The Theoretical Miniminum: Classical Physics

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Contents

Chapter 1

Lecture 1 - The nature of Classical Physics

1.1 Simple Dynamical Systems and the space of States

- The collection of all states occupied by a system is called its state-space.
- A world whose evolution is discrete could be called stroboscopic.
- A system that changes with time is called a dynamic system.
- The dynamical law is a rule that tells us the next state given the current state.
- The variables describing a system are called its degrees of freedom.

1.2 Rules that are not allowed: The Minus-First Law

- According to the principles of classical physics, a dynamic law must be reversible.
- It must obey the conservation of information, which tells you that every state has one arrow in and one arrow out.

1.3 Interlude 1: Spaces, trigonometry, and vectors - Vectors

- Magnitude of a vector

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2} \tag{1.1}$$

- Multiplication of vectors

$$\vec{A} \times \vec{B} = |\vec{A}| \times |\vec{B}| \cos\theta \tag{1.2}$$

- If the dot product of the vectors is 0, they are perpendicular.

1.4 Interlude 1 Exercises

Exercise 3: Show that the magnitude of a vector satisfies $|\vec{A}|^2 = \vec{A} \cdot \vec{A}$

$$\begin{aligned} &|\vec{A}| = \sqrt{A_x^2 + Ay^2 + A_z^2} \\ &|\vec{A}|^2 = Ax^2 + Ay^2 + Az^2 \\ &\vec{A} \cdot \vec{A} = A_x Ax + AyAy + AzAz = |\vec{A}|^2 \end{aligned} \tag{1.3}$$

Exercise 4: Let $(A_x = 2, \quad A_y = -3, A_z = 1)$ **and** $(B_x = -4, B_y = -3, B_z = 2)$**. Compute the mag- nitude of a** $\vec{A}$ **and** $\vec{B}$**, their dot product, and the angle between them.**

$$\begin{aligned}
&A_x = 2; A_y = -3; A_z = 1\\
&Bx = -4; By = -3; B_z = 2\\
&|\vec{A}| = \sqrt{4+9+1} = \sqrt{14}\\
&|\vec{B}| = \sqrt{16+9+4} = \sqrt{29}\\
&\vec{A}\cdot\vec{B} = 2\cdot(-4) + (-3)(-3) + 2 = -8 + 9 + 2 = 3\\
&|\vec{A}||\vec{B}|\cos\theta = 3 \Leftrightarrow \sqrt{14}\cdot\sqrt{29}\cdot\cos\theta = 3 \Leftrightarrow \cos\theta = \tfrac{3}{\sqrt{406}}\\
&\Leftrightarrow \theta = \cos^{-1}\left(\tfrac{3}{\sqrt{40^6}}\right) \cong 1, 4.2\text{rad}
\end{aligned} \tag{1.4}$$

Exercise 5: Determine which pair of vectors are orthogonal. $(1, 1, 1)(2, -1, 3)(3, 1, 0)(-3, 0, 2)$

$$\begin{aligned}
&(1,1,1)(2,-1,3) = x + -1 + 3 = 4\\
&(1,1,1)(3,1,0) = 3 + 2 + 0 = 4\\
&(1,1,1)(-3,0,2) = -3 + 0 + 2 = 1\\
&(2,-1,3)(3,1,0) = 6 - 1 + 0 = 5\\
&(2,-1,3)(-3,0,2) = -6 + 0 + 6 = -0\\
&(3,1,0)(-3,0,2) = -9 + 0 + 2 = -7
\end{aligned} \tag{1.5}$$

Exercise 6: Can you explain why the dot product of two vectors that are orthogonal is 0 ?

Because $\cos\left(\frac{\pi}{2}\right) = 0$.

Chapter 2

Lecture 2 - Motion

2.1 Mathematatial Interlude: Differential Calculus

- Definition of derivative

$$\frac{df(t)}{dt} = \lim_{\Delta t \to 0} \frac{\Delta f}{\Delta t} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} \tag{2.1}$$

- Sum rule

$$\frac{d(f+g)}{dt} = \frac{d(f)}{dt} + \frac{d(g)}{dt} \tag{2.2}$$

- Product rule

$$\frac{d(fg)}{dt} = f(t)\frac{d(g)}{dt} + g(t)\frac{d(f)}{dt} \tag{2.3}$$

- Chain rule

$$\frac{df}{dt} = \frac{df}{dg}\frac{dg}{dt} \tag{2.4}$$

  if f is a function of g.

- $\frac{d(t^n)}{dt} = nt^{n-1}$
- $\frac{d(\sin t)}{dt} = \cos t$
- $\frac{d(\cos t)}{dt} = -\sin t$
- $\frac{d\left(e^t\right)}{dt} = e^t$
- $\frac{d(\log t)}{dt} = \frac{1}{t}$

2.2 Particle Motion

- The position of a particle at time t can be described by $\vec{r}(t)$.
- The job of classical mechanics is to figure out $\vec{r}(t)$ from some initial condition and some dynamical law.
- The velocity is given by the rate of change of the position

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} \tag{2.5}$$

- The velocity vector has magnitude $|\vec{v}|$

$$|\vec{v}|^2 = v_x^2 + v_y^2 + v_z^2 \tag{2.6}$$

which represents how fast the particle is moving without regard to the direction. This is called *speed*.

