Single qubit error mitigation by simulating Non-Markovian dynamics

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Mirko Rossini, Dominik Maile, Joachim Ankerhold and Brecht I. C Donvil, "Single Qubit Error Mitigation by Simulating Non-Markovian Dynamics", Phys. Rev. Lett. 131, 110603 – Published 15 September, 2023

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General Dynamical Map

- General Dynamical Map [1] are
  - Linear
  - Trace preserving
  - Hermiticity preserving
  - **Not necessarily CP**

- Arise from
  - Intermediate steps in Non-Markovian dynamics
  - Correlated system-bath initial states
  - As *time reversed evolution*

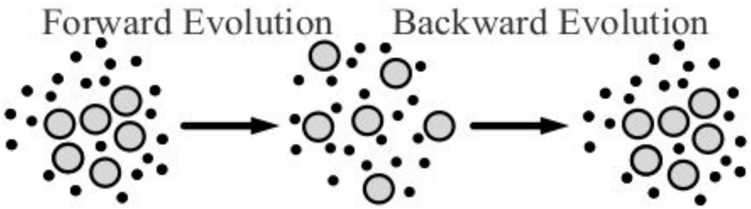
[1] *E. C. G. Sudarshan, P. M. Mathews and Jayaseetha Rau, "Stochastic Dynamics of Quantum-Mechanical Systems", Phys. Rev. 121, 920 – Published 1 February, 1961*

State recovery

- Consider the evolution

$$\frac{d}{dt}\rho_t = -i[H_t, \rho_t] + \sum_k \Gamma_{k,t}(L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\})$$

- The time reversed evolution is $\frac{d}{dt}\rho_t = -\mathcal{L}_t(\rho_t)$
- Evolving a state for a time t with and then for a time s $\leq$ t with its time reversed evolution results in the state obtained by just evolving with (6) for a time t — s



General Dynamical Map

- General Dynamical Map **as weighted sum of CPTP map:**

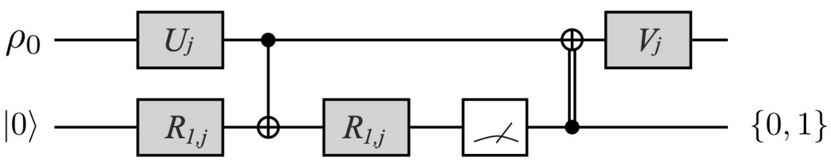
$$\Sigma(\rho) = (1 + p)\Lambda_+^* - p\Lambda_-^*$$

- CPTP map expressed as the sum of two extremal CP maps [1, 2]:

$$\Lambda(\rho) = \frac{1}{2}\Lambda_1(\rho) + \frac{1}{2}\Lambda_1(\rho)$$

$$\Lambda_j(\rho) = U_j\left(\sum_{i=1}^{2} F_{i,j}(V_j\rho V_j^\dagger)F_{i,j}^\dagger\right)U_j^\dagger$$

A simple circuit shown in to realise the action of the Λj using just one ancillary qubit and CNOT gates [3]



[1] *Mary Beth Ruskai, Stanislaw Szarek and Elisabeth Werner, "An analysis of completely-positive trace-preserving maps on M_2", Linear Algebra and its Applications, Vol. 347, Issues 1–3, 15 May 2002, Pages 159-187*

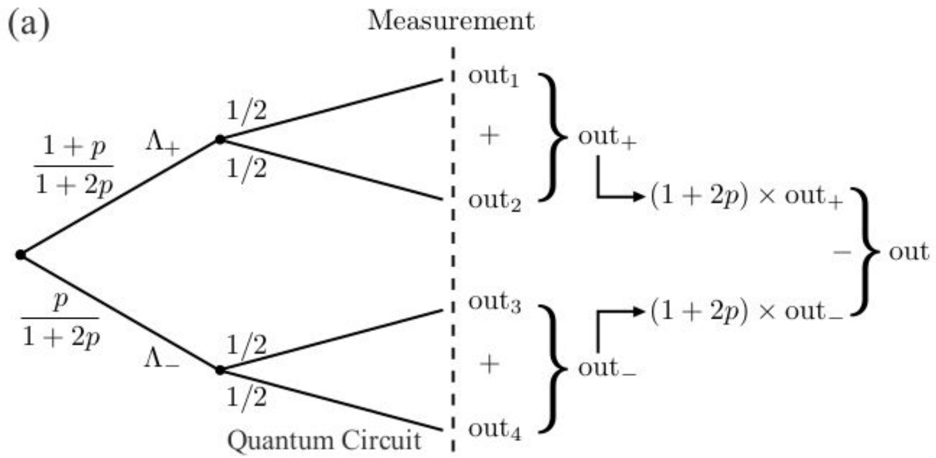
[2] *C. King and M. B. Ruskai, "Minimal entropy of states emerging from noisy quantum channels," in IEEE Transactions on Information Theory, vol. 47, no. 1, pp. 192-209, Jan. 2001, doi: 10.1109/18.904522.*

