Chapter 3

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Exercise 3.1

Compare the results of the closed($n = 1, 2, 3, 4$) and open (n=0,1,2,3) Newton-Cotes formulas when approximating

$$\int_0^{\pi/4} \sin x \mathrm{d}x = 1 - \frac{\sqrt{2}}{2} \approx 0.29289322$$

(**a**) For closed Newton-Cotes formula

$$\int_0^{\pi/4} \sin x \mathrm{d}x \approx \frac{\pi}{8}[f(\frac{\pi}{4}) - f(0)] = 0.27768017$$
$$\int_0^{\pi/4} \sin x \mathrm{d}x \approx \frac{\pi}{24}[f(0) + f(\frac{\pi}{8}) + f(\frac{\pi}{4})] = 0.29293268$$
$$\int_0^{\pi/4} \sin x \mathrm{d}x \approx \frac{3\pi/12}{8}[f(0) + 3f(\frac{\pi}{6}) + 3f(\frac{\pi}{12} + f(\frac{\pi}{4}))] = 0.29289432$$
$$\int_0^{\pi/4} \sin x \mathrm{d}x \approx \frac{2\pi/16}{45}[7f(0) + 32f(\frac{\pi}{16}) + 12f(\frac{\pi}{8}) + 32f(\frac{\pi}{16})) + 7f(\frac{\pi}{4}))] = 0.29289432$$

(**b**) For open Newton-Cotes formula

$$n = 0, \int_0^{\pi/4} \sin x \mathrm{d}x \approx 0.30055885$$
$$n = 1, \int_0^{\pi/4} \sin x \mathrm{d}x \approx 0.29798753$$
$$n = 2, \int_0^{\pi/4} \sin x \mathrm{d}x \approx 0.29285864$$
$$n = 3, \int_0^{\pi/4} \sin x \mathrm{d}x \approx 0.29286921$$

Exercise 3.2

Use Trapezoidal rule and Simpson's rule to compute

$$\int_0^1 \frac{\sin x}{x} \mathrm{d}x$$

Note that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(**a**) For Trapezoidal rule

$$\int_0^1 \frac{\sin x}{x} \mathrm{d}x \approx \frac{1}{2}[f(0) + f(1)] = 0.92073549$$

(**b**) For Simpson's rule

$$\int_0^1 \frac{\sin x}{x} \mathrm{d}x \approx \frac{1}{6}[f(0) + 4f(0.5) + f(1)] = 0.94614588$$

Exercise 3.3

Use composite Trapezoidal rule and composite Simpson's rule to approximate

$$\int_0^1 \frac{1}{1+x^3} \mathrm{d}x$$

and

$$\int_0^1 \frac{x}{\ln(1+x)} \mathrm{d}x$$

set $n = 8$

(**a**)

For composite Trapezoidal rule

$$\int_0^1 \frac{1}{1+x^3} \mathrm{d}x \approx 0.83466962$$

For composite Simpson's rule

$$\int_0^1 \frac{1}{1+x^3} \mathrm{d}x \approx 0.83556594$$

(**b**) Note that

$$\lim_{x \to 0} \frac{x}{\ln(1+x)} = 1$$

For composite Trapezoidal rule

$$\int_0^1 \frac{x}{\ln(1+x)} \mathrm{d}x \approx 1.22914661$$

For composite Simpson's rule

$$\int_0^1 \frac{x}{\ln(1+x)} \mathrm{d}x \approx 1.22927385$$

Exercise 3.4

Determine values of h that will ensure an approximation error of less than 0.00002 when approximating

$$\int_0^\pi \sin x \mathrm{d}x$$

and employing

- Composite Trapezoidal rule
- Composite Simpson's rule

(**a**) The error from the Composite Trapezoidal rule for $f(x) = \sin x$ on $[0, \pi]$ is

$$\left|\frac{b-a}{6}h^2 f''(\mu)\right| = \left|\frac{\pi h^2}{12}f''(\mu)\right| = \left|\frac{\pi h^2}{12}(-\sin\mu)\right| = \frac{\pi h^2}{12}|\sin\mu|$$

To ensure sufficient accuracy with this technique, we need to have

$$\frac{\pi h^2}{12}|\sin\mu| \le \frac{\pi h^2}{12} < 0.00002$$

Since $h = \pi/n$, we need

$$\frac{\pi^3}{12n^2} < 0.00002$$

which implies that

$$n > \left(\frac{\pi^3}{12(0.00002)}\right)^{1/2} \approx 359.44$$

and the Composite Trapezoidal rule requires $n \ge 360$

(**b**) The error form for the Composite Simpson's rule for $f(x) = \sin x$ on $[0, \pi]$ is

$$\left|\frac{b-a}{180}h^4 f^{(4)}(\mu)\right| = \frac{\pi h^4}{180}|\sin\mu| \le \frac{\pi h^4}{180} < 0.00002$$

Since $h = \pi/n$, we need

$$\frac{\pi^5}{180n^4} < 0.00002$$

which implies that

$$n > \left(\frac{\pi^5}{180(0.00002)}\right)^{1/2} \approx 17.07$$

So, the Composite Simpson's rule requires only $n \ge 18$