Computational Heat & Fluid Flow (ME605)

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Assignment 3

Notes:

- For each problem, please provide: (i) the grid details (with a neat sketch) and discretization steps, (ii) the boundary condition implementation details, (iii) a well-documented code, (iv) the required output (plots/any other such means).
- If you are using one generalized code for problem solving, please make sure the documentation/annotation in the code is clear, and be sure to point it out in your write-up.

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1. A one-dimensional slab of 1 m width and a constant thermal diffusivity of 1 m^2/hr is initially at a uniform temperature of 100 ˚C. The surface temperatures of the left (x = 0) and right (x = L) faces are suddenly increased and maintained at 300 ˚C. There are no sources. Determine the temperature distribution within the wall as a function of time. Specifically, plot the temperature distribution at each 0.1 hr interval from 0.0 to 0.5 hr. Use a grid size of 0.05 m.

a) Solve the problem with the fully explicit method. Demonstrate the stability criterion as discussed in class (this should involve the time step size and the consequent behaviour of the solution).

b) Solve the problem with the Crank-Nicolson method. Investigate the behaviour of the solution with a few choices of the time step. Do not exceed the time step beyond 0.1 hr.

c) Solve the problem with the fully implicit method. Investigate the behaviour of the solution with a few choices of the time step. Do not exceed the time step beyond 0.1 hr.

The analytical solution for this case is given by:

$$T = T_s + 2\left(T_i - T_s\right)\sum_{m=1}^{\infty} e^{-\left[\left(\frac{m\pi}{L}\right)^2 \alpha t\right]} \frac{1-\left(-1\right)^m}{m\pi} \sin\left(\frac{m\pi x}{L}\right)$$

where T_s denotes the equal surface temperature at the two faces, T_i is the initial temperature in the wall, and L is the width of the wall. Use this analytical solution for comparison. You will need to determine the appropriate number of terms required in the series. Choose a few times for comparison.

2. Consider a two-dimensional rectangular plate of dimension L = 1 m in the x direction and H = 2 m in the y direction. The plate material has constant thermal conductivity. The

steady-state temperature distribution within this plate is to be determined for the following imposed boundary conditions: (i) y = 0, T = 100 ˚C, (ii) x = 0, T = 0 ˚C, (iii) $y = H$, T = 0 ˚C, and (iv) $x = L$, T = 0 ˚C. Choose a uniform grid size of 0.05 m in both directions. Solve the problem using the point-by-point Gauss-Seidel iterative method. Experiment with the initial guess and comment on the number of iterations required for convergence in each case. Clearly explain your convergence criterion for the iterations and how it is implemented. Plot the temperature contours as the output.

The analytical solution for this case is given by:

$$T = T_1\left[2\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n\pi}\frac{\sinh\left[\frac{n\pi(H-y)}{L}\right]}{\sinh\left[\frac{n\pi H}{L}\right]}\sin\left(\frac{n\pi x}{L}\right)\right]$$

where T_1 = T (y=0) = 100 ˚C. Use this analytical solution for comparison with your numerical results.

3. Consider a two-dimensional rectangular plate of dimension L = 0.3 m in the x direction and H = 0.4 m in the y direction. The material of the plate has a thermal conductivity of 380 W/m-K and a thermal diffusivity of 11.234×10^{-5} m^2/s. Choose a uniform grid size of 0.01 m in both directions. The plate is initially at a uniform temperature of 0 ˚C. Subsequently, its surfaces are subjected to the following constant temperatures and these surface temperatures are maintained at these values: (i) y = 0, T = 40 ˚C, (ii) x = 0, T = 0 ˚C, (iii) $y = H$, T = 10 ˚C, and (iv) $x = L$, T = 0 ˚C. The transient temperature distribution in the plate is to be determined. Employ the fully implicit formulation, and solve the resultant system of equations using a line-by-line method. You can experiment with the (i) time step (although the scheme is unconditionally stable) and (ii) the sweep direction for the implementation of the line-by-line method. Compute the solution till steady state is reached. You will need to determine and implement a criterion for the determination of the "steady state" in addition to that for the convergence of iterations within a time step. For the output, you can plot the time history of the temperatures at a few points ("monitor" points) in the domain. The analytical solution for the steady-state temperature distribution in this case is given by:

$$T = T_A + T_B$$

where

$$T_A = T_{(y=0)} \times \left[2\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n\pi} \frac{\sinh\left[\frac{n\pi(H-y)}{L}\right]}{\sinh\left[\frac{n\pi H}{L}\right]} \sin\left(\frac{n\pi x}{L}\right) \right]$$

and

$$T_B = T_{(y=H)} \times \left[2\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n\pi} \frac{\sinh\left[\frac{n\pi y}{L}\right]}{\sinh\left[\frac{n\pi H}{L}\right]} \sin\left(\frac{n\pi x}{L}\right) \right]$$

You can use the steady-state analytical solution for comparison with the steady-state solution obtained with the numerical method. **Note that you must obtain the steady-state solution using the transient computation in the "large" time limit. Directly computing the steady-state temperature distribution is not permitted.**