Assignment 03

Submitted by
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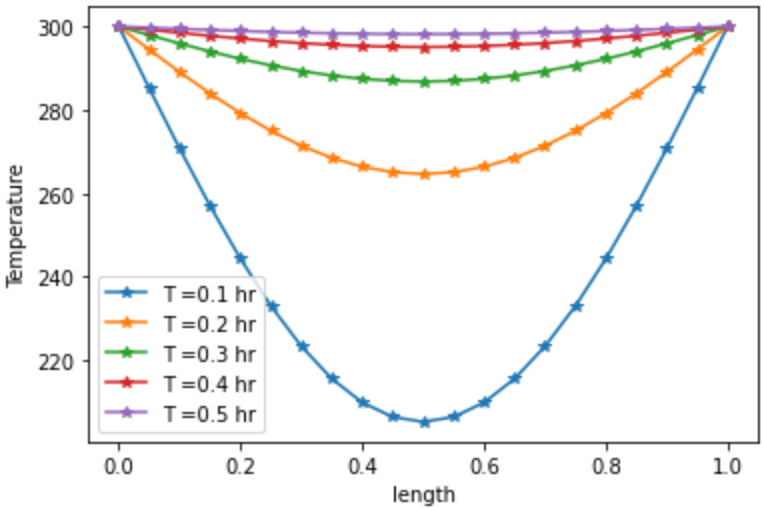
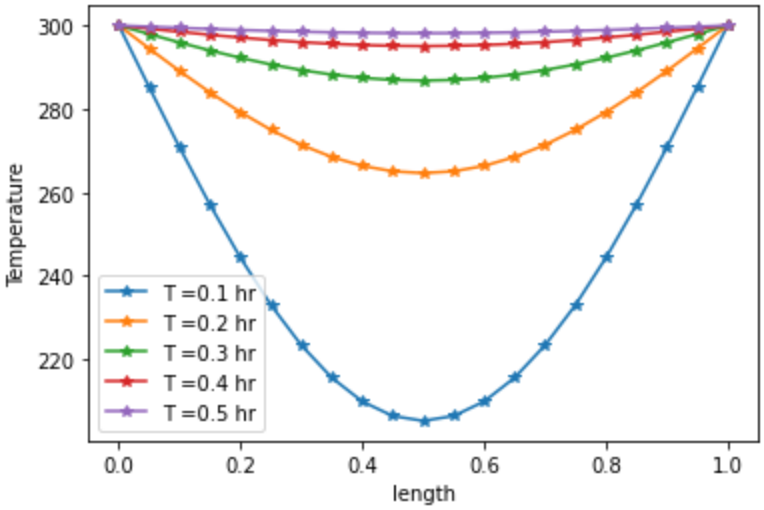
Question 01:

a) **Fully Explicit Method:**



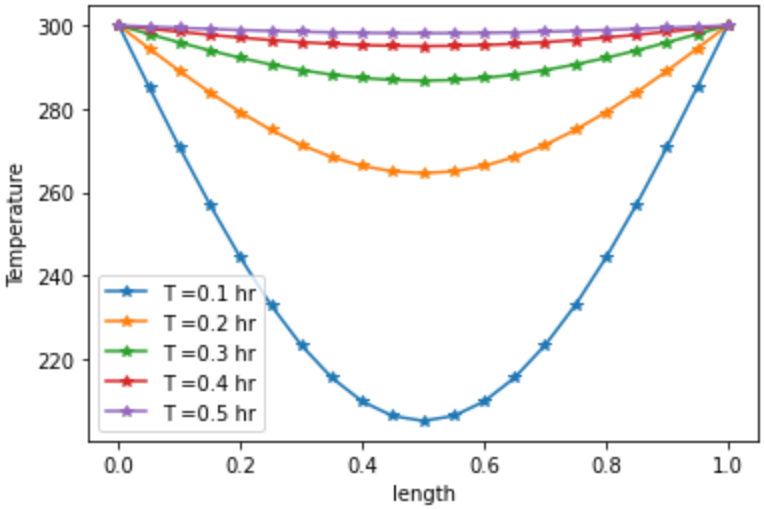
Here, the time step is 0.000125 hr(i.e 0.45 seconds). The graph is plotted at each 0.1hr interval till 0.5 hr time is reached. Grid size is 0.05

b) **Crank Nicolson Scheme:**



Here, the time step is 0.000125 hr(i.e 0.45 seconds). The graph is plotted at each 0.1hr interval till 0.5 hr time is reached. Grid size is 0.05

