Sub: Compiler Construction
Syntax Analysis PART 4

Compiled for: 7th Sem, CE, DDU

Compiled by: Niyati J. Buch

Ref. : Compilers: Principles, Techniques, and Tools, 2nd Edition
Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman

Topics Covered

- Bottom-up parsing
  - Introduction to LR Parsing
  - LR(0) automaton
  - LR Parsing Table
    - Structure of LR Parsing Table
    - LR Parsing Algorithm
  - Simple LR (SLR)
    - Constructing SLR parsing table
    - Construct SLR parsing table for expression grammar
    - Moves of an LR parser on id * id + id
    - An unambiguous grammar that is not SLR(1)
    - A grammar that is SLR(1) but not LL(1)
    - A grammar that is LL(1) but not SLR(1)

Introduction to LR Parsing

- The most prevalent type of bottom-up parser is based on a concept called **LR(k) parsing**
  - the **"L"** is for left-to-right scanning of the input,
  - the **"R"** for constructing a rightmost derivation in reverse,
  - and the **k** for the number of input symbols of lookahead that are used in making parsing decisions.
- The cases **k = 0 or k = 1** are of practical interest, and we shall only consider LR parsers with k = 1 here.
- The easiest method for constructing shift-reduce parsers, called **"simple LR" (or SLR, for short).**
- Two more complex methods are used in the majority of LR parsers:
  - **canonical-LR**
  - **LALR**

LR Parsers

- LR parsers are **table-driven**, much like the nonrecursive LL parsers.
- A grammar for which we can construct a parsing table is said to be an LR grammar.
- Intuitively, for a grammar to be LR it is sufficient that a left-to-right shift-reduce parser be able to recognize handles of right-sentential forms when they appear on top of the stack.

Why LR Parsers ???

- LR parsers can be constructed to **recognize virtually all programming language constructs** for which context-free grammars can be written. Non-LR context-free grammars exist, but these can generally be avoided for typical programming-language constructs.
- The LR-parsing method is the **most general nonbacktracking shift-reduce parsing method** known, yet it can be implemented as efficiently as other, more primitive shift-reduce methods .
- An LR parser can **detect a syntactic error as soon as** it is possible to do so on a left-to-right scan of the input.
- **The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive or LL methods**.

LR grammars can describe more languages than LL grammars.

Drawback

- The principal drawback of the LR method is that it is **too much work** to construct an LR parser by hand for a typical programming-language grammar.
- A specialized tool, an **LR parser generator (like YACC)**, is needed.
- Such a generator takes a context-free grammar and automatically produces a parser for that grammar.

How does a shift-reduce parser know when to shift and when to reduce?

- An LR parser makes shift-reduce decisions **by maintaining states** to keep track of where we are in a parse.
- **States represent sets of "items".**
- An LR(0) item (item for short) of a grammar G is **a production of G with a dot at some position of the body**.
- Thus, production A → XYZ yields the four items:

    1. A → · X Y Z
    2. A → X · Y Z
    3. A → X Y · Z
    4. A → X Y Z ·

Intuitively, an item indicates how much of a production we have seen at a given point in the parsing process.
For example, the item A → · X Y Z indicates that we hope to see a string derivable from XY Z next on the input.

- The production A → ∈ generates only one item, A → ·

LR(0) automaton

- One collection of sets of LR(0) items, called the **canonical LR(0) collection**, provides the basis for constructing a deterministic finite automaton that is used to make parsing decisions.
- Such an automaton is called an **LR(0) automaton**.
- In particular, each state of the LR(0) automaton represents a set of items in the canonical LR(0) collection.

Canonical LR(0) collection

- To construct the canonical LR(0) collection for a grammar, we define **an augmented grammar** and two functions, **CLOSURE and GOTO**.
- If G is a grammar with start symbol S, then G0 , the augmented grammar for G, is G with a **new start symbol S'** and production **S' → S**.
- The **purpose** of this new starting production is to indicate to the parser when it should stop parsing and announce acceptance of the input.
- That is, acceptance occurs when and only when the parser is about to reduce by S' → S

CLOSURE of Item Sets

- If I is a set of items for a grammar G, then CLOSURE(I) is the set of items constructed from I by the two rules:
  - Initially, add every item in I to CLOSURE(I).
  - If $A \rightarrow \alpha \cdot B \beta$ is in CLOSURE(I) and $B \rightarrow \gamma$ is a production, then add the item $B \rightarrow \cdot \gamma$ to CLOSURE(I), if it is not already there. Apply this rule until no more new items can be added to CLOSURE(I).
- For example: E' → E

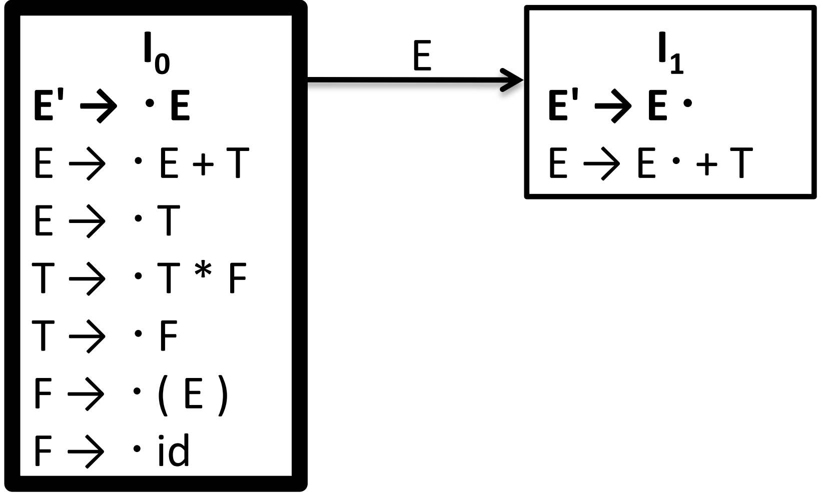
$E' \rightarrow \cdot E$
$E \rightarrow \cdot E + T$
$E \rightarrow \cdot T$
$T \rightarrow \cdot T * F$
$T \rightarrow \cdot F$
$F \rightarrow \cdot (E)$
$F \rightarrow \cdot \mathrm{id}$

GOTO

- The second useful function is GOTO(I, X) where I is a set of items and X is a grammar symbol.
- GOTO(I, X) is defined to be the closure of the set of all items $[A \rightarrow \alpha \; X \cdot \beta \;]$ such that $[A \rightarrow \alpha \cdot X \, \beta \;]$ is in I.
- Intuitively, **the GOTO function is used to define the transitions in the LR(0) automaton for a grammar**.
- The states of the automaton correspond to sets of items, and GOTO(I, X) specifies the transition from the state for I under input X.

