Homework

Tell me your understanding of "What is Guidance"

- Send to haichao.hong@sjtu.edu.cn
- Include "作业" and YOUR NAME in the subject line
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About me

Haichao Hong / 洪海超

Associate Professor

Email: haichao.hong@sjtu.edu.cn

Office: A227

Homepage: haichao.de

Contents

AE8120

- What is Guidance
- **Generic Formulation of Trajectory Optimization**
- …

Keywords:

- Trajectory / Maneuvering
- Motion of Aircraft (Translational Dynamics)
- Select Commands
- Current State to Desired State
- Constraints

Relationship between

(Computational) Guidance and Trajectory Optimization

Determine a trajectory	Select a trajectory
Steer the aircraft	Design the maneuver
Need to consider constraints	Can consider constraints
Optimality is not fundamental	Optimality is fundamental
Computational guidance mostly onboard	Not necessarily onboard

Trajectory Optimization

- Performance index / Objective function / Cost function
- Equations of motion (often translational dynamics)
- Initial and terminal conditions
- Path constraints

Constraints

Generic Formulation of Trajectory Optimization

Control history $\boldsymbol{u}(t) \in \mathbb{R}^{m\times 1}$

Corresponding state history $\boldsymbol{x}(t) \in \mathbb{R}^{n\times 1}$

Relevant parameters $\boldsymbol{p} \in \mathbb{R}^{s\times 1}$

Bolza cost function

$$J = \underbrace{\boxed{\varphi(\boldsymbol{x}(t_f))}}_{\text{Mayer}} + \underbrace{\boxed{\int_{t_0}^{t_f} L(\boldsymbol{x}(t), \boldsymbol{u}(t), t)dt}}_{\text{Lagrange}}$$

Aircraft dynamics

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t)$$

Equality and inequality constraints

$$\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) = 0 \qquad \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) \leq 0$$

Generic Formulation of Trajectory Optimization

Equality and inequality constraints $\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) = 0$ $\boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) \leq 0$

- Equality constraints -> Inequality constraints
- Not necessarily be the function of state, control, and parameter,

  but THINK TWICE!
- ≥ ⟺ ≤ $\boldsymbol{AX} \leq \boldsymbol{b}$ $\boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) \leq 0$
- Main reason causing infeasibility

A generic formulation:

$$
\begin{aligned}
\underset{\boldsymbol{u}(t),\boldsymbol{p}}{\text{minimize}} \quad & J = \varphi(\boldsymbol{x}(t_f)) + \int_{t_0}^{t_f} L(\boldsymbol{x}(t), \boldsymbol{u}(t), t)dt \\
\text{subject to} \quad & \dot{\boldsymbol{x}}(t) = \boldsymbol{f}\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), \boldsymbol{p}, t\right) \ , \\
& \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) = 0\,, \\
& \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) \leq 0
\end{aligned}
$$

Note:

- Constraints are imposed when necessary.
- t_f might or might not be given.

Generic Formulation of Trajectory Optimization

Before solving it

$$
\begin{aligned}
\underset{\boldsymbol{u}(t),\boldsymbol{p}}{\text{minimize}} \quad & J = \varphi(\boldsymbol{x}(t_f)) + \int_{t_0}^{t_f} L(\boldsymbol{x}(t), \boldsymbol{u}(t), t)dt \\
\text{subject to} \quad & \dot{\boldsymbol{x}}(t) = \boldsymbol{f}\left(\boldsymbol{x}\left(t\right), \boldsymbol{u}\left(t\right), \boldsymbol{p}, t\right)\ , \\
& \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) = 0\,, \\
& \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t) \leq 0
\end{aligned}
$$

1. Figure out your purpose
2. Know your objective (function)
3. Constraints = Practical concerns

Contents

Why discretization methods

Simulation

$$\boldsymbol{x}(t) = \boldsymbol{x}(0) + \int_0^{t_f} \dot{\boldsymbol{x}}(t)dt$$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t)$$

