$$\text{Ansatz} = (IX)CX(R2\ I)(HI)|00\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

$$IX = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (R2I) = \begin{bmatrix} e^{-i\theta/2} & 0 & 0 & 0 \\ 0 & e^{i\theta/2} & 0 & 0 \\ 0 & 0 & e^{-i\theta/2} & 0 \\ 0 & 0 & 0 & e^{i\theta/2} \end{bmatrix}$$

$$HI = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$(IX)CX = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(R2I)(HI) = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{-i\theta/2} & 0 & e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} & 0 & e^{i\theta/2} \\ e^{-i\theta/2} & 0 & -e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} & 0 & -e^{i\theta/2} \end{bmatrix}$$

multiply → $|00\rangle$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & e^{i\theta/2} & 0 & e^{i\theta/2} \\ e^{-i\theta/2} & 0 & e^{-i\theta/2} & 0 \\ e^{-i\theta/2} & 0 & -e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} & 0 & -e^{i\theta/2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \frac{\overset{H_1}{ZZ} + \overset{H_2}{XX} + \overset{H_3}{YY} - \overset{H_4}{II}}{2}$$

$$\langle H_1 \rangle = \langle \psi' | ZZ | \psi' \rangle$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\phi/2} & e^{i\phi/2} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -e^{i\phi/2} & -e^{i\phi/2} & 0 \end{bmatrix}\begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix} = 0 - 1 - 1 + 0$$

$$= -2/2$$

$$= -1$$

$$\langle H_2 \rangle = \langle \psi' | XX | \psi' \rangle$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\phi/2} & e^{i\phi/2} & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\phi/2} & e^{i\phi/2} & 0 \end{bmatrix}\begin{bmatrix} 0 \\ e^{-i\phi/2} \\ e^{-i\phi/2} \\ 0 \end{bmatrix} = \frac{2}{2} = 1$$

$$\langle H_3\rangle = \langle\psi'|YY|\psi'\rangle$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\theta/2} & e^{i\theta/2} & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ e^{-i\theta/2} \\ e^{-i\theta/2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\theta/2} & e^{i\theta/2} & 0 \end{bmatrix}\begin{bmatrix} 0 \\ e^{-i\theta/2} \\ e^{-i\theta/2} \\ 0 \end{bmatrix} = \frac{2}{2} = 1$$

$$\langle H_4\rangle = \langle\psi'|II|\psi'\rangle$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\theta/2} & e^{i\theta/2} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ e^{-i\theta/2} \\ e^{-i\theta/2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & e^{i\theta/2} & e^{i\theta/2} & 0 \end{bmatrix}\begin{bmatrix} 0 \\ e^{-i\theta_2} \\ e^{-i\theta/2} \\ 0 \end{bmatrix} = \frac{2}{2} = 1$$

$$\langle H\rangle = \cancel{\frac{-2+2+2+2}{2} = \frac{4}{2} = 2}$$

$$\boxed{\langle H\rangle = \frac{-1+1+1+1}{2} = 1}$$