I_0

E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

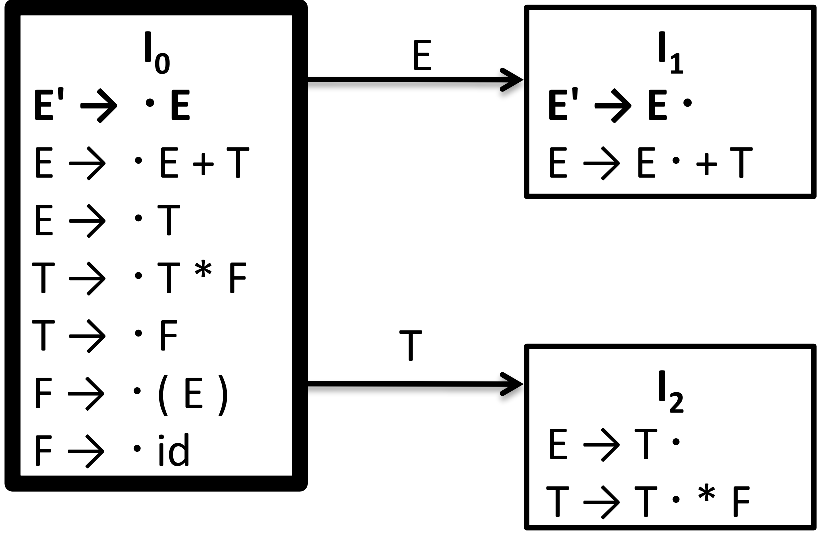
F → · id

E

I_1

E' → E ·

E → E · + T



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

E

I_1

E' → E ·

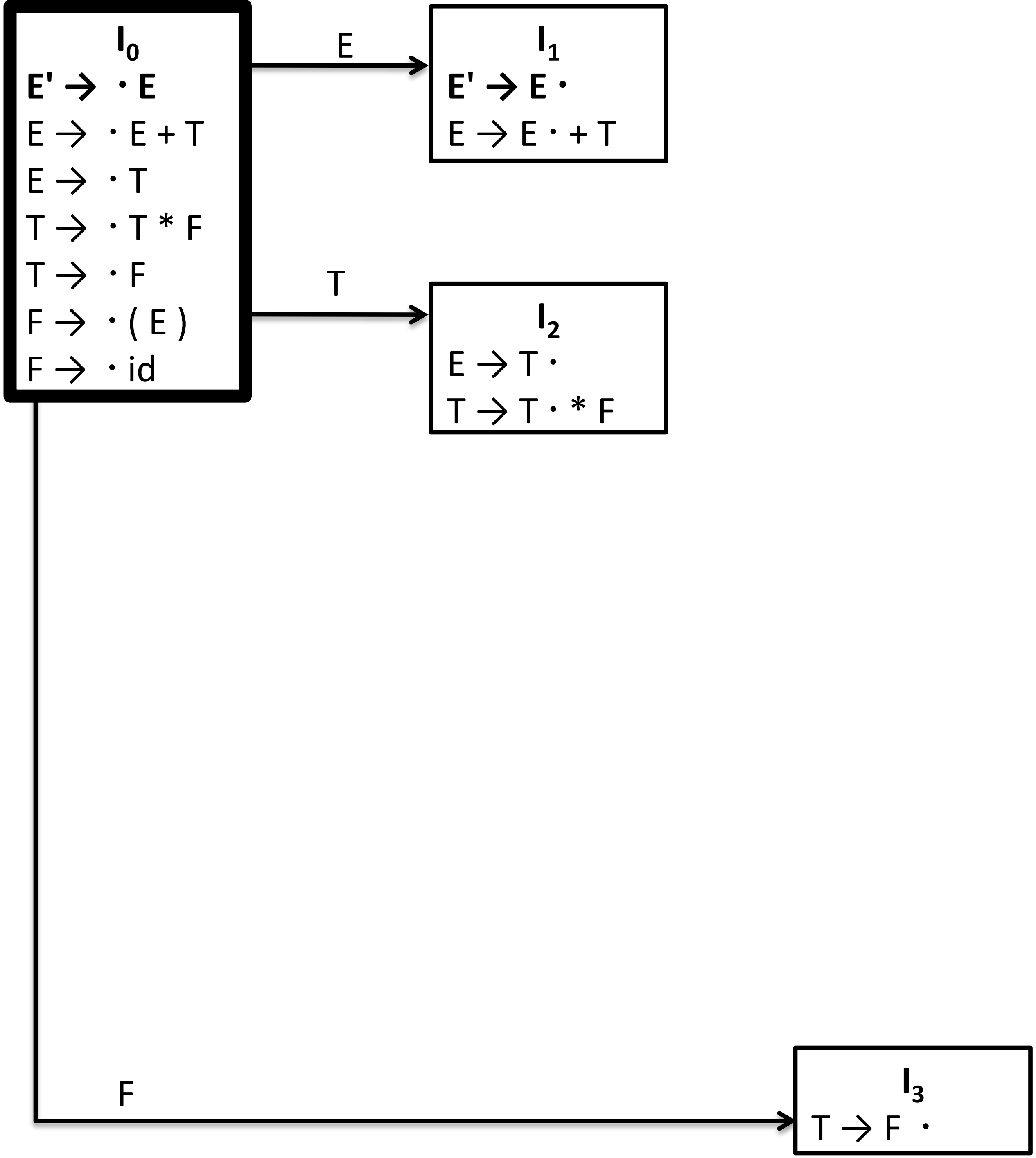
E → E · + T

T

I_2

E → T ·

T → T · * F



I_0

$E' \rightarrow \cdot E$

$E \rightarrow \cdot E + T$

$E \rightarrow \cdot T$

$T \rightarrow \cdot T * F$

$T \rightarrow \cdot F$

$F \rightarrow \cdot (E)$

$F \rightarrow \cdot id$

E

I_1

$E' \rightarrow E \cdot$

$E \rightarrow E \cdot + T$

T

I_2

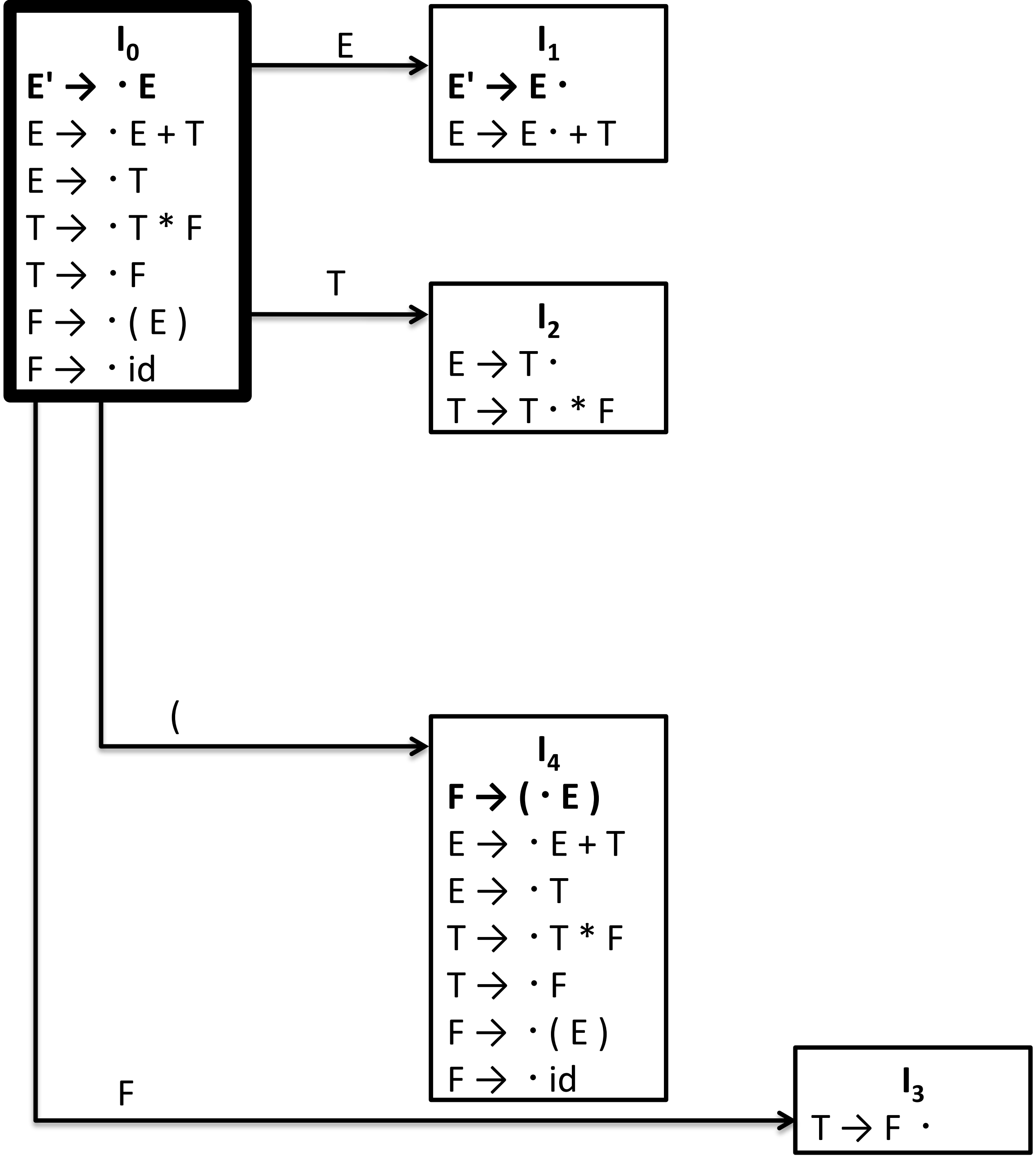
$E \rightarrow T \cdot$

$T \rightarrow T \cdot * F$

F

I_3

$T \rightarrow F \cdot$



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_2

E → T ·

T → T · * F

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

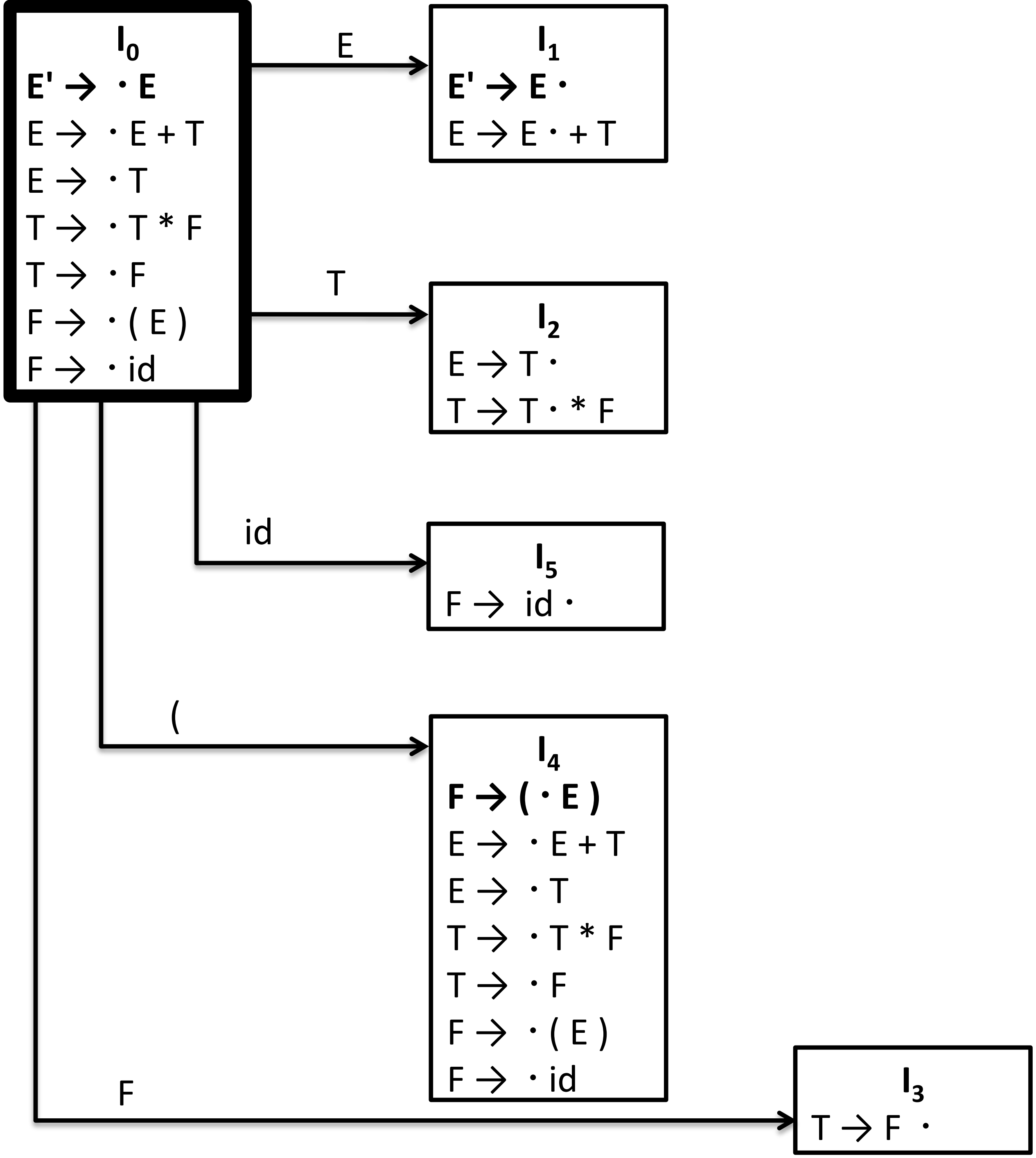
T → · F

F → · (E)

F → · id

I_3

T → F ·



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_2

E → T ·

T → T · * F

I_5

F → id ·

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

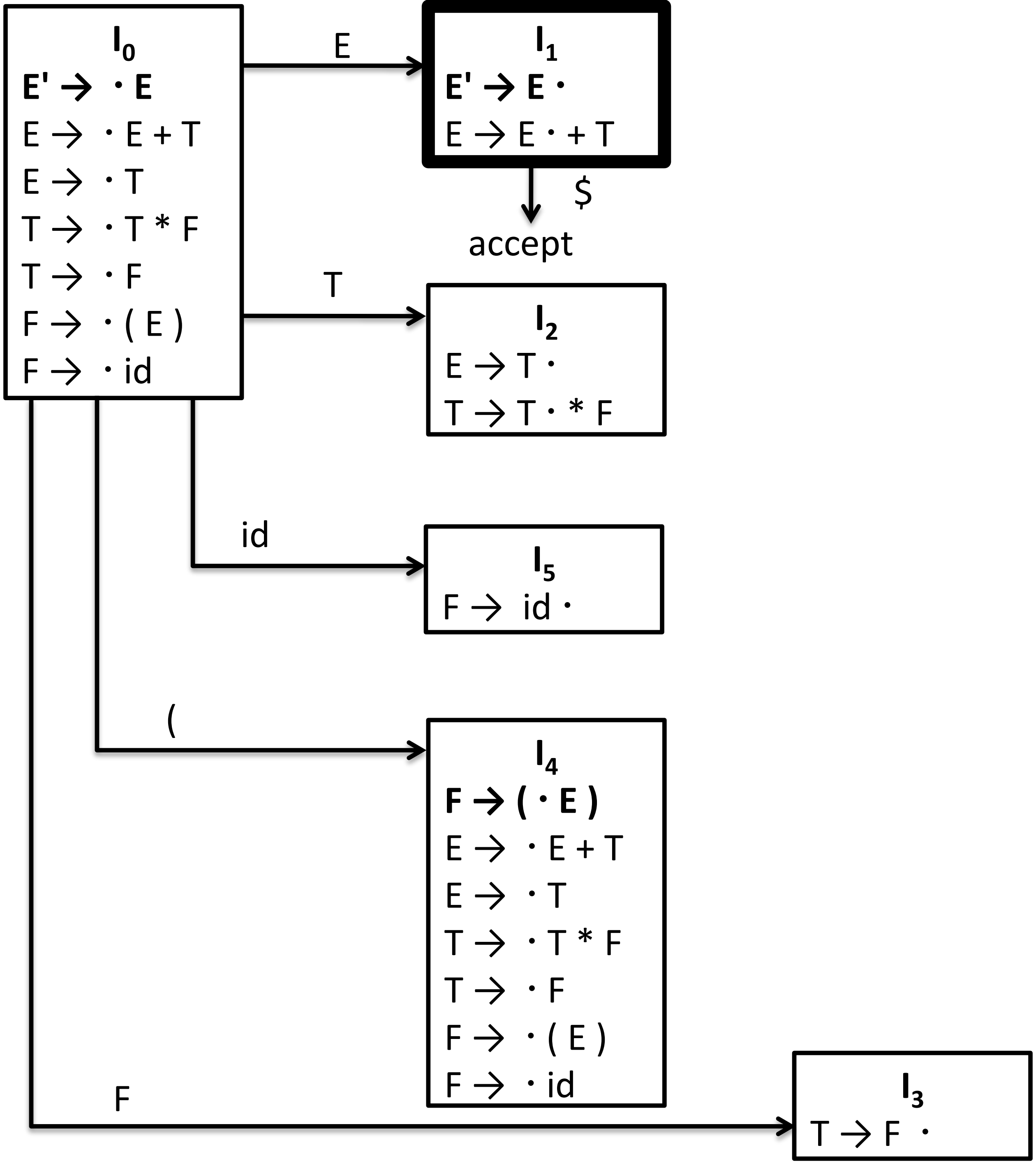
T → · F

F → · (E)

F → · id

I_3

T → F ·



I₀

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

E

I₁

E' → E ·

E → E · + T

$

accept

T

I₂

E → T ·

T → T · * F

id

I₅

F → id ·

(

I₄

F → (· E)

E → · E + T

E → · T

T → · T * F

T → · F

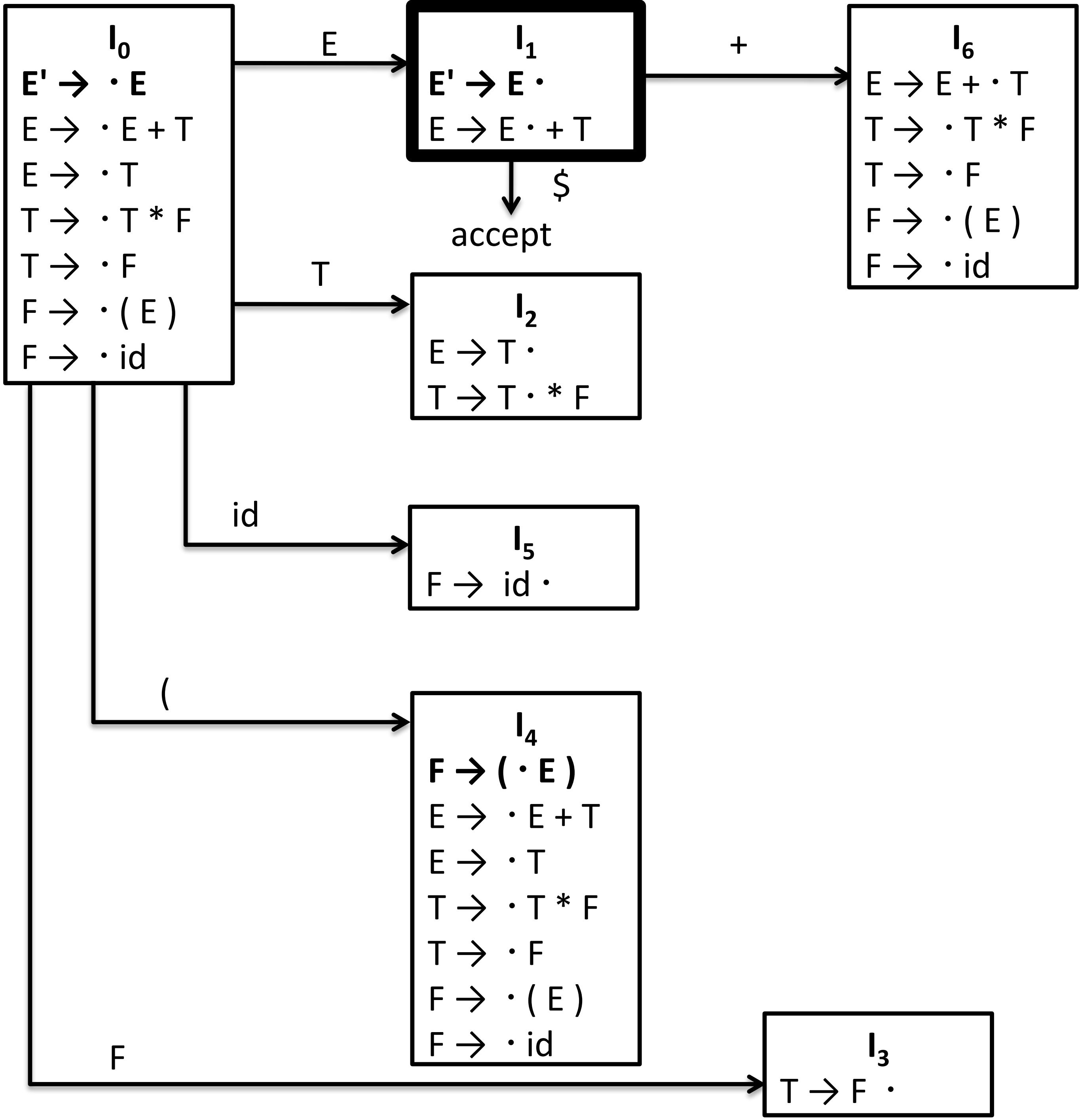
F → · (E)

F → · id

F

I₃

T → F ·



I₀

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I₁

E' → E ·

E → E · + T

I₆

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I₂

E → T ·

T → T · * F

I₅

F → id ·

I₄

F → (· E)

E → · E + T

E → · T

T → · T * F

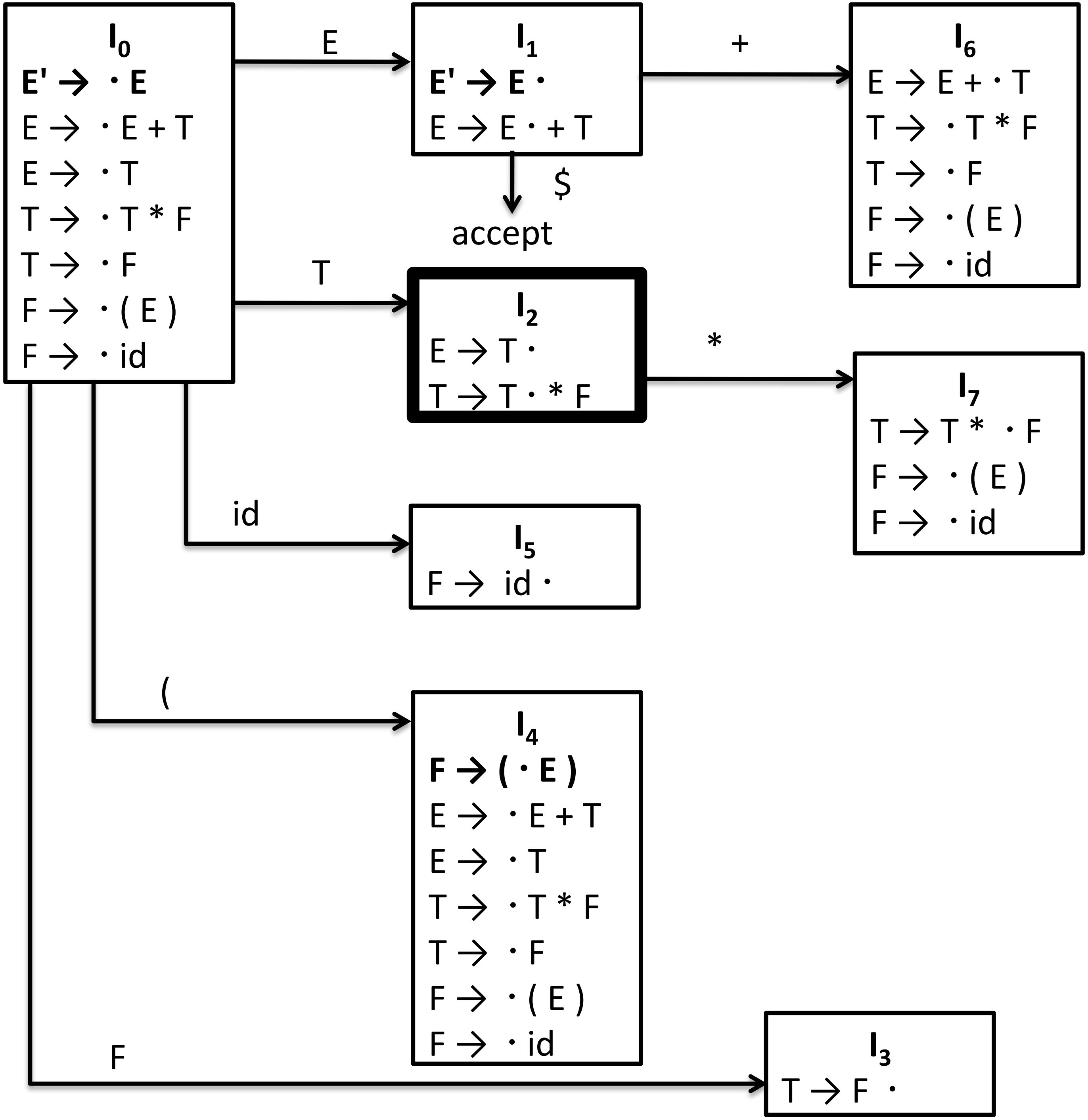
T → · F

F → · (E)

