Assignment 02

Submitted by

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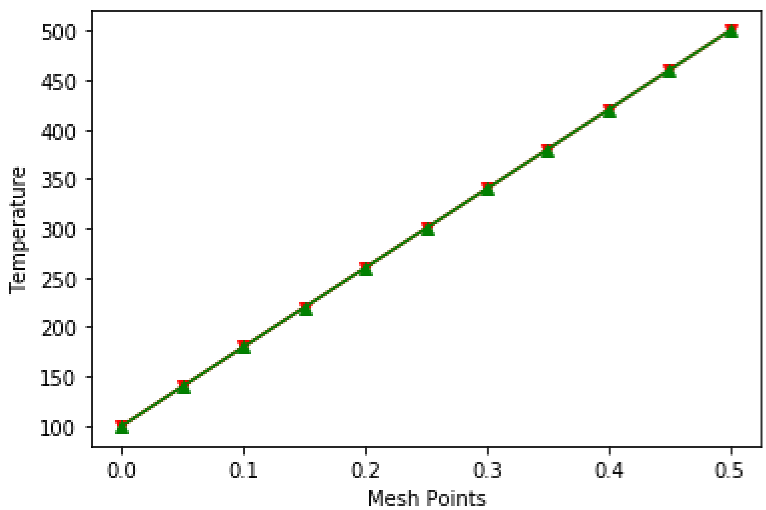
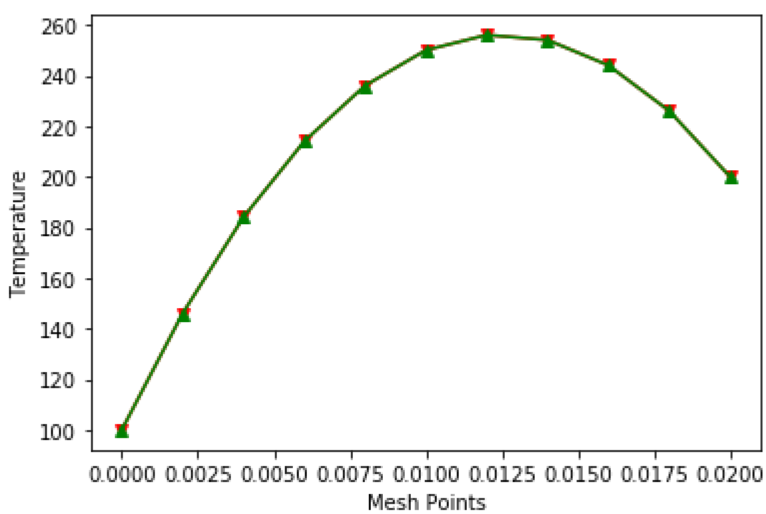
Question 1

Grid Points = 11



Question 2

Grid Points = 11



Discretization steps for question 01 and question 02

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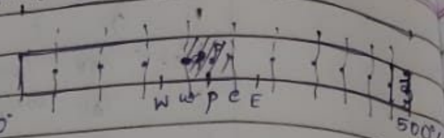
Assignment_02 (Discretization for Q1 & Q2)



Q① Given

$k = 1000\ W/mK$

$A = 10\times10^{-3}\ m^2$

and no sources

The steady state equation with source is given by

$$\frac{d}{dx}\left[\frac{k\,dT}{dx}\right] + S = 0$$

$$(\delta x)_e = (\delta x)_w = \Delta x$$

$$\int_w^e \frac{d}{dx}\left[\frac{k\,dT}{dx}\right] + \int_w^e S\,\Delta x = 0$$

$$\left(\frac{k\,dT}{dx}\right)_e - \left(\frac{k\,dT}{dx}\right)_w + \bar{S}\int_w^e \Delta x = 0$$

Assuming piece wise linear profile assumption

$$\frac{k_e(T_E - T_P)}{(\delta x)_e} - \frac{k_w(T_P - T_W)}{(\delta x)_w} + \bar{S}(\Delta x) = 0$$

$$T_P\left[\frac{k_e}{(\delta x)_e} + \frac{k_w}{(\delta x)_w}\right] = \frac{k_e T_E}{(\delta x)_e} + \frac{k_w T_W}{(\delta x)_w} + \bar{S}(\Delta x)$$

$$a_P T_P = a_E T_E + a_W T_W + b$$

~~① For simplicity, 4 grid point's~~

① For simplicity, 6 grid points are used, similarly, we can say that,

$$a_i T_i = a_{i-1} T_{i-1} + a_{i+1} T_{i+1}$$

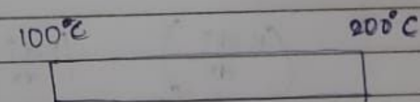
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -k/\Delta x & 2k/\Delta x & -k/\Delta x & 0 & 0 & 0 \\ 0 & -k/\Delta x & +2k/\Delta x & -k/\Delta x & 0 & 0 \\ 0 & 0 & -k/\Delta x & +2k/\Delta x & -k/\Delta x & 0 \\ 0 & 0 & 0 & -k/\Delta x & +2k/\Delta x & -k/\Delta x \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_L \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 500 \end{bmatrix}$$

