Lab 7

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11:59PM April 22, 2021

#Rcpp

We will get some experience with speeding up R code using C++ via the `Rcpp` package.

First, clear the workspace and load the `Rcpp` package.

```
pacman::p_load(Rcpp)
```

Create a variable `n` to be 10 and a vaiable `Nvec` to be 100 initially. Create a random vector via `rnorm` `Nvec` times and load it into a `Nvec` x `n` dimensional matrix.

```
n = 10
Nvec = 100
X = matrix(data = rnorm(Nvec), nrow = 100, ncol = 10)
head(X)
```

```
##            [,1]       [,2]       [,3]       [,4]       [,5]       [,6]
## [1,]  0.2635986  0.2635986  0.2635986  0.2635986  0.2635986  0.2635986
## [2,] -0.5530956 -0.5530956 -0.5530956 -0.5530956 -0.5530956 -0.5530956
## [3,]  1.0264043  1.0264043  1.0264043  1.0264043  1.0264043  1.0264043
## [4,] -0.7303913 -0.7303913 -0.7303913 -0.7303913 -0.7303913 -0.7303913
## [5,] -1.2155867 -1.2155867 -1.2155867 -1.2155867 -1.2155867 -1.2155867
## [6,] -0.9892604 -0.9892604 -0.9892604 -0.9892604 -0.9892604 -0.9892604
##            [,7]       [,8]       [,9]      [,10]
## [1,]  0.2635986  0.2635986  0.2635986  0.2635986
## [2,] -0.5530956 -0.5530956 -0.5530956 -0.5530956
## [3,]  1.0264043  1.0264043  1.0264043  1.0264043
## [4,] -0.7303913 -0.7303913 -0.7303913 -0.7303913
## [5,] -1.2155867 -1.2155867 -1.2155867 -1.2155867
## [6,] -0.9892604 -0.9892604 -0.9892604 -0.9892604
```

Write a function `all_angles` that measures the angle between each of the pairs of vectors. You should measure the vector on a scale of 0 to 180 degrees with negative angles coerced to be positive.

```
angle = function(u,v){
  acos(sum(u*v)/sqrt(sum(u^2)*sum(v^2))) * (180/pi)
}
all_angles = function(X){
  A = matrix(NA, nrow = nrow(X), ncol = nrow(X))
  for (i in 1:(nrow(X)-1)) {
    for (j in (i+1):nrow(X)) {
      A[i,j] = angle(X[i,], X[j,] )
```

```
    }
  }
  A
}
## all_angles(X)
```

Plot the density of these angles.

```
pacman::p_load(ggplot2)
ggplot(data.frame(angles = c(all_angles(X)))) +
  aes(x = angles) +
  geom_density()

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## Warning: Removed 6128 rows containing non-finite values (stat_density).
```

