homework of matrix analysis

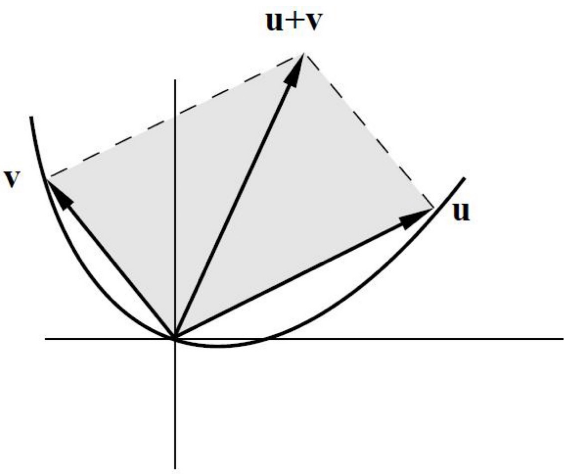
Student ID 122431910061

Name Liu Zhaohong

刘昭宏.

Lecture 1

work1: whether a set of curved lines through the origin of R^2 is a subspace of R^2



To define curved lines through $(0,0)$ in R^2
define set W as below:

$$W = \{(x, y) \mid y = x\sum_{j=0}^{n} a_j x^j, x, y \in R^2, a_0, a_1, \dots, a_n \in R\}$$

according to the definition of subspaces
if W is a subspace of R^2, then

$$\begin{cases} \text{if } (x_1, y_1), (x_2, y_2) \in W & \text{then } (x_1 + x_2, y_1 + y_2) \in W \\ \text{if } (x_1, y_1) \in W, k \in F, & \text{then } (kx_1, ky_1) \in W \end{cases}$$

$$y_1 = \sum_{j=0}^{n} a_j x_1^{j+1} \qquad y_2 = \sum_{j=0}^{n} b_j x_2^{j+1}$$

if $y_1 + y_2 = \sum_{j=0}^{n} C_j (x_1 + x_2)^{j+1}$

$$= C_0(x_1 + x_2) + C_1(x_1^2 + 2x_1x_2 + x_2^2) + \dots$$

while $y_1 + y_2 = a_0x_1 + b_0x_2 + a_1x_1^2 + b_1x_2^2 + \dots$

$\Rightarrow a_0 = b_0 = C_0, \quad C_1x_1x_2 \equiv 0$

x_1, x_2 can't equal to 0 forever
so $C_1 = 0$

As the same way:
$C_2, C_3 \dots C_n = 0$
$\Rightarrow y = a_0x, \ a_0 \in R$

Answer: A banch of straight lines are subspace of R^2
However, curved lines are not.

Lecture 1

Let $\boldsymbol{v}_1, \boldsymbol{v}_2 \in \boldsymbol{R}^3$, $\boldsymbol{W} = \{a\boldsymbol{v}_1 + b\boldsymbol{v}_2 | a \in \boldsymbol{R}, b \in \boldsymbol{R}\}$. Is $\boldsymbol{W}$ a subspace of $\boldsymbol{R}^3$?

According to the definition of subspace.

if W is a subspace of R^3, then

$$\begin{cases} \text{if } \alpha, \beta \in W, \text{ then } \alpha + \beta \in W \\ \text{if } \alpha \in W, k \in F, \text{ then } k\alpha \in W \end{cases}$$

① $\alpha = av_1 + bv_2$, $a, b \in R$

$\beta = cv_1 + dv_2$, $c, d \in R$

$\Rightarrow \alpha + \beta = (a+c)v_1 + (b+d)v_2 \in W$

② $\alpha = av_1 + bv_2$, $k \in F$

$k\alpha = kav_1 + kbv_2 \in W$

In conclusion, W is a subspace of R^3

Lecture 1

Let $\boldsymbol{W}_2 = \{\boldsymbol{B} | \boldsymbol{B} \in \boldsymbol{R}^{n\times n}, \det(\boldsymbol{B}) \neq 0\}$. Is $\boldsymbol{W}_2$ a subspace of $\boldsymbol{R}^{n\times n}$?

$\det(B) \neq 0 \Rightarrow 0 \notin W_2$

$\Rightarrow W_2$ is not a subspace of $R^{n\times n}$.