Characterizing Diffraction Gratings

JAIME DÍEZ GONZÁLEZ-PARDO

University of Cantabria
Experimental Optics

March 28, 2018

Abstract

It has been studied the diffraction phenomenon produced by a grating for the ligth emited from a Cd atomic lamp, obtainig its spectral lines. First of all it has been necessary to obtain the grating parameter with a value of $\bar{d} = (884.4 \pm 0.8)$ nm, from the diffraction angle θ of a well-known spectral line, in order to characterize the diffraction grating. In this way it has been able to obtain the wavelength of another unknown wavelength, with a result $\bar{\lambda} = (508.6 \pm 0.5)$ nm compatible with the real wavelength for that line.

1. INTRODUCTION

Gratings can produce diffraction on an incident ligth, as long as its wavelength and distances between slots (the grating parameter d) are in the same order of magnitud, due to interferences between beams which have been passed thru differents slots. If the difference between optics path is a multiple of wavelength, the interference will be constructive, obtaining a maximum. This condition is fulfilled only by some directions for a given grating parameter d as it is shown for a normal incident beam, in the equation 1, where θ is the angle formed by the normal to the surface grating and the diffracted direction.

$$2d \sin\theta = m\lambda \tag{1}$$

Following the equation 1, if either the wavelength λ or the grating parameter d is known, the other can be obtain by measure the angle θ.

Due to the parameter d of the grating used in the experimental set-up is unknown, it is going to try to obtain by measure the angles θ for the diffraction of a spectral line with well-known wavelength. Once d is known it is possible to obtain any wavelength of a diffracted line.

2. METHOD

The light source used is a spectral lamp of Cd atomic gas, with well-known wavelength for the clear blue emission line $\lambda = 480.0nm$ [1].

All measurements have been made by means of a goniometer with two differents arms. The collimator arm is placed just after the spectral lamp in order to drives the incident beam from the source onto the grating. The second arm contains a telescope attached to a rotating disk with a lateral scale that indicates its angular position with a precision of $1'$ arc. The zero of the scale is in a lateral window of another rotating disk. This other disk contains a platfom, that can be levelled with three screws, in which the grating has been put.

The different parts of the experimental set-up are shown in the Figure 1.

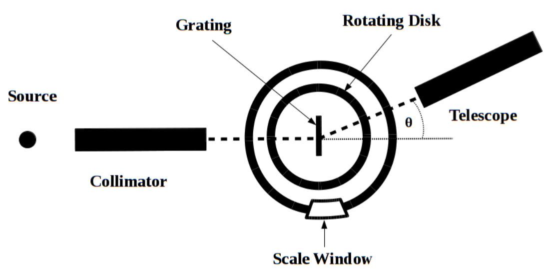

Figure 1: *Scheme of the experimental set-up used in the laboratory.*

Before measuring, it is important to check all the set-up, focusing the collimator to see the slit sharp, the telescope to infinity (or a far object), its eyepiece on the cross and setting that cross correctly vertical.

Another important thing to check is that the grating is as perpendicular as possible to the incident beam. To this purpose it has been measured the position of the telescope where the slit's image is centred θ_{middle}, and then the position of the clearly blue line of the spectrum diffracted in the first order at the right side θ_r and at the left side θ_l. If the grating is completely perpendicular to the incident beam, the angle between θ_{middle} and, both θ_r and θ_l, should be the same. In the case the difference between both angles is larger than $5'$ arc the platform has to be rotate slightly by means of the screws, and the measurements repeated.

Once the experimental set-up is correctly disposed it is able to start characterizing the diffraction grating. First, the diffraction grating parameter will be obtain by measure the positions θ_r and θ_l of the clearly blue line obtaining the diffraction angle $\theta = \frac{\theta_r - \theta_l}{2}$. In order to minimize the error it is going to be taken six measurements of θ. After calculate the average $\bar{\theta}$, $\bar{d}$ will be obtain by the equation 1.

In the last part of the experiment it has been tried to obtain the wavelength of another spectral line from the former value of $\bar{d}$ and the new average angle $\bar{\theta}$. The method used to obtain the new $\bar{\theta}$ is the same used for the clearly blue line but for the new spectral line.

3. Results

Most of the angle values shown in the report are express in therms of degrees using the conversion of the equation 2, although they were taken in degrees and minutes in order to simplify the results and the calculations.

$$1' = \left(\frac{1}{60}\right)^{o} \tag{2}$$

The first part of the experiment correspond to the alignment of the diffraction grating, which had to be rotated once to achieve a final value for the differences between both angles of $\delta\theta \approx 4'$ arc.

$$|\theta_r - \theta_{middle}| - |\theta_l - \theta_{middle}| \approx 4' < 5' \tag{3}$$

In the Table 1 appear all the values of θ_{middle}, θ_r, θ_l and $\delta\theta$ obtained before and after adjust the platform of the grating.