F → · id

I₃

T → F ·



I₀
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₁
E' → E ·
E → E · + T

I₆
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

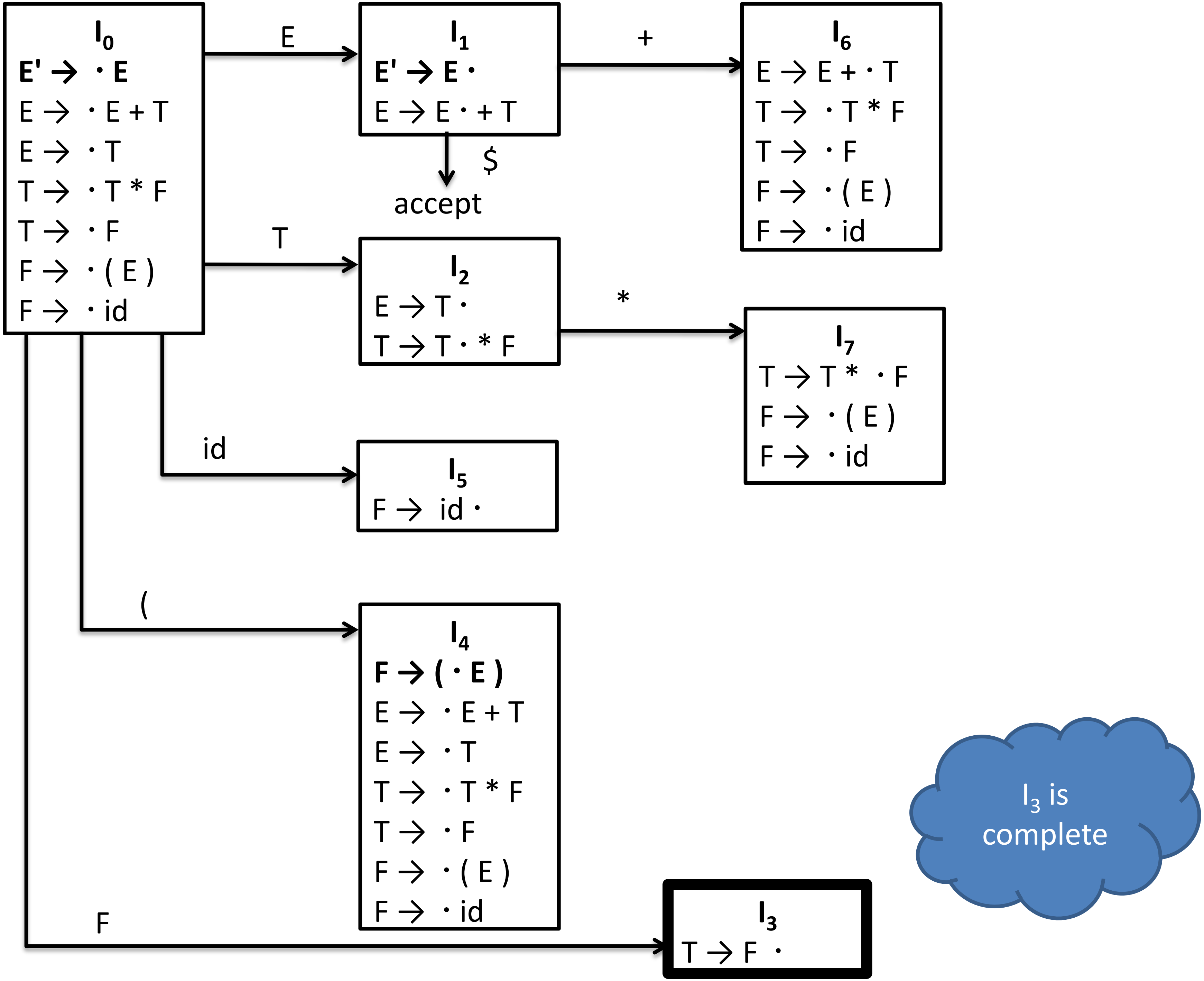
I₂
E → T ·
T → T · * F

I₇
T → T * · F
F → · (E)
F → · id

I₅
F → id ·

I₄
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₃
T → F ·



I_0
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I_1
E' → E ·
E → E · + T

I_6
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I_2
E → T ·
T → T · * F

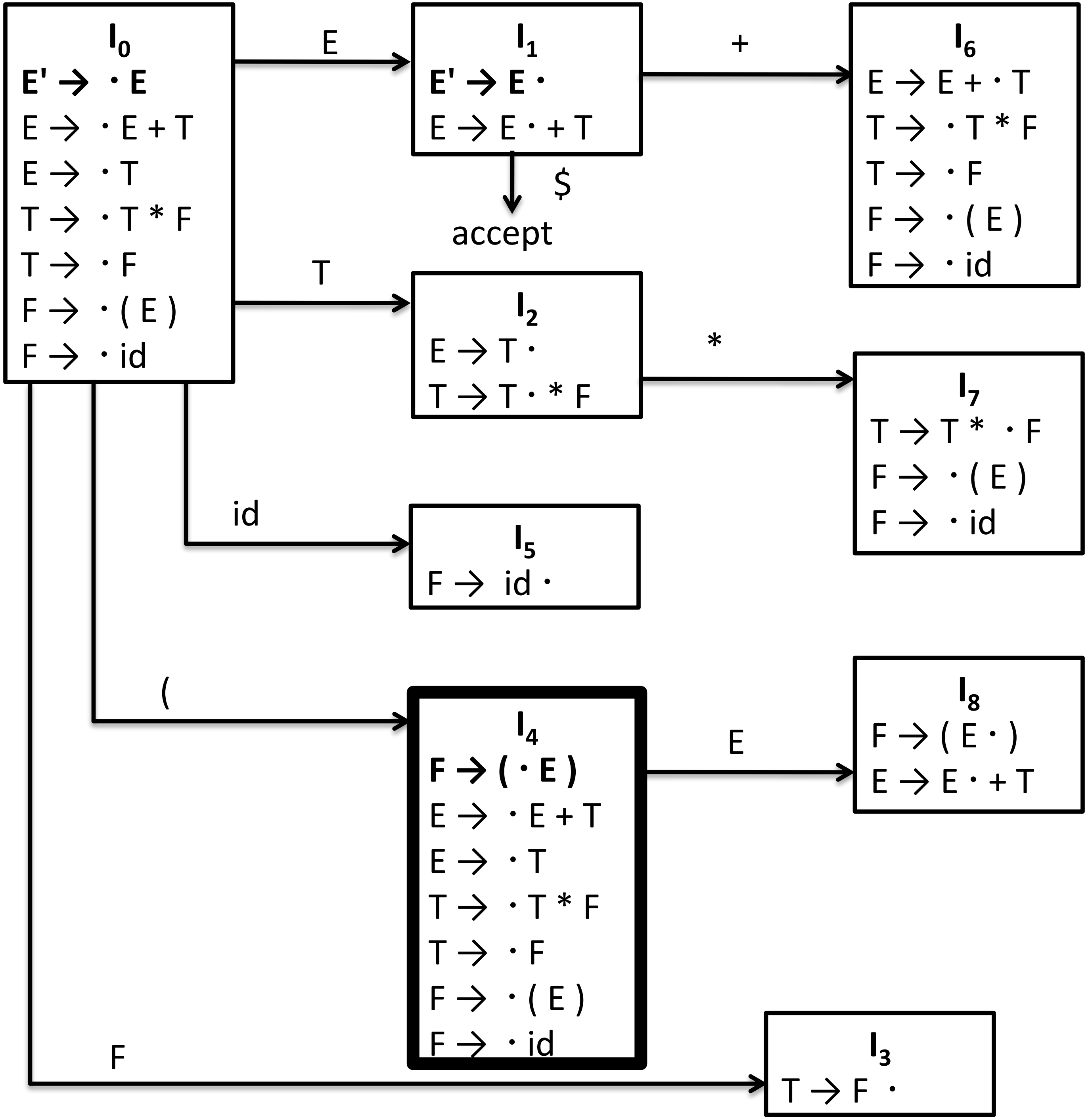
I_7
T → T * · F
F → · (E)
F → · id

I_5
F → id ·

I_4
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I_3
T → F ·

I_3 is complete



I0
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I1
E' → E ·
E → E · + T

I6
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I2
E → T ·
T → T · * F

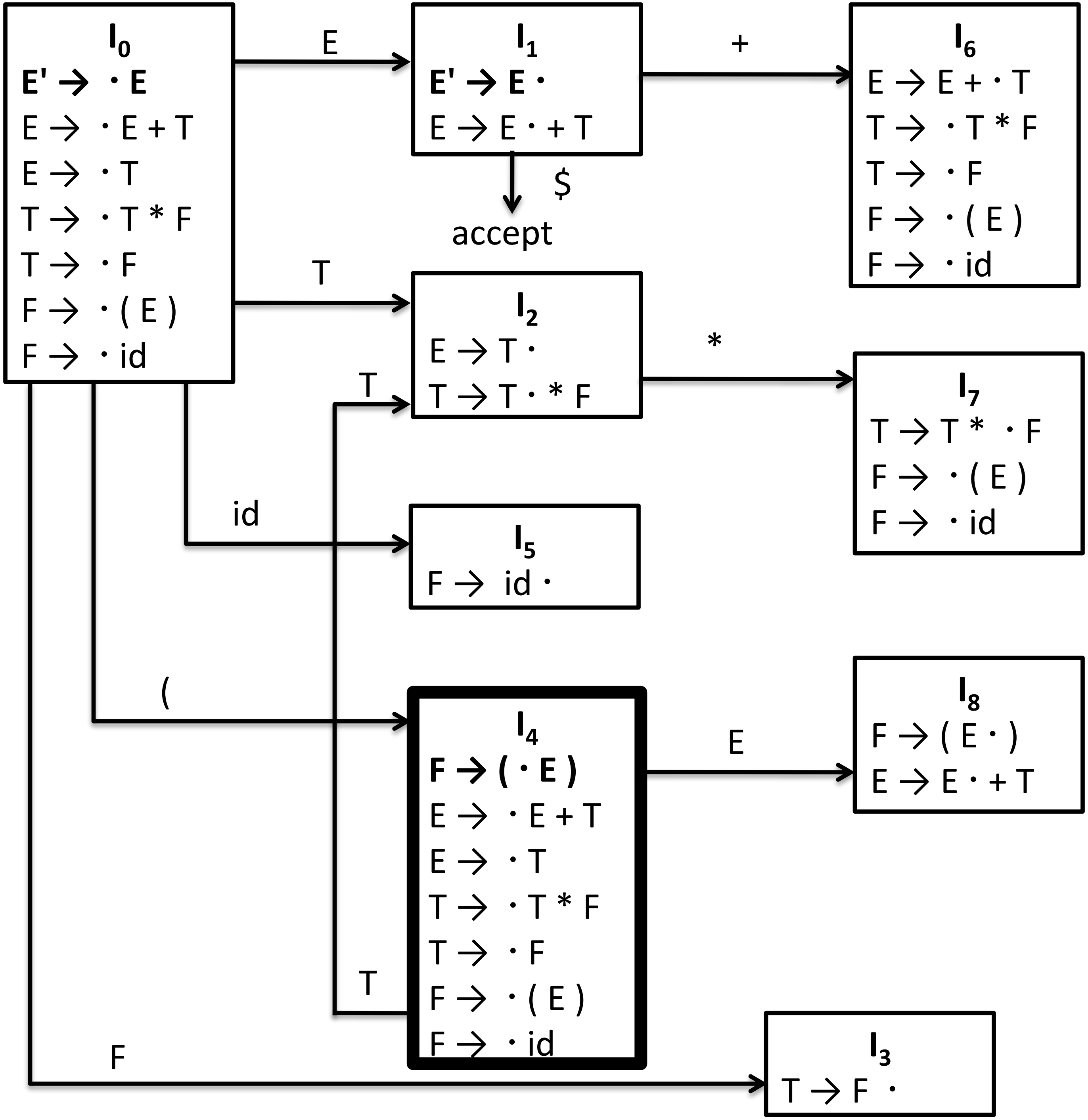
I7
T → T * · F
F → · (E)
F → · id

I5
F → id ·

I4
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I8
F → (E ·)
E → E · + T

I3
T → F ·



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_2

E → T ·

T → T · * F

I_7

T → T * · F

F → · (E)

F → · id

I_5

F → id ·

I_8

F → (E ·)

E → E · + T

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

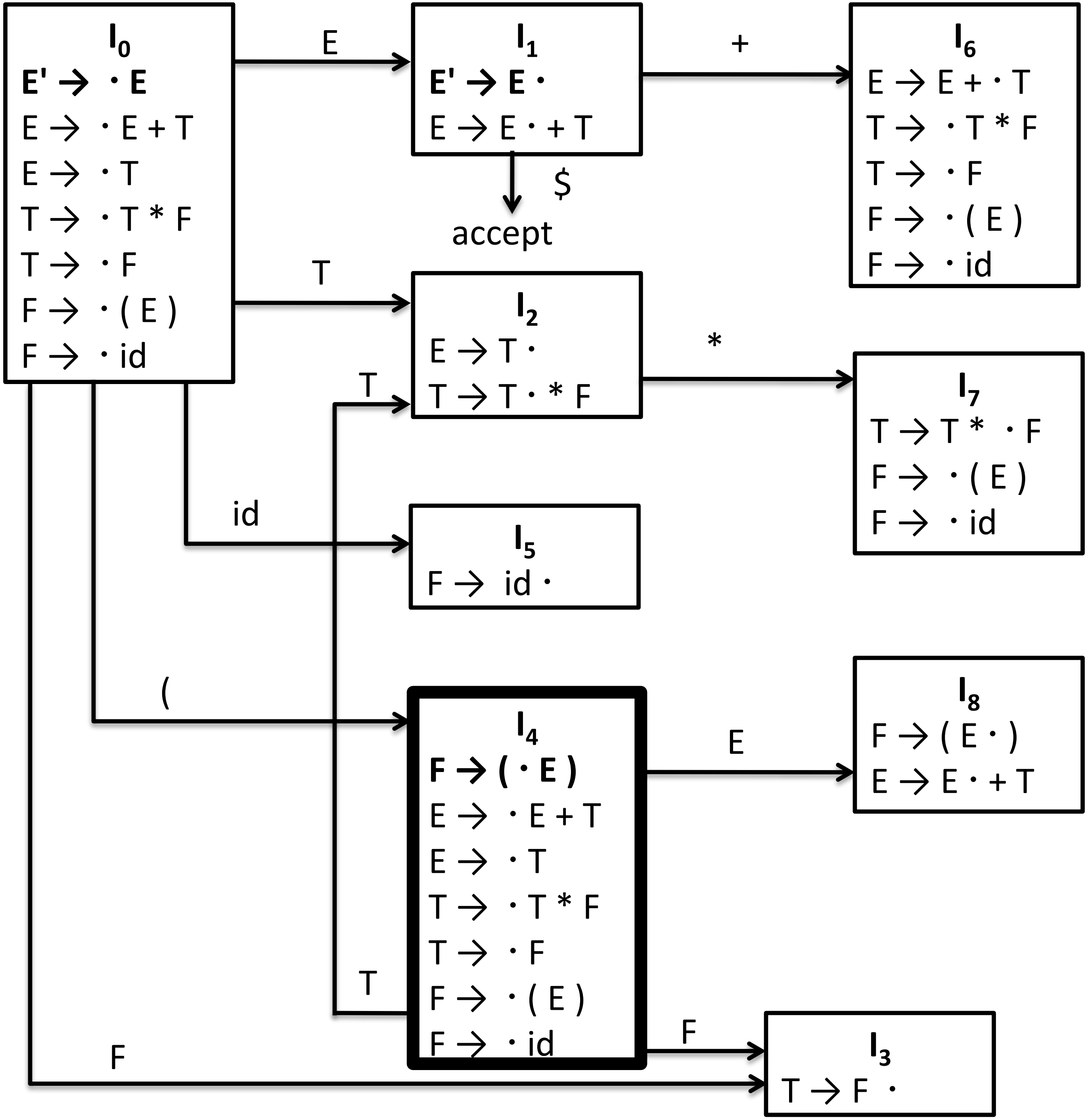
T → · F

F → · (E)

F → · id

I_3

T → F ·



I$_0$
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_1$
E' → E ·
E → E · + T

I$_6$
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_2$
E → T ·
T → T · * F