c) **Fully Implicit Scheme:**

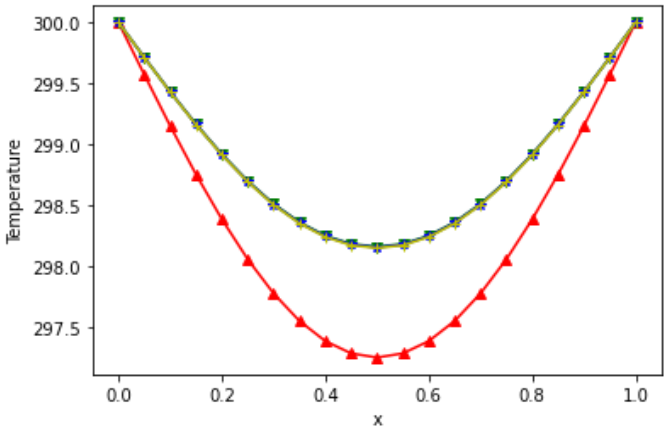


Here, the time step is 0.000125 hr(i.e 0.45 seconds). The graph is plotted at each 0.1hr interval till 0.5 hr time is reached. Grid size is 0.05

Number of terms of series are chosen to be 1000
Grid size = 0.05 m

1) **Minimum number of iterations required for convergence of Fully Explicit Scheme is 400(Time step should not exceed 0.00125 hr)**
2) **Minimum number of iterations required for convergence of Crank Nicolson Scheme is 200(Time step should not exceed 0.0025 hr)**
3) **Any number of iterations can be performed for Fully Implicit Scheme as this Scheme is Unconditionally Stable**

Graph of Analytical + different Schemes at 0.5 hr:



Assignment-03

Question 01:

For fully Explicit scheme: ($f = 0$)

Equation is given by

$$T_p' = \frac{\alpha \Delta t}{(\Delta x)^2} T_W^0 + \frac{\alpha \Delta t}{(\Delta x)^2} T_E^0 + \left(1 - \frac{2\alpha \Delta t}{(\Delta x)^2}\right) T_p^0$$

So, For convergence,

$$1 - \frac{2\alpha \Delta t}{(\Delta x)^2} \geq 0$$

$$\Rightarrow \Delta t \leq \frac{(\Delta x)^2}{2\alpha}$$

$$\Rightarrow \Delta t \leq 0.00125 \text{ hr}$$

So, I chose $\Delta t = 0.000125$ hr

Hence, no. of iterations required to reach 0.5hr with $\Delta t = 0.000125$ hr is $= \frac{0.5}{0.000125} = 4000$ iterations.

For crank Nicolson scheme ($f = 1/2$):

Equation is given by

$$\left[1 + \frac{\alpha \Delta t}{(\Delta x)^2}\right] T_p' = \frac{\alpha \Delta t}{2(\Delta x)^2} T_E' + \frac{\alpha \Delta t}{2(\Delta x)^2} T_W' + \frac{\alpha \Delta t}{2(\Delta x)^2} T_E^0 + \frac{\alpha \Delta t}{2(\Delta x)^2} T_W^0 + \left[1 - \frac{\alpha \Delta t}{(\Delta x)^2}\right] T_p^0$$

For convergence,

$$1 - \frac{\alpha \Delta t}{(\Delta x)^2} \geq 0$$

$$\Rightarrow \Delta t \leq \frac{(\Delta x)^2}{\alpha}$$

$$\Rightarrow \Delta t \leq 0.0025 \text{ hr}$$

So, I chose $\Delta t = 0.000125$ hr

Hence, no. of iterations $= \frac{0.5}{0.000125} = 4000$ iterations.

For fully implicit scheme ($f = 1$):

$$\left[1 + \frac{2\alpha \Delta t}{(\Delta x)^2}\right] T_p' = \frac{\alpha \Delta t}{(\Delta x)^2} T_W' + \frac{\alpha \Delta t}{(\Delta x)^2} T_E' + \frac{\alpha \Delta t}{(\Delta x)^2} T_p^0$$

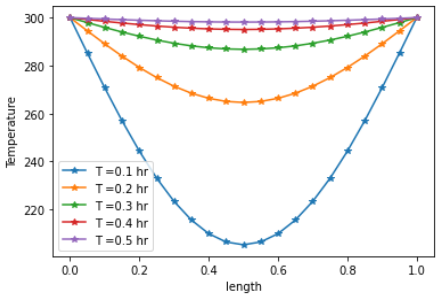
So, as this scheme is unconditionally stable.

