Lecture 2 homework 4-5

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homework 4

For a set of vectors $S = \{v_1, v_2, \ldots v_n\}$. Prove that span $\{S\}$ is the intersection of all subspaces that contain S.

> Hint: for $M = \cap_{S \subseteq V} V$, V is any subspace contains S, prove that span $\{S\} \subseteq M$ and $M \subseteq$ span $\{S\}$

Situation 1: if S is nonempty

According to the definition of span,

span $\{S\} = \{a_1 v_1 + a_2 v_2 + \ldots + a_k v_k | \, v_1, v_2 \ldots v_k \in S, a_1, a_2 .. a_k \in F, \text{and } k = 1, 2, \ldots\}$

S is a subset of a vector space V over a field $\mathbf{F}$.

to prove span $(S) = \cap\{M, \text{a subspace of } V | S \subseteq M\}$;

- span(S) is a vector space that contains all of S, so it's one of spaces M in the intersection
- span(S) only has linear combinations of vectors in S, so every vector in span(S) has to be in every vector space M that contains all of S
- therefore span(S) is a subset of all the spaces M in the intersection

Situation 2: if S is empty

- if S is empty, it is contained in every subspace of V
- the intersection of every subspace of V is the subspace $\{0\}$
- so the definition ensures that span$\{S\} = 0$

In conclusion, span $\{S\}$ is the intersection of all subspaces that contain S.

homework 5

For any $A = [a_{ij}] \in M_{m,n}(C)$, show that tr $A^* A = 0$ if and only if $A = 0$.

noticed that $A = [a_{ij}] \in M_{m,n}(C)$, so $A^* = A^T$

if tr $A^T A = \sum_{i=1,j=1}^{m,n} a_{ij}^2 = 0$, then every a_{ij} should equal to 0, which means $A = 0$