- Acceleration is the quantity that tells you how the velocity is changing. A constant velocity vector not onbly implies constant speed, but also a constant direction.

$$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} \tag{2.7}$$

2.3 Interlude 2: Integral Calculus

- Differential Calculus studies the rates of change. Integral Calculus has to do with sums of many tiny incremental quantities. These are connected.
- Consider a rectangle located at value t, with width Δt and height $f(t)$. It follows that the area of a single rectangle δA is

$$\delta A = f(t)\Delta t \tag{2.8}$$

- Adding up all the incremental areas

$$A = \sum_i f(t_i)\Delta t \tag{2.9}$$

- To get the exact answer, we take the limit in which Δt shrinks to zero, and the number of rectangles increases to infinity

$$A = \int_a^b f(t)dt = \lim_{\Delta t \to 0} f(t_i)\Delta t \tag{2.10}$$

which is the definition of an integral between a and b.

- The indefinite integral of $f(t)$ is

$$F(T) = \int_a^T f(t)dt \tag{2.11}$$

where T is an unknown variable which can take any value of t.

- The *fundamental theorem of calculus* states that

$$f(t) = \frac{dF(t)}{dt} \tag{2.12}$$

- The process of integration and differentiation are reciprocal: The derivative of the integral is the original integrand.
- Can we completely determine $F(t)$ knowing that its derivative is $f(t)$. Almost, because adding a costant to $F(t)$ doesnt change its derivative.
- Given $f(t)$, its indefinite integral is ambiguous up to a constant.
- An alternative way to express the fundamental theorem is

$$\int_a^b f(t)dt = F(t)|_a^b = F(b) - F(a) \tag{2.13}$$

or

$$\int \frac{df}{dt}dt = f(t) + c \tag{2.14}$$

2.4 Lecture 2 Exercises

Exercise 1: Calculate the derivatives of each of these functions.
$f(t) = t^4 + 3t^3 - 12t^2 + t - 6$
$g(x) = \sin x - \cos x$
$\theta(\alpha) = e^{\alpha} + \alpha \ln \alpha$
$x(t) = \sin^2 x - \cos x$

a)

- $$\frac{d(f+g)}{dt} = \frac{df}{dt} + \frac{dg}{dt}$$
- $$\frac{df}{dt} = 4t^3 + 9t^2 - 24t + 1$$

b)

- $$\frac{dg}{dt} = \cos x + \operatorname{sen} x$$

c)

- $$\frac{d\theta}{dt} = e^{\alpha} + \alpha\frac{1}{\alpha} = e^{\alpha}$$

d)

- $$\frac{d(fg)}{dt} = f(t)\frac{dg}{dt} + g(t)\frac{dt}{dt}$$
- $$\sin^2 t = \sin t \cdot \sin t$$
- $$\frac{d\sin^2}{dt} = \sin t \cos t + \sin t \cos t = 2 \sin t \cos t$$
- $$\frac{dx}{dt} = 2 \sin t \cos t + \sin t$$

Chapter 3

Theoretical Minimum - Dynamics

3.1 Aristotle's Law of Motion

- The velocity of any object is proportional to the total applied force.

$$\vec{F} = m\vec{v} \tag{3.1}$$

- m is a quantity describing the resistance of the body to being moved.
- Aristotle's law is wrong.

3.2 Mass, Acceleration and Force

- Aristotle's mistake was to think that a net applied force is needed to keep an object moving.
- The right idea is that the applied force is needed to overcome the force of friction.
- An isolated object movign in free space, with no forces acting on it, requires nothing to keep it moving. This is the *law of inertia*.
- This involves a change in the velocity of an object, and therefore an acceleration.
- Newton's second law of motion

$$\vec{F} = m\vec{a} \tag{3.2}$$

3.3 An Interlude on Units

- Velocity

$$[v] = \left[\frac{length}{time}\right] = \frac{m}{s} \tag{3.3}$$

- Acceleration

$$[a] = \left[\frac{length}{time}\right]\left[\frac{1}{time}\right] = \frac{m}{s^2} \tag{3.4}$$

- Force

$$[F] = \left[\frac{mass \times length}{time^2}\right] = \frac{kg \times m}{s^2} = N(Newton) \tag{3.5}$$

3.4 Some Simple Examples of Solving Newton's Equations

- $$m\frac{d\vec{v}}{dt} = 0 \implies \vec{v}(t) = \vec{v}(0) \tag{3.6}$$

  This is newtons first law of motion: *Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.*

- The first law is simply a special case of the second law when the force is zero.

3.5 Interlude 3: Partial Differentiation - Partial Derivatives

- Assume that for every value of x, y and z, there is a unique value $V(x, y, z)$ that varies smoothly as we vary the coordinates.
- Multivariable differential calculus revolves around the concept of *partial derivatives*.
- If we want to change x while keeping y and z fixed

  $$\frac{dV}{dx} = \lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \frac{\partial V}{\partial x} \tag{3.7}$$

  where

  $$\Delta V = V([x + \Delta x], y, z) - V(x, y, z) \tag{3.8}$$

- The second derivative is

  $$\frac{\partial^2 V}{\partial x^2} = \partial_x \left(\frac{\partial V}{\partial x}\right) = \partial_{x,x} V \tag{3.9}$$

- And a mixed partial derivative

  $$\frac{\partial^2 V}{\partial x\, \partial y} = \partial_x \left(\frac{\partial V}{\partial y}\right) = \partial_{x,y} V \tag{3.10}$$

  which does not depend on the order in which the derivatives are carried out on.

Bibliography