Hence, I chose no. of iterations = 4000 and

$\Delta t = 0.000125$ hr (i.e 0.45 s)

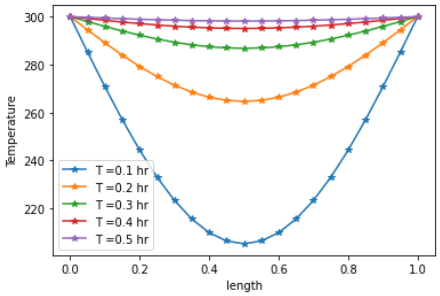
Solution of Crank Nicolson Scheme at different timesteps:(With temperature arrays)

1) **At Number of Iterations = 4000 (timestep = 0.000125 hr)**



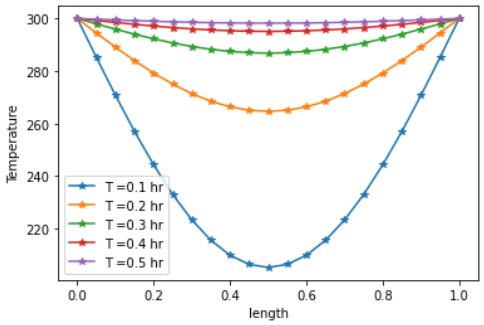

```
[300.         299.71153781 299.43017852 299.16285012 298.91613513
 298.69610849 298.50818799 298.35700085 298.2462698  298.17872141
 298.15601894 298.17872141 298.2462698  298.35700085 298.50818799
 298.69610849 298.91613513 299.16285012 299.43017852 299.71153781
 300.        ]
```

2) **At Number of iterations = 2000 (timestep = 0.00025 hr)**




```
[300.         299.71189354 299.43088122 299.16388249 298.91747175
 298.69771645 298.51002769 298.35902699 298.2484325  298.18096741
 298.15829294 298.18096741 298.2484325  298.35902699 298.51002769
 298.69771645 298.91747175 299.16388249 299.43088122 299.71189354
 300.]
```

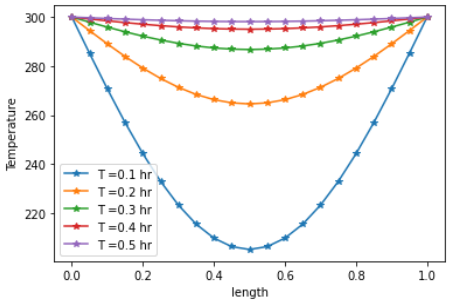
3) **At Number of iterations = 250 (timestep = 0.002 hr)**




```
[300.         299.71688387 299.440739   299.17836499 298.93622236
 298.72027347 298.53583569 298.38745049 298.27877162 298.2124751
 298.19019338 298.2124751  298.27877162 298.38745049 298.53583569
 298.72027347 298.93622236 299.17836499 299.440739   299.71688387
 300.        ]
```

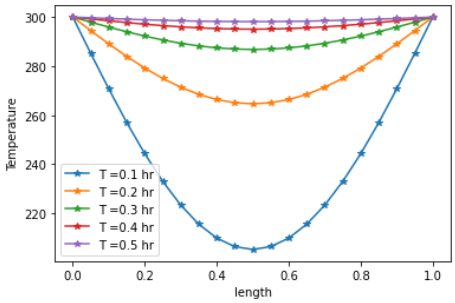
Solution of Fully Implicit Scheme at different timesteps:(With temperature arrays)

1) **At Number of Iterations = 4000 (timestep = 0.000125 hr)**



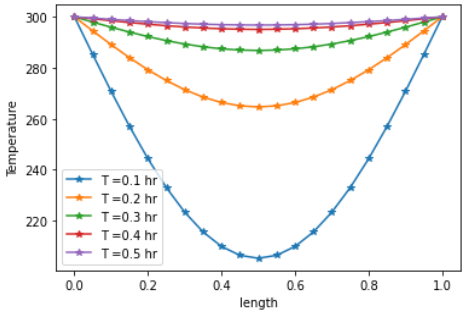

```
[300.         299.7106621  299.42844866 299.16030871 298.91284475
 298.69215015 298.50365916 298.35201305 298.24094585 298.17319239
 298.15042101 298.17319239 298.24094585 298.35201305 298.50365916
 298.69215015 298.91284475 299.16030871 299.42844866 299.7106621
 300.        ]
```

