Course Project of Numerical Analysis

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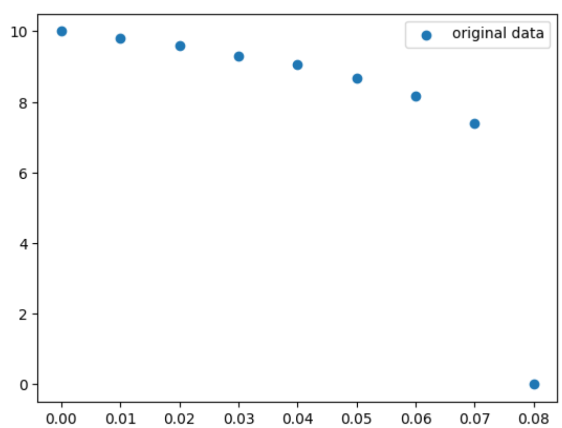
I. Numerical Integration

given that

$$\int_0^R \rho v 2\pi r \mathrm{d}r$$

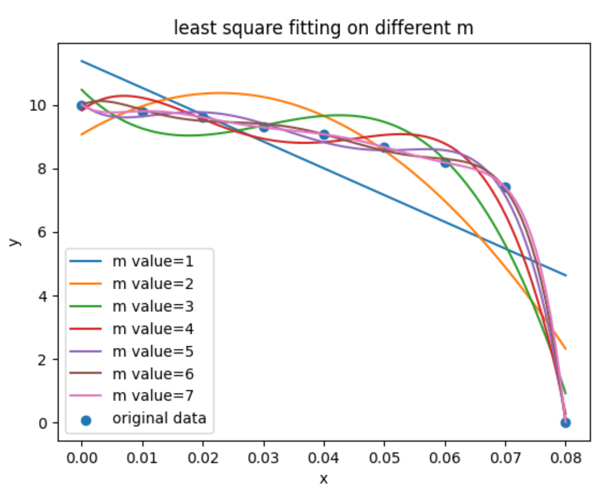
with $\rho = 1.2kg/m^3$

The first thing is to change unit from m to cm for r, then we use scatter to see the relation between r and v.



It can be concluded that we can use interpolation methods to construct a polynomial, which can be used in numerical integration.

So we use least square method to compose a fitting polynomial,



The order can be as high as 7 if we wanted. However, higher order doesn't equal to better efficiency, so order 5 is decided here.

Then we get the function, also the relation between v and r.

$$v(r) = 10 - 153r + 17677r^2 - 70205r^3 + 13323135r^4 - 80032051r^5$$

The next step is to compose $\rho v(r) 2\pi r$, and coefficients for this function is:

[0.00000000e+00 7.57145786e+01 -1.15339684e+03 1.33281077e+05 -5.80721195e+06 1.00454071e+08 -6.03427440e+08]

Finally we use integration methods:

- Composite trapezoidal rule: 0.1853
- Composite Simpson's rule: 0.1855
- Romberg's method(order 10): 0.1855

II. ODE Initial Value Problem

given that

$$\frac{\partial k}{\partial t} = -\epsilon$$

and

$$\frac{\partial \epsilon}{\partial t} = -C\frac{\epsilon^2}{k}$$

where $C = 1.83$. At $t_0 = 1, \quad k = 1, \quad \epsilon = 0.2176$

predict k at $t = 5$

For each step, firstly we use $\partial\epsilon/\partial t$ to get $\epsilon(t)$; secondly send this $\epsilon(t)$ into $\partial k/\partial t$ to acquire the k.

The final is:

- Euler's method: 0.490872
- Modified Euler's method: 0.472633
- Runge-Kutta 4: 0.473380

Considering both convergence and accuracy, euler's method should be the last one of the three methods, while modified euler method works better than euler's method;

The most fast-converge and accurate method of the three should be 4-th runge kutta method.

III. Non Linear Equations

given that

$$y_+ = U_+ + e^{-kB}[e^{kU_+} - 1 - kU_+ - \frac{1}{2}(kU_+)^2 - \frac{1}{6}(kU_+)^3 - \frac{1}{24}(kU_+)^4]$$

where

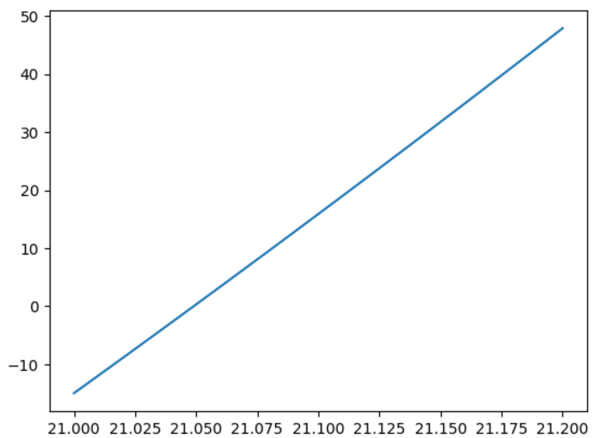
$$y_+ = \frac{u_r y}{v}, \quad U_+ = \frac{U}{u_r}$$

$$k = 0.41, \quad B = 5.1, \quad v = 1.5 \times 10^{-5}, \quad \rho = 1.25kg/m^3$$

$y = 0.01m$, the velocity is $21m/s$

$$u_\tau = \sqrt{\frac{\tau_{wall}}{\rho}}$$

The first step is to use matplotlib.pyplot to find box of root.



so the initial guess value can be set as 21.

