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# Fuzzy rule based inference

Mamdani style fuzzy inference process

Ex: Estimate the level of risk involved in a software Engineering project

Two i/p's : Project funding
Project staffing

→ sets defined for project funding: inadequate, marginal and adequate

→ sets defined for project staffing: small and large

Rules

1. Project funding is adequate or project staffing is small then risk is low.

2. If project funding is marginal and project staffing is large then risk is normal.

3. If project funding is inadequate then risk is high.

* Mamdani-style fuzzy inference process is performed in four steps

  1. Fuzzification of i/p variable
  2. Rule Evaluation
  3. Aggregation of rule outputs
  4. Defuzzification

① fuzzification of i/p variable

- Take crisp i/p and determine the degree to which these i/p's belong to each of the appropriate fuzzy sets

Step1

→ first step to convert the crisp i/p to fuzzy one

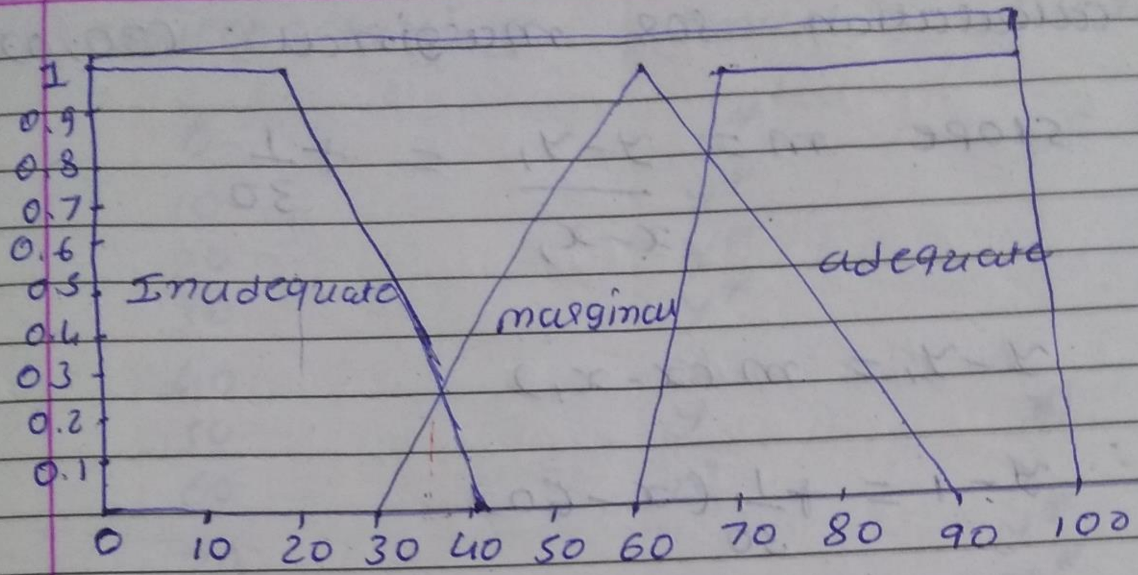
- suppose our i/p's are project-funding = 35% and project-starting = 60%.

| Project funding | Inadequate | marginal | adequate |
|---|---|---|---|
| 0 | y* | | |
| 10 | y* | | |
| 20 | y* | | |
| 30 | y | N | |
| 40 | N | y | |
| 50 | | y | |
| 60 | | y* | N |
| 70 | | y | y* |
| 80 | | y | y* |
| 90 | | N | y* |
| 100 | | | y* |

y*: value is ideal member to set
$\mu_A(x) = 1$

N: value is not a member of a set,
$\mu_A(x) = 0$

y: value belongs to set: $0 < \mu_A(x) < 1$




project funding = 35%

calculation for Inadquate : $\overset{x\ y}{(20,1)}$ $\overset{x_1\ y_1}{(40,0)}$

slope $m = \frac{y - y_1}{x - x_1} = \frac{1-0}{20-40} = -\frac{1}{20}$

$y - y_1 = m(x - x_1)$

$\therefore y - 0 = \frac{-1}{20}(x - 40)$

$\therefore y = \frac{-1}{20}(x-40)$

$x = 35, 50$

$\therefore y = -\frac{1}{20}(35-40)$

$= \frac{5}{20} = \boxed{0.25}$

calculation for marginal : (30, 0) (60, 1)