2) **At Number of Iterations = 2000 (timestep = 0.00025 hr)**




```
[300.         299.71014183 299.42742092 299.15879881 298.91088987
 298.68979844 298.50096851 298.34904972 298.2377828  298.16990751
 298.14709518 298.16990751 298.2377828  298.34904972 298.50096851
 298.68979844 298.91088987 299.15879881 299.42742092 299.71014183
 300.        ]
```

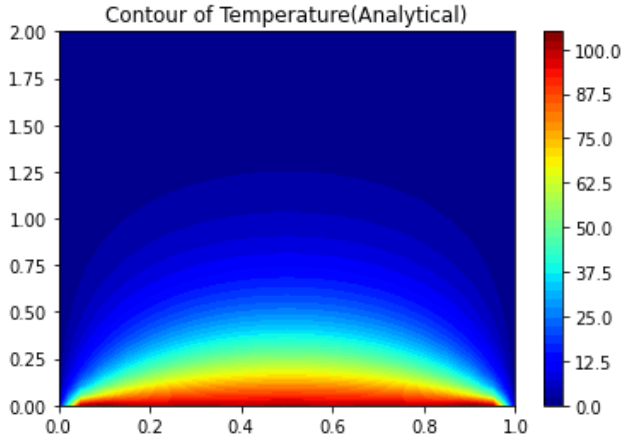
3) **At Number of Iterations =10 (timestep = 0.05 hr)**



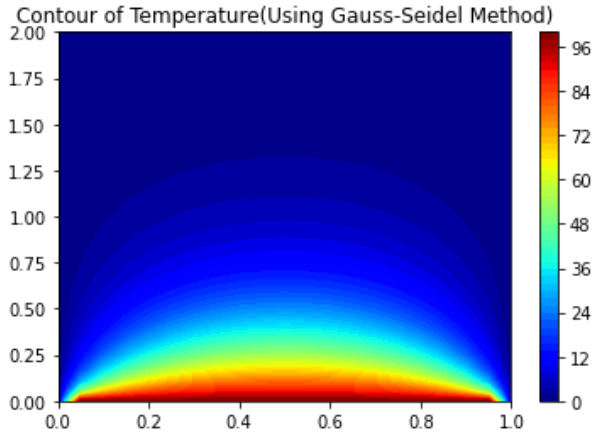

```
[300.         299.49422552 299.00090505 298.53218589 298.09960953
 297.71382742 297.38433871 297.11925637 296.92510745 296.80667238
 296.76686732 296.80667238 296.92510745 297.11925637 297.38433871
 297.71382742 298.09960953 298.53218589 299.00090505 299.49422552
 300.        ]
```

Question 02:

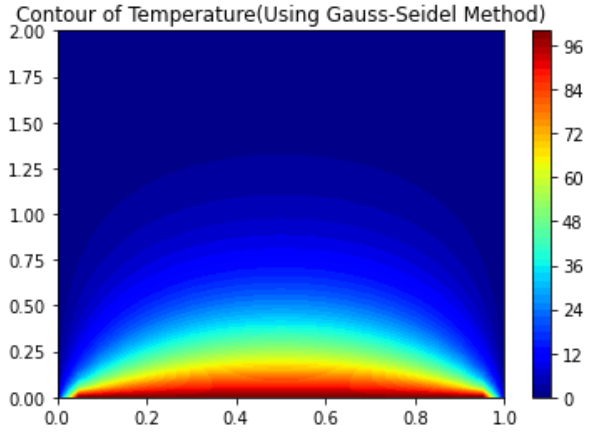
1) **Analytical Solution Graph:**



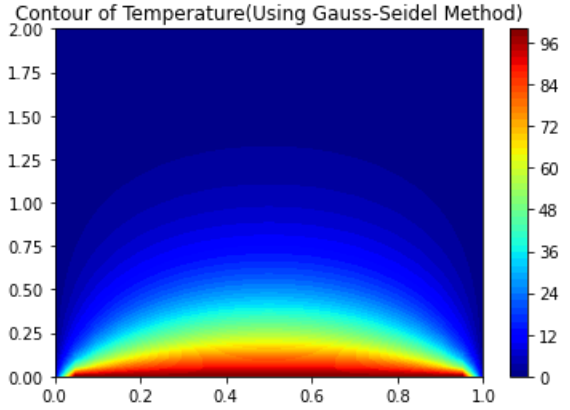
2) **Tguess = 0 ° C**
Number of iterations obtained are 559