Discretization Methods

Discretization methods

- Single Shooting
- Multiple Shooting
- Collocation – Full Discretization

Single Shooting

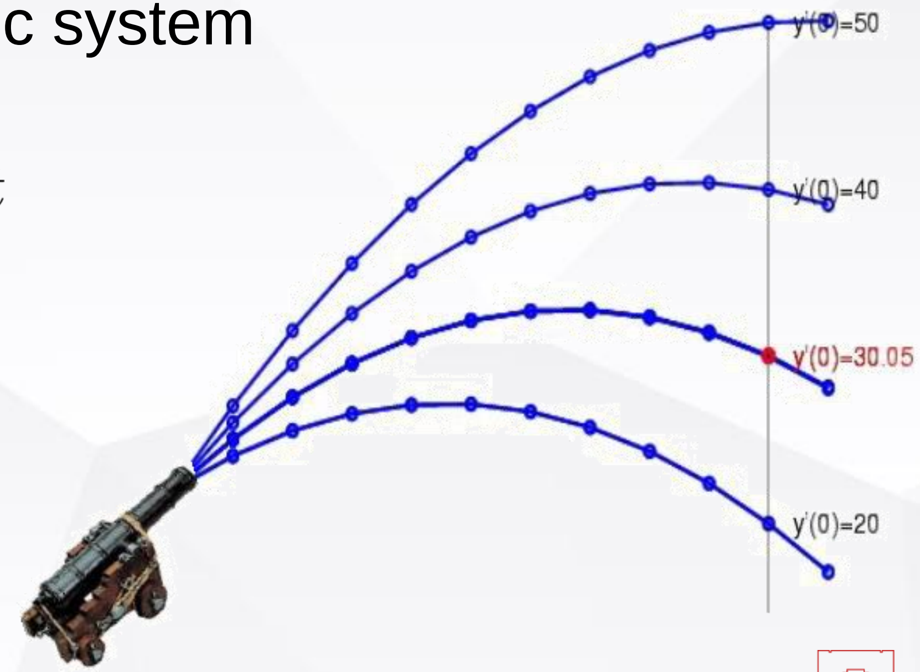
- Integration over complete time history in one single sweep

- Restoration of states by integration of dynamic system

$$\boldsymbol{x}(t) = \boldsymbol{x}(t_0) + \int_{t_0}^{t_f} \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}, t)dt$$

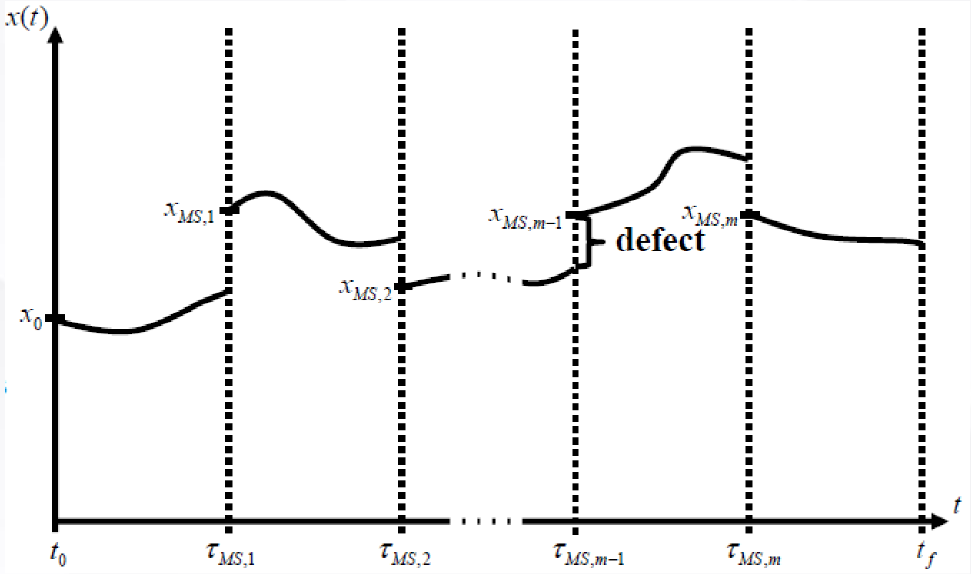
- Adjust **only the controls**

- **Least variables**



Multiple Shooting

- Separation of time history into m segments (shooting intervals)
- Propagation of differential equations on each interval
- Adjust **the controls and some states**



