

### Exercise 3.5

$$\int_{-1}^1 1 + \sin e^{3x} dx, \text{ Tol} = 0.005$$

$$\text{set } f(x) = 1 + \sin e^{3x}$$

for trapezoidal rule

$$\textcircled{1} S(-1, 1) = 2.994238$$

$$S(-1, 0) = 1.445619$$

$$S(0, 1) = 1.892971$$

$$\Delta = |S(-1, 1) - S(-1, 0) - S(0, 1)| = 0.344352$$

$$\Delta > 15 \text{ Tol}$$

$$\textcircled{2} \text{ Tol} = 0.005/2 = 0.0025$$

for  $(-1, 0)$

$$S(-1, 0) = 1.445619$$

$$S(-1, -0.5) = 0.567762$$

$$S(-0.5, 0) = 0.765689$$

$$\Delta = 0.765689$$

$$\Delta > 15 \text{ Tol}$$

for  $(0, 1)$

$$S(0, 1) = 1.892971$$

$$S(0, 0.5) = 0.466991$$

$$S(0.5, 1) = 0.492741$$

$$\Delta = 0.933239$$

$$\Delta > 15 \text{ Tol}$$

for Simpson's rule

$$\textcircled{1} S(-1, 1) = 3.453374$$

$$S(-1, 0) = 1.296062$$

$$S(0, 1) = 0.648653$$

$$\Delta = 1.508659$$

$$\Delta > 15 \text{ Tol}$$

$$\textcircled{2} \text{ Tol} = 0.005/2 = 0.0025$$

for  $(-1, 0)$

$$S(-1, 0) = 1.296062$$

$$S(-1, -0.5) = 0.557656$$

$$S(-0.5, 0) = 0.740228$$

$$\Delta = 1.508659$$

$$\Delta > 15 \text{ Tol}$$

for  $(0, 1)$

$$S(0, 1) = 0.648653$$

$$S(0, 0.5) = 0.773831$$

$$S(0.5, 1) = 0.476608$$

$$\Delta = 0.601787$$

$$\Delta > 15 \text{ Tol}$$

### Exercise 3.7

Romberg

$$\int_1^2 \ln x dx \quad n=3$$

$$\text{set } f(x) = \ln x$$

$$R_{11} = \frac{1}{2} (f(1) + f(2)) = 0.34657359$$

$$R_{21} = \frac{1}{2} [R_{11} + h_1 f(1+h_1)] = 0.37601935$$

$$R_{31} = \frac{1}{2} [R_{21} + h_2 f(1+h_2)] = 0.38367951$$

$$R_{k,j} = R_{k,j-1} + \frac{R_{k,j-1} - R_{k-1,j-1}}{4^{j-1} - 1}$$

$$\Rightarrow R_{22} = 0.3858346$$

$$R_{32} = 0.38625956$$

$$R_{33} = 0.38628789$$

### Exercise 3.8

$$\int_{-1}^1 e^x dx \quad \int_1^2 \ln x dx$$

$$\int_a^b f(x) dx = \int_{-1}^1 f\left(\frac{(b-a)t + b + a}{2}\right) \frac{b-a}{2} dt$$

$$\int_{-1}^1 e^x dx = \frac{5}{9} e^{0.774576692} + \frac{8}{9} e^0$$

$$+ \frac{5}{9} e^{-0.774576692}$$

$$= 2.35033695$$

$$\int_1^2 \ln x dx = \int_{-1}^1 \frac{1}{2} \ln \frac{t+3}{2} dt$$

$$= \frac{5}{9} \cdot \frac{1}{2} \cdot \ln \frac{3+0.774576692}{2} +$$

$$\frac{8}{9} \cdot \frac{1}{2} \cdot \ln \frac{3}{2} +$$

$$\frac{5}{9} \cdot \frac{1}{2} \cdot \ln \frac{3-0.774576692}{2}$$

$$= 0.38630042$$