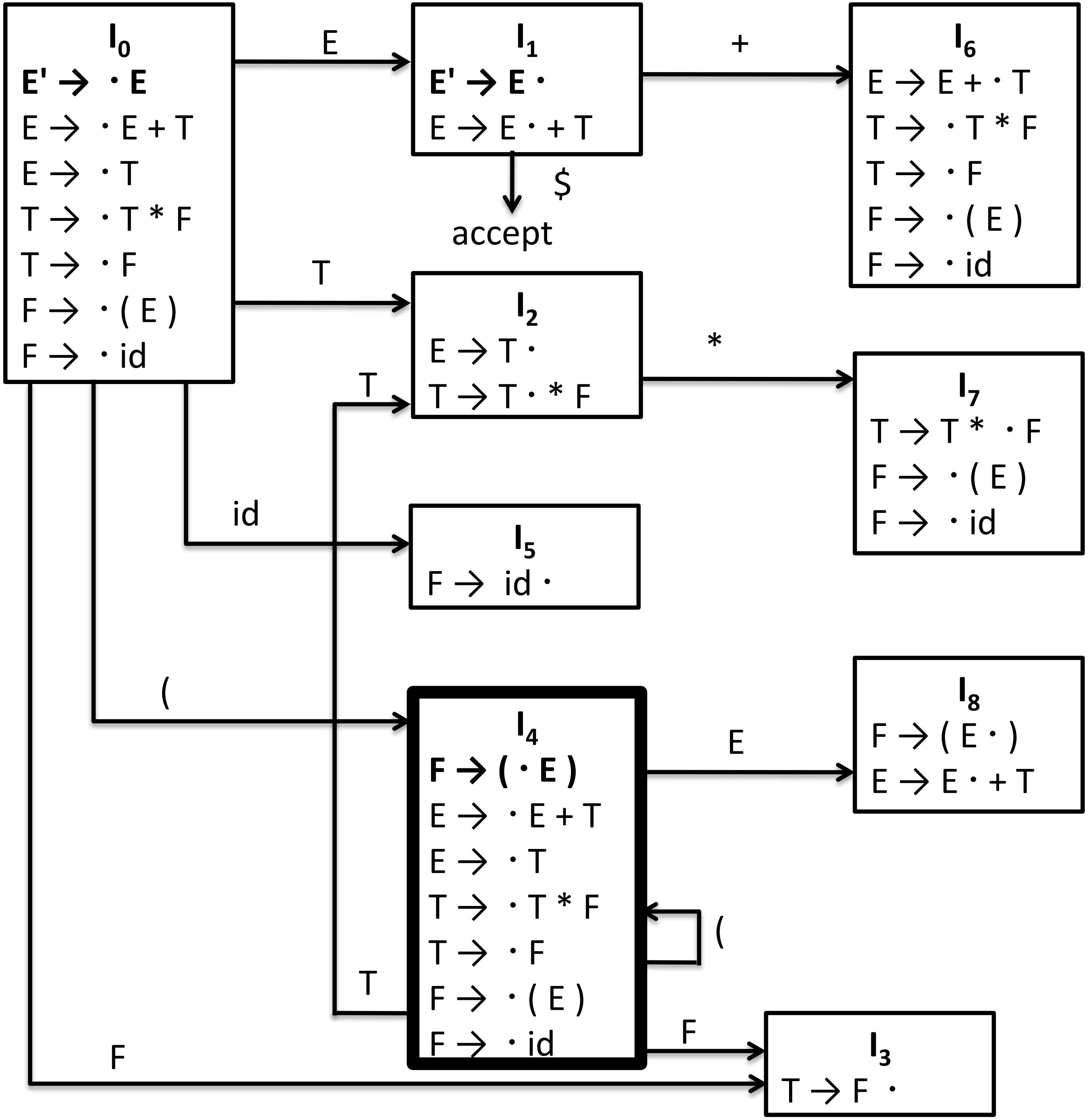
I$_7$
T → T * · F
F → · (E)
F → · id

I$_5$
F → id ·

I$_4$
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_8$
F → (E ·)
E → E · + T

I$_3$
T → F ·



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_2

E → T ·

T → T · * F

I_3

T → F ·

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_5

F → id ·

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_7

T → T * · F

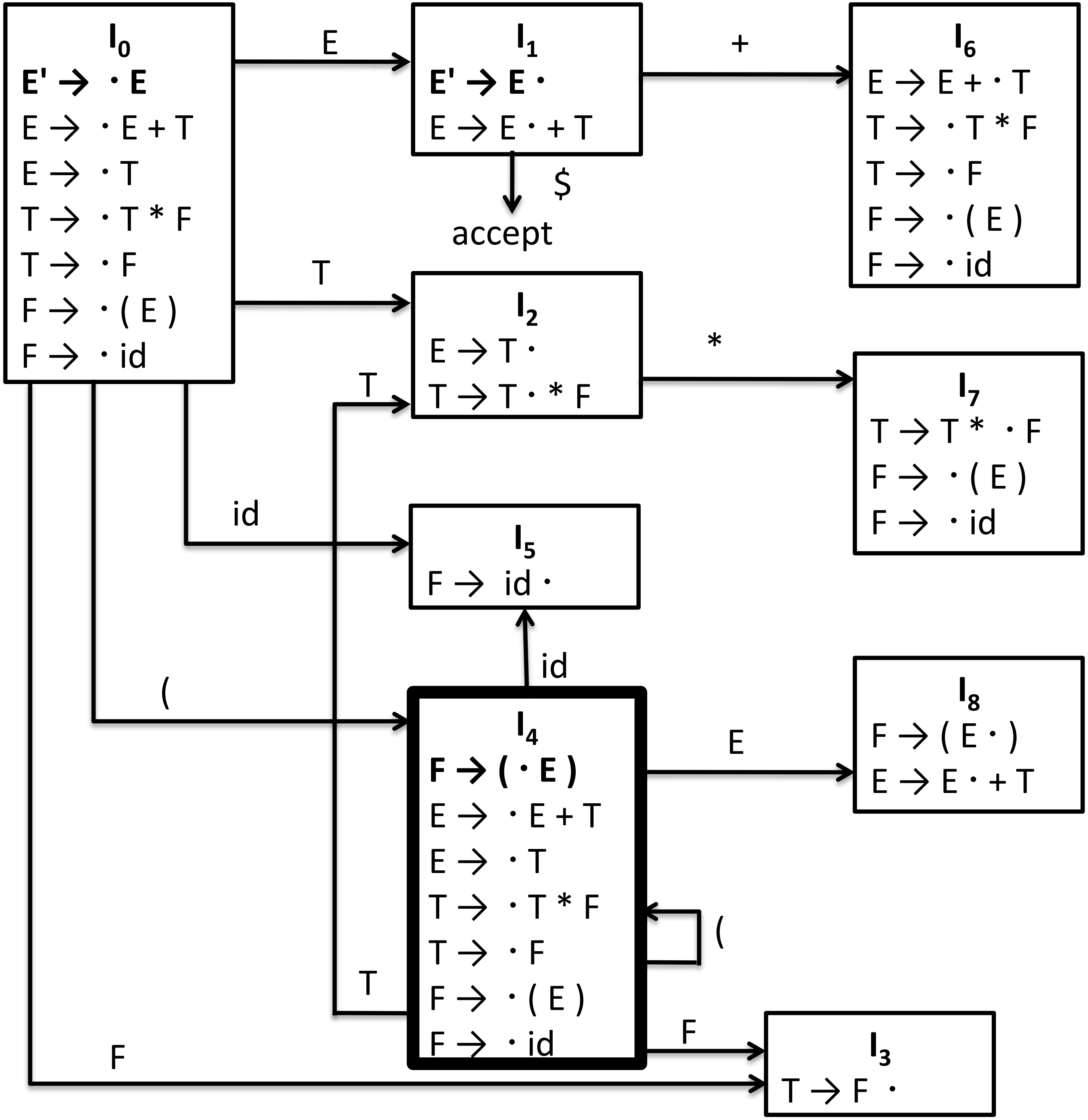
F → · (E)

F → · id

I_8

F → (E ·)

E → E · + T



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_2

E → T ·

T → T · * F

I_7

T → T * · F

F → · (E)

F → · id

I_5

F → id ·

I_8

F → (E ·)

E → E · + T

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

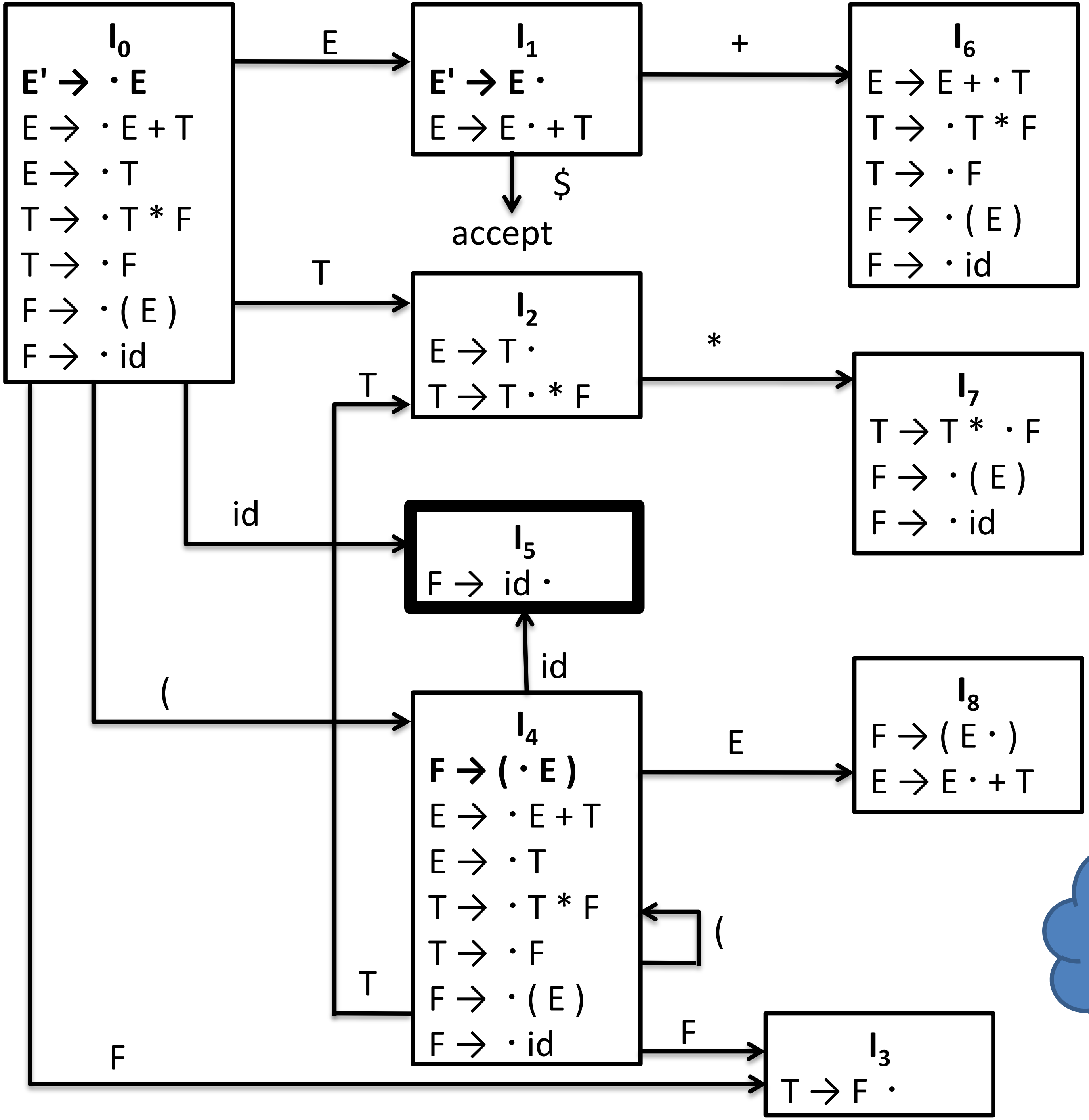
T → · F

F → · (E)

F → · id

I_3

T → F ·



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_2

E → T ·

T → T · * F

I_7

T → T * · F

F → · (E)

F → · id

I_5

F → id ·

I_8

F → (E ·)

E → E · + T

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

T → · F

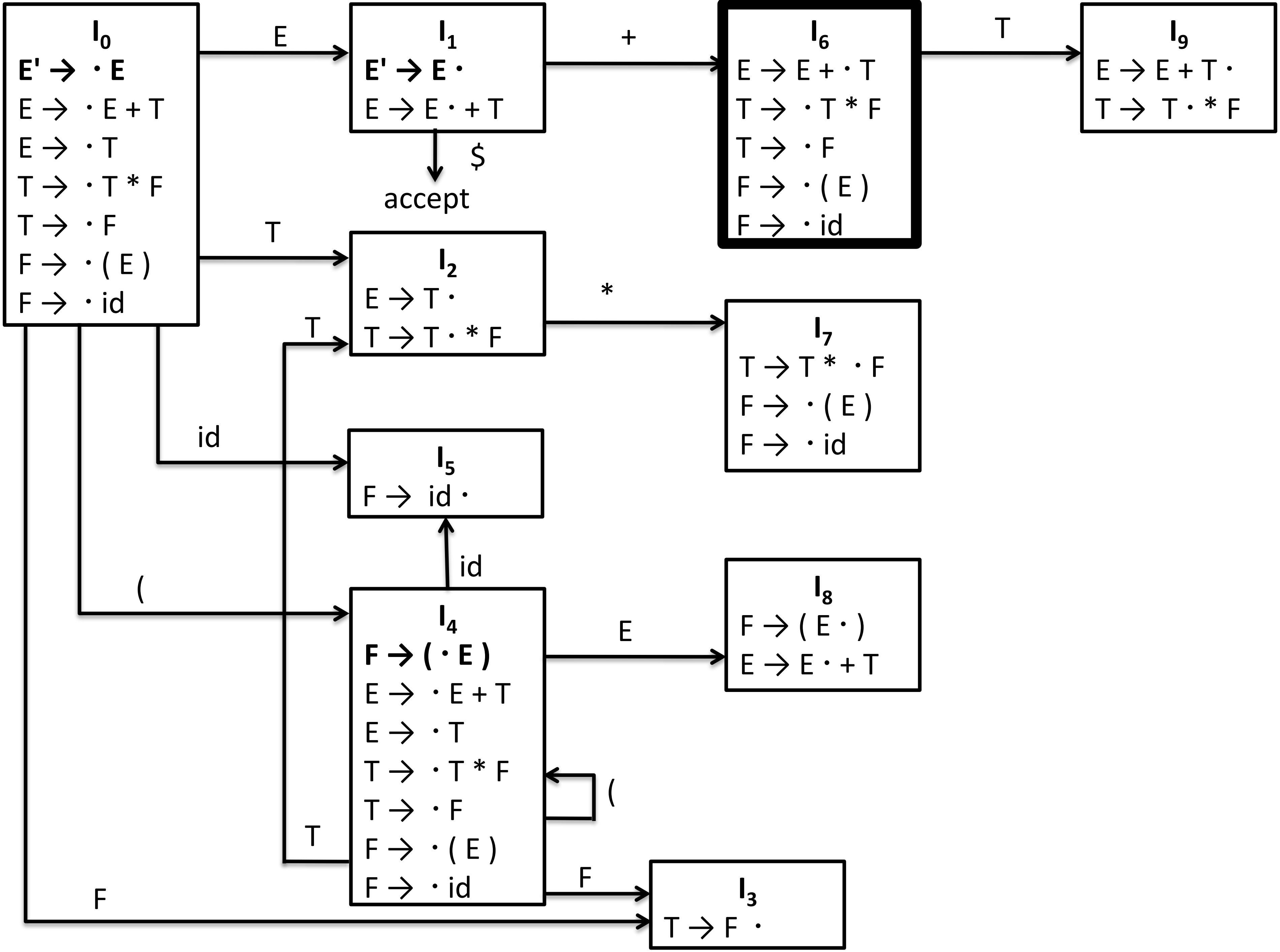
F → · (E)

F → · id

I_3

T → F ·

I_5 is complete



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_9

E → E + T ·

T → T · * F

I_2

E → T ·

T → T · * F

I_7

T → T * · F

F → · (E)

F → · id

I_5

F → id ·

I_8

F → (E ·)

E → E · + T

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

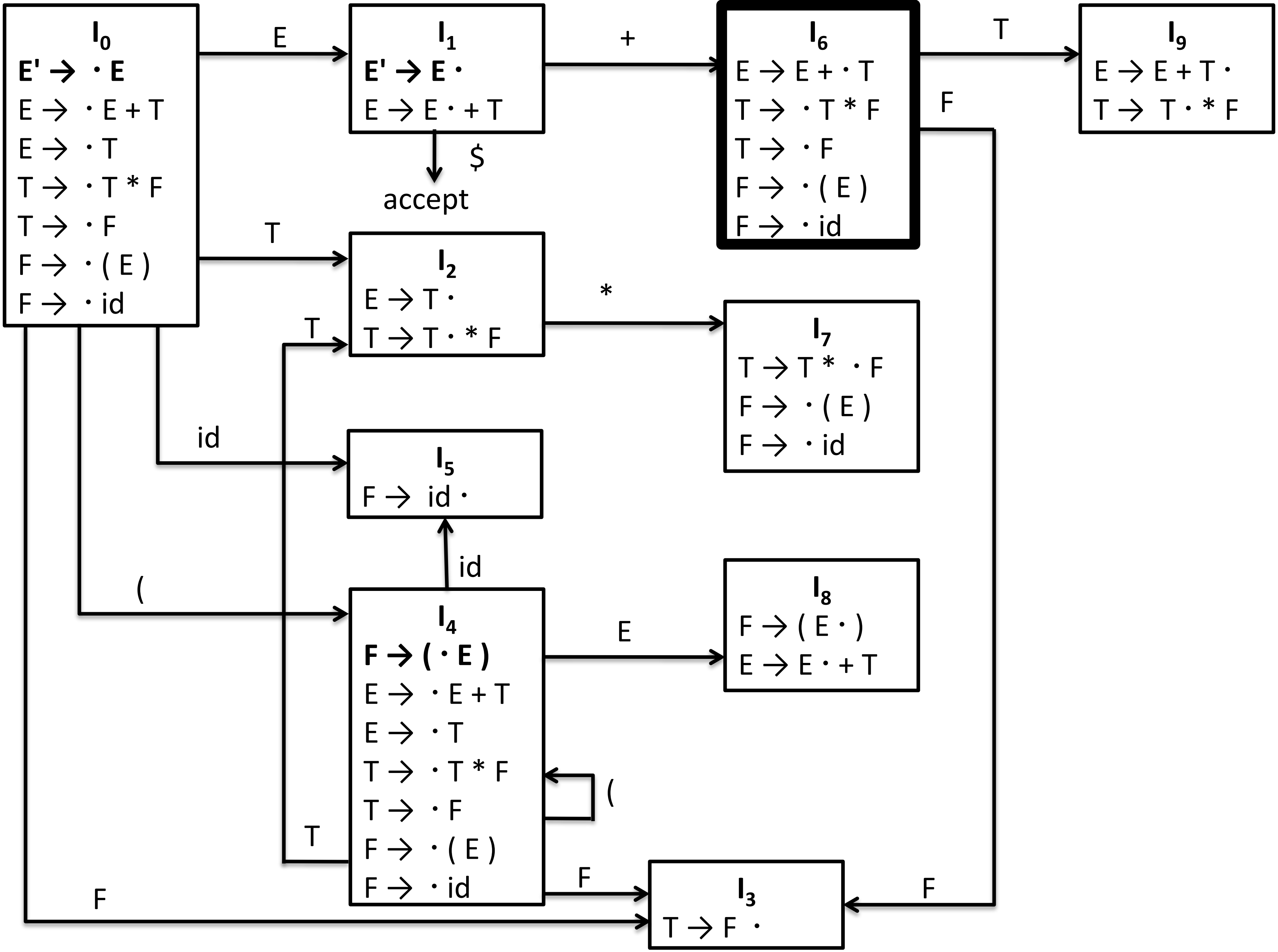
T → · F

F → · (E)

