Chapter 2

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Exercise 2.1

Given the following 4 data points

$$\{(0,1),(1,2),(3,6),(5,7)\}$$

find a polynomial in Lagrange form to interpolate these data

we know that the formula for Lagrange interpolaiton is:

$$L_{n,k}(x)=\prod_{i=0,i\neq k}^{n}\frac{x-x_i}{x_k-x_i}$$

so the Lagrange polynomial for the 4 points should be:

$$\begin{aligned}\mathcal{L}_0 &= -\frac{-15+23x-9x^2+x^3}{15}\\ \mathcal{L}_1 &= \frac{15x-8x^2+x^3}{4}\\ \mathcal{L}_2 &= -\frac{5x-6x^2+x^3}{2}\\ \mathcal{L}_3 &= \frac{7(3x-4x^2+x^3)}{40}\end{aligned}$$

so the final polynomial should be sum of the 4 equations:

$$\mathcal{L}=1+\frac{29x}{120}+\frac{9x^2}{10}-\frac{17x^3}{120}$$

Exercise 2.5

Consider a function $f(x)=\sin(\pi x)+3x$ at 6 distinct nodes in the interval $[-1,1]$ to determine the Newton's divided difference formula. The data are given in Table below:

x	**-1.0**	**-0.6**	**-0.2**	**0.2**	**0.6**	**1.0**
$f(x)$	-3.0	-2.7511	-1.1878	1.1878	2.7511	3.0

the whole coefficient matrix can be presented below:

$$\begin{bmatrix} p=0 & p=1 & p=2 & p=3 & p=4 & p=5\\ -3 & & & & & \\ -2.7511 & 0.62225 & & & & \\ -1.1878 & 3.90825 & 4.1075 & & & \\ 1.1878 & 5.939 & 2.5384375 & -1.30755208 & & \\ 2.7511 & 3.90825 & -2.5384375 & -4.23072917 & -1.82698568 & \\ 3 & 0.62225 & -4.1075 & -1.30755208 & 1.82698568 & 1.82698568\end{bmatrix}$$

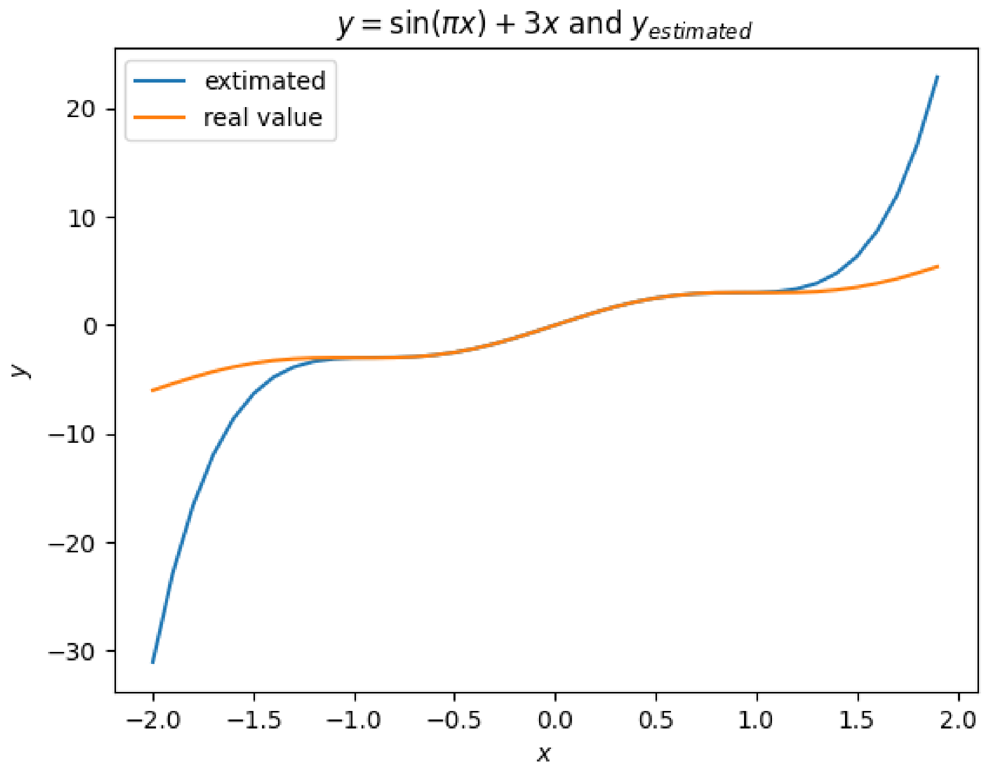
according to the Newton's divided difference formula,

$$P_n(x)=F_{0,0}+\sum_{i=1}^{n}F_i\prod_{j=0}^{i-1}(x-x_j)$$

so the polynomial should be:

$$
\begin{aligned}
P(x) = & -3 \\
& + 0.62225(x+1.0) \\
& + 4.1075(x+1.0)(x+0.6) \\
& - 1.30755208(x+1.0)(x+0.6)(x+0.2) \\
& - 1.82698568(x+1.0)(x+0.6)(x+0.2)(x-0.2) \\
& + 1.82698568(x+1.0)(x+0.6)(x+0.2)(x-0.2)(x-0.6)
\end{aligned}
$$

compare the polynomial we get and the real function $f(x) = \sin(\pi x) + 3x$



we can see that **inside** the interval $[-1, 1]$, the polynomial performs well, but the error will exceed greatly **beyond** the interval.