Single shooting vs. Multiple shooting:

Single Shooting

- States are sensitive w.r.t. (i.e. depend on) the initial states
- States are sensitive w.r.t. preceding control variables

Large, non-linear variations can lead to severe convergence problems

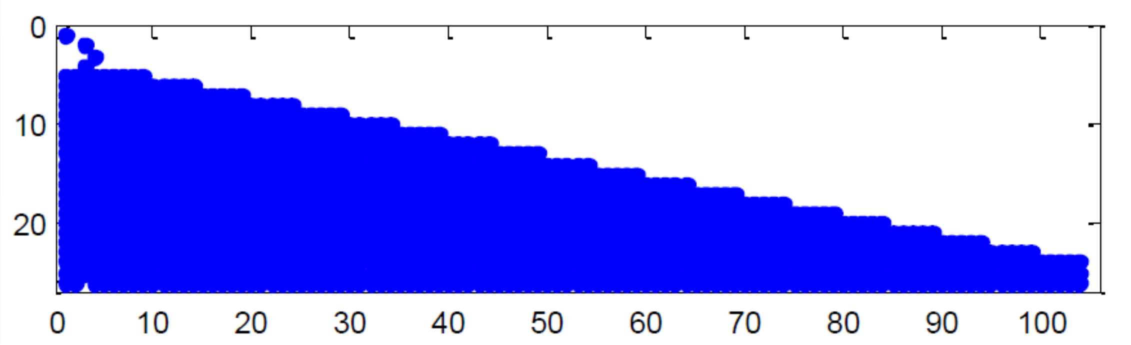
Multiple Shooting

- States are only sensitive w.r.t. states at preceding multiple shooting nodes
- States are only sensitive w.r.t. preceding control variables within multiple shooting interval

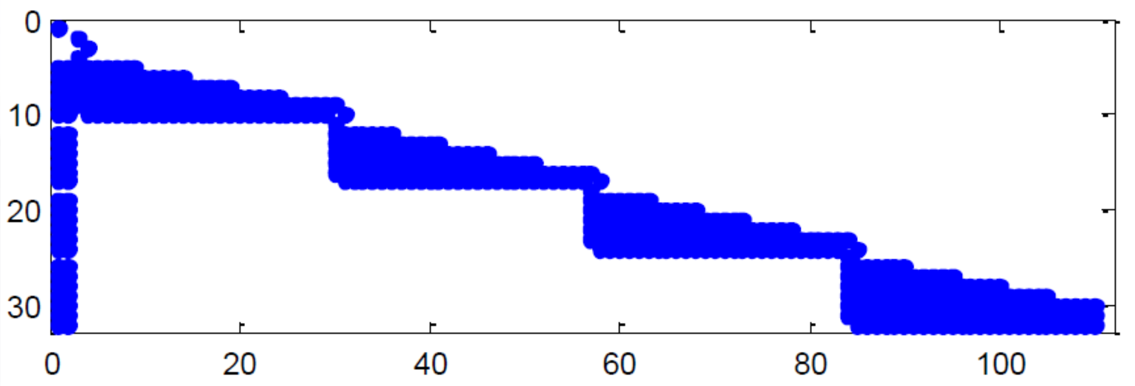
Large, non-linear variations can be avoided – improved convergence

Single shooting vs. Multiple shooting: Gradient Structure

Single Shooting



Multiple Shooting



Collocation – Full Discretization

- Discretization of controls and states at time discretization points:
- Propagation of differential equations on EVERY interval
- Adjust **all states and controls**
- **Most variables**
- **Sparse gradient structure**