~~dia~~

$$\left.\begin{aligned} a\_p[i] &= 2k/\Delta x \\ a\_e[i] &= -k/\Delta x \\ a\_w[i] &= -k/\Delta x \end{aligned}\right] \quad i = 1, 5$$

Question 02

Given:



$L = 0.02m$

$q = 1000\ kW/m^3$

$k = 0.5\ W/mK$

The steady state ~~eqn~~ equation is given by

$$\frac{d}{dx}\left[k\frac{dT}{dx}\right] + S = 0$$

$$\int_w^e \frac{d}{dx}\left[k\frac{dT}{dx}\right] + \int_w^e S\,dx = 0$$

$$\left[\frac{kdT}{dx}\right]_e - \left(\frac{kdT}{dx}\right)_w + \bar{S}(\Delta x) = 0$$

using linear profile assumption.

$$K_e\frac{(T_E - T_P)}{(\delta x)_e} - K_w\frac{(T_P - T_W)}{(\delta x)_w} + \bar{S}(\Delta x) = 0$$

$$T_P\left[\frac{K_e}{(\delta x)_e} + \frac{K_w}{(\delta x)_w}\right] = \frac{K_e T_E}{(\delta x)_e} + \frac{K_w T_W}{(\delta x)_w} + \bar{S}(\Delta x)$$

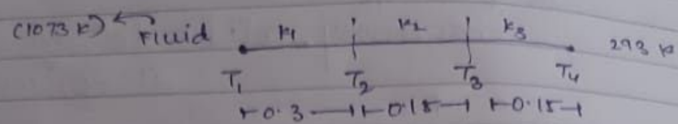
$$a_P T_P = a_E T_E + a_W T_W + \ell$$

For simplicity, assume six nodes

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -k/\Delta x & 2k/\Delta x & -k/\Delta x & 0 & 0 & 0 \\ 0 & -k/\Delta x & 2k/\Delta x & -k/\Delta x & 0 & 0 \\ 0 & 0 & -k/\Delta x & 2k/\Delta x & -k/\Delta x & 0 \\ 0 & 0 & 0 & -k/\Delta x & 2k/\Delta x & -k/\Delta x \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_\ell \end{bmatrix} = \begin{bmatrix} 100 \\ q\Delta x \\ q\Delta x \\ q\Delta x \\ q\Delta x \\ 200 \end{bmatrix}$$

For Question 03

~~As~~ question Q3



For exact solution,

heat transfer ~~conduct~~ convection = conduction through heat transfer

$$hA(\Delta T) = \frac{k \Delta T A}{l}$$

$$hA(T_f - T_1) = \frac{(T_2 - T_1) k_1 A}{0.3} = \frac{(T_3 - T_2) k_2 A}{0.15} = \frac{(T_4 - T_3) k_3 A}{0.15}$$

As steady state is given, so flux will be same

$$25(1073 - T_1) = \frac{(T_2 - T_1) 20}{0.3} = \frac{(T_3 - T_2)(1.5)}{0.15} = \frac{(T_4 - T_3) 50}{0.15}$$

on solving,

we get, $T_1 = 875.531$ K
$T_2 = 801.481$ K
$T_3 = 307.810$ K

and
conductivity_1 = 20
conductivity_2 = 1.5
conductivity_3 = 50

For exact solution

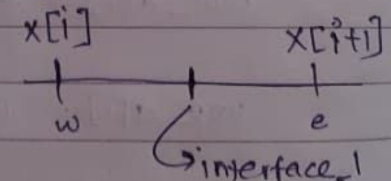
$$T = T_1 + \frac{(T_2 - T_1)x}{0.3} \qquad x \in [0, 0.3]$$