slope $m = \frac{y - y_1}{x - x_1} = +\frac{1}{30}$

$y - y_1 = m(x - x_1)$

$\therefore y - 1 = +\frac{1}{30}(x - 60)$

$\therefore y - 1 = \frac{x}{30} - 2$

$\therefore y = \frac{x}{30} - 1$

$x = 35$ so,

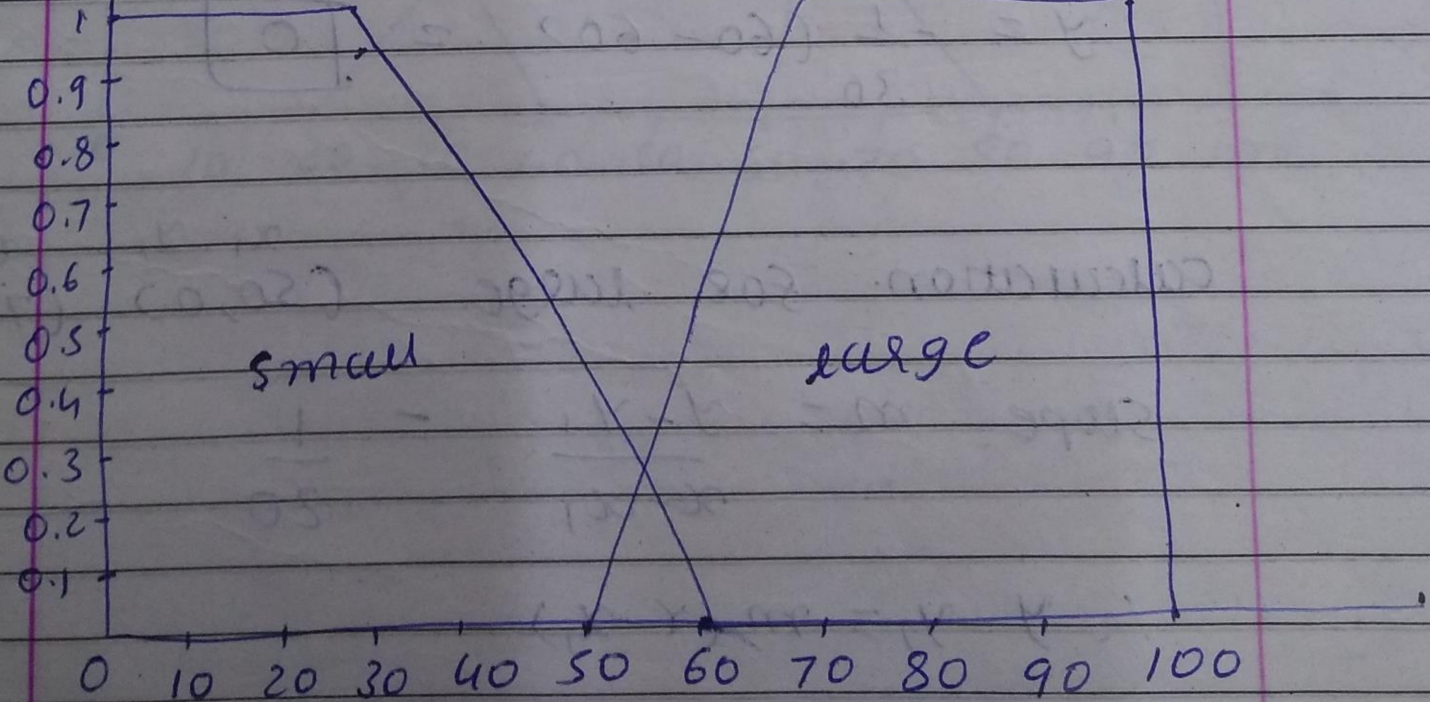
$y = \frac{35}{30} - 1$

$\therefore y = \frac{35 - 30}{30} = \frac{5}{30} = 0.16$

$\therefore$ Project funding : 35%

inadequate : 0.25
marginal : 0.16
adequate : 0

| Project staffing | small | large |
|---|---|---|
| 0 | y* | |
| 10 | y* | |
| 20 | y* | |
| 30 | y* | |
| 40 | y | |
| 50 | y | N |
| 60 | N | y |
| 70 | | y* |
| 80 | | y* |
| 90 | | y* |
| 100 | | y* |