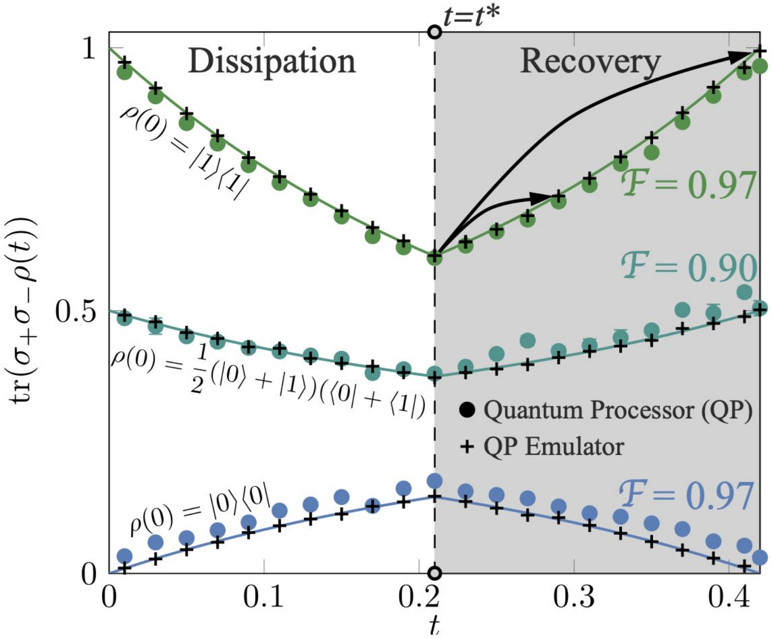
[3] *Dong-Sheng Wang, Dominic W. Berry, Marcos C. de Oliveira and Barry C. Sanders, "Solovay-Kitaev Decomposition Strategy for Single-Qubit Channels", Phys. Rev. Lett. 111, 130504 – Published 25 September, 2013*

Protocol

$$\frac{d}{dt}\rho_t = -i[H_t,\rho_t] + \sum_k \Gamma_{k,t}\left(L_k\rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k,\rho_t\}\right)$$

- Dissipation: $\Gamma_{k,t}$ are positive definite
- Recovery: $\Gamma_{k,t}$ are not necessarily positive definite

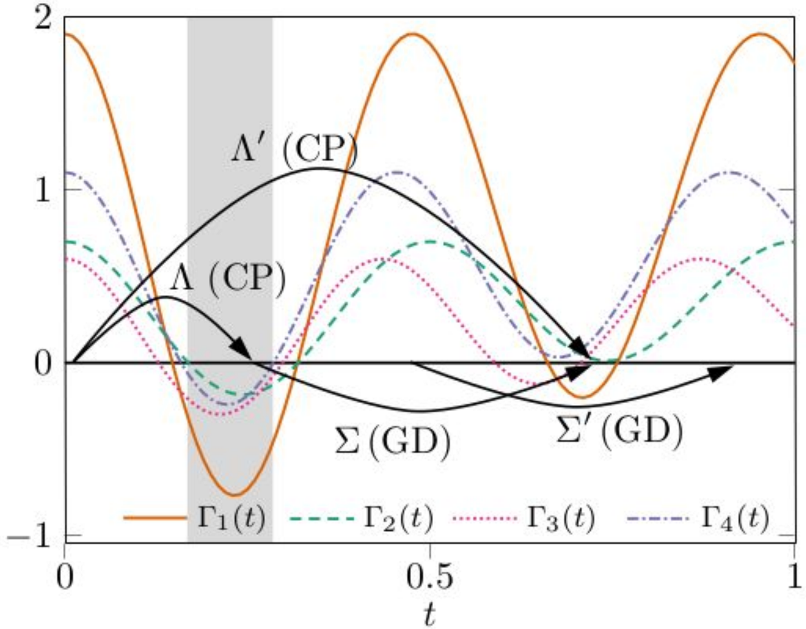


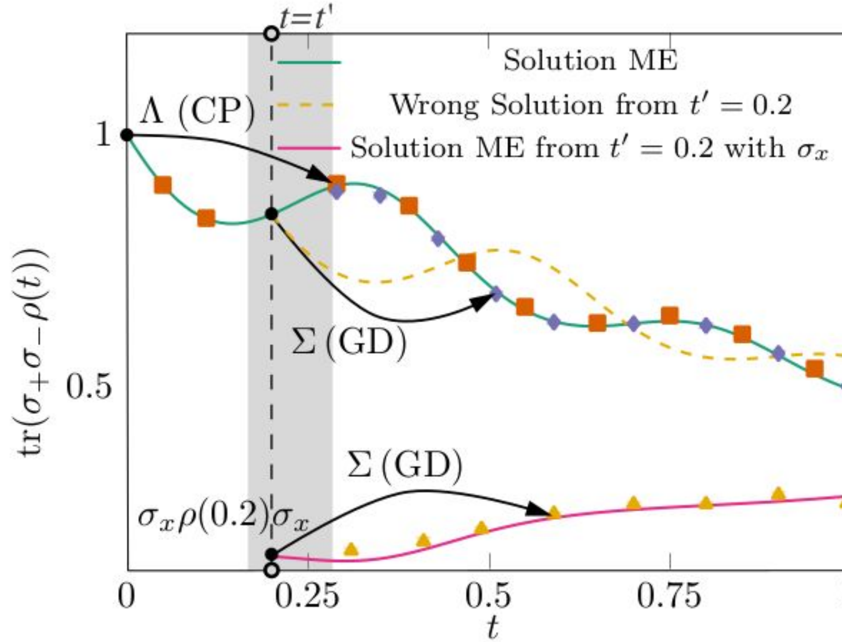


A qubit weakly coupled to a thermal bath

Extra slides

Example GD evolution





$$\frac{d}{dt}\rho_t = -i[H_t, \rho_t] + \sum_k \Gamma_{k,t}(L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\})$$