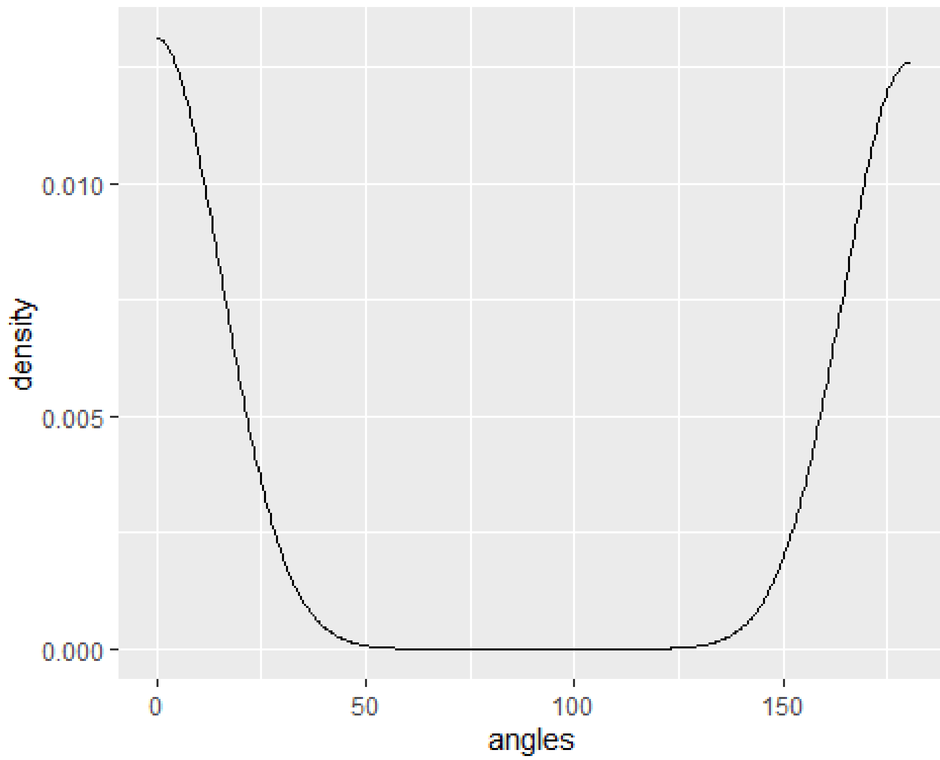

Write an Rcpp function `all_angles_cpp` that does the same thing. Use an IDE if you want, but write it below in-line.

```
"
cppFunction(
  '
  NumericMatrix all_angles_cpp(NumericMatrix X) {
    int n = X.nrow();
    int p = X.ncol();
    NumericMatrix A(n, n);
    std::fill(A.begin(), A.end(), NA_REAL);
    for (int i_1 = 0; i_1 < (n - 1); i_1++){

      for (int i_2 = i_1 + 1; i_2 < n; i_2++){
        double sum_sqd_u = 0;
        double sum_sqd_v = 0;
        double sum_u_v = 0;
```

```
        for (int j = 0; j < p; j++){
        sum_sqd_u += pow(X(i_1, j), 2);
        sum_sqd_v += pow(X(i_2, j), 2);
        sum_u_v += X(i_1, j) * X(i_2, j);

        }
        A(i_1, i_2) = acos(sum_u_v)/sqrt((sum_sqd_u)*(sum_sqd_v)) *
(180/M_PI);
      }
    }
    return A;
  }
  '
)
all_angles_cpp(X)
"
```

```
## [1] "\ncppFunction(\n  ' \n  NumericMatrix all_angles_cpp(NumericMatrix X)
{\n    int n = X.nrow();\n    int p = X.ncol();\n    NumericMatrix A(n, n);\n
std::fill(A.begin(), A.end(), NA_REAL);\n    for (int i_1 = 0; i_1 < (n - 1);
i_1++){\n\n      for (int i_2 = i_1 + 1; i_2 < n; i_2++){\n        double
sum_sqd_u = 0;\n        double sum_sqd_v = 0;\n        double sum_u_v = 0;\n
for (int j = 0; j < p; j++){\n        sum_sqd_u += pow(X(i_1, j), 2);\n
sum_sqd_v += pow(X(i_2, j), 2);\n        sum_u_v += X(i_1, j) * X(i_2, j);\n
\n        }\n        A(i_1, i_2) =
acos(sum_u_v)/sqrt((sum_sqd_u)*(sum_sqd_v)) * (180/M_PI); \n      }\n    }\n
return A;\n  }\n  '\n)\nall_angles_cpp(X)\n"
```

Test the time difference between these functions for `n = 1000` and `Nvec = 100, 500, 1000, 5000` using the package `microbenchmark`. Store the results in a matrix with rows representing `Nvec` and two columns for base R and Rcpp.

```
pacman::p_load(microbenchmark)
n = 1000
Nvec = c(100, 500 , 1000, 5000)
"
  for (i in 1:4) {
    X = matrix(data = rnorm(Nvec[i]), nrow = Nvec[i])
    microbenchmark(all_angles(X), all_angles_cpp(X), times = 10)
  }
"
```

```
## [1] "\n  for (i in 1:4) {\n    X = matrix(data = rnorm(Nvec[i]), nrow =
Nvec[i])\n    microbenchmark(all_angles(X), all_angles_cpp(X), times = 10)\n
}\n"
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot. We wil see later how to create "long" matrices that make such plots easier.

Let `Nvec = 10000` and vary `n` to be 10, 100, 1000. Plot the density of angles for all three values of `n` on one plot using color to signify `n`. Make sure you have a color legend. This is not easy.

```
n = c(10, 100, 1000)
Nvec = 10000
"  for (i in 1:4) {
    X = matrix(data = rnorm(Nvec), nrow = Nvec)
    microbenchmark(all_angles(X), all_angles_cpp(X), times = n[1])
}
  "

## [1] "  for (i in 1:4) {\n    X = matrix(data = rnorm(Nvec), nrow = Nvec)\n
microbenchmark(all_angles(X), all_angles_cpp(X), times = n[1])\n}\n  "
```

Write an R function `nth_fibonnaci` that finds the nth Fibonnaci number via recursion but allows you to specify the starting number. For instance, if the sequency started at 1, you get the familiar 1, 1, 2, 3, 5, etc. But if it started at 0.01, you would get 0.01, 0.01, 0.02, 0.03, 0.05, etc.

```
f = c(NA)
f[1]=0.01
f[2]=0.01
nth_fibonnaci = function(n){
  for (i in 3:n) {
    f[i] = f[i-2]+ f[i-1]
  }
  f[n]
}
nth_fibonnaci(6)

## [1] 0.08
```

Write an Rcpp function `nth_fibonnaci_cpp` that does the same thing. Use an IDE if ou want, but write it below in-line.

```
"
cppFunction(
  'vector<int> f;
  f[0] = 0.01;
  f[1] = 0.01;
  NumericMatrix nth_fibonnaci_cpp(NumericMatrix n) {
    for(i = 2; i<=n; i++){
      f[i] = f[i-2]+ f[i-1];
    }
    f[n];
  }')
"
```

```
## [1] "\ncppFunction(\n  'vector<int> f;\n  f[0] = 0.01;\n  f[1] = 0.01;\n
NumericMatrix nth_fibonnaci_cpp(NumericMatrix n) {\n    for(i = 2; i<=n;
i++){\n      f[i] = f[i-2]+ f[i-1];\n    }\n    f[n];\n  }')\n"
```

Time the difference in these functions for n = 100, 200, ..., 1500 while starting the sequence at the smallest possible floating point value in R. Store the results in a matrix.

