拉瓦尔喷管 7 cases

给定压比 $\frac{P_B}{P_0}$, PR, $\frac{A_e}{A_t}$, AR

$AR \rightarrow$ Table A $A/A^* \rightarrow M$, $\frac{P}{P_0}$ — case 2 and case 6

case 1 — $PR > PR_2$, all subsonic flow

case 2 — $P_B = P_e$, $P_e/P_0 = P/P_0\ (M = M\,\text{case 2})$

case 6 — $P_B = P_e$, $P_e/P_0 = P/P_0\ (M = \text{case 6})$

$PR \in [PR_6, PR_2) \rightarrow$ case 3、4、5

case 4 — NS at exit $\rightarrow$ Table B $M_2\ (M_1 = \text{case6 } M)$

$\rightarrow \frac{P_2}{P_1}$ Table B $(M_1 = \text{case6 } M)$ and $\frac{P_1}{P_0} = \frac{P}{P_0}$ Table A $(M = \text{case6 } M)$

$\rightarrow \frac{P_2}{P_0} = \frac{P_2}{P_1} \cdot \frac{P_1}{P_0} = PR_4 (M = \text{case4 } M)$

$PR \in (PR_4, PR_2) \rightarrow$ **case 3** — NS between t→e

$AR \cdot PR \xrightarrow{\text{Table A}} M_e \rightarrow \frac{P_e}{P_{02}} = \frac{P}{P_0}$ Table A $(M = M_e) \rightarrow$

$\frac{P_{02}}{P_{01}} = \frac{P_{02}}{P_e} \cdot \frac{P_e}{P_{01}} = \frac{P_{02}}{P_e} \cdot PR_3 \xrightarrow[P_{02}/P_{01}]{\text{Table B}} M_1, M_2, M_{t(1)}$

$\frac{A}{A^*}$ Table A $(M = M_1) = \frac{A}{A_t}$, A is where NS happens

$PR \in (PR_6, PR_4) \rightarrow$ **case 5** — Compression oblique shock

$\frac{P_e}{P_0} = \frac{P}{P_0}\ (M = \text{case6 } M) \rightarrow \frac{P_B}{P_e} = PR_5 \Big/ \frac{P_e}{P_0} = \frac{P_2}{P_1}$ Table B

$\rightarrow M_{n1} = M_1 (P_2/P_1)$, $M_{n2} = M_2 (P_2/P_1)$, $M_1 = \text{case6 } M$

$\rightarrow \sin\beta = M_{n1}/M_1 \rightarrow \beta \xrightarrow[M = M_1]{\theta-\beta-M} \theta \rightarrow M_2 = \frac{M_{n2}}{\sin(\beta-\theta)}$

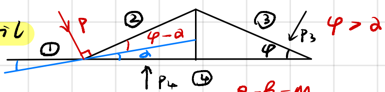
$PR \in (0, PR_6) \rightarrow$ **case 7** — expansion wave

A hypothesis for Plandtl Mayer: $P_{01} = P_{02}$

$PR_7 = \frac{P_2}{P_0} \xrightarrow[P/P_0]{\text{Table A}} M\,(P/P_0 = PR) = M_2$, $M_1 = \text{case6 } M$

$\xrightarrow{\text{Table C}} \omega_1 (M = M_1)$, $\omega_2 (M = M_2) \longrightarrow \theta_1 = 0$, $\theta_2 = -(\omega_2 - \omega_1)$

Supersonic Airfoil



if $\varphi < \alpha$, $\alpha - \varphi$

①→② compression wave $\theta_1 = \varphi - \alpha \xrightarrow[\text{weak soln}]{\theta-\beta-M_1} \beta_1$

$\rightarrow M_{n1} = M_1 \sin\beta_1 \rightarrow M_{n2} = M_2$ Table B $(M_1 = M_1)$, $\frac{P_2}{P_1} (M_1 = M_1)$

$\rightarrow M_2 = \frac{M_{n2}}{\sin(\beta_1 - \theta_1)}$

②→③ expansion wave, $\theta_2 = -2\varphi < 0 \rightarrow$

$\theta_2 = -(\omega_3 - \omega_2)$, $\omega_2 = \omega_{(M = M_2)}$ Table C $\rightarrow \omega_3 = \omega_2 - \theta_2$

