2.3 STAGGERED QUANTUM WALK

The staggered quantum walk (SQW) model aims to spread a transition probability to neighboring vertices with discrete time steps. The notion of adjacency comes from cliques[3], and the initial stage of this walk consists of partitioning the graph into several different cliques. This is known as the *tessellation* process. An element of a tessellation $\mathscr{T}$ is called a polygon, and it is only valid if all of its vertices belong to the clique in $\mathscr{T}$. The set of polygons of each tessellation must cover all vertices of the graph, and the set of tessellations $\{\mathscr{T}_1,\mathscr{T}_2,...,\mathscr{T}_k\}$ must cover all the edges.

These definitions allow the construction of operators $H_1,H_2,...,H_k$ to propagate the probability amplitude locally, in each polygon. The state associated to each polygon is

$$\left|u_j^k\right\rangle = \frac{1}{\sqrt{|\alpha_j^k|}} \sum_{l\in\alpha_j^k} |l\rangle\,, \tag{16}$$

where α_j^k is the j^{th} polygon in the k^{th} tessellation.

The unitary, local and Hermitian operator H_k, associated to each tessellation is defined in Portugal et al. (2017) as

$$H_k = 2\sum_{j=1}^{p} \left|u_j^k\right\rangle \left\langle u_j^k\right| - I. \tag{17}$$

Solving the time-independent Schrodinger equation for this Hamiltonian gives the evolution operator

$$U = e^{i\theta_k H_k}...e^{i\theta_2 H_2}e^{i\theta_1 H_1}, \tag{18}$$

where

$$e^{i\theta_k H_k} = \cos(\theta_k)I + i\sin(\theta_k)H_k, \tag{19}$$

since $H_k^2 = I$, meaning that the Hamiltonian is a reflection operator that, when expanded in a Taylor series, generates a local operator.

The simplest use case of this quantum walk model is the one-dimensional lattice, where the minimum tessellations are

$$\mathscr{T}_\alpha = \{\{2x, 2x+1\} \colon x \in \mathbb{Z}\}, \tag{20}$$

$$\mathscr{T}_\beta = \{\{2x+1, 2x+2\} \colon x \in \mathbb{Z}\}. \tag{21}$$

Each element of the tessellation has a corresponding state, as can be seen in figure 6, and

3 A clique is defined as the subset of vertices of an undirected graph such that every two distinct vertices in each clique are adjacent.

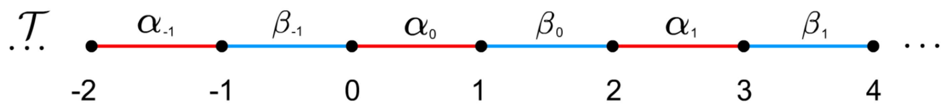

Figure 6: Tessellation of a line graph.

the uniform superposition of these states is

$$|\alpha_x\rangle = \frac{|2x\rangle + |2x+1\rangle}{\sqrt{2}}, \tag{22}$$

$$|\beta_x\rangle = \frac{|2x+1\rangle + |2x+2\rangle}{\sqrt{2}}. \tag{23}$$

One can now define Hamiltonians H_α and H_β as

$$H_\alpha = 2\sum_{x=-\infty}^{+\infty} |\alpha_x\rangle\langle\alpha_x| - I, \tag{24}$$

$$H_\beta = 2\sum_{x=-\infty}^{+\infty} |\beta_x\rangle\langle\beta_x| - I. \tag{25}$$

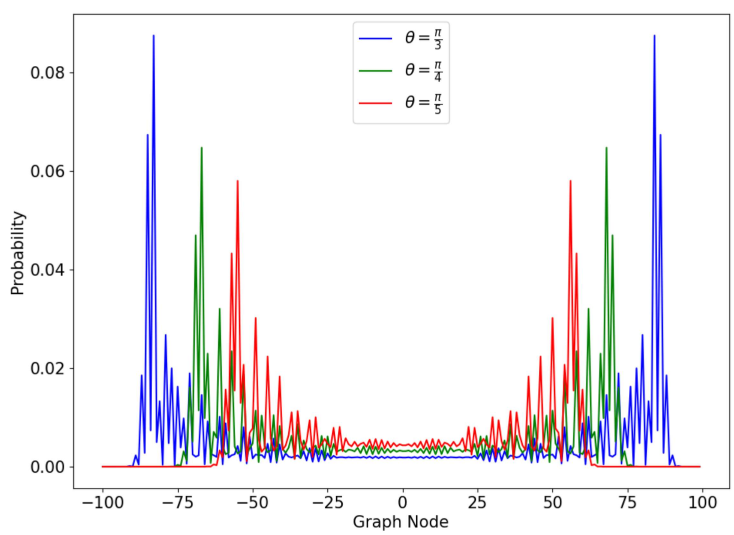

Figure 7: Probability distribution for the staggered quantum walk on a line after 50 steps, with initial condition $|\psi(0)\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, for multiple values of θ.