```
f = c(NA)
A = c(NA)
f[1]=0.01
f[2]=0.01

nth_fibonnaci = function(n){

  for (i in 3:n) {
    f[i] = f[i-2]+ f[i-1]
    }

    A[i-2] = f[100]

}
```

Plot the divergence of performance (in log seconds) over n using a line geometry. Use two different colors for the R and CPP functions. Make sure there's a color legend on your plot.

```
#TO-DO
```

Data Wrangling / Munging / Carpentry

Throughout this assignment you can use either the `tidyverse` package suite or `data.table` to answer but not base R. You can mix `data.table` with `magrittr` piping if you wish but don't go back and forth between `tbl_df`'s and `data.table` objects.

```
pacman::p_load(dplyr, magrittr, data.table)
```

Load the `storms` dataset from the `dplyr` package and investigate it using `str` and `summary` and `head`. Which two columns should be converted to type factor? Do so below.

```
data(storms)
str(storms)

## tibble [10,010 x 13] (S3: tbl_df/tbl/data.frame)
##  $ name        : chr [1:10010] "Amy" "Amy" "Amy" "Amy" ...
##  $ year        : num [1:10010] 1975 1975 1975 1975 1975 ...
##  $ month       : num [1:10010] 6 6 6 6 6 6 6 6 6 6 ...
##  $ day         : int [1:10010] 27 27 27 27 28 28 28 28 29 29 ...
##  $ hour        : num [1:10010] 0 6 12 18 0 6 12 18 0 6 ...
##  $ lat         : num [1:10010] 27.5 28.5 29.5 30.5 31.5 32.4 33.3 34 34.4
34 ...
##  $ long        : num [1:10010] -79 -79 -79 -79 -78.8 -78.7 -78 -77 -75.8 -
```

```
74.8 ...
##  $ status     : chr [1:10010] "tropical depression" "tropical depression"
"tropical depression" "tropical depression" ...
##  $ category   : Ord.factor w/ 7 levels "-1"<"0"<"1"<"2"<..: 1 1 1 1 1 1 1
1 2 2 ...
##  $ wind       : int [1:10010] 25 25 25 25 25 25 25 30 35 40 ...
##  $ pressure   : int [1:10010] 1013 1013 1013 1013 1012 1012 1011 1006 1004
1002 ...
##  $ ts_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
##  $ hu_diameter: num [1:10010] NA NA NA NA NA NA NA NA NA NA ...
```

```
summary(storms)
```

```
##      name                year          month             day
##  Length:10010       Min.   :1975   Min.   : 1.000   Min.   : 1.00
##  Class :character   1st Qu.:1990   1st Qu.: 8.000   1st Qu.: 8.00
##  Mode  :character   Median :1999   Median : 9.000   Median :16.00
##                     Mean   :1998   Mean   : 8.779   Mean   :15.86
##                     3rd Qu.:2006   3rd Qu.: 9.000   3rd Qu.:24.00
##                     Max.   :2015   Max.   :12.000   Max.   :31.00
##
##       hour             lat             long             status
##  Min.   : 0.000   Min.   : 7.20   Min.   :-109.30   Length:10010
##  1st Qu.: 6.000   1st Qu.:17.50   1st Qu.: -80.70   Class :character
##  Median :12.000   Median :24.40   Median : -64.50   Mode  :character
##  Mean   : 9.114   Mean   :24.76   Mean   : -64.23
##  3rd Qu.:18.000   3rd Qu.:31.30   3rd Qu.: -48.60
##  Max.   :23.000   Max.   :51.90   Max.   :  -6.00
##
##  category       wind           pressure       ts_diameter
hu_diameter
##  -1:2545   Min.   : 10.00   Min.   : 882.0   Min.   :   0.00   Min.   :
0.00
##  0 :4373   1st Qu.: 30.00   1st Qu.: 985.0   1st Qu.:  69.05   1st Qu.:
0.00
##  1 :1685   Median : 45.00   Median : 999.0   Median : 138.09   Median :
0.00
##  2 : 628   Mean   : 53.49   Mean   : 992.1   Mean   : 166.76   Mean   :
21.41
##  3 : 363   3rd Qu.: 65.00   3rd Qu.:1006.0   3rd Qu.: 241.66   3rd Qu.:
28.77
##  4 : 348   Max.   :160.00   Max.   :1022.0   Max.   :1001.18   Max.
:345.23
##  5 :  68                                     NA's   :6528      NA's
:6528
```

```
head(storms)
```

```
## # A tibble: 6 x 13
##   name   year month   day  hour   lat  long status       category  wind
pressure
```

```
##   <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>        <ord>    <int>
<int>
## 1 Amy    1975     6    27     0  27.5 -79   tropical de~ -1          25
1013
## 2 Amy    1975     6    27     6  28.5 -79   tropical de~ -1          25
1013
## 3 Amy    1975     6    27    12  29.5 -79   tropical de~ -1          25
1013
## 4 Amy    1975     6    27    18  30.5 -79   tropical de~ -1          25
1013
## 5 Amy    1975     6    28     0  31.5 -78.8 tropical de~ -1          25
1012
## 6 Amy    1975     6    28     6  32.4 -78.7 tropical de~ -1          25
1012
## # ... with 2 more variables: ts_diameter <dbl>, hu_diameter <dbl>
```

Reorder the columns so name is first, status is second, category is third and the rest are the same.

