0.1 STAGGERED QUANTUM WALK

Similarly to the continuous-time quantum walk, the staggered case aims to spread a transition probability to neighboring vertices but with discrete time steps. The notion of adjacency now comes from cliques[1], and the initial stage of this walk consists in partitioning the graph in several different cliques. This is called tessellation, and it is defined as the division of the set of vertices into disjoint cliques. An element of a tessellation $\mathscr{T}$ is called a polygon, and it's only valid if if all of its vertices belong to the clique in $\mathscr{T}$. The set of polygons of each tessellation must cover all vertices of the graph, and the set of tessellations $(\mathscr{T}_1, \mathscr{T}_2, ..., \mathscr{T}_k)$ must cover all edges.

These definitions allow the construction of operators $H_1, H_2, ..., H_k$ that will be used to propagate the probability amplitude locally, in each polygon. The state associated to each polygon is

$$\left|u_j^k\right\rangle = \frac{1}{\sqrt{|\alpha_j^k|}} \sum_{l \in \alpha_j^k} |l\rangle \tag{1}$$

where α_j^k is the $j-th$ polygon in the $k-th$ tessellation.

The unitary and Hermitian operator H_k, associated to each tessellation is defined in **?** as

$$H_k = 2 \sum_{j=1}^{p} \left|u_j^k\right\rangle \left\langle u_j^k\right| - I \tag{2}$$

Solving the time-independent Schrodinger equation for this Hamiltonian gives the evolution operator

$$U = e^{i\theta_k H_k} ... e^{i\theta_2 H_2} e^{i\theta_1 H_1} \tag{3}$$

where

$$e^{i\theta_k H_k} = \cos(\theta_k) I + i \sin(\theta_k) H_k \tag{4}$$

since $H_k^2 = I$.trocar and por since. Posso referir livro do nielsen

The simplest use case of this quantum walk model is the one-dimensional lattice, where the minimum tessalations are

$$\mathscr{T}_\alpha = \{\{2x, 2x+1\} : x \in Z\} \tag{5}$$

$$\mathscr{T}_\beta = \{\{2x+1, 2x+2\} : x \in Z\} \tag{6}$$

1 A clique is defined as the subset of vertices of an undirected graph such that every two distinct vertices in each clique are adjacent.

Each element of the tessellation has a corresponding state, and the uniform superposition of these states is

$$|\alpha_x\rangle = \frac{|2x\rangle + |2x+1\rangle}{\sqrt{2}} \tag{7}$$

$$|\beta_x\rangle = \frac{|2x+1\rangle + |2x+2\rangle}{\sqrt{2}} \tag{8}$$

One can now define Hamiltonians H_α and H_β as trocar operators por hamiltonians, talvez

$$H_\alpha = 2\sum_{x=-\infty}^{+\infty} |\alpha_x\rangle\langle\alpha_x| - I \tag{9}$$

$$H_\beta = 2\sum_{x=-\infty}^{+\infty} |\beta_x\rangle\langle\beta_x| - I \tag{10}$$

The Hamiltonian evolution operator reduces to

$$U = e^{i\theta H_\beta} e^{i\theta H_\alpha} \tag{11}$$

and applying it to an initial condition $|\Psi(0)\rangle$ results in the time evolution operator

$$U|\Psi(t)\rangle = U^t|\Psi(0)\rangle \tag{12}$$

Having defined the time evolution operator, the walk is ready to be coded with a certain initial condition and θ value, to better understand how the probability distribution spreads through time. você pode acrescentar que o θ terá um papel similar ao γ no controle do desvio-padrão

For the first case study, the initial condition will be a uniform superposition of states $|0\rangle$ and $|1\rangle$ and the θ value will be varied in order to understand how this parameter impacts the walk

img/StagQuantumWalk/stagqwMultiple.png

Figure 1: Probability distribution for the staggered quantum walk on a line after 50 steps, with initial condition $|\Psi(0)\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, for multiple angles.

The overall structure of the probability distribution remains the same, the difference is that the walker is more likely to be found further away from the origin as the angle increases.

Another interesting case study is to see how the initial condition affects the dynamics of the system

Similarly to the coined case, each initial condition results in asymmetric probability distributions, $|\Psi(0)\rangle = |0\rangle$ leads to a peak in the left-hand side while condition $|\Psi(0)\rangle = |1\rangle$

img/StagQuantumWalk/stagqwSingle0.png

Figure 2: $|\Psi(0)\rangle = |0\rangle$

img/StagQuantumWalk/stagqwSingle1.png

Figure 3: $|\Psi(0)\rangle = |1\rangle$

results in a peak in the right-hand side. As was shown in **??**, the uniform superposition of both these conditions results in a symmetric probability distribution. acho que você pode explicar um pouco mais o papel de θ através dos gráficos e podemos pensar se fazemos gráficos do desvio-padrão para este e o contínuo