Example of Genetic algorithm

The MAXONE problem: Suppose we want to maximize the number of ones in a string of L binary digits.

An individual is encoded (naturally) as a string of L binary digits. Let's say L = 10.
Then, L = 0000000001 (10 bits)

The fitness function is:

f(chromosome) = COUNTONE(S_i)

We start with a population of n random strings. Suppose that L = 10 and n = 6
We toss a fair coin 60 times and get the following initial population:

S1 = 1111010101
S2 = 0111000101
S3 = 1110110101
S4 = 0100010011
S5 = 1110111101
S6 = 0100110000

Solution No.	Solution	f(chromosome) = COUNTONE(Si)	%pi=fi/ ∑fi *100
S1	1111010101	7	20.59
S2	0111000101	5	14.71
S3	1110110101	7	20.50
S4	0100010011	4	11.76
S5	1110111101	8	23.52
S6	0100110000	3	8.82
∑		34	100

Reproduction:
Roulette Wheel selection is used for generating four chromosomes for the next generation.
Suppose that, after performing selection, we get the following population:

S1` = 1111010101 (S1)
S2` = 1110110101 (S3)
S3` = 1110111101 (S5)
S4` = 0111000101 (S2)
S5` = 0100010011 (S4)
S6` = 1110111101 (S5)

Crossover
Suppose that we decide to actually perform crossover only for couples (S1`, S2`) and (S5`, S6`).For each couple, we randomly extract a crossover point, for instance 2 for the first and 5 for the second.

Before Crossover:

S1’ = 1111010101
S2’ = 1110110101

S5’ = 0100010011
S6’ = 1110111101

After Crossover:

S1’’ = 1110110101
S2’’ = 1111010101

S5’’ = 0100011101
S6’’ = 1110110011

Mutation

Before applying mutation:

S1`` = 11101**1**0101
S2`` = 1111**0**1010**1**
S3`` = 1110111101
S4`` = 0111000101
S5`` = 0**1**00011101
S6`` = 111**0**110**0**11

After applying mutation:

S1``` = 11101**0**0101 (6)
S2``` = 1111**1**1010**0** (7)
S3``` = 1110111101 (8)
S4``` = 0111000101 (5)
S5``` = 0**0**00011101 (4)
S6``` = 111**1**110**1**11 (9)

In one generation, the total population fitness changed from 34 to 39, thus improved by ~13%.

At this point, we go through the same process all over again, until a stopping criterion is met.

Solution No.	Solution	f(chromosome) = COUNTONE(Si)	%pi=fi/ ∑fi *100
S1	1110100101	6	**15.38**
S2	1111110100	7	**17.95**
S3	1110111101	8	**20.51**
S4	0111000101	5	12.82
S5	0000011101	4	**10.26**
S6	1111110111	9	**23.08**
∑		39	100

Reproduction:
Roulette Wheel selection is used for generating four chromosomes for the next generation. Suppose that, after performing selection, we get the following population:

S1` = 1110100101 (S1)
S2` = 1110111101 (S3)
S3` = 1111110111 (S6)
S4` = 0000011101 (S5)
S5` = 1111110111 (S6)
S6` = 1111110100 (S2)

Crossover
Suppose that we decide to actually perform crossover only for couples (S1`, S2`) and (S5`, 6`).For each couple, we randomly extract a crossover point, for instance 2 for the first and 5 for the second.

Before Crossover:

S1` = 1110100101
S2` = 1110111101

S5` = 1111110111
S6` = 1111110100

After Crossover:

S1`` = 1110111101
S2``= 1110100101

S5`` = 1111110100
S6`` = 1111110111

Mutation:

Before applying mutation:

S1`` = 11101**1**1101
S2`` = 1110100111
S3`` = 1111110111
S4`` = 0000011101
S5`` = 1111110**1**0
S6`` = 111111**0**111

After applying mutation:

S1``` = 11101**0**1101 (7)
S2``` = 1110100111 (7)
S3``` = 1111110111 (9)
S4``` = 0000011101 (4)
S5``` = 1111110**1**0 (8)
S6``` = 111111**1**111 (10)

Thus the solution S6``` is {1111111111} that has highest fitness value has been found.

Example 2

Using genetic algorithm maximize $f(x) = x^2$ over $\{0, 1, 2, \ldots 15\}$ with initial x value of $\{1, 6, 8, 10\}$

	x-value	Initial	$f(x) = x^2$	$\%Pi = fi / \Sigma fi * 100$
S_1	1	0001	1	0.49
S_2	6	0110	36	17.91
S_3	8	1000	64	31.84
S_4	10	1010	100	49.75
Σ			201	

Reproduction

- Roulette wheel selection is used for generating four chromosomes for the next generation
- suppose after performing selection, we get the following population

$S_1' = 1000$ (S_3)

$S_2' = 1010$ (S_4)

$S_3' = 0110$ (S_2)

$S_4' = 1010$ (S_4)

Crossover

- suppose that we decide to perform crossover for couples (s_2', s_3')

⇒ Before crossover

$s_2' = 1 | 0\ 1\ 0$

$s_3' = 0 | 1\ 1\ 0$

⇒ After crossover

$s_2'' = 1\ 1\ 1\ 0$

$s_3'' = 0\ 0\ 1\ 0$

Mutation

Before applying mutation

$s_1'' = 1\ \underline{0}\ 0\ 0$

$s_2'' = 1\ 1\ 1\ 0$

$s_3'' = 0\ \underline{0}\ 1\ 0$

$s_4'' = 1\ 0\ 1\ 0$

After applying mutation		X-value	$f(x) = x^2$	$\%Pi = fi/\Sigma fi * 100$
s_1''' =	1 1 0 0	12	144	30.25
s_2''' =	1 1 1 0	14	196	41.18
s_3''' =	0 1 1 0	6	36	7.56
s_4''' =	1 0 1 0	10	100	21
			476	