```
storms %>%
  select(name, status, category, everything())
```

```
## # A tibble: 10,010 x 13
##    name  status      category  year month   day  hour   lat  long  wind
pressure
##    <chr> <chr>       <ord>    <dbl> <dbl> <int> <dbl> <dbl> <dbl> <int>
<int>
##  1 Amy   tropical d~ -1        1975     6    27     0  27.5 -79      25
1013
##  2 Amy   tropical d~ -1        1975     6    27     6  28.5 -79      25
1013
##  3 Amy   tropical d~ -1        1975     6    27    12  29.5 -79      25
1013
##  4 Amy   tropical d~ -1        1975     6    27    18  30.5 -79      25
1013
##  5 Amy   tropical d~ -1        1975     6    28     0  31.5 -78.8    25
1012
##  6 Amy   tropical d~ -1        1975     6    28     6  32.4 -78.7    25
1012
##  7 Amy   tropical d~ -1        1975     6    28    12  33.3 -78      25
1011
##  8 Amy   tropical d~ -1        1975     6    28    18  34   -77      30
1006
##  9 Amy   tropical s~ 0         1975     6    29     0  34.4 -75.8    35
1004
## 10 Amy   tropical s~ 0         1975     6    29     6  34   -74.8    40
1002
## # ... with 10,000 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storms only in the 1970’s.

```r
storms %>%
  filter(year >= 1970 & year <= 1979)
```

```
## # A tibble: 546 x 13
##    name   year month   day  hour   lat  long status      category  wind
pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>
<int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25
1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25
1013
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25
1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25
1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25
1012
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25
1012
##  7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25
1011
##  8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30
1006
##  9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35
1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40
1002
## # ... with 536 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Find a subset of the data of storm observations only with category 4 and above and wind speed 100MPH and above.

```r
storms %>%
  filter(category >= 4 & wind >= 100)
```

```
## # A tibble: 416 x 13
##    name   year month   day  hour   lat  long status    category  wind
pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>     <ord>    <int>
<int>
##  1 Anita  1977     9     2     0  24.6 -96.2 hurricane 5          140
931
##  2 Anita  1977     9     2     6  24.2 -97.1 hurricane 5          150
926
##  3 Anita  1977     9     2    12  23.7 -98   hurricane 4          120
940
##  4 David  1979     8    28     0  12.2 -52.9 hurricane 4          115
947
##  5 David  1979     8    28     6  12.5 -54.4 hurricane 4          125
```

```
941
##  6 David  1979     8    28    12  12.8 -55.7 hurricane 4          130
938
##  7 David  1979     8    28    18  13.2 -56.9 hurricane 4          125
941
##  8 David  1979     8    29     0  13.7 -58   hurricane 4          120
944
##  9 David  1979     8    29     6  14.2 -59.2 hurricane 4          120
942
## 10 David  1979     8    29    12  14.8 -60.3 hurricane 4          125
938
## # ... with 406 more rows, and 2 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>
```

Create a new feature `wind_speed_per_unit_pressure`.

```
storms %>%
  mutate(wind_speed_per_unit_pressure = wind / pressure)
```

```
## # A tibble: 10,010 x 14
##    name   year month   day  hour   lat  long status      category  wind
pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>
<int>
##  1 Amy    1975     6    27     0  27.5 -79   tropical d~ -1          25
1013
##  2 Amy    1975     6    27     6  28.5 -79   tropical d~ -1          25
1013
##  3 Amy    1975     6    27    12  29.5 -79   tropical d~ -1          25
1013
##  4 Amy    1975     6    27    18  30.5 -79   tropical d~ -1          25
1013
##  5 Amy    1975     6    28     0  31.5 -78.8 tropical d~ -1          25
1012
##  6 Amy    1975     6    28     6  32.4 -78.7 tropical d~ -1          25
1012
##  7 Amy    1975     6    28    12  33.3 -78   tropical d~ -1          25
1011
##  8 Amy    1975     6    28    18  34   -77   tropical d~ -1          30
1006
##  9 Amy    1975     6    29     0  34.4 -75.8 tropical s~ 0           35
1004
## 10 Amy    1975     6    29     6  34   -74.8 tropical s~ 0           40
1002
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, wind_speed_per_unit_pressure <dbl>
```

Create a new feature: `average_diameter` which averages the two diameter metrics. If one is missing, then use the value of the one that is present. If both are missing, leave missing.

