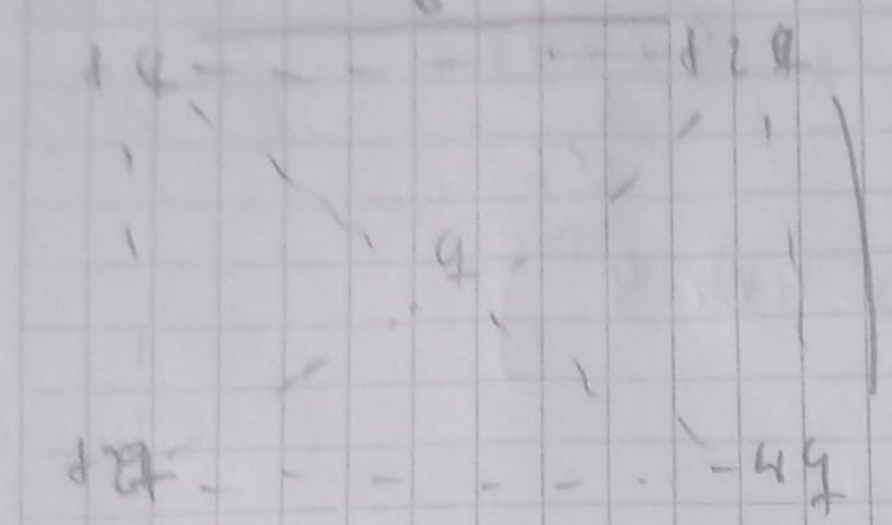
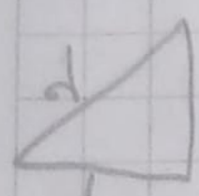
Encontrar la energía potencial (U) del siguiente sistema



Formula para este ejercicio

$$U_{q_0 j} = \frac{K\, q_0 q_j}{r_{q_0 q_j}}$$

1) encontrar la distancia diagonal i su mitad



$$d^2 = b^2 + b^2 \rightarrow d = \sqrt{2b^2} \rightarrow \boxed{d = b\sqrt{2}}$$

$$\frac{d}{2} = \frac{b\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2b}{2\sqrt{2}} \rightarrow \boxed{\frac{d}{2} = \frac{b}{\sqrt{2}}}$$

2) ahora si hay que encontrar la energía potencial del sistema

i)

$$U(q_i q_j) = K\frac{q\,2q}{b} + K\frac{q\,2q}{b} + \frac{q\,q}{b\sqrt{2}} + \frac{q(-4)q}{b\sqrt{2}}$$

$$= \frac{K}{b}\left(2q^2 + 2q^2 + \sqrt{2}q^2 - \frac{4q^2}{\sqrt{2}}\right)$$

$$= \frac{K}{b}\left(4q^2 + \sqrt{2}\,q^2 - \frac{4q^2}{\sqrt{2}}\right)$$

$$= \frac{K}{b}\left(\frac{4\sqrt{2}q^2 + \sqrt{2}\sqrt{2}\,q^2 - 4q^2}{\sqrt{2}}\right)$$

$$= \frac{K\,q^2}{b\sqrt{2}}\left(4\sqrt{2} + 2 - 4\right) = \frac{K\,q^2}{\sqrt{2}b}\left(4\sqrt{2} - 2\right)$$

$$\boxed{U(q, q_j) = K\,\frac{(4\sqrt{2} - 2)\,q^2}{\sqrt{2}\,b}}$$

ii) q pero el de centro

$$U(q, q_j) = k\frac{qq}{\frac{b}{\sqrt{2}}} + \frac{q2q}{\frac{b}{\sqrt{2}}} + \frac{q2q}{\frac{b}{\sqrt{2}}} + \frac{q(-4q)}{\frac{b}{\sqrt{2}}}$$