F → · id

I_3

T → F ·



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_2

E → T ·

T → T · * F

I_3

T → F ·

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_5

F → id ·

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_7

T → T * · F

F → · (E)

F → · id

I_8

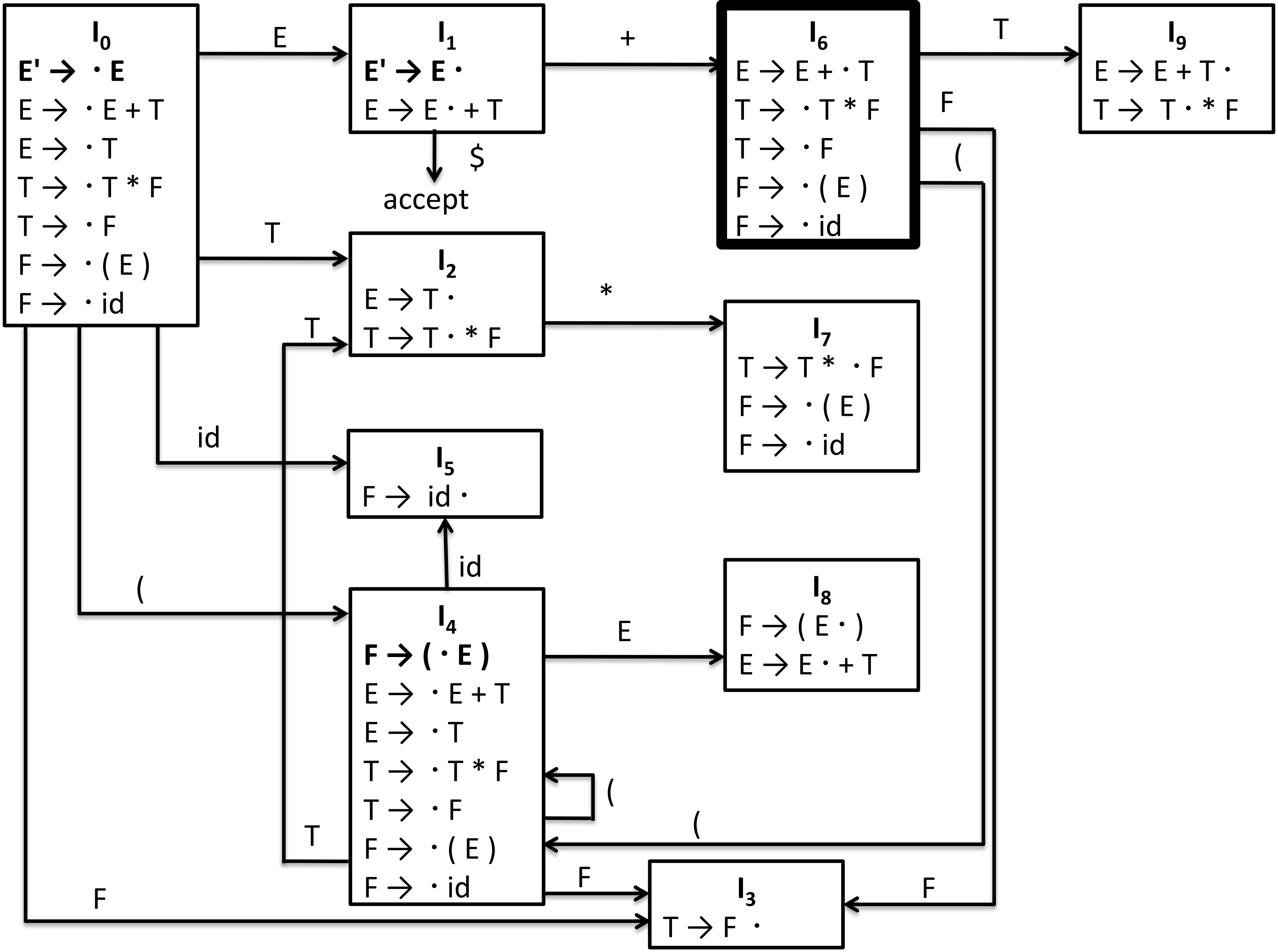
F → (E ·)

E → E · + T

I_9

E → E + T ·

T → T · * F



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_9

E → E + T ·

T → T · * F

I_2

E → T ·

T → T · * F

I_7

T → T * · F

F → · (E)

F → · id

I_5

F → id ·

I_8

F → (E ·)

E → E · + T

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

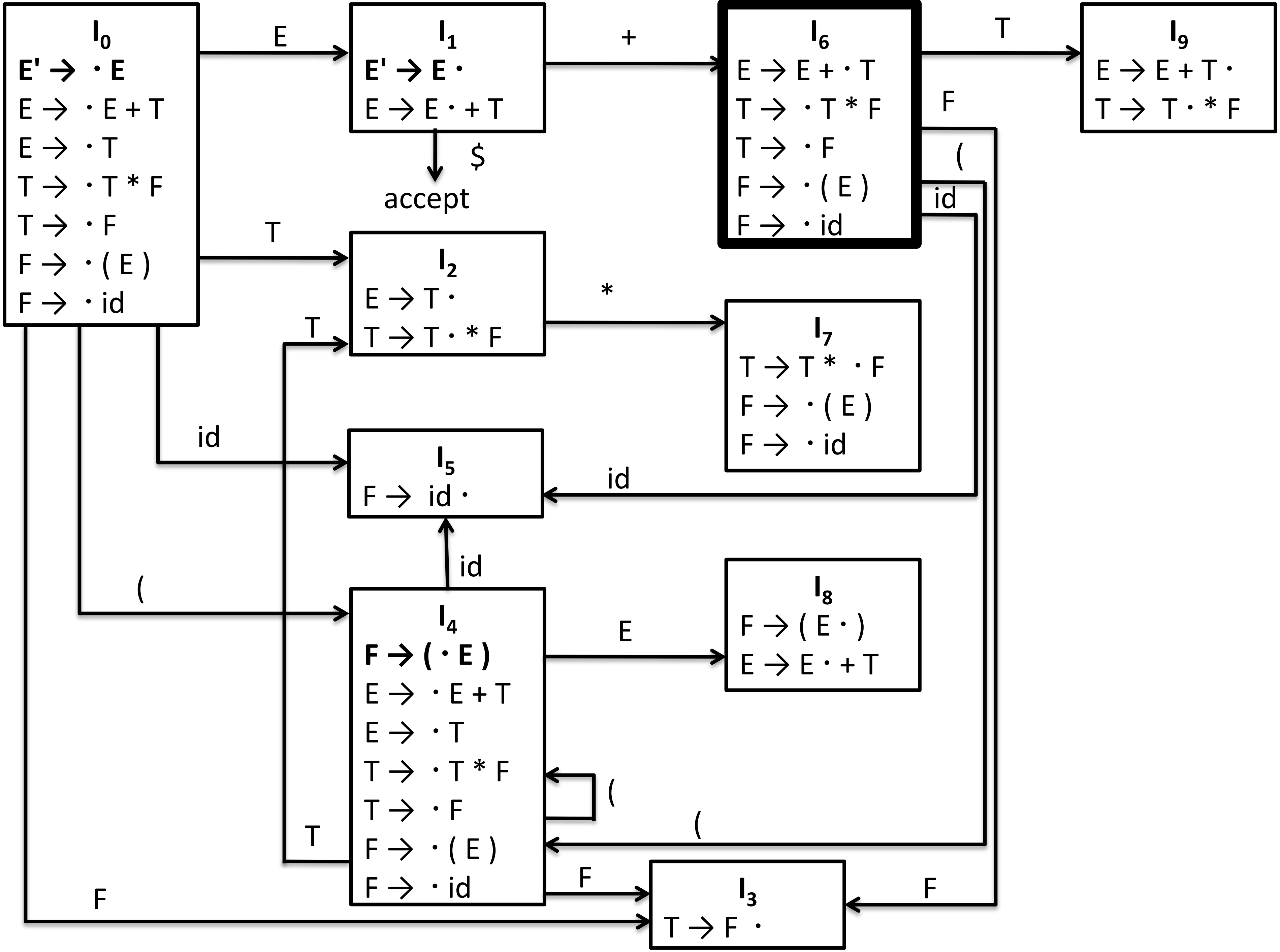
T → · F

F → · (E)

F → · id

I_3

T → F ·



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_2

E → T ·

T → T · * F

I_3

T → F ·

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_5

F → id ·

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_7

T → T * · F

F → · (E)

F → · id

I_8

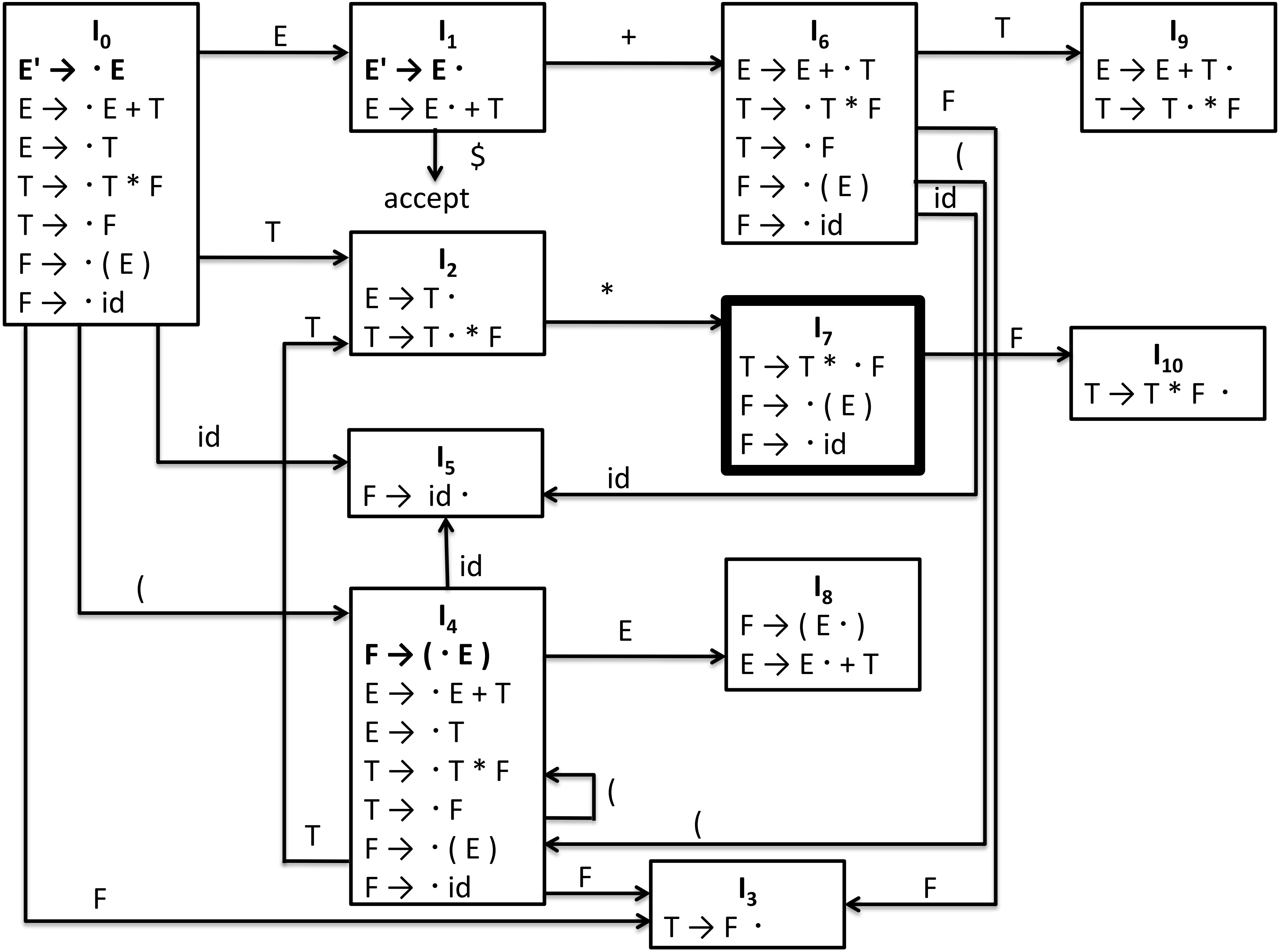
F → (E ·)

E → E · + T

I_9

E → E + T ·

T → T · * F



I$_0$
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_1$
E' → E ·
E → E · + T

I$_2$
E → T ·
T → T · * F

I$_3$
T → F ·

I$_4$
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_5$
F → id ·

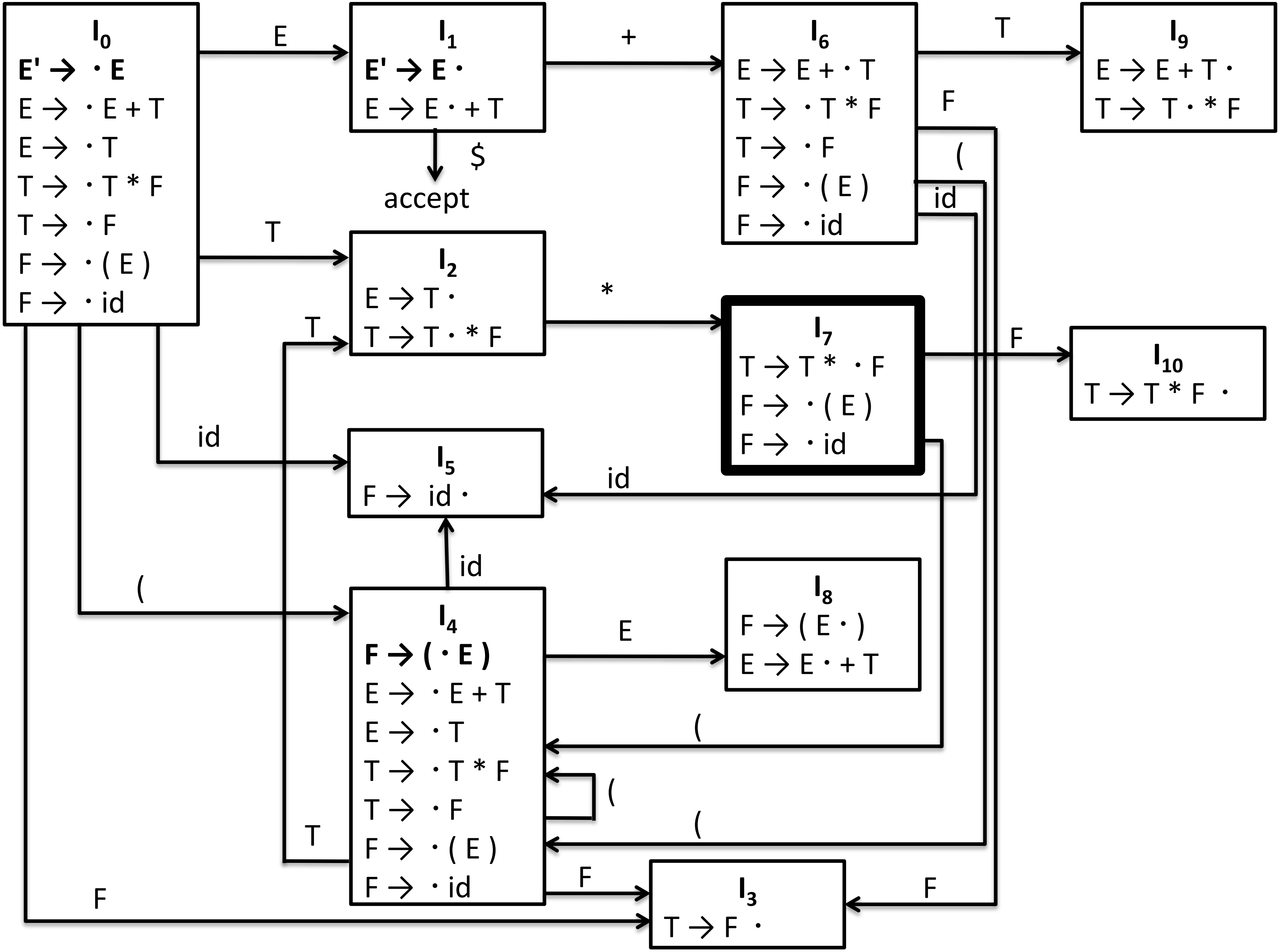
I$_6$
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_7$
T → T * · F
F → · (E)
F → · id