$\xrightarrow[\omega = \omega_3]{\text{Table C}} M_3\ (\omega = \omega_3) \longrightarrow P_{02} = P_{03}$, $\frac{P_2}{P_{02}} = \frac{P}{P_0}$ Table A $(M = M_2)$

$\frac{P_3}{P_{03}} = \frac{P}{P_0}$ Table A $(M = M_3) \rightarrow \frac{P_3}{P_2} = \frac{P_3}{P_{03}} \Big/ \frac{P_2}{P_{02}} \rightarrow \frac{P_3}{P_1} = \frac{P_3}{P_2} \cdot \frac{P_2}{P_1}$

①→④ compression wave, $\theta_3 = \alpha \xrightarrow[\text{weak soln}]{\theta-\beta-M} \beta_3$

$\rightarrow M_{n1} = M_1 \sin\beta_3 \rightarrow M_{n4} = M_2$ Table B $(M_1 = M_{n1}) \rightarrow M_4 = \frac{M_{n4}}{\sin(\beta_3 - \theta_3)}$

$\frac{P_4}{P_1} = \frac{P_2}{P_1}$ Table B $(M_1 = M_{n1})$

$$C_L = \frac{1}{\gamma M_1^2 \cos\varphi}\left[\frac{P_4}{P_1} \cdot 2\cos\alpha - \frac{P_2}{P_1} \cdot \cos(\varphi - \alpha) - \frac{P_3}{P_1}\cos(\varphi + \alpha)\right]$$

$$C_D = \frac{1}{\gamma M_1^2 \cos\varphi}\left[\frac{P_4}{P_1} 2\sin\alpha + \frac{P_2}{P_1}\sin(\varphi - \alpha) - \frac{P_3}{P_1}\sin(\varphi + \alpha)\right]$$

for ◇ one ①→④ $\theta_3 = \varphi + \alpha$

④→⑤ expansion wave, $\theta_5 = -2\varphi = -(\omega_5 - \omega_4)$

$\omega_4 = \omega_{(M = M_4)}$ Table C $\rightarrow \omega_5 = \omega_4 - \theta_5 \xrightarrow[\omega = \omega_5]{\text{Table C}} M_5$

$\frac{P_5}{P_{05}} = \frac{P}{P_0}\ (M = M_5)$, $\frac{P_4}{P_{04}} = \frac{P}{P_0}\ (M = M_4)$, $P_{05} = P_{04} \rightarrow \frac{P_5}{P_4}$

$\rightarrow \frac{P_5}{P_1} = \frac{P_5}{P_4} \cdot \frac{P_4}{P_1}$

$$C_L = \frac{1}{\gamma M_1^2 \cos\varphi}\left[\cos(\varphi - \alpha)\left(\frac{P_5}{P_1} - \frac{P_2}{P_1}\right) + \cos(\varphi + \alpha)\left(\frac{P_4}{P_1} - \frac{P_3}{P_1}\right)\right]$$

$$C_D = \frac{1}{\gamma M_1^2 \cos\varphi}\left[\sin(\varphi + \alpha)\left(\frac{P_4}{P_1} - \frac{P_3}{P_1}\right) + \sin(\varphi - \alpha)\left(\frac{P_2}{P_1} - \frac{P_5}{P_1}\right)\right]$$

Chemical Rocket Engine

$F_T = F_{int} - F_{ext} = F_{int} - P_a A_e$, $C_F \equiv \frac{F_t}{P_0 A_t}$

$$C_F = \gamma M_e^2 \frac{A_e}{A_t}\frac{P_e}{P_0} + \left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right)\frac{A_e}{A_t}$$

$$= \gamma\sqrt{\frac{2}{\gamma - 1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\sqrt{1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}} + \left(\frac{P_e}{P_0} - \frac{P_a}{P_0}\right)\frac{A_e}{A_t}$$

$F_{TV} - F_{TSL} = A_e P_a \Rightarrow A_e$ (RS-68)

$$C_{F,opt}\Big|_{P_e = P_a} = \gamma\sqrt{\frac{2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}\sqrt{\left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Q1D FSDC Solution

$$\frac{P}{P_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\gamma/\gamma-1}$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1/\gamma-1}$$

$$\frac{T}{T_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{1+\gamma}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\tan\theta = 2\cot\beta\,\frac{M_1^2\sin^2\beta - 1}{2 + (\gamma + \cos 2\beta)M_1^2}$$

