Optimization Method & Optimal Guidance

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Contents

Trajectory Optimization Practice

FALCON.m is the *FSD optimAL CONtrol tool for MATLAB* that has been developed at the Institute of *Flight System Dynamics* of *Technische Universität München*.

Trajectory Optimization Practice

FALCON.m is able to solve optimal control problems of the following form:
Minimize the cost function

$$\min \; J(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p})$$

subject to a set of constraints, formed by the differential algebraic equation

$$\begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \boldsymbol{y}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}) \\ \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}) \end{bmatrix}$$

where $\boldsymbol{x}(t)$ specifies the states, $\dot{\boldsymbol{x}}(t)$ the state derivatives and $\boldsymbol{y}(t)$ additional model outputs.

Remark: A maximization of the cost function $\bar{J}$ can be achieved by simply choosing

$$J = -\bar{J}.$$

The states $\boldsymbol{x}(t)$, the controls $\boldsymbol{u}(t)$ and the parameters $\boldsymbol{p}$ are limited by a lower and an upper bound:

$$
\begin{aligned}
\boldsymbol{x}_{lb} \leq& \boldsymbol{x}(t) \leq \boldsymbol{x}_{ub} \\
\boldsymbol{u}_{lb} \leq& \boldsymbol{u}(t) \leq \boldsymbol{u}_{ub} \\
\boldsymbol{p}_{lb} \leq& \boldsymbol{p} \leq \boldsymbol{p}_{ub}
\end{aligned}
$$

The problem is considered on the time interval $[t_0, t_f]$ with each of the two either being fixed or free. In the formulation presented here, t_0 and t_f are seen to be part of the parameter vector $\boldsymbol{p}$.

Additionally, an arbitrary number of nonlinear constraints of the form

$$\boldsymbol{g}_{lb} \leq \boldsymbol{g}(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{p}) \leq \boldsymbol{g}_{ub}$$

may be imposed. A special type of constraints appearing in many problems are initial and final boundary conditions specifying a start and an end state condition of the form

$$\begin{aligned} \boldsymbol{x}_{0,lb} &\leq \boldsymbol{x}(t_0) \leq \boldsymbol{x}_{0,ub} \\ \boldsymbol{x}_{f,lb} &\leq \boldsymbol{x}(t_f) \leq \boldsymbol{x}_{f,ub} \end{aligned}$$

For all constraints, equality conditions can be achieved by simply setting the upper and the lower limits to the same values.

