Computational Heat & Fluid Flow (ME605)

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Assignment 4

Notes:

- For each problem, please provide: (i) the grid details (with a neat sketch) and discretization steps, (ii) the boundary condition implementation details, (iii) a well-documented code, (iv) the required output (plots/any other such means).
- If you are using one generalized code for problem solving, please make sure the documentation/annotation in the code is clear, and be sure to point it out in your write-up.

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1. Consider a one-dimensional unsteady convection-diffusion problem in a domain of L = 1. Two separate cases are to be considered: (i) $\rho = 1,\ \Gamma = 1,\ u = 1$ and (ii) $\rho = 1,\ \Gamma = 0.01,\ u = 3$. The boundary conditions for the transported variable ϕ are: (i) $x = 0,\ \phi = 0$ (ii) $\phi = L,\ \phi = 1$. Assume that initially $\phi = 0.5$ everywhere. Using the fully implicit formulation, solve for the distribution of ϕ within the domain for each case, as a function of time. Use $\Delta t = 0.1$. Implement both CDS and upwind scheme for each case. Perform the calculation till the steady state is reached. Plot the numerical results in the domain of interest for this purpose. Comment on the accuracy of the results, and on the suitability of CDS and upwind scheme with respect to the cell Peclet number.
2. In a steady two-dimensional situation, the variable ϕ is governed by

$$\nabla \cdot \left(\rho \vec{V} \phi\right) = \nabla \cdot \left(\Gamma \nabla \phi\right) + a - b\phi$$

where $\rho = 1,\ \Gamma = 1,\ a = 10,\ b = 2$. The flow field is such that $u = 1$ and $v = 4$ everywhere. Consider a square domain divided using control volumes of size $\Delta x = \Delta y = 1$. Use 20 control volumes in each direction. The boundary conditions are given as: (i) $x = 0,\ \phi = 100$, (ii) $x = L,\ \phi = 0$, (iii) $y = 0,\ \phi = 100$ and (iv) $y = L,\ \phi = 0$. Solve for the ϕ-field within this domain using

   a) The CDS
   b) The upwind scheme
   c) The hybrid scheme

   In each case, present your results using filled contour plots of ϕ.

3. Consider a steady hydrodynamically fully-developed flow entering a two-dimensional channel formed by two horizontal plates. The vertical distance between the plates is $H = 1$. The plates are subjected to a constant temperature of 100, and the flow entering the channel is at a uniform temperature of 50. Thus, this is a hydrodynamically fully-developed but thermally developing problem. Let the length of the channel be $L = 20$. The velocity profile everywhere is given by $u = 1.5\left(1-4y^2\right),\ v = 0$ where y is measured from the centerline of the channel.

   The steady flow energy equation neglecting viscous dissipation is given by

$$\nabla\cdot\left(\rho c\vec{V}T\right) = \nabla\cdot\left(k\nabla T\right),$$

   where the two-dimensional form is to be used. Use $\rho = 1,\ c = 100,\ k = 1$. Using the Hybrid scheme, solve for the temperature field in the channel domain. Use a uniform grid and $\Delta x = \Delta y$. Use the condition $\partial T/\partial x = 0$ at the outflow boundary. Plot the temperature profiles at various axial locations to demonstrate the development of the temperature field. Additionally, try to do the following :

a) From the numerical solution, determine the bulk mean temperature at each axial location. You will need to employ an integration method to do this.

b) Using the bulk mean temperature data, determine the heat transfer coefficient at each axial location. You need to use the expression

$$h = \frac{\left(-k\dfrac{\partial T}{\partial x}\right)_w}{T_w - T_b}$$

   where T_w denotes the wall temperature and T_b is the bulk mean temperature. Note that you will need a prescription to evaluate the temperature gradient at the wall.

c) Convert the heat transfer coefficient data to Nusselt number data. Use the definition

$$Nu = \frac{hD_h}{k},$$

   where $D_h = 2H$ for this situation. Plot the Nusselt number as a function of the axial distance, and see if the Nusselt number asymptotes to a constant value of 7.54.