TC | SC

Simple Sieve — $N \log\log N$ | 1 | N
Segmented Sieve — " | 1 | $\sqrt{N}$
Incremental Sieve — " | 1 | N

Euler Phi — $\sqrt{n}\log n$ | 1

$$\phi(n) = \begin{cases} n-1, & \text{if } n \text{ is prime} \\ n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots, & \text{otherwise} \end{cases}$$

$p_1, p_2, \ldots$ are prime factors

CRT — $n \log m$ | 1

Strobogrammatic number — N | 1

0 1 6 8 9
↓ ↓ ↓ ↓ ↓
0 1 9 8 6

0, 1, 8 → Strobo + Palindrome

Binary Palindrome — $\log n$ | 1
Booth's Algorithm

Block Swap / Rotating array — n | 1

Euclid's Algorithm — $\log(\min(a,b))$ | 1
Valid for only INTEGERS.
$ax + by = \gcd(a,b)$
$x = y_1$
$y = x_1 - (a/b)y_1$

• $b = 0$ then a
• $\gcd(a,b) = \gcd(b, a\%b)$

Karatsuba Algorithm — $n^{1.585}$ or $n^{\log_2 3}$ | 1
result = $ac \times 10^m + (abcd - ac - bd)10^{m/2} + bd$

Longest sequence of 1s after flip — n | 1
Max Product Subarray — n | 1
Swap 2 nibbles — 1 | 1
Leaders in Array — n | n
Lexicographically First Palindrome — n | 1
Majority Element — n | 1
Boyre-Moore Solution
Majority Vote
Max Equilibrium Sum — n | 1
Max Sum - Hour Glass — mn | 1 — $(R-2) \times (C-2)$ = no. of hour glasses
Selection Sort — n^2 | 1
Quick Sort — $n^2, n\log n$ | 1

Weighted Substring — n^n | 1
Move Hypens to Beginning — n | 1
★ Manacher's / Longest Palindromic Substring — n | n
Using DP → n^2 | n^2
Maneuvering Problem — n | 1
Josephus Problem — n | 1
josephus(n, k) = (josephus(n-1, k) % k - 1) % n + 1
and josephus(1, k) = 1
Activity Selection — n | 1
★ Permutations — $n \times n!$ | n
★ Combinations — ${}^nC_n$ | 2
Maze — $2^{n \times n}$ | $n \times n$
N-queen

Huffman Coding

Message: ABBCDBCCDAABBEEEBEAB

Length = 20 letters

Total = 20 × 8 = 160 bits

A - 65 → 01000001

B - 66 → 01000010

C - 67

D - 68

E - 69

(8 bits)

(i) Fixed Length Coding

$2^3 \Rightarrow$ 8 diff. combinations from 3 bits

000

001

010 - A

011 - B

100 - C

101 - D

110 - E

111

20 × 3 = 60 bits → Message*

5 × 3 + 5 × 8 = 55 bits → Table**

Total = 60 + 55 = 115 bits (Answer)***

"better than 160 bits"

(ii) Variable Length Coding

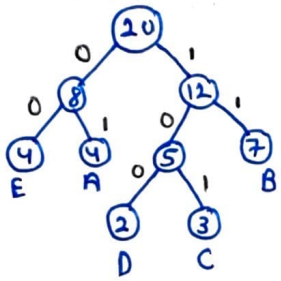
Char	Frequency / Count	
A	4	01
B	7	11
C	3	101
D	2	100
E	4	00

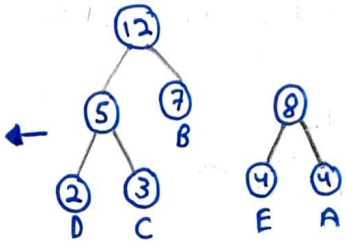
Step 1: Arrange in ascending order

2 3 4 4 7

D C E A B

Step 2: Draw Tree, merge 2 min.





Step 3: Assign code [0 / 1]

left right

• Traverse root to leaf

Char	Code
(4) A	101
(7) B	11
(3) C	101
(2) D	1001
(4) E	00

Step 4: Calculations

* Message → 4×2 + 7×2 + 3×3 + 3×2 + 2×4 = 8 + 14 + 9 + 6 + 8 = 45 bits

** Table → 8×5 + 2 + 2 + 2 + 3 + 3 = 40 + 12 = 52 bits

Total = 45 + 52 = 97 bits (Answer)***

Note: No code is prefix of another code. → Prefix rule

• David Huffman in 1951

•• Encoding follows the prefix rule

∴ Most generated character will get the small code and least generated will get big code.

:: Time complexity - O(n log n)

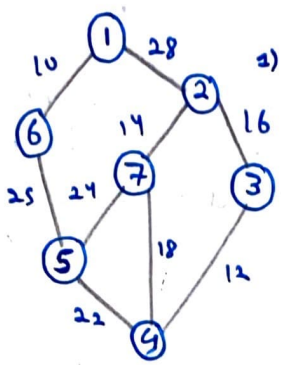
Minimum Cost Spanning Tree
↳ Kruskal's and Prims algo.

No. of MST formed from a graph with E, V = $E C_{V-1}$ - cycles

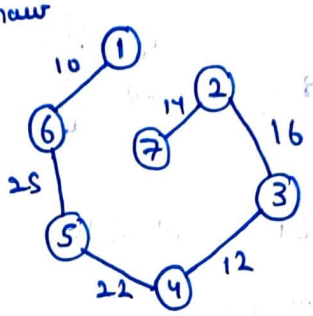
$\{ {}^{n}C_r = n!/r!(n-r)! \}$

$|E| = 6 \quad \therefore {}^{6}C_5 = 6!/5!(6-1)! = 6$

* Kruskal [Always select min cost edge]



1) Ascending order → 10, 12, 14, 16, 18, 22, 24, 25, 28
2) Pick min, make connections, no cycles
3) Draw



Answers ⇒ 10 + 12 + 14 + 16 + 22 + 25 = 99 ***

3 **Graph Coloring** {*** Welsh Powell algo}

↳ Chromatic number * - smallest no. of colors needed to color a graph such that no to adjacent nodes have same colors.

Applications - Map coloring, Sodoku, Time Table, Bipartite graphs.

+ 4 Hamiltonian Cycle

5 The Knight's Tour Problem (Warnsdorff's Rule)