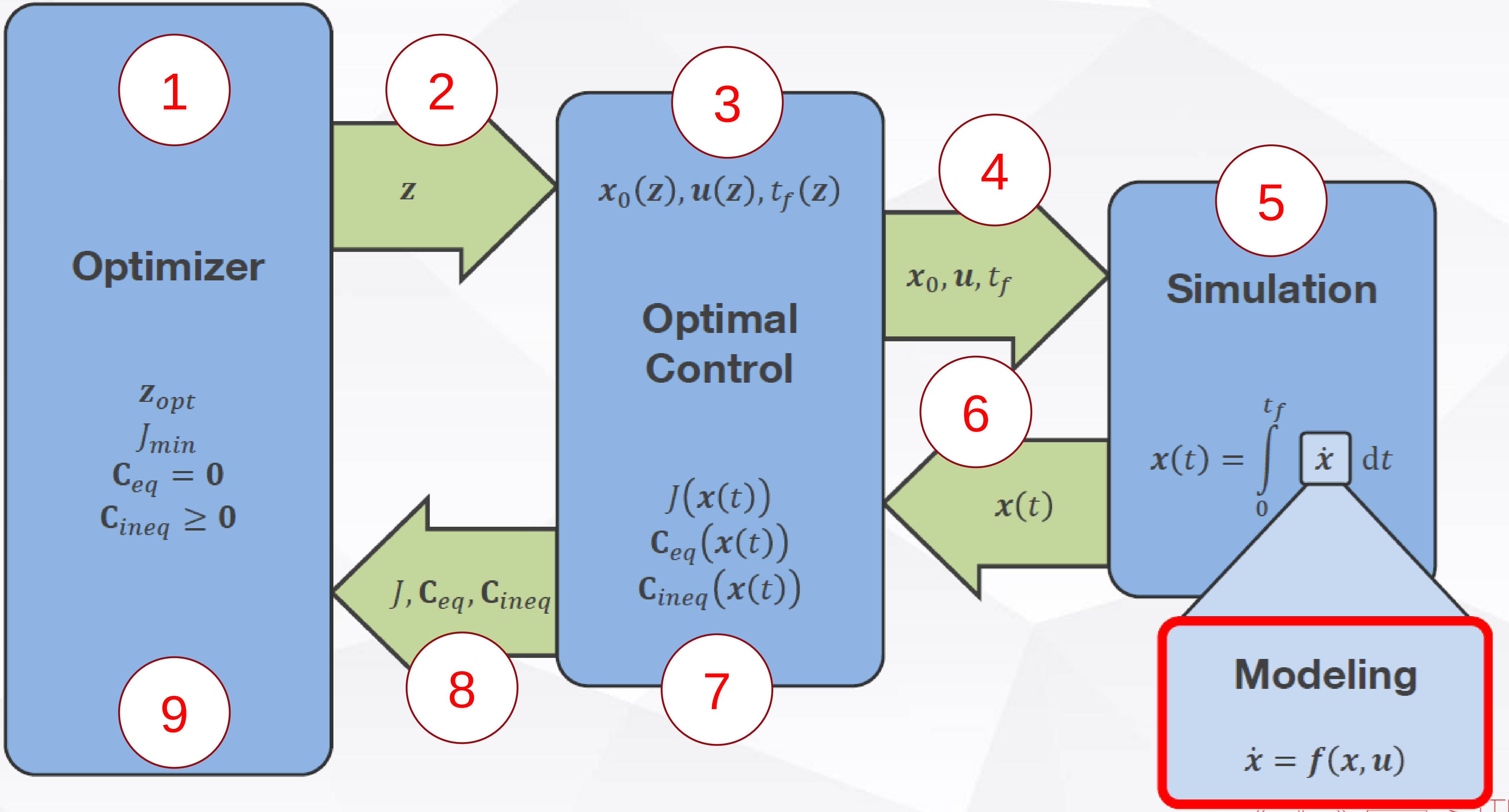
$$\square_{lb} = \square_{ub}$$

Run StartupCheck.m

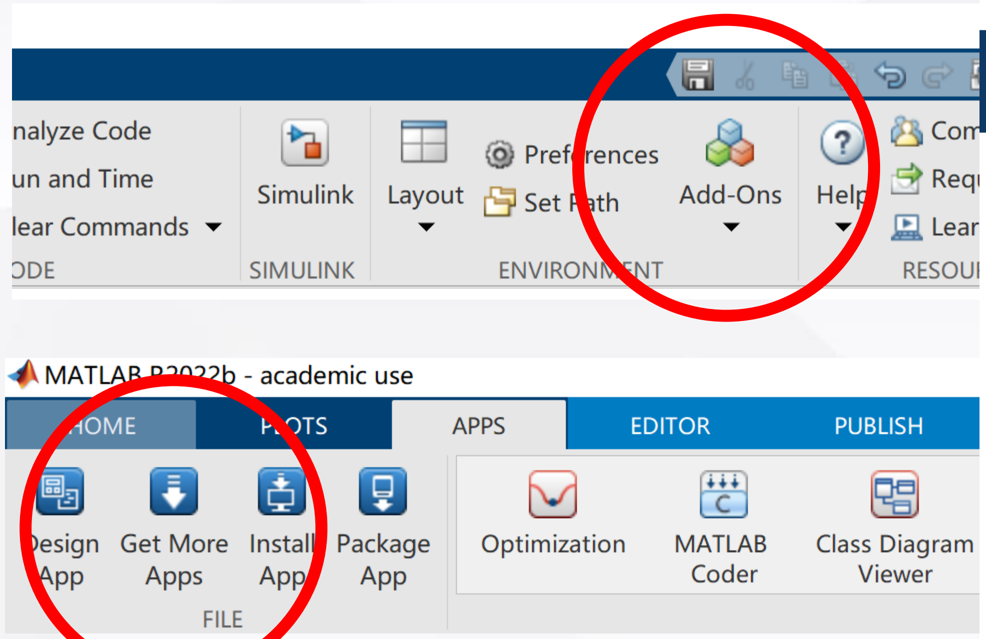
```
-> Copyright Institute of Flight System Dynamics, TUM, 2014 - 2022.
-> Checking MATLAB Version........OK
-> Checking JAVA Version..........OK
-> Checking Available Toolboxes
   - MATLAB Coder.................NOT INSTALLED
   - MATLAB Coder.................Licensed
   - Symbolic Math Toolbox........NOT INSTALLED
   - Symbolic Math Toolbox........Licensed
   - Compiler (CPP)...............NOT FOUND
   - A suitable compiler must be installed and configured.
   - No supported compiler found on your system.
   - Please see the list of supported compilers: LINK
   - After installing a compiler, configure it by clicking here and following the instructions.
-> Checking Optimizers
   - fmincon......................OK
   - snopt........................not found -> you may download it from here
   - ipopt........................not found -> see prompt for download and installation
   - Downloading IPOPT from COIN-OR.
```