Exercise 2.6

Construct an approximating polynomial for the following data using Hermite interpolation with Newton's forward difference

Table 1:

x	0.1	0.2	0.3
$f(x)$	-0.29004996	-0.56079734	-0.81401972
$f'(x)$	-2.8019975	-2.6159201	-2.9734038

Table 2:

x	-1.0	-0.5	0.0	0.5
$f(x)$	0.86199480	0.95802009	1.0986123	1.2943767
$f'(x)$	0.15536240	0.23269654	0.33333333	0.45186776

for data in Table 1:

$$
\begin{array}{llllllll}
z_0 = 0.1 & f[z_0] = -0.29004996 & -0.29004996 & & & & & \\
z_1 = 0.1 & f[z_1] = -0.29004996 & -0.29004996 & -2.8019975 & & & & \\
z_2 = 0.2 & f[z_2] = -0.56079734 & -0.56079734 & -2.7074738 & 0.945237 & & & \\
z_3 = 0.2 & f[z_3] = -0.56079734 & -0.56079734 & -2.6159201 & 0.915537 & -0.297 & & \\
z_4 = 0.3 & f[z_4] = -0.81401972 & -0.81401972 & -2.5322238 & 0.836963 & -0.39287 & -0.47935 & \\
z_5 = 0.3 & f[z_5] = -0.81401972 & -0.81401972 & -2.9734038 & -4.4118 & -52.48763 & -260.4738 & -1299.97225
\end{array}
$$

for data in Table 2:

$$
\begin{array}{cccccccc}
0.8619948 & & & & & & & \\
0.8619948 & 0.1553624 & & & & & & \\
0.95802009 & 0.19205058 & 0.07337636 & & & & & \\
0.95802009 & 0.23269654 & 0.08129192 & 0.01583112 & & & & \\
1.0986123 & 0.28118442 & 0.09697576 & 0.01568384 & -0.00014728 & & & \\
1.0986123 & 0.33333333 & 0.10429782 & 0.01464412 & -0.00103972 & -0.00089244 & & \\
1.2943767 & 0.3915288 & 0.11639094 & 0.01209312 & -0.002551 & -0.00100752 & -0.00007672 & \\
1.2943767 & 0.45186776 & 0.12067792 & 0.00857396 & -0.00351916 & -0.00096816 & 0.00002624 & 0.00006864
\end{array}
$$

so the polynomial 1 should be:

$$
\begin{aligned}
P(x) = & -0.29004996 \\
& -2.8019975(x-0.1) \\
& +0.945237(x-0.1)^2 \\
& -0.297(x-0.1)^2(x-0.2) \\
& -0.47935(x-0.1)^2(x-0.2)^2 \\
& -1299.97225(x-0.1)^2(x-0.2)^2(x-0.3)
\end{aligned}
$$

the polynomial 2 should be:

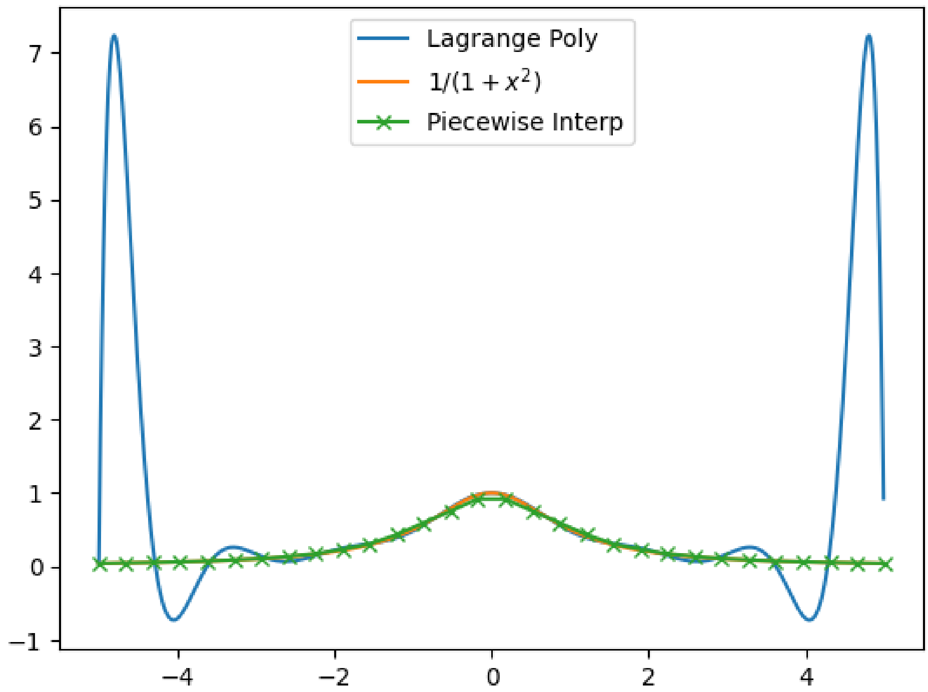
$$
\begin{aligned}
P(x) = & 0.8619948 \\
& +0.1553624(x+1) \\
& +0.07337636(x+1)^2 \\
& +0.01583112(x+1)^2(x+0.5) \\
& -0.00014728(x+1)^2(x+0.5)^2 \\
& -0.00089224(x+1)^2(x+0.5)^2x \\
& -0.00007672(x+1)^2(x+0.5)^2x^2 \\
& +0.00006864(x+1)^2(x+0.5)^2x^2(x-0.5)
\end{aligned}
$$

Exercise 2.7

Choose equally-spaced 15 points in $[-5,5]$ and get the data points

$$\{x_i, f(x_i)\}_{i=0}^{14}$$

by $f(x)=1/(1+x^2)$. Using piecewise linear interpolation to get the approximation of $f(x)$ in $[-5,5]$ and, compare the result with that obtained by Lagrange interpolation.



It can be seen that Lagrange interpolation keeps a good tracing trend in the center area; Piecewise interpolation keeps a good trending in all area;