```
storms %>%
  rowwise() %>%
  arrange(desc(year)) %>%
  mutate(average_diameter = mean(c(ts_diameter, hu_diameter), na.rm = TRUE))
```

```
## # A tibble: 10,010 x 14
## # Rowwise:
##    name   year month   day  hour   lat  long status      category  wind
pressure
##    <chr> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>       <ord>    <int>
<int>
##  1 Ana    2015     5     9     6  32.2 -77.5 tropical s~ 0           50
998
##  2 Ana    2015     5     9    12  32.5 -77.8 tropical s~ 0           50
1001
##  3 Ana    2015     5     9    18  32.7 -78   tropical s~ 0           45
1001
##  4 Ana    2015     5    10     0  33.1 -78.3 tropical s~ 0           45
1001
##  5 Ana    2015     5    10     6  33.5 -78.6 tropical s~ 0           40
1002
##  6 Ana    2015     5    10    10  33.8 -78.8 tropical s~ 0           40
1002
##  7 Ana    2015     5    10    12  33.9 -78.8 tropical s~ 0           35
1002
##  8 Ana    2015     5    10    18  34.3 -78.7 tropical d~ -1          30
1006
##  9 Ana    2015     5    11     0  34.7 -78.5 tropical d~ -1          30
1009
## 10 Ana    2015     5    11     6  35.5 -78   tropical d~ -1          30
1010
## # ... with 10,000 more rows, and 3 more variables: ts_diameter <dbl>,
## #   hu_diameter <dbl>, average_diameter <dbl>
```

For each storm, summarize the maximum wind speed. "Summarize" means create a new dataframe with only the summary metrics you care about.

```
storms %>%
  group_by(name) %>%
  summarise(max_wind_speed = max(wind, na.rm = TRUE))
```

```
## # A tibble: 198 x 2
##    name     max_wind_speed
##  * <chr>             <int>
##  1 AL011993             30
##  2 AL012000             25
##  3 AL021992             30
##  4 AL021994             30
##  5 AL021999             30
##  6 AL022000             30
##  7 AL022001             25
```

```
##   8 AL022003             30
##   9 AL022006             45
##  10 AL031987             40
## # ... with 188 more rows
```

Order your dataset by maximum wind speed storm but within the rows of storm show the observations in time order from early to late.

```
storms %>%
  group_by(name) %>%
  mutate(max_wind_speed = max(wind, na.rm = TRUE)) %>%
  select(name, max_wind_speed, everything()) %>%
  arrange(desc(max_wind_speed), year, month, day, hour)
```

```
## # A tibble: 10,010 x 14
## # Groups:   name [198]
##    name   max_wind_speed  year month   day  hour   lat  long status
category
##    <chr>           <int> <dbl> <dbl> <int> <dbl> <dbl> <dbl> <chr>
<ord>
##  1 Gilbe~            160  1988     9     8    18  12    -54   tropical ~ -1
##  2 Gilbe~            160  1988     9     9     0  12.7  -55.6 tropical ~ -1
##  3 Gilbe~            160  1988     9     9     6  13.3  -57.1 tropical ~ -1
##  4 Gilbe~            160  1988     9     9    12  14    -58.6 tropical ~ -1
##  5 Gilbe~            160  1988     9     9    18  14.5  -60.1 tropical ~ 0
##  6 Gilbe~            160  1988     9    10     0  14.8  -61.5 tropical ~ 0
##  7 Gilbe~            160  1988     9    10     6  15    -62.8 tropical ~ 0
##  8 Gilbe~            160  1988     9    10    12  15.3  -64.1 tropical ~ 0
##  9 Gilbe~            160  1988     9    10    18  15.7  -65.4 tropical ~ 0
## 10 Gilbe~            160  1988     9    11     0  15.9  -66.8 hurricane  1
## # ... with 10,000 more rows, and 4 more variables: wind <int>, pressure
<int>,
## #   ts_diameter <dbl>, hu_diameter <dbl>
```

Find the strongest storm by wind speed per year.

```
storms %>%
  group_by(year) %>%
  arrange(year, desc(wind)) %>%
  slice(1) %>%
  select(name,year, wind)
```

```
## # A tibble: 41 x 3
## # Groups:   year [41]
##    name       year  wind
##    <chr>     <dbl> <int>
##  1 Caroline  1975   100
##  2 Belle     1976   105
##  3 Anita     1977   150
##  4 Cora      1978    80
##  5 David     1979   150
```

```
##  6 Ivan      1980    90
##  7 Harvey    1981   115
##  8 Debby     1982   115
##  9 Alicia    1983   100
## 10 Diana     1984   115
## # ... with 31 more rows
```

For each named storm, find its maximum category, wind speed, pressure and diameters. Do not allow the max to be NA (unless all the measurements for that storm were NA).

```
maximum_category = max(storms$category)
maximum_wind_speed = max(storms$wind)
maximum_pressure= max(storms$pressure)
maximum_ts_diameter = max(storms$ts_diameters)

## Warning: Unknown or uninitialised column: `ts_diameters`.

## Warning in max(storms$ts_diameters): no non-missing arguments to max;
returning
## -Inf

maximum_hu_diameter = max(storms$hu_diameter)
maximum_hu_diameter

## [1] NA

maximum_pressure

## [1] 1022

maximum_wind_speed

## [1] 160

maximum_category

## [1] 5
## Levels: -1 < 0 < 1 < 2 < 3 < 4 < 5

maximum_ts_diameter

## [1] -Inf
```

For each year in the dataset, tally the number of storms. "Tally" is a fancy word for "count the number of". Plot the number of storms by year. Any pattern?

```
storms %>%
  group_by(year) %>%
  tally()

## # A tibble: 41 x 2
##     year     n
##   * <dbl> <int>
```

```
## 1  1975    86
## 2  1976    52
## 3  1977    53
## 4  1978    54
## 5  1979   301
## 6  1980   161
## 7  1981   164
## 8  1982   105
## 9  1983    79
## 10 1984   236
## # ... with 31 more rows
```

For each year in the dataset, tally the storms by category.