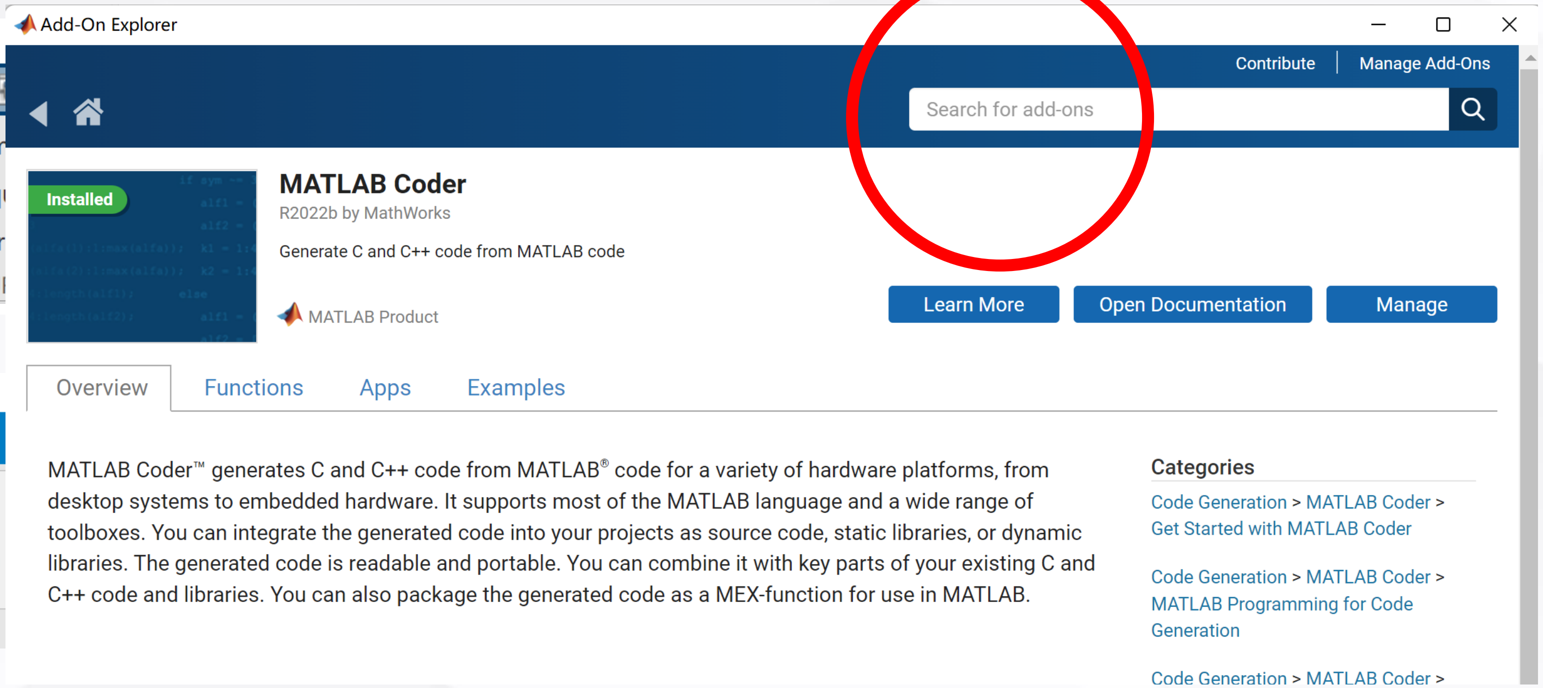
Add-on

Run Assistant


1
2
3
4
5
6
7
8
9
Optimizer
z_{opt}
J_{min}
$C_{eq} = 0$
$C_{ineq} \geq 0$
z
$x_0(z), u(z), t_f(z)$
Optimal Control
$J(x(t))$
$C_{eq}(x(t))$
$C_{ineq}(x(t))$
x_0, u, t_f
$x(t)$
J, C_{eq}, C_{ineq}
Simulation
$x(t) = \int_0^{t_f} \boxed{\dot{x}} \, \mathrm{d}t$
Modeling
$\dot{x} = f(x, u)$

Trajectory Optimization Practice





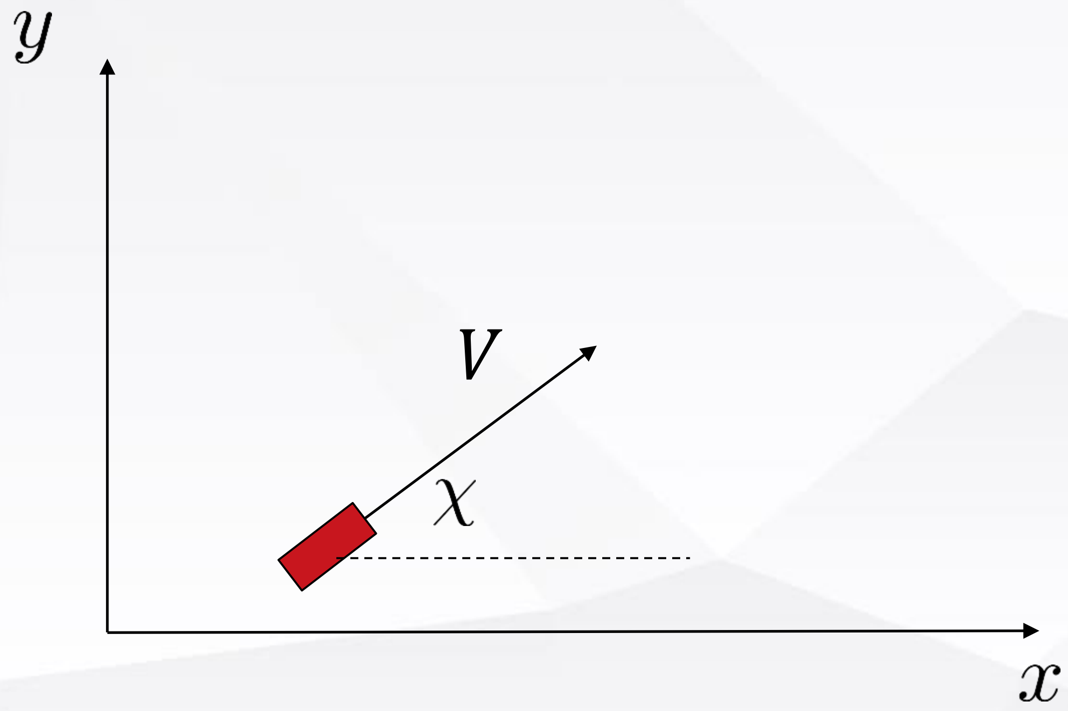
- **MATLAB Support for MinGW-w64 C/C++ Compiler**
- **MATLAB Coder**
- **Symbolic Toolbox**

Run StartupCheck.m

Numerical Benchmark – 2D Time Optimal Trajectory

- \falcon\examples\SimpleCarProblem

Numerical Benchmark – 2D Time Optimal Trajectory



The dynamic model is given as

$$\dot{x}(t) = V(t)\cos\chi(t) \ ,$$
$$\dot{y}(t) = V(t)\sin\chi(t) \ ,$$
$$\dot{V}(t) = \dot{V}_{cmd}(t) \ ,$$
$$\dot{\chi}(t) = \dot{\chi}_{cmd}(t) \ ,$$

and the state and control vectors are defined as

$$\boldsymbol{x} = [x, y, V, \chi]^{\mathrm{T}}$$
$$\boldsymbol{u} = \left[\dot{V}_{cmd}, \dot{\chi}_{cmd}\right]^{\mathrm{T}} .$$