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calculation for small $(30,1)$ $(60,0)$

slope $m = \dfrac{y-y_1}{x-x_1} = \dfrac{1-0}{30-60} = \dfrac{-1}{30}$

$y - y_1 = m(x - x_1)$

$\therefore y - 0 = \dfrac{-1}{30}(x - 60)$

$\therefore y = \dfrac{-1}{30}(x - 60)$

$x = 60$ so,

$y = \dfrac{-1}{30}(60 - 60) = \boxed{0}$

calculation for large $\overset{x_1 \; y_1}{(50,0)}$ $\overset{x \; y}{(70,1)}$

slope $m = \dfrac{y-y_1}{x-x_1} = \dfrac{1}{20}$

$\therefore y - y_1 = m(x - x_1)$

$\therefore y - 1 = \dfrac{1}{20}(x - 50)$

$\therefore ~~y = \dfrac{x}{20} - 69~~$ $\quad x = 60$ so,

$\therefore y - 0 = \dfrac{1}{20}(60 - 50)$

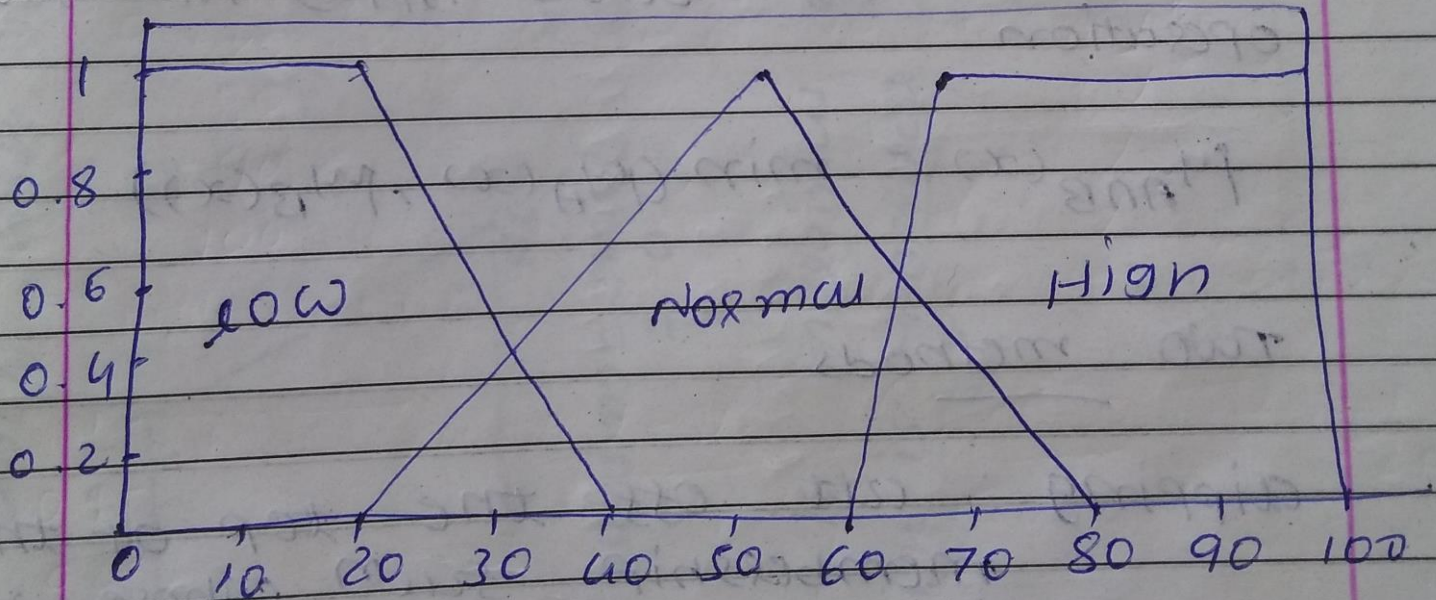
$\therefore y - 0 = \dfrac{1}{20}(10) = \boxed{0.5}$

Project staffing : 60%

small : 0
large : 0.5

| Project risk | low | Normal | High |
|---|---|---|---|
| 0 | Y* | | |
| 10 | Y* | | |
| 20 | Y* | N | |
| 30 | Y | Y | |
| 40 | N | Y | |
| 50 | | Y* | |
| 60 | | Y | N |
| 70 | | Y | Y* |
| 80 | | N | Y* |
| 90 | | | Y* |
| 100 | | | Y* |