$\theta_{middle}/^o$	$\theta_l/^o$	$\theta_r/^o$	$\delta\theta$
91.583	75.750	107.300	$7'$
91.383	75.567	107.133	$4'$

Table 1: *Positions of the telescope for the slit's image θ_{middle}, the first order diffracted clearly blue line at the left side θ_l, at the right side θ_r, and the difference between angles, $\delta\theta$ obtained by the equation 3. The error of the positions is $\Delta\theta = 0.017^o$.*

3.1. Grating Parameter d

In order to obtain the diffraction grating parameter d it has been taken six measurements of the positions θ_r and θ_l of the clearly blue line obtaining the diffraction angle by $\theta = \frac{\theta_r - \theta_l}{2}$. This values appear in the Table 2.

$\theta_l/^o$	$\theta_r/^o$	$\theta/^o$
75.567	107.133	15.783
75.667	107.067	15.700
75.583	107.100	15.758
75.617	107.050	15.717
75.550	107.117	15.783
75.617	107.083	15.733

Table 2: *Positions θ_r and θ_l of the clearly blue line and the diffraction angle $\theta = \frac{\theta_r - \theta_l}{2}$. The error of θ_l and θ_r is $\Delta\theta_{l,r} = 0.017^o$. The error of θ calculated by the equation 8 on the appendix A is $\Delta\theta = 0.012$.*

With those values the average angle $\bar{\theta}$ is calculated obtaining a value:

$$\bar{\theta} = \frac{1}{N}\sum_{i=1}^{N}\theta_i = (15.746 \pm 0.014)^o \tag{4}$$

The error of $\bar{\theta}$ correspond to the standard deviation calculated in the appendix A.1 by the equation 9.

Once $\bar{\theta}$ is calculated, obtain the diffraction parameter is just substitute the known values in the equation 1.

$$\bar{d} = \frac{m\lambda}{2 \cdot \sin(\bar{\theta})} = (884.4 \pm 0.8)\ \text{nm} \tag{5}$$

Where $m = 1$ accordingly to first order diffraction lines, and $\lambda = 480.0$ nm [1]. The error of $\bar{d}$ is calculated in the appendix A.1 in the equation 10

3.2. Unknown Wavelength

The same method as the used for the diffraction grating parameter has been used to obtain the wavelength for another spectral line, the green one for this experiment. The Table 3 shown the measurements for the positions.

$\theta_l/^o$	$\theta_r/^o$	$\theta/^o$
74.667	108.067	16.700
74.667	108.083	16.708
74.683	108.100	16.708
74.700	108.100	16.700
74.633	108.083	16.725
74.600	108.050	16.725

Table 3: *Positions θ_r and θ_l of the clearly blue line and the diffraction angle $\theta = \frac{\theta_r - \theta_l}{2}$. The error of θ_l and θ_r is $\Delta\theta_{l,r} = 0.017^o$. The error of θ calculated by the equation 8 on the appendix A is $\Delta\theta = 0.012^o$.*

As in the former method, the average angle $\bar{\theta}$ is calculated obtaining a value:

$$\bar{\theta} = \frac{1}{N}\sum_{i=1}^{N}\theta_i = (16.711 \pm 0.005)^o \tag{6}$$

The error of $\bar{\theta}$ correspond to the standard deviation calculated in the appendix A.2 by the equation 11.

Once the $\bar{\theta}$ is calculated, obtain the the diffraction parameter is just substitute the known values in the equation 1.

$$\bar{\lambda} = 2\bar{d}\sin(\bar{\theta}) = (508.6 \pm 0.5)\ \text{nm} \tag{7}$$

Where $m = 1$ and $\bar{d}$ is the value obtained in the section 3.1. The error of $\bar{\lambda}$ is calculated in the appendix A.2 in the equation 12

4. Discussion

A diffraction grating has been characterizing by measured its grating parameter d from the angle $\bar{\theta}$, formed by the normal to the surface grating and the diffracted direction of the spectral clearly blue line, obtaining a value for it of $\bar{d} = (884.4 \pm 0.8)$ nm. In order to try to minimized the error it has been taken six measurements for the angle θ using the average to obtain d. By this way, a relative error of about 0.09% has been obtained. This error means a very precise value for d, which does not necessary implate that the value is correct, due to the manufacture grating parameter value is unknown.