```
storms %>%
  group_by(year, category) %>%
  tally()
```

```
## # A tibble: 233 x 3
## # Groups:   year [41]
##     year category     n
##    <dbl> <ord>    <int>
##  1  1975 -1          30
##  2  1975 0           33
##  3  1975 1           12
##  4  1975 2            9
##  5  1975 3            2
##  6  1976 -1          10
##  7  1976 0           20
##  8  1976 1           10
##  9  1976 2            9
## 10  1976 3            3
## # ... with 223 more rows
```

For each year in the dataset, find the maximum wind speed per status level.

```
storms %>%
  group_by(year, wind) %>%
  tally()
```

```
## # A tibble: 837 x 3
## # Groups:   year [41]
##     year  wind     n
##    <dbl> <int> <int>
##  1  1975    20     2
##  2  1975    25    25
##  3  1975    30     3
##  4  1975    35     2
##  5  1975    40     2
##  6  1975    45     2
##  7  1975    50     7
```

```
## 8  1975    55     9
## 9  1975    60    11
## 10 1975    65     5
## # ... with 827 more rows
```

For each storm, summarize its average location in latitude / longitude coordinates.

```
storms %>%
  group_by(name, lat) %>%
  tally()
```

```
## # A tibble: 8,170 x 3
## # Groups:   name [198]
##    name       lat     n
##    <chr>    <dbl> <int>
##  1 AL011993  21.5     1
##  2 AL011993  22.3     1
##  3 AL011993  23.2     1
##  4 AL011993  24.5     1
##  5 AL011993  25.4     1
##  6 AL011993  26.1     1
##  7 AL011993  26.7     1
##  8 AL011993  27.8     1
##  9 AL012000  20.7     1
## 10 AL012000  20.8     1
## # ... with 8,160 more rows
```

For each storm, summarize its duration in number of hours (to the nearest 6hr increment).

```
storms %>%
  group_by(name, hour<=6) %>%
  tally()
```

```
## # A tibble: 396 x 3
## # Groups:   name [198]
##    name     `hour <= 6`     n
##    <chr>    <lgl>       <int>
##  1 AL011993 FALSE           4
##  2 AL011993 TRUE            4
##  3 AL012000 FALSE           2
##  4 AL012000 TRUE            2
##  5 AL021992 FALSE           3
##  6 AL021992 TRUE            2
##  7 AL021994 FALSE           3
##  8 AL021994 TRUE            3
##  9 AL021999 FALSE           1
## 10 AL021999 TRUE            3
## # ... with 386 more rows
```

For storm in a category, create a variable `storm_number` that enumerates the storms 1, 2, … (in date order).

```
storms %>%
  group_by(category, name) %>%
  tally()
```

```
## # A tibble: 687 x 3
## # Groups:   category [7]
##    category name         n
##    <ord>    <chr>    <int>
##  1 -1       AL011993     8
##  2 -1       AL012000     4
##  3 -1       AL021992     5
##  4 -1       AL021994     6
##  5 -1       AL021999     4
##  6 -1       AL022000    12
##  7 -1       AL022001     5
##  8 -1       AL022003     4
##  9 -1       AL022006     1
## 10 -1       AL031987    28
## # ... with 677 more rows
```

Convert year, month, day, hour into the variable `timestamp` using the `lubridate` package. Although the new package `clock` just came out, `lubridate` still seems to be standard. Next year I'll probably switch the class to be using `clock`.

```
pacman::p_load("lubridate")
storms_date = storms %>%
  mutate(Date = paste(year, month, day, hour, sep = "-")) %>%
  select(name, Date)

storms_date
```

```
## # A tibble: 10,010 x 2
##    name  Date
##    <chr> <chr>
##  1 Amy   1975-6-27-0
##  2 Amy   1975-6-27-6
##  3 Amy   1975-6-27-12
##  4 Amy   1975-6-27-18
##  5 Amy   1975-6-28-0
##  6 Amy   1975-6-28-6
##  7 Amy   1975-6-28-12
##  8 Amy   1975-6-28-18
##  9 Amy   1975-6-29-0
## 10 Amy   1975-6-29-6
## # ... with 10,000 more rows
```

Using the `lubridate` package, create new variables `day_of_week` which is a factor with levels "Sunday", "Monday", … "Saturday" and `week_of_year` which is integer 1, 2, …, 52.

