**Probability, Statistics and Stochastics Processes (MAL2010)**

# Lecture 1

$$\text{Data} \rightarrow \text{Analysis} \rightarrow \text{Utilizing Data}$$

We begin by considering two primary modelling approaches:

- **Deterministic Models**: No uncertainty is involved. Examples include:

$$V = IR, \quad F = ma$$

- **Statistical Models**: Incorporate uncertainty. Examples include:

  Relation between Temperature and Atmospheric Pressure
  Relation between Weight and Height
  Relation between Income and Expenditure

  A statistical formulation can be expressed as:

$$w = f(u) + \epsilon$$

  where $\epsilon$ represents **uncertainty**.

**Remark 1.** *A **mathematical structure** is used to define a statistical model.*

As behaviour changes, uncertainty tends to increase. The target in modelling is to:

1. Reduce error
2. Increase accuracy

# 1 Statistics

Statistics can be divided into three broad categories:

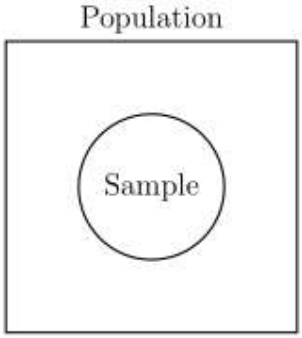
## 1.1 Descriptive Statistics

Provides a historical perspective of data. Examples include:

- Measures of central tendency: **Mean**, **Median**, **Mode**
- Measures of dispersion: **Range**, **Variance**, **Covariance**
- Data visualization: Bar Chart, Pie Chart, Box-plot, Scatter diagram

## 1.2 Predictive Statistics

- Analyzing historical data for prediction
- Using a **mathematical structure** for prediction to understand the behaviour of the entire population




After modelling, we test the model to ensure reliability.

## 1.3 Prescriptive / Inferential Statistics

Focused on:

- **Testing**
- **Validation**

# 2 Probability Theory

## 2.1 Understanding Probability

Probability is synonymous with **chance**.

**Definition 1** (Empirical Definition)**.** *Probability is defined as:*

$$\frac{\textit{Favourable outcomes}}{\textit{Total outcomes}}$$

This is the **classical** definition, not the rigorous mathematical one.

## 2.2 Mathematical Structure of Probability

A **Probability Function** can be illustrated using examples:

**Example 1.** *Tossing a coin* $\Rightarrow P(\textit{Head}) = \frac{1}{2}$ *Throwing a die* $\Rightarrow P(\textit{Each face}) = \frac{1}{6}$

## 2.3 Random Experiment

- Number of possible outcomes is fixed.
- In a **deterministic experiment**, the number of outcomes is fixed and known.
- For a random experiment, the set of outcomes is fixed, but the actual outcome is not fixed.
- Repetition under identical conditions is possible.

## 2.4 Sample Space

**Definition 2.** *The **Sample Space** $S$ is the set of all possible outcomes.*

**Example 2.** *1. Weekday of your birthday:* $S = \{Mon, \ldots, Sun\}$

*2. Height of a student in a class*

*3. Set of all even numbers*

$S$ may be finite, countably infinite, or uncountable. Example: $S = \{2, 4, 6, \ldots\}$.

## 2.5 Events

**Definition 3.** *An **Event** is any subset of the sample space $S$. The empty set $\varnothing$ is also an event.*

## 2.6 Axiomatic Definition of Probability

A probability is a function:

$$P : F \to [0, 1]$$

satisfying:

1. $P(S) = 1$
2. If $A \cap B = \varnothing$, then:
$$P(A \cup B) = P(A) + P(B)$$

## 2.7 Power Set

The **Power Set** $\mathcal{P}(S)$ is the set of all subsets of $S$.

- Contains all subsets of $S$
- Contains $\varnothing$ and $S$
- If $A \in \mathcal{P}(S)$, then $A^c \in \mathcal{P}(S)$