Lecture 1 homework 1-3

homework 1

whether a set of curved lines through the origin of $\mathbf{R^2}$ is a subspace of $\mathbf{R^2}$?

define a set W as a set of curved lines through the origin of $\mathbf{R^2}$:

$$W = \{(x, y) | y = \sum_{j=0}^{n} a_j x^{j+1}, x, y \in \mathbf{R^2}, a_0, a_1, \ldots a_n \in R\}$$

according to the definition of subspaces, if W is a subspace of $\mathbf{R^2}$, then:

$$\begin{cases} (x_1, y_1), (x_2, y_2) \in W, \implies (x_1 + x_2, y_1 + y_2) \in W \\ x_1, y_1) \in W, k \in F, \implies (kx_1, ky_1) \in W \end{cases}$$

- $y_1 = \sum_{j=0}^{n} a_j x_1^{j+1}$, $y_2 = \sum_{j=0}^{n} b_j x_2^{j+1}$, $y_1 + y_2 = \sum_{j=0}^{n} c_j (x_1 + x_2)^{j+1}$
- $y_1 + y_2 = a_0 x_1 + b_0 x_2 + a_1 x_1^2 + b_1 x_2^2$

$\implies c_1 x_1 x_2 \equiv 0$

while x_1, x_2 can't be 0 forever, so $c_1 = 0$

As the same way, it can be concluded that $c_2, c_3, \ldots, c_n = 0$

so a set of straight lines are subspace of $\mathbf{R^2}$. However, curved lines are not.

homework 2

let $v_1, v_2 \in \mathbf{R^3}$, $W = \{av_1 + bv_2 | a, b \in \mathbf{R}\}$. Is W a subspace of $\mathbf{R^3}$?

According to the definition of subspace, if W is a subspace of $\mathbf{R^3}$, then:

for $\alpha, \beta \in W$,

- $\alpha = av_1 + bv_2, a, b \in R$
- $\beta = cv_1 + dv_2, c, d \in R$

$\implies \alpha + \beta = (a + c)v_1 + (b + d)v_2 \in W$

- $\alpha = av_1 + bv_2, k \in F$

$\implies k\alpha = kav_1 + kbv_2 \in W$

In conclusion, W is a subspace of $\mathbf{R^2}$

homework 3

let $W_2 = B | B \in R^{n \times n}. \det(B) \neq 0$. Is W_2 a subspace of $R^{n \times n}$?

$\det(B) \neq 0 \implies 0 \notin W_2$

In conclusion, W_2 is not a subspace of $R^{n \times n}$