Project risk



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(2) Rule evaluation

→ To evaluate the disjuction of rule antecedents we use OR fuzzy operation

$$\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x))$$

In order to evaluate the conjuction of rule antecendents apply AND fuzzy operation

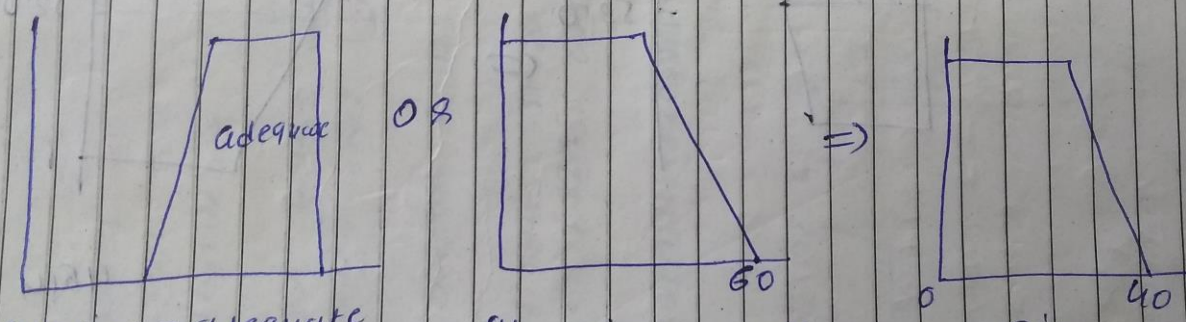
$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Two methods

Clipping : cut off the top of the membership fun$^n$ whose value is higher than matching degree

Scaling : scales down the membership fun$^n$ in proportion to the matching degree

① Project funding is adequate or project staffing is small then risk is low



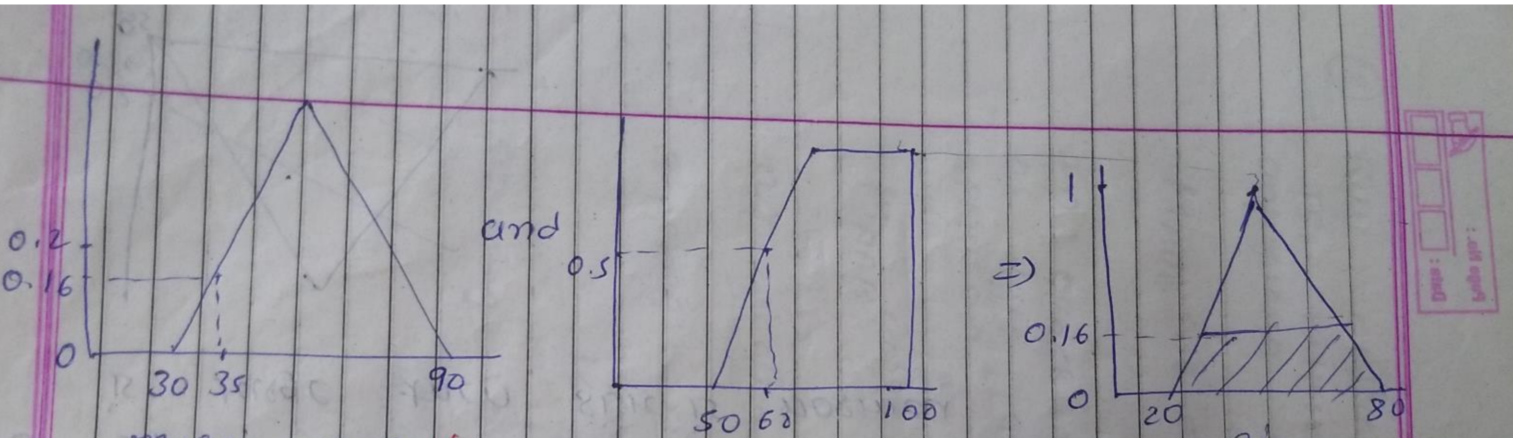

μ funding = adequate = 0

μ staffing = small = 0

Risk low

Rules.