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$$

Unit Conversion:

1 atm = 1.01325 × 10^5 Pa

1 psi = 6894.76 Pa   1 bar = 100 kPa

1 lb = 0.453592 kg   1 inch = 0.0254 m

1 ft = 0.3048 m

T = 0 + 273.15 K

SL: $T_0 = 288$ K $\rho_0 = 1.225\ kg/m^3$

R = 287.14

Viscous Flows

$\tau = \mu \frac{\partial u}{\partial y}$

Couette Flow $\tau = \mu \frac{u_e}{2h}$ $C_f \equiv \frac{\tau}{\frac{1}{2}\rho u_e^2}$

skin friction coef $C_{f,lw} = \frac{\mu}{\rho u_e h} = \frac{2}{Re_{2h}}$

T 分布 $T(y) = \frac{-\mu u_e^2}{2k}\left[\left(\frac{y+h}{2h}\right)^2 - \frac{y+h}{2h}\right] + (T_e - T_w) \times \frac{y+h}{2h} + T_w$, $T(-h) = T_w$, $T(h) = T_e$ (普适)

$\tau u' + (kT')' = 0$, $T'' = -\frac{\tau u'}{k}$, heat flux:

$$\dot{q}(y) = -k\frac{\partial T}{\partial y} = \frac{\mu u_e^2}{2h}\left(\frac{y+h}{2h} - \frac{1}{2}\right) - k\frac{T_e - T_w}{2h}$$

Hagen-Poiseuille Flow

mass flow rate $\dot{m} = \frac{-P'\rho\pi R^4}{8\mu}$, $U = \frac{\dot{m}}{\rho\pi R^2} = \frac{-P'R^2}{8\mu}$

$P' = (P_o - P_{in})/L$

wall sheer stress $\tau = \mu\frac{du}{dr} = \frac{4\mu U}{R}$, $C_f = \frac{2\tau}{\rho U^2}$

$C_f = \frac{8\mu}{\rho UR} = \frac{8}{Re_R} = \frac{16}{Re_D}$ (circular pipe)

$Re = \frac{\rho UL}{\mu}$ (Boundary layer)

Add Blasius + Couette

$T = -\frac{\tau u'}{2k}y^2 + ay + b$

$\dot{q}(y) = -k\frac{\partial T}{\partial y}$ $u = \frac{y+h}{2h}u_e$

$D = \frac{1}{2}\rho u_\infty^2 \cdot L \cdot b \cdot C_D$

Turbulent B-L Solution

Prandtl: $5 \times 10^5 < Re_x$, $Re_c < 10^7$

$C_f(x) = 0.0592\,Re_x^{-0.2}$ $C_D(c) = 0.074\,Re_c^{-0.2}$

Von Karman: $\frac{1}{\sqrt{C_f(x)}} = 4.15\log_{10}(Re_x C_f) + 1.7$

$\frac{1}{\sqrt{C_D(c)}} = 4.13\log_{10}(Re_c C_D)$, Schlichting:

$C_f(x) = (2\log_{10}Re_x - 0.65)^{-2.3}$, $C_D(c) = \frac{0.455}{(\log_{10}Re_c)^{2.58}}$

White: $C_f(x) = \frac{0.455}{\ln^2(0.06\,Re_x)}$, $C_D(c) = \frac{0.523}{\ln^2(0.06\,Re_c)}$

Blasius solution

B-L 厚度 $\delta = 5.0x/\sqrt{Re_x}$

$\tau_w = 0.332\,\rho U_\infty^2 \frac{1}{\sqrt{Re_x}}$, skin friction coef $C_f = \frac{0.664}{\sqrt{Re_x}}$

$C_D = \frac{1.328}{\sqrt{Re_l}}$, BL thickness displacement $\delta^* = 1.721\frac{x}{\sqrt{Re_x}}$

BL thickness momentum $\theta = \int_0^\infty \frac{\rho u}{\rho_e u_e}\left(1 - \frac{u}{u_e}\right)dy = 0.664\frac{x}{\sqrt{Re_x}}$