I$_8$
F → (E ·)
E → E · + T

I$_9$
E → E + T ·
T → T · * F

I$_{10}$
T → T * F ·



I$_0$
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_1$
E' → E ·
E → E · + T

I$_6$
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_9$
E → E + T ·
T → T · * F

I$_2$
E → T ·
T → T · * F

I$_7$
T → T * · F
F → · (E)
F → · id

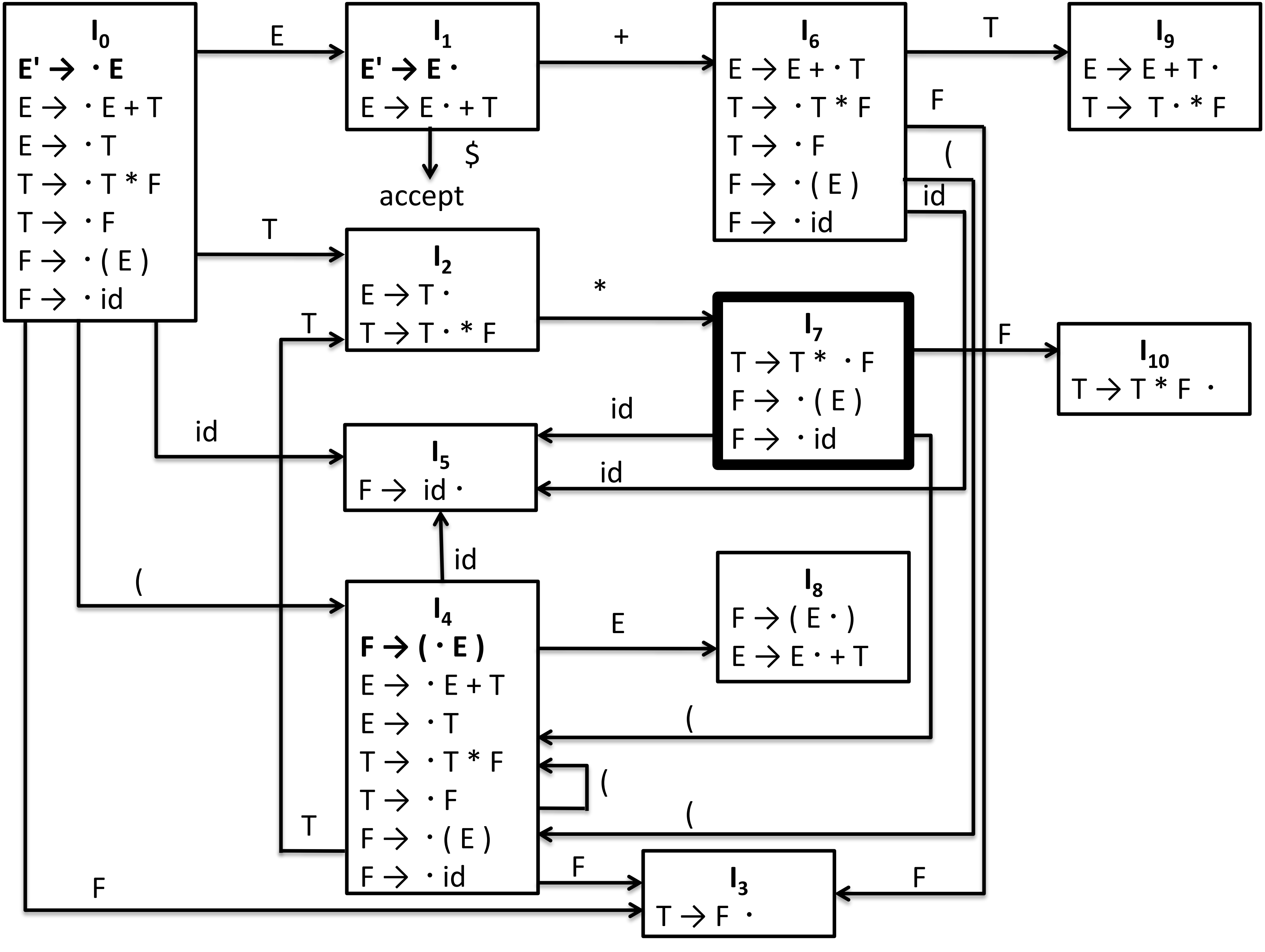
I$_{10}$
T → T * F ·

I$_5$
F → id ·

I$_8$
F → (E ·)
E → E · + T

I$_4$
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I$_3$
T → F ·



I₀
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₁
E' → E ·
E → E · + T

I₂
E → T ·
T → T · * F

I₃
T → F ·

I₄
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₅
F → id ·

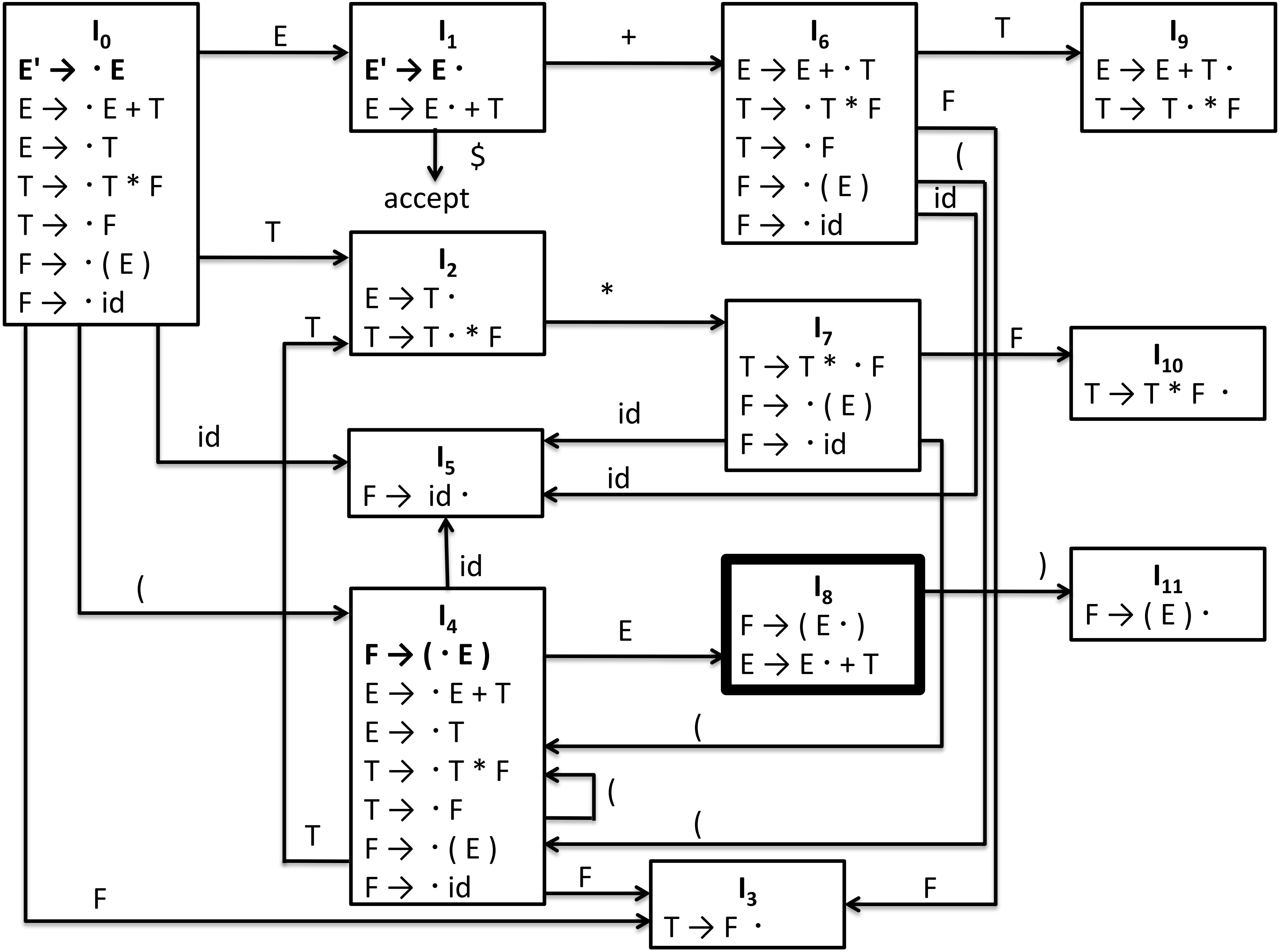
I₆
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I₇
T → T * · F
F → · (E)
F → · id

I₈
F → (E ·)
E → E · + T

I₉
E → E + T ·
T → T · * F

I₁₀
T → T * F ·



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_9

E → E + T ·

T → T · * F

I_2

E → T ·

T → T · * F

I_7

T → T * · F

F → · (E)

F → · id

I_{10}

T → T * F ·

I_5

F → id ·

I_8

F → (E ·)

E → E · + T

I_{11}

F → (E) ·

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

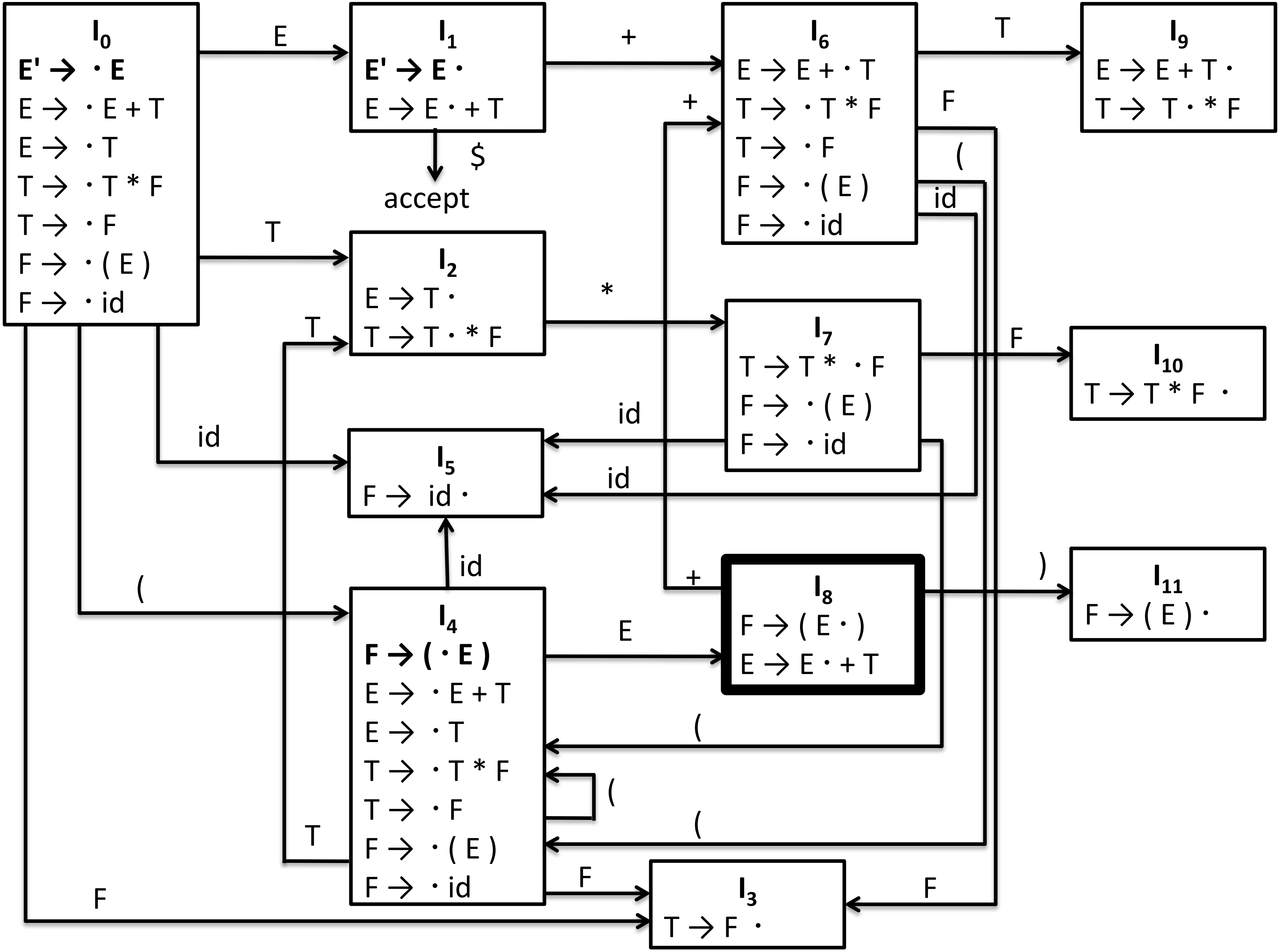
T → · F

F → · (E)