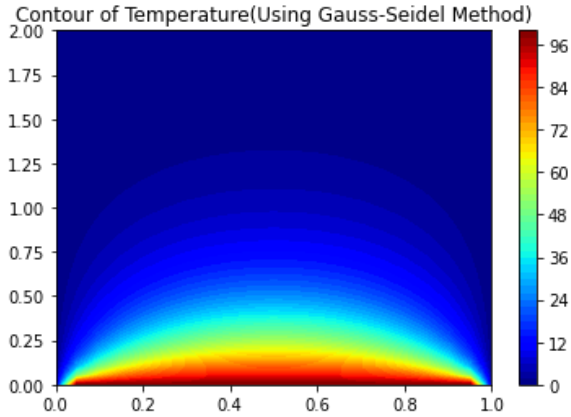
3) **Tguess = 9° C**
Number of iterations obtained are 382



4) **Tguess = 20° C**
Number of iterations obtained are 577



5) **Tguess = 50° C**
Number of iterations obtained are 660

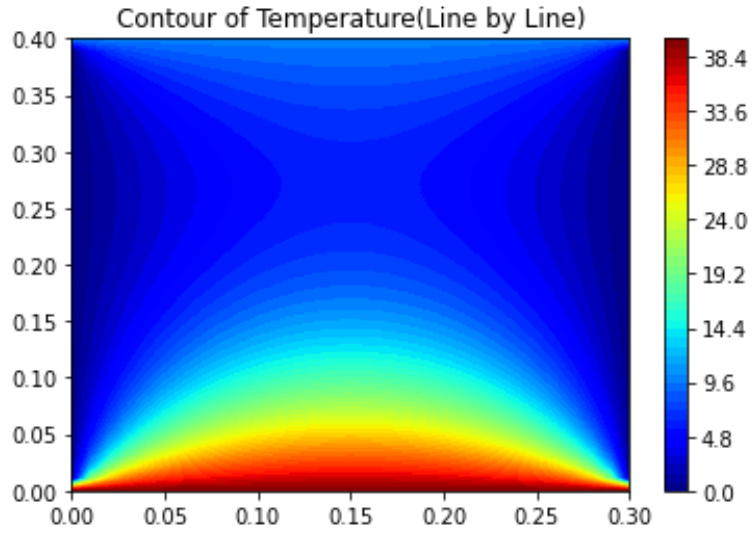


Explanation for Convergence Criterion:

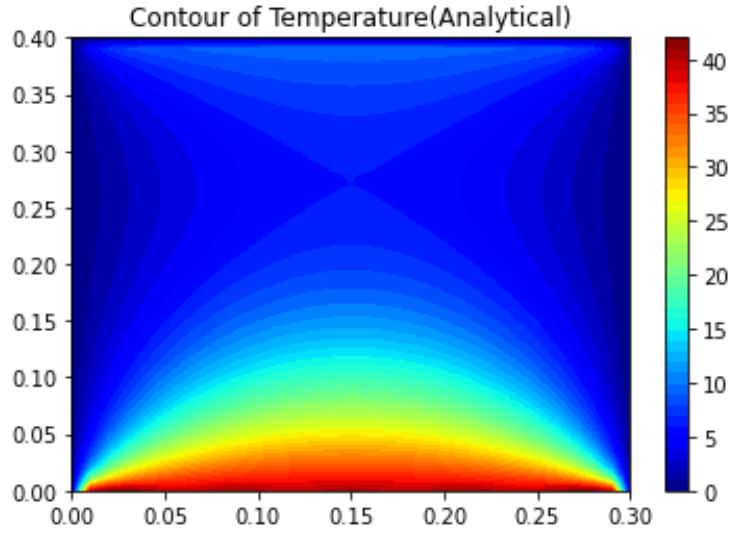
The convergence loop which is while loop is running until the norm of Error between the Temperature of previous timestep and Temperature of current timestep is less than 0.001. The norm is the maximum sum of absolute values of row elements in a 2D array. If the norm is less than 0.001, then the converged boolean variable becomes true and the while loop is terminated. We can also notice that the number of iterations are lowest when Tguess is 9° C because the steady state solution converges at a temperature close to 9° C.

Question 03:

Steady State Solution:



Analytical Solution:



Total iterations required for convergence(for delta_t = 1 sec) is 1165. According to these iterations, the time history of temperature of whole slab is shown below

Time history of temperature of whole slab :