$$\text{Ansatz} = (IX)(X(R2\ I)(HI)|\infty\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{bmatrix}$$

$$IX = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (R2I) = \begin{bmatrix} e^{-i\phi/2} & 0 & 0 & 0 \\ 0 & e^{i\phi/2} & 0 & 0 \\ 0 & 0 & e^{-i\phi/2} & 0 \\ 0 & 0 & 0 & e^{i\phi/2} \end{bmatrix}$$

$$HI = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$(IX)CX = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (R2)(HI) = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{-i\phi/2} & 0 & e^{i\phi/2} & 0 \\ 0 & e^{i\phi/2} & 0 & e^{i\phi/2} \\ e^{-i\phi/2} & 0 & -e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} & 0 & -e^{i\phi/2} \end{bmatrix}$$

multiply →

$$|\infty\rangle \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & e^{i\phi/2} & 0 & e^{i\phi/2} \\ e^{-i\phi/2} & 0 & e^{i\phi/2} & 0 \\ e^{-i\phi/2} & 0 & -e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} & 0 & -e^{i\phi/2} \end{bmatrix}\begin{bmatrix} \phi \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|\psi'\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{array}{cccc} H_1 & H_2 & H_3 & H_4 \end{array}$$

$$\frac{ZZ + XX + YY - II}{2}$$

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$\langle H_1 \rangle$   $\langle \psi' | ZZ | \psi' \rangle$

$$= \frac{1}{2} \begin{bmatrix} 0 & e^{i\phi/2} & e^{i\phi/2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & -e^{i\phi/2} & -e^{i\phi/2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix} = 0 \;-1\; -1 + 0$$

$$= -2/2$$
$$= -1$$

$\langle H_2 \rangle = \langle \psi' | XX | \psi' \rangle$

$$= \frac{1}{2} \begin{bmatrix} 0 & e^{i\phi/2} & e^{i\phi/2} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & e^{i\phi/2} & e^{i\phi/2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix} = \frac{2}{2} = 1$$

$$\langle H_3 \rangle = \langle \psi' | YY | \psi' \rangle$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\theta/2} & e^{i\theta/2} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-i\theta/2} \\ e^{-i\theta/2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\theta/2} & e^{i\theta/2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-i\theta/2} \\ e^{-i\theta/2} \\ 0 \end{bmatrix} = \frac{2}{2} = 1$$

$$\langle H_4 \rangle = \langle \psi' | II | \psi' \rangle$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\theta/2} & e^{i\theta/2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-i\theta/2} \\ e^{-i\theta/2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\theta/2} & e^{i\theta/2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-i\theta_2} \\ e^{-i\theta/2} \\ 0 \end{bmatrix} = \frac{2}{2} = 1$$

$$\langle H \rangle = \frac{-2 + 2 + 2 + 2}{2} = \frac{4}{2} = 2$$

$$\boxed{\langle H \rangle = \frac{-1+1+1-1}{2} = 0}$$