F → · id

I_3

T → F ·



I₀
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₁
E' → E ·
E → E · + T

I₂
E → T ·
T → T · * F

I₃
T → F ·

I₄
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₅
F → id ·

I₆
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

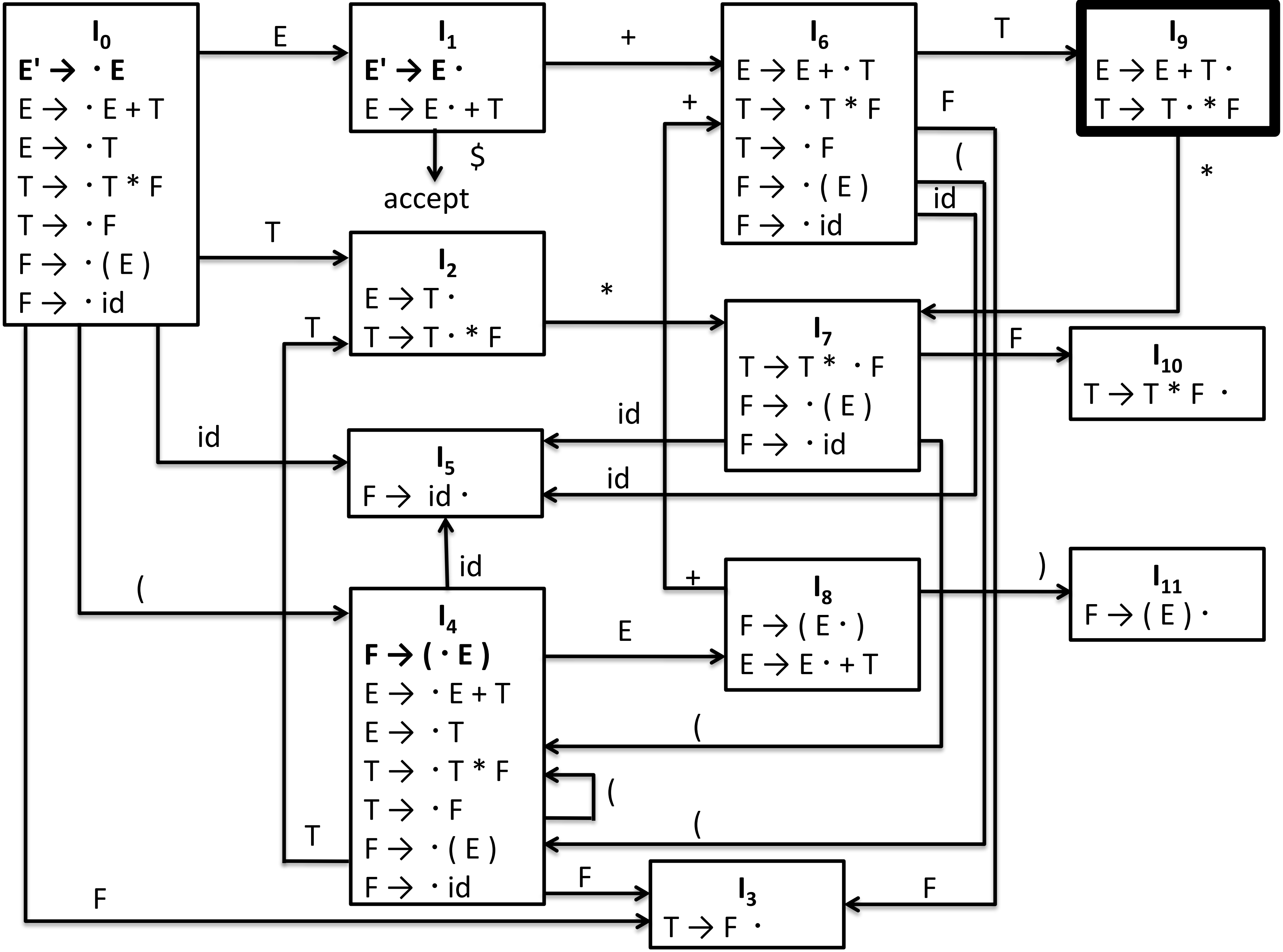
I₇
T → T * · F
F → · (E)
F → · id

I₈
F → (E ·)
E → E · + T

I₉
E → E + T ·
T → T · * F

I₁₀
T → T * F ·

I₁₁
F → (E) ·



I₀
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₁
E' → E ·
E → E · + T

I₂
E → T ·
T → T · * F

I₃
T → F ·

I₄
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₅
F → id ·

I₆
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

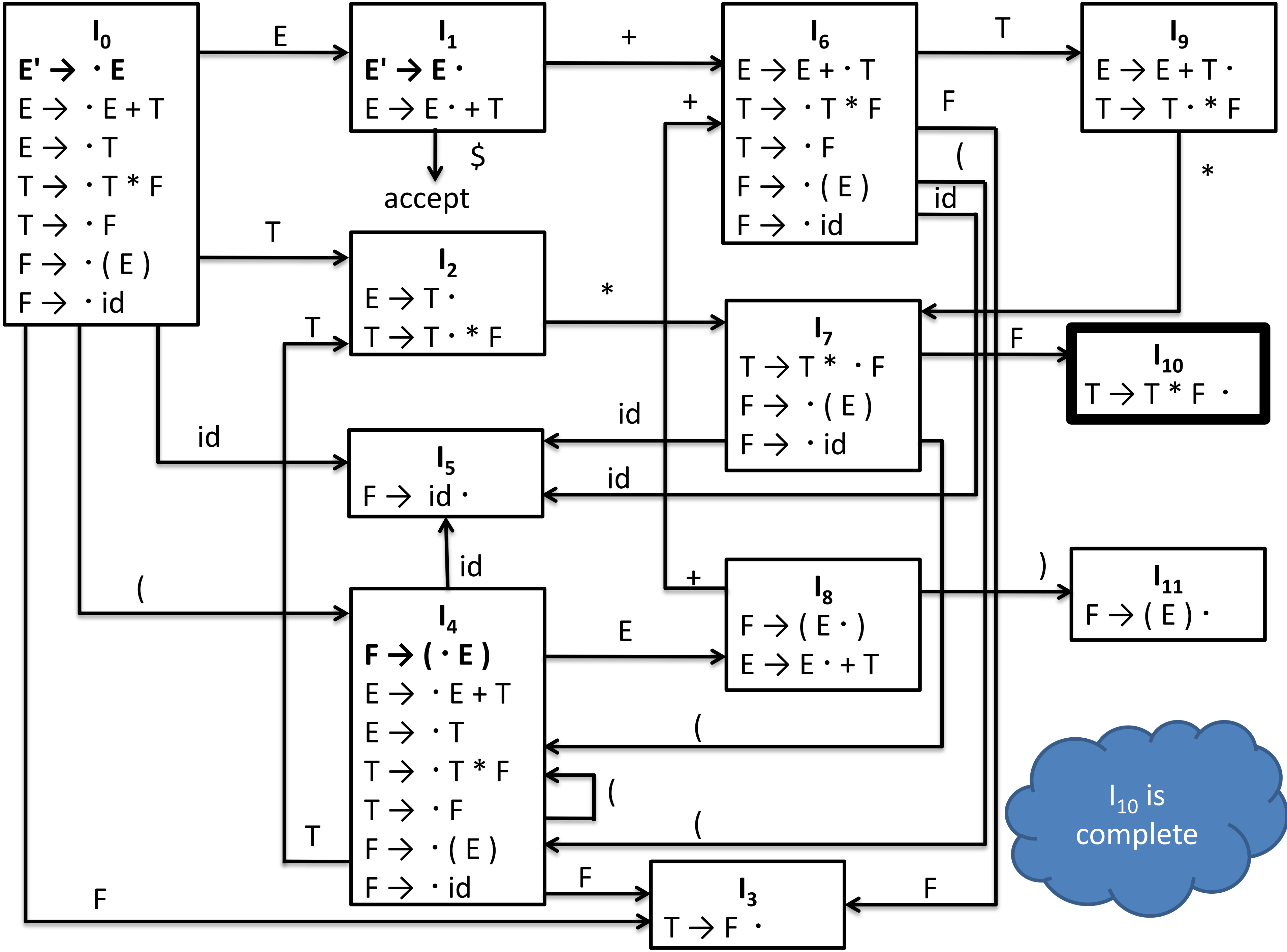
I₇
T → T * · F
F → · (E)
F → · id

I₈
F → (E ·)
E → E · + T

I₉
E → E + T ·
T → T · * F

I₁₀
T → T * F ·

I₁₁
F → (E) ·



I_0
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I_1
E' → E ·
E → E · + T

I_2
E → T ·
T → T · * F

I_3
T → F ·

I_4
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I_5
F → id ·

I_6
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I_7
T → T * · F
F → · (E)
F → · id

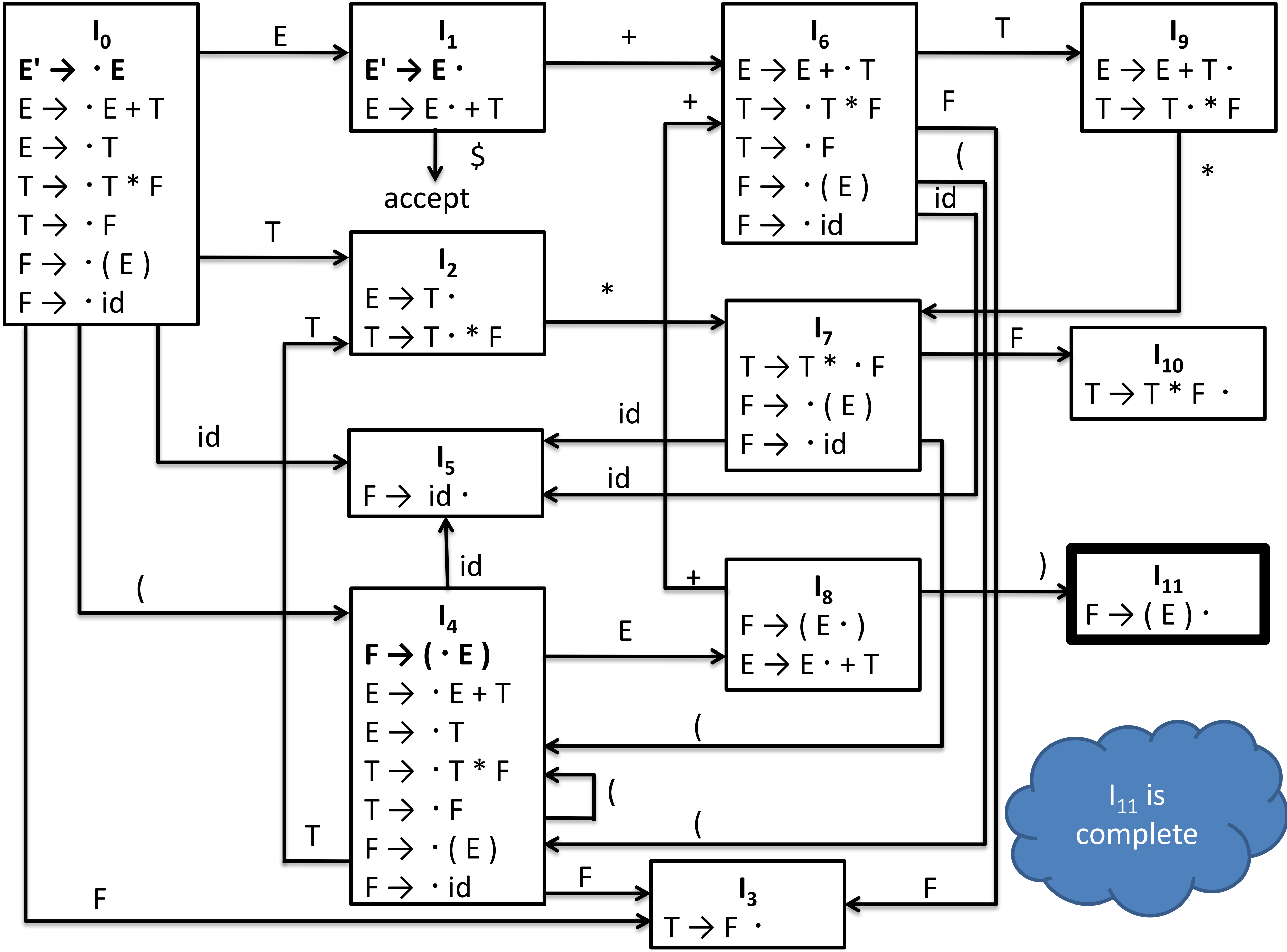
I_8
F → (E ·)
E → E · + T

I_9
E → E + T ·
T → T · * F

I_{10}
T → T * F ·

I_{11}
F → (E) ·

I_{10} is complete



I_0

E' → · E

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_1

E' → E ·

E → E · + T

I_2

E → T ·

T → T · * F

I_3

T → F ·

I_4

F → (· E)

E → · E + T

E → · T

T → · T * F

T → · F

F → · (E)

F → · id

I_5

F → id ·

I_6

E → E + · T

T → · T * F

T → · F

F → · (E)

F → · id

I_7

T → T * · F

F → · (E)

F → · id

I_8

F → (E ·)

E → E · + T

I_9

E → E + T ·

T → T · * F

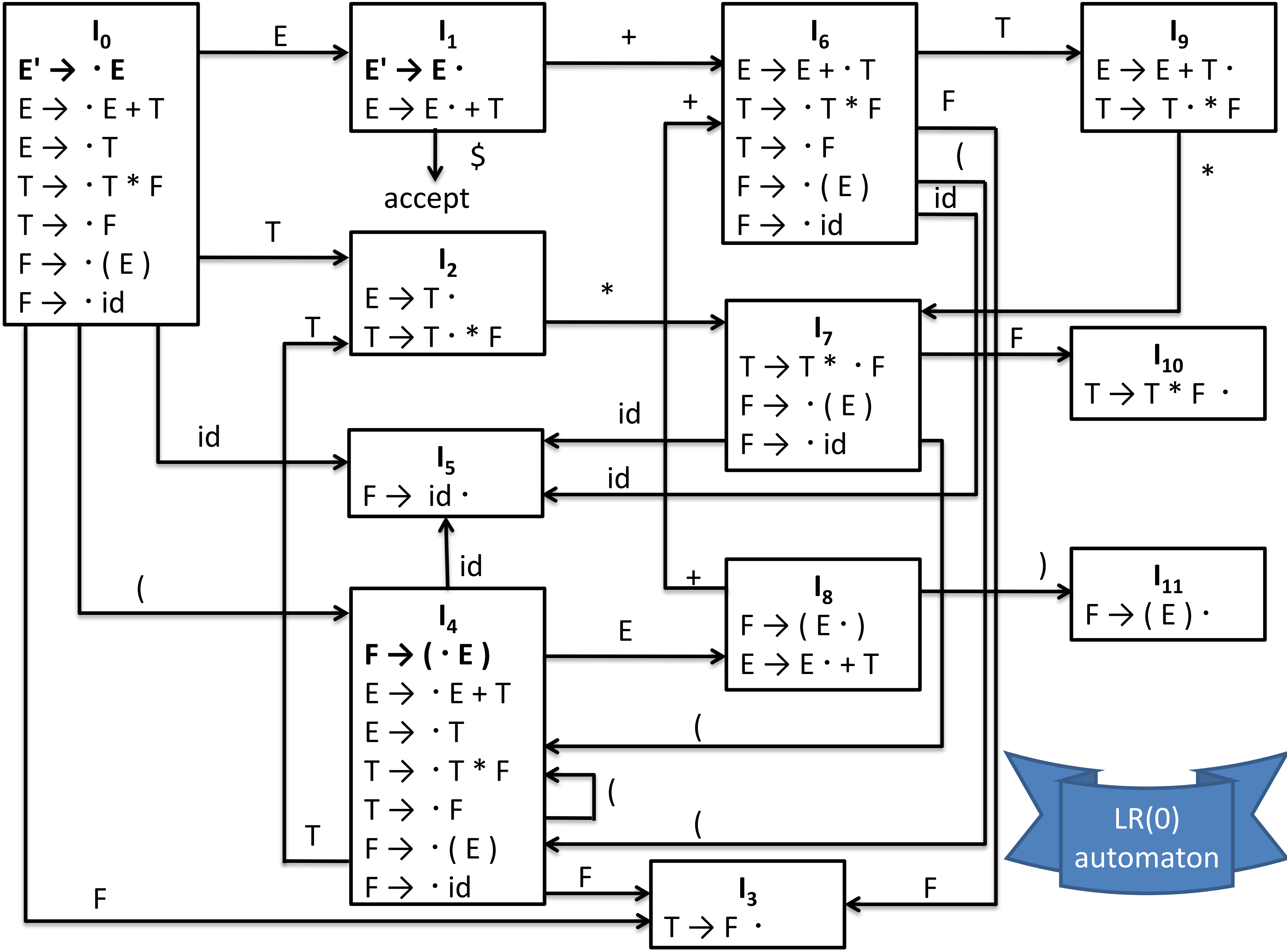
I_{10}

T → T * F ·

I_{11}

F → (E) ·

I_{11} is complete



I₀
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₁
E' → E ·
E → E · + T

accept

I₂
E → T ·
T → T · * F

I₃
T → F ·

I₄
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I₅
F → id ·

I₆
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I₇
T → T * · F
F → · (E)
F → · id

I₈
F → (E ·)
E → E · + T

I₉
E → E + T ·
T → T · * F

I₁₀
T → T * F ·

I₁₁
F → (E) ·

LR(0) automaton

Use of the LR(0) Automaton

- The central idea behind "Simple LR", or SLR, parsing is the construction from the grammar of the LR(0) automaton.
- The states of this automaton are the sets of items from the canonical LR(0) collection, and the transitions are given by the GOTO function.
- The start state of the LR(0) automaton is CLOSURE({[S' → ·S]}), where S' is the start symbol of the augmented grammar.
- All states are accepting states.
- We say "state j" to refer to the state corresponding to the set of items I_j .

How can LR(0) automata help with shift-reduce decisions?

- Shift-reduce decisions can be made as follows.
  - Suppose that the string of grammar symbols takes the LR(0) automaton from the start state 0 to some state j.
  - Then, **shift** on next input symbol a **if state j has a transition on a**.
  - Otherwise, we choose to **reduce**; **the items in state j will tell us which production to use**.

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	

The next input symbol is id and state 0 has a transition on id to state 5. We therefore shift.

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	reduce by F → id
(3)	0 3	$ F	* id $	

At line (2), state 5 (symbol id) has been pushed onto the stack. There is no transition from state 5 on input , so we reduce. From item [F → id ·] in state 5, the reduction is by production F → id.
With symbols, a reduction is implemented by popping the body of the production from the stack (on line (2), the body is id) and pushing the head of the production (in this case, F). With states, we pop state 5 for symbol id, which brings state 0 to the top and look for a transition on F , the head of the production. In LR(0) automaton, state 0 has a transition on F to state 3, so we push state 3, with corresponding symbol F ; see line (3).

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	reduce by F → id
(3)	0 3	$ F	* id $	reduce by T → F
(4)	0 2	$ T	* id $	

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	reduce by F → id
(3)	0 3	$ F	* id $	reduce by T → F
(4)	0 2	$ T	* id $	shift to 7
(5)	0 2 7	$ T *	id $	

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	reduce by F → id
(3)	0 3	$ F	* id $	reduce by T → F
(4)	0 2	$ T	* id $	shift to 7
(5)	0 2 7	$ T *	id $	shift to 5
(6)	0 2 7 5	$ T * id	$	

consider line (5), with state 7 (symbol) on top of the stack. This state has a transition to state 5 on input id, so we push state 5 symbol id). State 5 has no transitions, so we reduce by F → id. When we pop state 5 for the body id, state 7 comes to the top of the stack. Since state 7 as a transition on F to state 10, we push state 10 (symbol F).