Numerical Benchmark – 2D Time Optimal Trajectory

Variable	LB	UB	Scaling	Final
x	0	100	0.01	100
y	0	100	0.01	100
V	0	5	1	5
chi	-2*pi	2*pi	1	0
Vdot	-0.1	0.1	1	n/a
chidot	-pi/8	pi/8	1	n/a

The dynamic model is given as

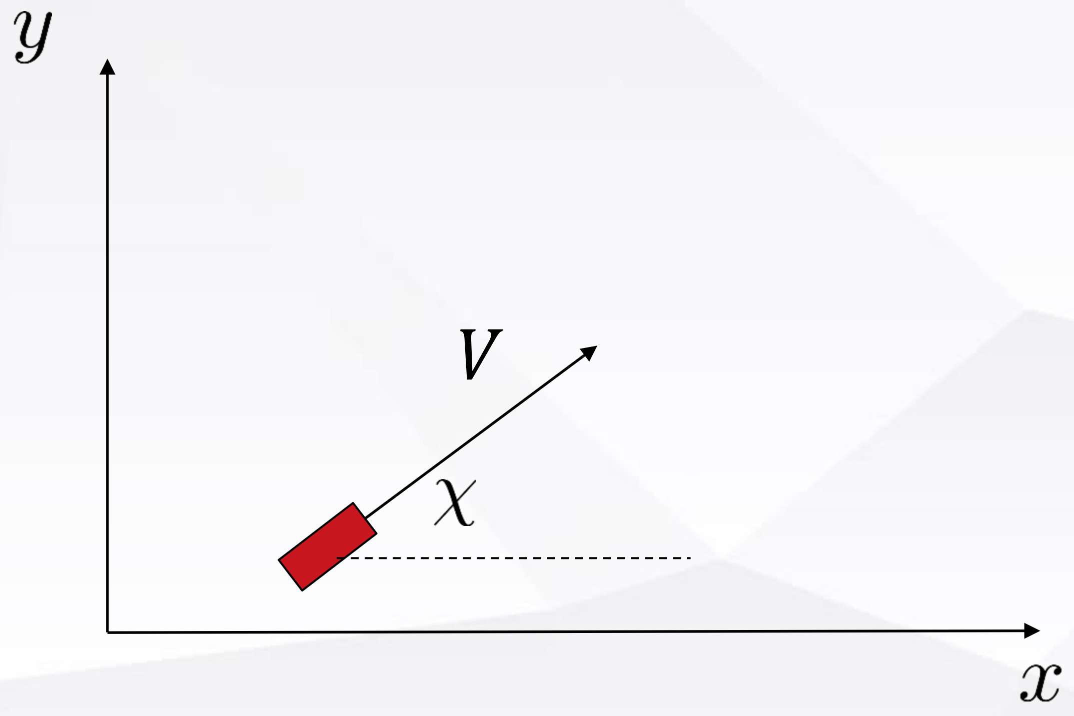
$$\dot{x}(t) = V(t)\cos\chi(t) \ ,$$
$$\dot{y}(t) = V(t)\sin\chi(t) \ ,$$
$$\dot{V}(t) = \dot{V}_{cmd}(t) \ ,$$
$$\dot{\chi}(t) = \dot{\chi}_{cmd}(t) \ ,$$

and the state and control vectors are defined as

$$\boldsymbol{x} = [x, y, V, \chi]^{\mathrm{T}}$$
$$\boldsymbol{u} = \left[\dot{V}_{cmd}, \dot{\chi}_{cmd}\right]^{\mathrm{T}} .$$

Tasks:
- **Complete x_vec**
- **Complete u_vec**
- **Complete source_car**
- **Add final boundary constraints**

Numerical Benchmark – 2D Time Optimal Trajectory



$$-\frac{1}{2\cdot V}\leq\dot{\chi}_{cmd}\leq\frac{1}{2\cdot V}$$

$$c_{lb}=-\frac{1}{2\cdot V}-\dot{\chi}_{cmd}\leq 0$$

$$c_{ub}=\dot{\chi}_{cmd}-\frac{1}{2\cdot V}\leq 0$$

Tasks:

- **Complete source_path**
- **Run the optimization**

Trajectory Optimization Practice

Flight Trajectory Optimization

- \falcon\examples\AircraftExample

Flight Trajectory Optimization 2D

Variable	LB	UB	Scaling
x	-inf	Inf	1e-3
z	-12e3	-304	1e-3
V	60	200	1e-2
Gamma	-0.15	-0.15	1
C_T	0	1	10
C_L	0	1	10

$$\dot{x} = V \cdot \cos\gamma$$

$$\dot{z} = -V \cdot \sin\gamma$$

$$\dot{V} = \frac{1}{m} \cdot \left(T - \left(\frac{\rho}{2} \cdot V^2 \cdot S \cdot (C_{D0} + k \cdot C_L^2)\right)\right) - g\sin\gamma$$

$$\dot{\gamma} = \frac{1}{m \cdot V} \cdot \left(\frac{\rho}{2} \cdot V^2 \cdot S \cdot C_L\right) - \frac{g}{V}\cos\gamma$$

Tasks:

- **Complete x_vec**
- **Complete u_vec**
- **Complete source_aircraft**

$$T = C_T * T_{max}$$

Flight Trajectory Optimization 3D

Variable	LB	UB	Scaling
x	-inf	Inf	1e-3
y	-inf	Inf	1e-3
z	-12e3	0	1e-3
V	60	300	1e-2
Chi	-2*pi	2*pi	1
Gamma	-0.15	-0.15	1
C_T	0	1	10
C_L	0	1	10
Mu	-30 deg	30 deg	10

$$\dot{x} = V \cdot \cos\chi \cdot \cos\gamma$$

$$\dot{y} = V \cdot \sin\chi \cdot \cos\gamma$$

$$\dot{z} = -V \cdot \sin\gamma$$

$$\dot{V} = \frac{1}{m} \cdot (T - D - W \sin\gamma)$$

$$\dot{\chi} = \frac{1}{m \cdot V \cdot \cos\gamma} \cdot L \cdot \sin\mu$$

$$\dot{\gamma} = \frac{1}{m \cdot V} \cdot (L - W \cos\gamma)$$

$$L = \frac{\rho}{2} \cdot V^2 \cdot S \cdot C_L$$

$$D = \frac{\rho}{2} \cdot V^2 \cdot S \cdot C_D(C_L)$$

$$W = m \cdot g$$

Tasks:

- **Complete x_vec**
- **Complete u_vec**
- **Complete source_aircraft**

Flight Trajectory Optimization 3D

Variable	LB	UB	Scaling	Initial	Final
x	-inf	Inf	1e-3	0	4000
y	-inf	Inf	1e-3	0	2000
z	-12e3	0	1e-3	0	-400
V	60	300	1e-2	100	100
Chi	-2*pi	2*pi	1	0	0
Gamma	-0.15	-0.15	1	0	0
C_T	0	1	10	n/a	n/a
C_L	0	1	10	n/a	n/a
Mu	-30 deg	30 deg	10	n/a	n/a

$$\dot{x} = V \cdot \cos\chi \cdot \cos\gamma$$

$$\dot{y} = V \cdot \sin\chi \cdot \cos\gamma$$

$$\dot{z} = -V \cdot \sin\gamma$$

$$\dot{V} = \frac{1}{m} \cdot (T - D - W \sin\gamma)$$

$$\dot{\chi} = \frac{1}{m \cdot V \cdot \cos\gamma} \cdot L \cdot \sin\mu$$

$$\dot{\gamma} = \frac{1}{m \cdot V} \cdot (L - W \cos\gamma)$$

$$L = \frac{\rho}{2} \cdot V^2 \cdot S \cdot C_L$$

$$D = \frac{\rho}{2} \cdot V^2 \cdot S \cdot C_D(C_L)$$

$$W = m \cdot g$$

Tasks:

- **Set initial and final boundaries**
- **Add an time optimal cost**
- **Run the optimization**

Flight Trajectory Optimization 3D - Path

Task:

- **Distance to (2100,1200) of a minimum 200 m**