33,75	33,33	32,30	33,31	33,30	26,10	32,91	32,46	35,17	31,61	33,39
	4	0,67	53,50	69,08	62,73	67,45	64,59	67,58	67,51	67,11	65,97	67,74	63,47	67,18	65,32
	5	0,33	40,50	35,00	35,86	51,84	34,50	34,88	33,93	35,40	34,91	34,25	39,54	34,67	37,10
	5	0,67	64,6	78,1	75,1	72,9	67,4	71,9	69,5	69,4	68,7	71,4	71,63	70,16	70,89