```
storms_date %>%
  mutate(day_of_week = wday(storms_date$Date, label = TRUE))
```

```
## # A tibble: 10,010 x 3
##    name  Date          day_of_week
##    <chr> <chr>         <ord>
##  1 Amy   1975-6-27-0   Fri
##  2 Amy   1975-6-27-6   Fri
##  3 Amy   1975-6-27-12  Fri
##  4 Amy   1975-6-27-18  Fri
##  5 Amy   1975-6-28-0   Sat
##  6 Amy   1975-6-28-6   Sat
##  7 Amy   1975-6-28-12  Sat
##  8 Amy   1975-6-28-18  Sat
##  9 Amy   1975-6-29-0   Sun
## 10 Amy   1975-6-29-6   Sun
## # ... with 10,000 more rows
```

```
storms_date
```

```
## # A tibble: 10,010 x 2
##    name  Date
##    <chr> <chr>
##  1 Amy   1975-6-27-0
##  2 Amy   1975-6-27-6
##  3 Amy   1975-6-27-12
##  4 Amy   1975-6-27-18
##  5 Amy   1975-6-28-0
##  6 Amy   1975-6-28-6
##  7 Amy   1975-6-28-12
##  8 Amy   1975-6-28-18
##  9 Amy   1975-6-29-0
## 10 Amy   1975-6-29-6
## # ... with 10,000 more rows
```

For each storm, summarize the day in which is started in the following format “Friday, June 27, 1975”.

```
storms_date %>%
  mutate(day_of_week = wday(storms_date$Date, label = TRUE)) %>%
  mutate(Month = month(storms_date$Date, label = TRUE))
```

```
## # A tibble: 10,010 x 4
##    name  Date          day_of_week Month
##    <chr> <chr>         <ord>       <ord>
##  1 Amy   1975-6-27-0   Fri         Jun
##  2 Amy   1975-6-27-6   Fri         Jun
##  3 Amy   1975-6-27-12  Fri         Jun
##  4 Amy   1975-6-27-18  Fri         Jun
##  5 Amy   1975-6-28-0   Sat         Jun
##  6 Amy   1975-6-28-6   Sat         Jun
##  7 Amy   1975-6-28-12  Sat         Jun
##  8 Amy   1975-6-28-18  Sat         Jun
##  9 Amy   1975-6-29-0   Sun         Jun
```

```
## 10 Amy   1975-6-29-6  Sun         Jun
## # ... with 10,000 more rows
```

```
storms_date
```

```
## # A tibble: 10,010 x 2
##    name  Date
##    <chr> <chr>
##  1 Amy   1975-6-27-0
##  2 Amy   1975-6-27-6
##  3 Amy   1975-6-27-12
##  4 Amy   1975-6-27-18
##  5 Amy   1975-6-28-0
##  6 Amy   1975-6-28-6
##  7 Amy   1975-6-28-12
##  8 Amy   1975-6-28-18
##  9 Amy   1975-6-29-0
## 10 Amy   1975-6-29-6
## # ... with 10,000 more rows
```

Create a new factor variable `decile_windspeed` by binning wind speed into 10 bins.

```
#TO-DO
```

Create a new data frame `serious_storms` which are category 3 and above hurricanes.

```
storms %>%
  mutate(serious_storms =category >=3) %>%
  select(name, serious_storms)
```

```
## # A tibble: 10,010 x 2
##    name  serious_storms
##    <chr> <lgl>
##  1 Amy   FALSE
##  2 Amy   FALSE
##  3 Amy   FALSE
##  4 Amy   FALSE
##  5 Amy   FALSE
##  6 Amy   FALSE
##  7 Amy   FALSE
##  8 Amy   FALSE
##  9 Amy   FALSE
## 10 Amy   FALSE
## # ... with 10,000 more rows
```

In `serious_storms`, merge the variables lat and long together into `lat_long` with values `lat / long` as a string.

```
storms %>%
  mutate(lat_long = paste(lat, long, sep = ", ")) %>%
  select(name, lat_long)
```

```
## # A tibble: 10,010 x 2
##    name  lat_long
##    <chr> <chr>
##  1 Amy   27.5, -79
##  2 Amy   28.5, -79
##  3 Amy   29.5, -79
##  4 Amy   30.5, -79
##  5 Amy   31.5, -78.8
##  6 Amy   32.4, -78.7
##  7 Amy   33.3, -78
##  8 Amy   34, -77
##  9 Amy   34.4, -75.8
## 10 Amy   34, -74.8
## # ... with 10,000 more rows
```

Let's return now to the original storms data frame. For each category, find the average wind speed, pressure and diameters (do not count the NA's in your averaging).

```
  Average_wind = mean(storms$wind)
  Average_pressure = mean(storms$pressure)
  Average_ts_diamenter = mean(storms$ts_diameter)
  Average_hu_diamter= mean(storms$hu_diameter)
```

For each named storm, find its maximum category, wind speed, pressure and diameters (do not allow the max to be NA) and the number of readings (i.e. observations).

```
## this a repeat question from above in line 259
```

Calculate the distance from each storm observation to Miami in a new variable `distance_to_miami`. This is very challenging. You will need a function that computes distances from two sets of latitude / longitude coordinates.

```
MIAMI_LAT_LONG_COORDS = c(25.7617, -80.1918)
distance_to_miami = c(NA)
distance_to_miami_left = c(NA)
distance_to_miami_right =c(NA)
distance = function(d) {
  for (i in 1:d) {

      distance_to_miami_left[i] = MIAMI_LAT_LONG_COORDS[1]-storms$lat[i]
      distance_to_miami_right[i] = MIAMI_LAT_LONG_COORDS[2] - storms$long[i]
    }
    distance_to_miami[i] = c(distance_to_miami_left[i],
distance_to_miami_right[i])
  }
```

For each storm observation, use the function from the previous question to calculate the distance it moved since the previous observation.