② If project funding is marginal and project staffing is large then risk is normal

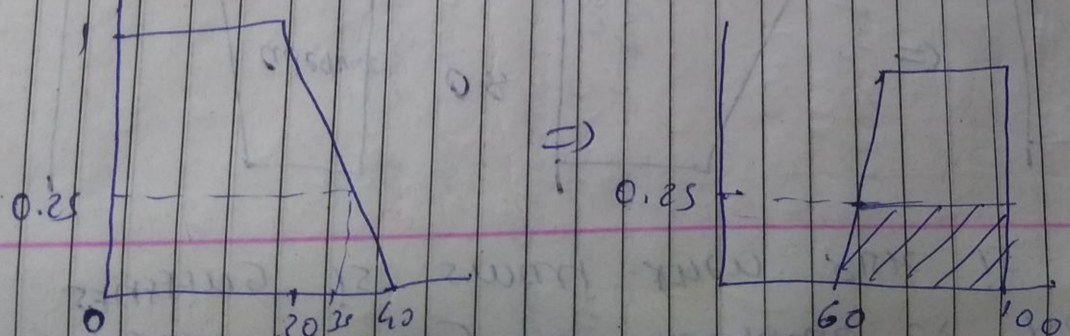



marginal = 0.16

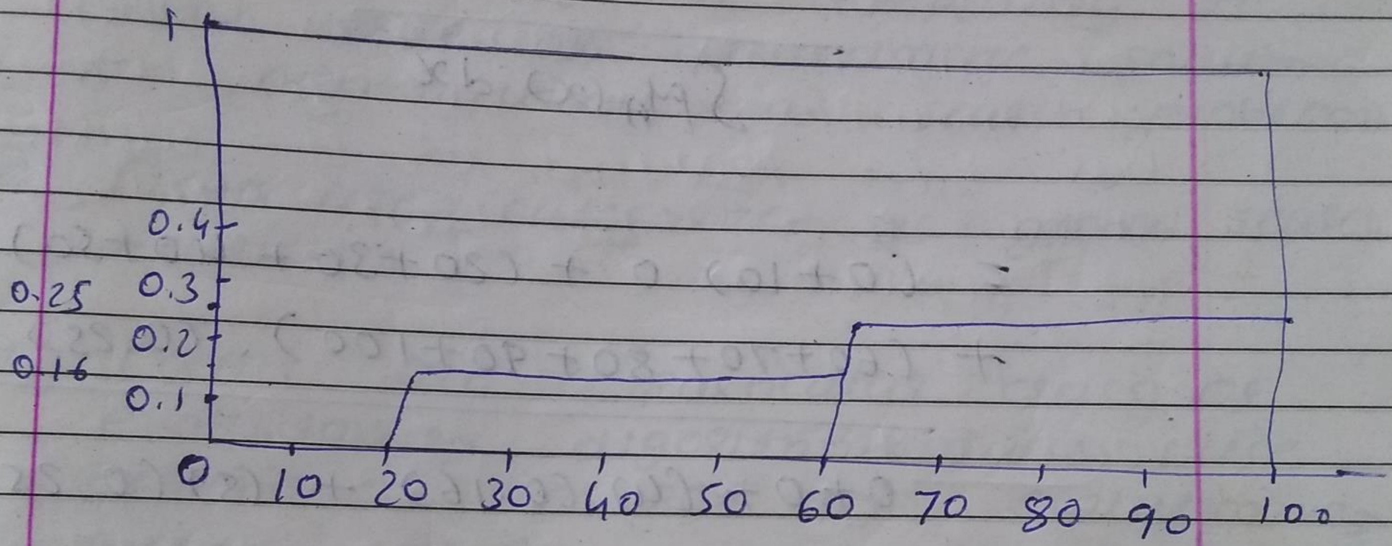
large(60) = 0.5

risk = normal = 0.16

③ If project funding is inadequate then risk is high



③ Aggregation of rule output




④ Defuzzification

- for fuzzy system whose final o/p needs to be in a crisp form

Two major techniques

① COG (center of gravity)
② mean of maximum

- calculate the avg of all variable values with max$^m$ membership degree

$$\text{COA} = \frac{\int \mu_A(x) \cdot x \, dx}{\int \mu_A(x) \, dx}$$

$$= \frac{(0+10) \cdot 0 + (20+30+40+50)(0.16) + (60+70+80+90+100) \cdot (0.25)}{0+0+(4)(0.16)+(5)(0.25)}$$

$$= \frac{22.4 + 100}{0.64 + 1.25}$$

$$= \frac{122.4}{1.89}$$

$$\boxed{\text{COA} = 64.76}$$