$$T = T_2 + \frac{(T_3 - T_2)(x - 0.3)}{(0.15)} \qquad x \in [0.3, 0.45]$$

$$T = T_3 + \frac{(T_4 - T_3)(x - 0.45)}{(0.15)} \qquad x \in [0.45, 0.6]$$

For arithmetic mean,

$$f_e = \frac{x[i+1] - interface\_1}{x[i+1] - x[i]}$$



and conductivity at interface will be given by

conductivity_a[i] = fe* conductivity_2 + (1 - fe)* conductivity_1

else if

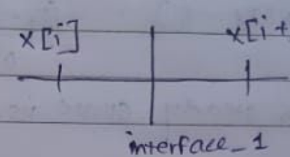
x[i+1] < interface_1

then

conductivity_a[i] = conductivity_1

similarly for other interfaces we follow same formulation

For Harmonic mean,



$$fe = \frac{x[i+1] - interface\_1}{x[i+1] - x[i]}$$

and

if x[i] < interface_1 and x[i+1] > interface_2

$$conductivity\_h[i] = \frac{1}{\frac{fe}{conductivity\_2} + \frac{(1-fe)}{conductivity\_1}}$$

else if

x[i+1] < interface_1

then,

conductivity_h[i] = conductivity_1

similarly, for other interfaces we follow same formulation.

• Now, ~~creat~~ creating diagonal elements,

$$low\_a[i] = \cancel{(-1)(conductivity[i-1]} \quad \frac{(-1)\,k_{i-1}}{\Delta x}$$

$$upp\_a[i] = \frac{(-1)(k_i)}{\Delta x}$$

$$dia\_a[i] = \frac{k_i}{\Delta x} + \frac{k_{i-1}}{\Delta x}$$

similarly, for harmonic mean.

The matrix ~~with~~ is given by

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
low1 & dia1 & upp1 & 0 & 0 & 0 & \cdots & 0 \\
0 & low2 & dia2 & upp2 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & & & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_n
\end{bmatrix}
=
\begin{bmatrix}
Temp\_fluid \\ 0 \\ 0 \\ 0 \\ \vdots \\ 293
\end{bmatrix}
$$

- Also, it is observed that, error is minimum for

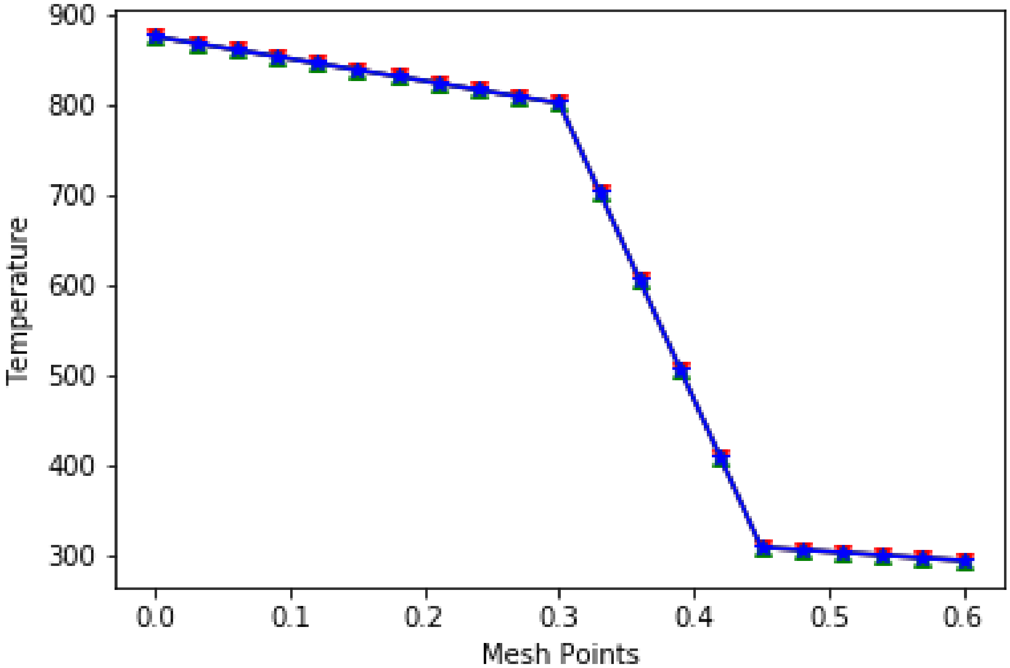
  no. of nodes = $4n+1$

  For $n \geq 5$

calculations are done for $n=5$, i.e

no. of nodes = 21

Graph:



900
800
700
600
500
400
300
Temperature
0.0
0.1
0.2
0.3
0.4
0.5
0.6
Mesh Points