```
#TO-DO
```

For each storm, find the total distance it moved over its observations and its total displacement. "Distance" is a scalar quantity that refers to "how much ground an object has covered" during its motion. "Displacement" is a vector quantity that refers to "how far out of place an object is"; it is the object's overall change in position.

```
#TO-DO
```

For each storm observation, calculate the average speed the storm moved in location.

```
#TO-DO
```

For each storm, calculate its average ground speed (how fast its eye is moving which is different from windspeed around the eye).

```
#TO-DO
```

Is there a relationship between average ground speed and maximum category attained? Use a dataframe summary (not a regression).

```
#TO-DO
```

Now we want to transition to building real design matrices for prediction. This is more in tune with what happens in the real world. Large data dump and you convert it into X and y how you see fit.

Suppose we wish to predict the following: given the first three readings of a storm, can you predict its maximum wind speed? Identify the `y` and identify which features you need $x_1, \ldots x_p$ and build that matrix with `dplyr` functions. This is not easy, but it is what it's all about. Feel free to "featurize" as creatively as you would like. You aren't going to overfit if you only build a few features relative to the total 198 storms.

```
#TO-DO
```

Fit your model. Validate it.

```
#TO-DO
```

Assess your level of success at this endeavor.

#TO-DO

The Forward Stepwise Procedure for Probability Estimation Models

Set a seed and load the `adult` dataset and remove missingness and randomize the order.

```
set.seed(1)
pacman::p_load_gh("coatless/ucidata")
data(adult)
adult = na.omit(adult)
adult = adult[sample(1 : nrow(adult)), ]
```

Copy from the previous lab all cleanups you did to this dataset.

```
#TO-DO
```

We will be doing model selection. We will split the dataset into 3 distinct subsets. Set the size of our splits here. For simplicitiy, all three splits will be identically sized. We are making it small so the stepwise algorithm can compute quickly. If you have a faster machine, feel free to increase this.

```
Nsplitsize = 1000
```

Now create the following variables: `Xtrain`, `ytrain`, `Xselect`, `yselect`, `Xtest`, `ytest` with `Nsplitsize` observations. Binarize the y values.

```
Xtrain = adult[1 : Nsplitsize, ]
Xtrain$income = NULL
ytrain = ifelse(adult[1 : Nsplitsize, "income"] == ">50K", 1, 0)
Xselect = adult[(Nsplitsize + 1) : (2 * Nsplitsize), ]
Xselect$income = NULL
yselect = ifelse(adult[(Nsplitsize + 1) : (2 * Nsplitsize), "income"]
==">50K", 1, 0)
Xtest = adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), ]
Xtest$income = NULL
ytest = ifelse(adult[(2 * Nsplitsize + 1) : (3 * Nsplitsize), "income"] ==
">50K", 1, 0)
```

Fit a vanilla logistic regression on the training set.

```
logistic_mod = glm(ytrain ~ ., Xtrain, family = "binomial")

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

and report the log scoring rule, the Brier scoring rule.

```
#TO-DO
```

We will be doing model selection using a basis of linear features consisting of all first-order interactions of the 14 raw features (this will include square terms as squares are interactions with oneself).

Create a model matrix from the training data containing all these features. Make sure it has an intercept column too (the one vector is usually an important feature). Cast it as a data frame so we can use it more easily for modeling later on. We're going to need those model matrices (as data frames) for both the select and test sets. So make them here too (copy-paste). Make sure their dimensions are sensible.

```
#TO-DO
"
dim(Xmm_train)
dim(Xmm_select)
dim(Xmm_test)
"
```

```
## [1] "\ndim(Xmm_train)\ndim(Xmm_select)\ndim(Xmm_test)\n"
```

Write code that will fit a model stepwise. You can refer to the chunk in the practice lecture. Use the negative Brier score to do the selection. The negative of the Brier score is always positive and lower means better making this metric kind of like s_e so the picture will be the same as the canonical U-shape for oos performance.

Run the code and hit "stop" when you begin to the see the Brier score degrade appreciably oos. Be patient as it will wobble.

```
pacman::p_load(Matrix)
"
p_plus_one = ncol(Xmm_train)
predictor_by_iteration = c() #keep a growing list of predictors by iteration
in_sample_brier_by_iteration = c() #keep a growing list of briers by
iteration
oos_brier_by_iteration = c() #keep a growing list of briers by iteration
i = 1

repeat {

  #TO-DO
  #wrap glm and predict calls with use suppressWarnings() so the console is
clean during run

  if (i > Nsplitsize || i > p_plus_one){
    break
  }
}
"

## [1] "\np_plus_one = ncol(Xmm_train)\npredictor_by_iteration = c() #keep a
growing list of predictors by iteration\nin_sample_brier_by_iteration = c()
#keep a growing list of briers by iteration\noos_brier_by_iteration = c()
#keep a growing list of briers by iteration\ni = 1\n\nrepeat {\n\n  #TO-DO \n
#wrap glm and predict calls with use suppressWarnings() so the console is
clean during run\n  \n  if (i > Nsplitsize || i > p_plus_one){\n    break\n
}\n}\n"
```

Plot the in-sample and oos (select set) Brier score by p. Does this look like what's expected?

```
#TO-DO
```