Once the diffraction grating parameter d has been obtained, it has been tried to calculate the wavelength of the green spectral line of the Cd atomic gas. The wavelength's value obtained is $\bar{\lambda} = (508.6 \pm 0.5)$ nm. It has been used the average angle of six differents measurements of θ such as in the former method. The relative error obtained is about 1%. This error is little bigger than the error obtained for d, due to it depends on it and on the standard deviation of $\bar{\theta}$, but again very small, showing that the method used in the experiment provide very precise values.

The experimental wavelength obtained is compatible with the known value of the green spectral line for the Cd I (508.58217 nm) [2]. With a result as good as the obtained for the $\bar{\lambda}$ and due to it has been obtained from the experimental value of $\bar{d}$, it could be said that the manufacture value of d should be near the experimental one, $\bar{d}$.

Knowing the diffraction grating parameter it is possible to obtain the wavelength of any spectral line by measurements of the diffracted angle θ, as it has been done for the green spectral line of the Cd. This method could be used in order to determine the spectrum of a certain emitting gas by characterized its spectral lines, been able to identify the gas.

However, the diffraction grating used in this experiment could only be used to identify gases with its spectrum in the visible range. This is due to the diffraction phenomenon only occures when the grating parameter and the wavelength of the incident beam have the same order of magnitude.

A similar method could be used using a glass prism instead the diffraction grating. In this case the prism will dispers the light, been able to observe (and measure) the spectral lines clearly separated. The dispersion produce by the prism is not based in the diffraction phenomenon as in the grating, but the index of refraction of the glass depends on the wavelength, so the ligth will be transmitted from one medium to the other with differents angles depends on the wavelength, separating the spectral lines.

5. Conclussions

It has been verify the expression 1 for the diffraction produced by a grating, obtaining the grating parameter $\bar{d}$ and an unknown wavelength $\bar{\lambda}$. It can be said that this experimental set-up could be used to identify a certain gas by characterizing its spectral lines as long as that gas emits in the visible range.

References

[1] Experimental Optics. Guide of Experiment N.5. Characterizing diffraction gratings: measurement of an atomic tansition wavelength.

[2] A. Kramida, Yu. Ralchenko, J. Reader, and and NIST ASD Team. NIST Atomic Spectra Database (ver. 5.5.3), [Online]. Available: `https://physics.nist.gov/asd` [2018, March 23]. National Institute of Standards and Technology, Gaithersburg, MD., 2018.

A. Error Propagation

In the sections 3.1 and 3.2 the angles θ have been obtained from the positions θ_r and θ_l by $\theta = \frac{\theta_r - \theta_l}{2}$. The error of those values is calculated by the equation 8 and depends only on the error of θ_r and θ_l which are the error given by the instrument ($\pm 0.017^o$).

$$\Delta\theta = \sqrt{\left(\frac{\Delta\theta_r}{2}\right)^2 + \left(\frac{\Delta\theta_l}{2}\right)^2} = \frac{0.017}{\sqrt{2}} = 0.012^o \tag{8}$$

A.1. Grating Parameter d

In the section 3.1 it has been calculated the average angle $\bar{\theta}$. Its error correspond to the standard deviation calculated in the equation 9.

$$\sigma_{\bar{\theta}}^2 = \frac{1}{N(N-1)} \sum_{i=1}^{N} (\theta_i - \bar{\theta})^2 \quad \rightarrow \quad \sigma_{\bar{\theta}} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (\theta_i - \bar{\theta})^2} = 0.014^o \tag{9}$$

The error of the $\bar{d}$ calculated in the equation 5 is obtained by the following equation 10.

$$\sigma_{\bar{d}} = \sqrt{\left[\frac{\lambda \cos(\bar{\theta})}{2\sin^2(\bar{\theta})}\right]^2 \sigma_{\bar{\theta}}^2} = 0.78nm \tag{10}$$

A.2. Unknow Wavelength

In the section 3.2 it has been calculated the average angle $\bar{\theta}$. Its error correspond to the standard deviation calculated in the equation 9.

$$\sigma_{\bar{\theta}}^2 = \frac{1}{N(N-1)} \sum_{i=1}^{N} (\theta_i - \bar{\theta})^2 \quad \rightarrow \quad \sigma_{\bar{\theta}} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (\theta_i - \bar{\theta})^2} = 0.005^o \tag{11}$$

The error of the $\bar{d}$ calculated in the equation 7 is obtained by the following equation 12.

$$\sigma_{\bar{\lambda}} = \sqrt{(2\sin(\bar{\theta}))^2 \sigma_{\bar{d}}^2 + (2\bar{d}\cos(\bar{\theta}))^2 \sigma_{\bar{\theta}}^2} = 0.47nm \tag{12}$$