Computational Heat & Fluid Flow (ME 605)

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Assignment 1 – Finite difference methods

Write a computer program (in any programming language of your choice) to solve the Poisson equation within the domain [0,1]

$$-\frac{\partial^2 u}{\partial x^2} = \pi^2 \sin(\pi x)$$

The boundary conditions are given as,

$$u(0) = 0$$
$$u(1) = 0$$

Discretize the equation using second order finite difference scheme, and solve the resulting system of linear algebraic equations using Thomas algorithm, Jacobi, Gauss-Seidel, and Successive over-relaxation method. Submit a short report containing the following tasks.

1. Plot the distribution of the computational solution and the exact solution for 11 and 51 grid points
2. Estimate the order of accuracy by plotting the L_∞-norm of solution error against the number of mesh intervals 4, 8, 16, 32, 64, and 128. The L_∞-norm can be computed as,

$$L_\infty = \max_i \left| u_i - u_i^{\text{exact}} \right|$$

3. Plot the convergence (error vs. iteration count) of all three iterative schemes.
4. Perform the above two operations by using fourth order discretization. (Remember to take care of the nodes that are adjacent to the boundary) [This is optional; you can also update the assignment later on. Additional points will be given if this task is completed. Use SOR for the solution of linear algebraic equations.]