$$= \frac{k}{b}\left(q^2 + 2\sqrt{2}q^2 + 2\sqrt{2}q^2 - 4\sqrt{2}q^2\right)$$

$$\boxed{U(q, q_j) = k\frac{q^2}{b}}$$

iii)

$$U(2q, q_j) = k\frac{2qq}{b} + k\frac{2qq}{\frac{b}{\sqrt{2}}} + k\frac{2q(-4q)}{b} + k\frac{2q2q}{b\sqrt{2}}$$

$$= \frac{k}{b}\left(2q^2 + 2\sqrt{2}q^2 - 8q^2 + \frac{4q^2}{\sqrt{2}}\right)$$

$$= \frac{kq^2}{b}\left(\frac{2\sqrt{2}q^2 + 2\sqrt{2}\sqrt{2}q^2 - 8\sqrt{2}\ 4}{\sqrt{2}}\right)$$

$$= \frac{kq^2}{\sqrt{2}b}\left(2\sqrt{2} + 4 - 8\sqrt{2} + 4\right) = \frac{kq^2}{\sqrt{2}b}\left(8 - 6\sqrt{2}\right)$$

$$\boxed{U = (2q, q_j) = k\frac{(8 - 6\sqrt{2})q^2}{\sqrt{2}b}}$$

iv) Todas las distancias son iguales al iii) así el resultado será el ~~tmi~~ mismo

$$\boxed{U(2q, q_j) = k\frac{(8 - 6\sqrt{2})q^2}{\sqrt{2}b}}$$

v)

$$U(-4q, q_j) = k\frac{(-4q)2q}{b} + k\frac{(-4)(2q)}{b} + \frac{(-4q)q}{b\sqrt{2}} + \frac{(-4q)q}{\frac{b}{\sqrt{2}}}$$

$$= k\left(-\frac{8q^2}{b} + \frac{8q^2}{b} - \frac{4q^2}{\sqrt{2}b} - \frac{4\sqrt{2}q^2}{b}\right)$$

$$= \frac{kq^2}{b}\left(-8 - 8 - \frac{4}{\sqrt{2}} - 4\sqrt{2}\right)$$

$$U(-4q,q) = \frac{kq^2}{6}\left(-\frac{16\sqrt{2}}{\sqrt{2}} - \frac{4}{\sqrt{2}} - \frac{4\sqrt{2}\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{kq^2}{\sqrt{2}6}\left(-16\sqrt{2} - 4 - 8\right) = \frac{kq^2}{\sqrt{2}6} = \left(-12 - 16\sqrt{2}\right)$$

$$U(-4q,q) = k\,\frac{(-12-16\sqrt{2})\,q^2}{\sqrt{2}6}$$

La suma total de la energia potencial seria

$$U_{Total} = k\,\frac{(4\sqrt{2}-2)}{\sqrt{2}6}q^2 + k\,\frac{(8-6\sqrt{2})\,q^2}{\sqrt{2}6} + k\,\frac{(8-6\sqrt{2})\,q^2}{\sqrt{2}6} + k\,\frac{q^2}{6}$$

$$+ k\,\frac{(-12-16\sqrt{2})\,q^2}{\sqrt{2}6}$$

$$= \frac{kq^2}{6}\left(\frac{(4\sqrt{2}-2)}{\sqrt{2}} + \frac{(8-6\sqrt{2})}{\sqrt{2}} + \frac{(8-6\sqrt{2})}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}} + \frac{(-12-16}{\sqrt{2}}\right.$$

$$= \frac{kq^2}{\sqrt{2}6}\left(4\sqrt{2} - 2 + 8 - 6\sqrt{2} + 8 - 6\sqrt{2} + \sqrt{2} - 12 - 16\sqrt{2}\right.$$

$$\frac{kq^2}{\sqrt{2}6}\left((2 - 23\sqrt{2}\right)$$

$$U_{Total} = k\,\frac{(2-23\sqrt{2})\,q^2}{\sqrt{2}6}$$