Lecture 2 homework 6-7

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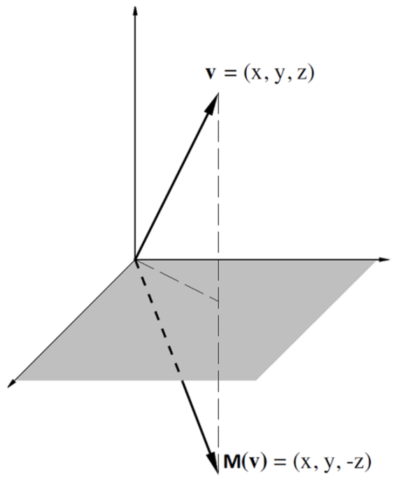
homework 6

The reflector M that maps each vector $v = (x, y, z)$ in $\mathbf{R^3}$ to its reflection $M(v) = (x, y, -z)$ about the xoy plane.

Show that the reflector $M(v)$ is a linear transformation on $\mathbf{R^3}$

Homework 6:

The reflector M that maps each vector $v = (x, y, z)$ in $\mathbf{R}^3$ to its reflection $M(v) = (x, y, -z)$ about the xy-plane. Show that the reflector $M(v)$ is a linear transformation on $\mathbf{R}^3$.



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\implies Q(u) = Au$

$$Q(\alpha u_1 + \beta u_2 + \gamma u_3) = A(\alpha u_1 + \beta u_2 + \gamma u_3) = \alpha Q(u_1) + \beta Q(u_2) + \gamma Q(u_3)$$

so the reflector $M(v)$ is a linear transformation on $\mathbf{R^3}$.

homework 7

Homework 7:

Consider two bases

$$B_U = \left\{ u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, u_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, u_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \quad \text{and} \quad B_V = \left\{ v_1 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$$

For the projector defined by $P(x, y, z) = (x, y, 0)$, determine the representation of the projector $[P]_{B_U B_V}$.

for B_u

$$P(u_1) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1u_1 + 0u_2 + 0u_3$$

$$P(u_2) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0u_1 + 1u_2 + 0u_3$$

$$P(u_3) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0u_1 + 1u_2 + 0u_3$$

$$[P]_{Bu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

for B_v

$$P(v_1) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = 1v_1 + 0v_2 + 0v_3$$

$$P(v_2) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0v_1 + 1v_2 + 0v_3$$

$$P(v_3) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0v_1 + 1v_2 + 0v_3$$

$$[P]_{Bv} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

so

$$P_{BuBv} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$