Then applying three methods to get U_+:

- Bisection method: 21.0498
- Newton method: 21.0494
- Fix point method: 21.0498

we can then conclude that $\tau_{wall} = 1.2441$ and the formula for Fix point method is:

$$U_+ = g(U_+) = ln((y_+ + U_+)e^{kB} + 1 + kU_+ + \frac{1}{2}(kU_+)^2$$

$$+\frac{1}{6}(kU_+)^3 + \frac{1}{24}(kU_+)^4)/k$$

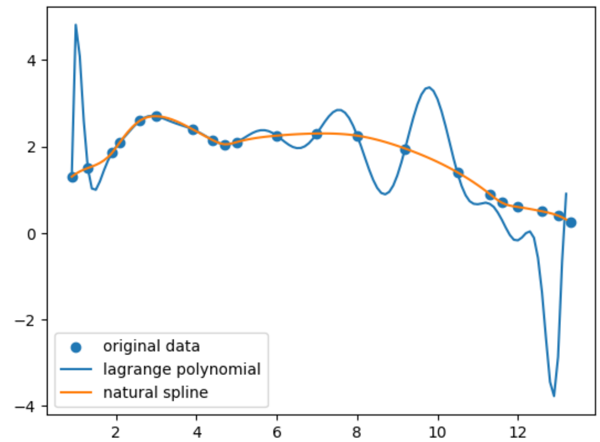
IV. Interpolation

With many as 21 points, the lagrange method could not follow the trend of original data points.

However, the natural spline function basically keeps the same changing pattern as original data points go.

V. Linear Equations

gaussian elimination: [-9.62900109e+03 5.23308734e+04 -1.28353328e+05 1.89495703e+05 -1.89338391e+05 1.36397396e+05 -7.36542206e+04 3.05862459e+04 -9.93812152e+03 2.55593429e+03 -5.24043290e+02 8.59478924e+01 -1.12743546e+01 1.17827153e+00 -9.72945654e-02 6.25917429e-03 -3.06784319e-04 1.10555985e-05 -2.75922145e-07 4.25747987e-09 -3.05797637e-11]



LU decomposition: [-9.60454784e+03 5.21953487e+04 -1.28013353e+05 1.88980865e+05 -1.88809167e+05 1.36003965e+05 -7.34343002e+04 3.04914228e+04 -9.90603943e+03 2.54731981e+03 -5.22194910e+02 8.56299984e+01 -1.12305538e+01 1.17345714e+00 -9.68760682e-02 6.23081593e-03 -3.05319883e-04 1.09999978e-05 -2.74460563e-07 4.23373827e-09 -3.04003669e-11]

gauss-seidel: [[-5.34167848e+04] [1.51689637e+05] [-1.81222598e+05] [1.81139810e+05] [-1.86383242e+05] [1.42524271e+05] [-7.54223407e+04] [3.04691827e+04] [-9.96721192e+03] [2.58850592e+03] [-5.25063148e+02] [8.49677686e+01] [-1.11779782e+01] [1.18017590e+00] [-9.78315352e-02] [6.30218484e-03] [-3.09533404e-04] [1.10590396e-05] [-2.74274036e-07] [4.44400390e-09] [-2.96942589e-11]]

SOR: [[-5.34167848e+04] [1.51689637e+05] [-1.81222598e+05] [1.81139810e+05] [-1.86383242e+05] [1.42524271e+05] [-7.54223407e+04] [3.04691827e+04] [-9.96721192e+03] [2.58850592e+03] [-5.25063148e+02] [8.49677686e+01] [-1.11779782e+01] [1.18017590e+00] [-9.78315352e-02] [6.30218484e-03] [-3.09533404e-04] [1.10590396e-05] [-2.74274036e-07] [4.44400390e-09] [-2.96942589e-11]]

VI. Eigen Value

$$A = \begin{bmatrix} 52 & 30 & 49 & 28 \\ 30 & 50 & 8 & 44 \\ 49 & 8 & 46 & 16 \\ 28 & 44 & 16 & 22 \end{bmatrix}$$

Power iteration: Max eigen value = [132.94900421]

Rayleigh quotient: Max eigen value = [[132.62787533]]

QR decomposition, eigen values = [132.62759169 52.4425832 -11.54109674 -3.52907815]