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	reduce by F → id
(3)	0 3	$ F	* id $	reduce by T → F
(4)	0 2	$ T	* id $	shift to 7
(5)	0 2 7	$ T *	id $	shift to 5
(6)	0 2 7 5	$ T * id	$	reduce by F → id
(7)	0 2 7 5 10	$ T * F	$	

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	reduce by F → id
(3)	0 3	$ F	* id $	reduce by T → F
(4)	0 2	$ T	* id $	shift to 7
(5)	0 2 7	$ T *	id $	shift to 5
(6)	0 2 7 5	$ T * id	$	reduce by F → id
(7)	0 2 7 5 10	$ T * F	$	reduce by T → T * F
(8)	0 2	$ T	$	

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	reduce by F → id
(3)	0 3	$ F	* id $	reduce by T → F
(4)	0 2	$ T	* id $	shift to 7
(5)	0 2 7	$ T *	id $	shift to 5
(6)	0 2 7 5	$ T * id	$	reduce by F → id
(7)	0 2 7 5 10	$ T * F	$	reduce by T → T * F
(8)	0 2	$ T	$	reduce by E → T
(9)	0 1	$ E	$	

The actions of a shift-reduce parser on input **id * id**, using the LR(0) automaton

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$	id * id $	shift to 5
(2)	0 5	$ id	* id $	reduce by F → id
(3)	0 3	$ F	* id $	reduce by T → F
(4)	0 2	$ T	* id $	shift to 7
(5)	0 2 7	$ T *	id $	shift to 5
(6)	0 2 7 5	$ T * id	$	reduce by F → id
(7)	0 2 7 5 10	$ T * F	$	reduce by T → T * F
(8)	0 2	$ T	$	reduce by E → T
(9)	0 1	$ E	$	accept

LR-Parsing

- The driver program is the same for all LR parsers; only **the parsing table changes from one parser to another**.
- The parsing program reads characters from an input buffer one at a time.
- **Where a shift-reduce parser would shift a symbol, an LR parser shifts a state.**
- Each state summarizes the information contained in the stack below it. The stack holds a sequence of states, $s_0 s_1 \ldots s_m$, where s_m is on top.
- **In the SLR method, the stack holds states from the LR(0) automaton.**

Structure of LR Parsing Table

The parsing table consists of two parts: a parsing-action function ACTION and a goto function GOTO.

1. The ACTION function takes as arguments a state i and a terminal a (or $, the input endmarker). The value of ACTION[i, a] can have one of four forms:
   a) Shift j, where j is a state. The action taken by the parser effectively shifts input a to the stack, but uses state j to represent a.
   b) Reduce A → β . The action of the parser effectively reduces on the top of the stack to head A.
   c) Accept. The parser accepts the input and finishes parsing.
   d) Error. The parser discovers an error in its input and takes some corrective action.
2. We extend the GOTO function, defined on sets of items, to states: if GOTO[I_i , A] = I_j , then GOTO also maps a state i and a nonterminal A to state j.

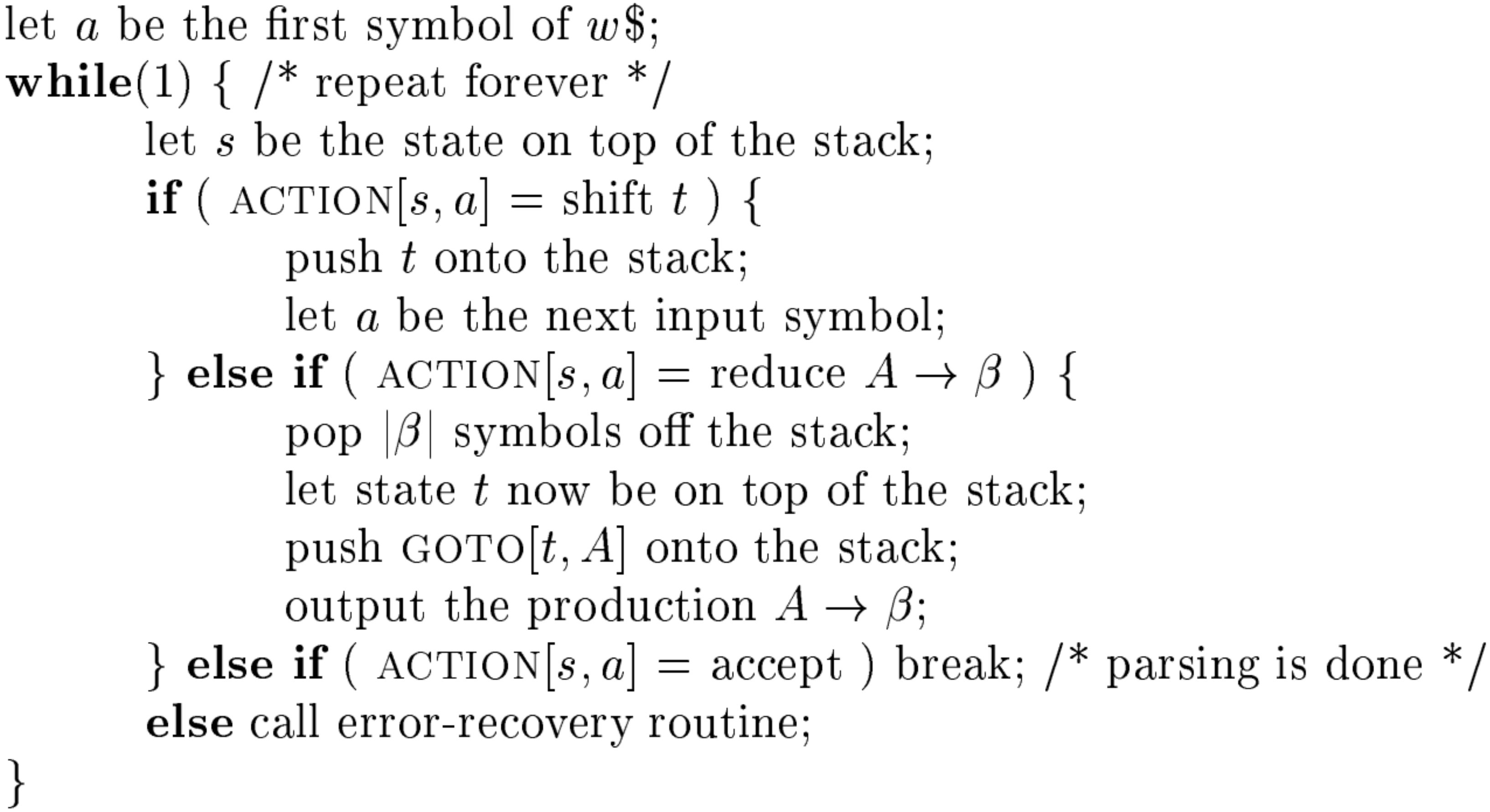
LR Parsing Algorithm

INPUT: An input string w and an LR-parsing table with functions ACTION and GOTO for a grammar.

OUTPUT: If w is in L(G), the reduction steps of a bottom-up parse for w_i otherwise, an error indication.

METHOD: Initially, the parser has s_0 on its stack, where s_0 is the initial state, and w$ in the input buffer.

LR Parsing Algorithm



```
let $a$ be the first symbol of $w\$$;
**while**(1) { /* repeat forever */
    let $s$ be the state on top of the stack;
    **if** ( ACTION$[s, a]$ = shift $t$ ) {
        push $t$ onto the stack;
        let $a$ be the next input symbol;
    } **else if** ( ACTION$[s, a]$ = reduce $A \rightarrow \beta$ ) {
        pop $|\beta|$ symbols off the stack;
        let state $t$ now be on top of the stack;
        push GOTO$[t, A]$ onto the stack;
        output the production $A \rightarrow \beta$;
    } **else if** ( ACTION$[s, a]$ = accept ) break; /* parsing is done */
    **else** call error-recovery routine;
}
```

Constructing an SLR-parsing table

INPUT: An augmented grammar G'.

OUTPUT: The SLR-parsing table functions ACTION and GOTO for G'.

METHOD:

1. Construct C ={ I_0, I_1, …, I_n}, the collection of sets of LR(0) items for G'.
2. State i is constructed from I_i. The parsing actions for state i are determined as follows:
   a) If [A → α·aβ] is in I_i and GOTO(I_i, a) = I_j , then set ACTION[i, a] to "shift j." Here a must be a terminal.
   b) If [A → α·] is in I_i , then set ACTION[i, a] to "reduce A → α" for all a in FOLLOW(A); here A may not be S'.
   c) If [S → S'·] is in I_i , then set ACTION[i, $] to "accept."

   If any conflicting actions result from the above rules, we say the grammar is not SLR(1). The algorithm fails to produce a parser in this case.
3. The goto transitions for state i are constructed for all nonterminals A using the rule: If GOTO(I_i , A) = I_j , then GOTO[i, A] = j.
4. All entries not defined by rules (2) and (3) are made "error."
5. 5. The initial state of the parser is the one constructed from the set of items containing [S → S'·].

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0									
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0							1	2	3
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0							1	2	3
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0							1	2	3
1									
2									
3									
4							8	2	3
5									
6									
7									
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0							1	2	3
1									
2									
3									
4							8	2	3
5									
6									
7									
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0							1	2	3
1									
2									
3									
4							8	2	3
5									
6								9	3
7									
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0							1	2	3
1									
2									
3									
4							8	2	3
5									
6								9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0							1	2	3
1									
2									
3									
4							8	2	3
5									
6								9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1									
2									
3									
4							8	2	3
5									
6								9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2									
3									
4							8	2	3
5									
6								9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4							8	2	3
5									
6								9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4							8	2	3
5									
6								9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4	s5			s4			8	2	3
5									
6								9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4	s5			s4			8	2	3
5									
6								9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4	s5			s4			8	2	3
5									
6	s5			s4				9	3
7									10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4	s5			s4			8	2	3
5									
6	s5			s4				9	3
7	s5			s4					10
8									
9									
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4	s5			s4			8	2	3
5									
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9									
10									
11									

LR(0) Automaton

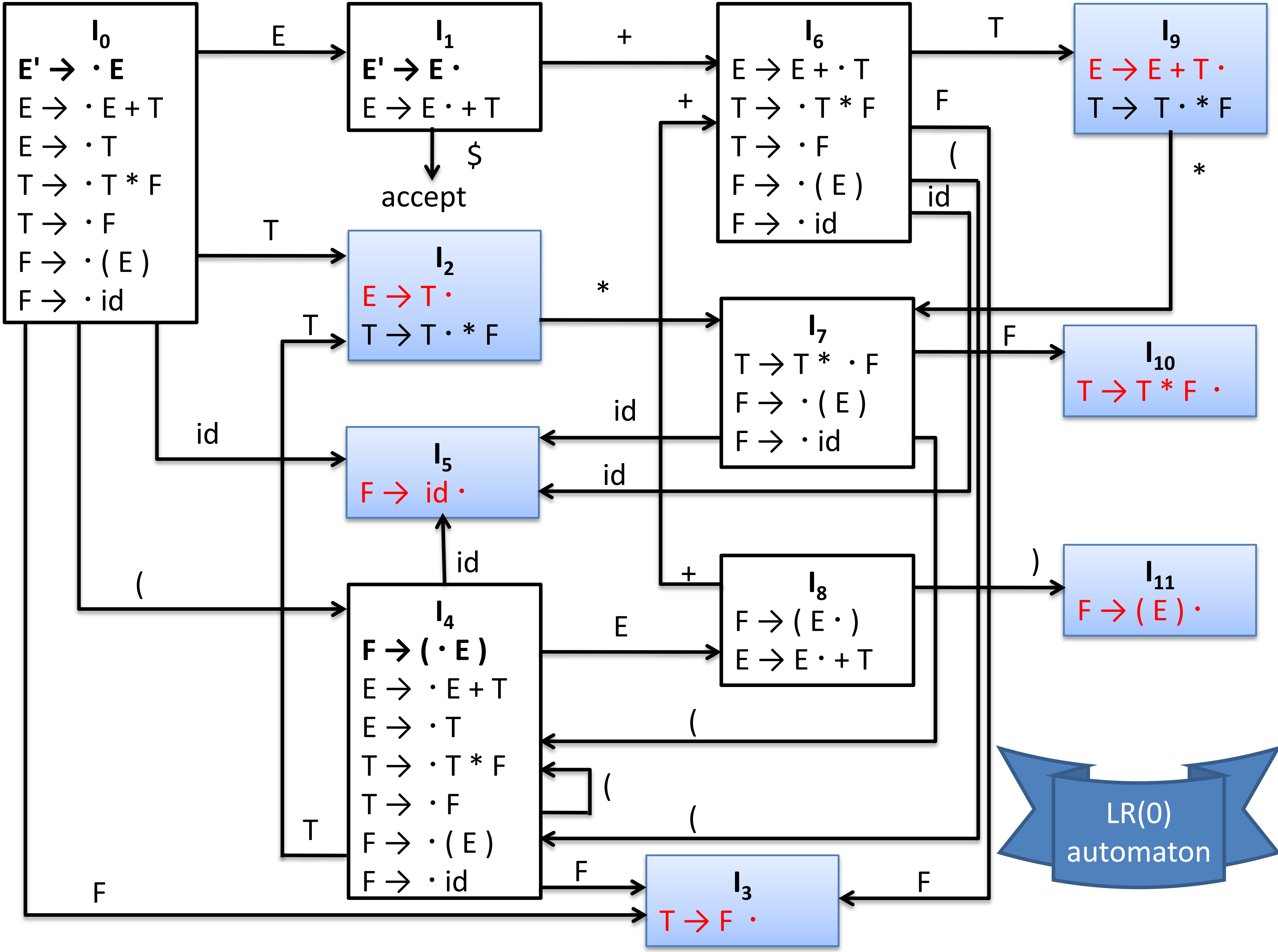
Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4	s5			s4			8	2	3
5									
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9			s7						
10									
11									

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4	s5			s4			8	2	3
5									
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9			s7						
10									
11									



I_0
E' → · E
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I_1
E' → E ·
E → E · + T

I_2
E → T ·
T → T · * F

I_3
T → F ·

I_4
F → (· E)
E → · E + T
E → · T
T → · T * F
T → · F
F → · (E)
F → · id

I_5
F → id ·

I_6
E → E + · T
T → · T * F
T → · F
F → · (E)
F → · id

I_7
T → T * · F
F → · (E)
F → · id

I_8
F → (E ·)
E → E · + T

I_9
E → E + T ·
T → T · * F

I_{10}
T → T * F ·

I_{11}
F → (E) ·

LR(0)
automaton

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2			s7						
3									
4	s5			s4			8	2	3
5									
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9			s7						
10									
11									

I_2
E → T ·
T → T · * F

FOLLOW(E) = {+,), $}
FOLLOW(T) = {*, +,), $}
FOLLOW(F) = {*, +,), $}

1. E → E + T
2. E → T
3. T → T * F
4. T → F
5. F → (E)
6. F → id

LR(0) Automaton

Parsing table for e on grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3									
4	s5			s4			8	2	3
5									
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9			s7						
10									
11									

I_3

T → F ·

FOLLOW(E) = {+,), $}
FOLLOW(T) = {*, +,), $}
FOLLOW(F) = {*, +,), $}

1. E → E + T
2. E → T
3. T → T * F
4. T → F
5. F → (E)
6. F → id

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5									
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9			s7						
10									
11									

I_5
F → id ·

FOLLOW(E) = {+,), $}
FOLLOW(T) = {*, +,), $}
FOLLOW(F) = {*, +,), $}

1. E → E + T
2. E → T
3. T → T * F
4. T → F
5. F → (E)
6. F → id

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9			s7						
10									
11									

I$_9$

E → E + T ·

T → T · * F

FOLLOW(E) = {+,), $}

FOLLOW(T) = {*, +,), $}

FOLLOW(F) = {*, +,), $}

1. E → E + T
2. E → T
3. T → T * F
4. T → F
5. F → (E)
6. F → id

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10									
11									

I_{10}

T → T * F ·

FOLLOW(E) = {+,), $}

FOLLOW(T) = {*, +,), $}

FOLLOW(F) = {*, +,), $}

1. E → E + T
2. E → T
3. T → T * F
4. T → F
5. F → (E)
6. F → id

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11									

I_{11}

F → (E) ·

FOLLOW(E) = {+,), $}
FOLLOW(T) = {*, +,), $}
FOLLOW(F) = {*, +,), $}

Parsing table for expression grammar

1. E → E + T
2. E → T
3. T → T * F
4. T → F
5. F → (E)
6. F → id

LR(0) Automaton

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

LR(0) Automaton

Parsing table for expression grammar

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Moves of an LR parser on id * id + id

1. E → E + T
2. E → T
3. T → T * F
4. T → F
5. F → (E)
6. F → id

	STACK	SYMBOLS	INPUT	ACTION
1	0		id * id + id $	Shift (s5)
2	0 5	id	* id + id $	reduce by F → id (r6)
3	0 3 (0 → 3 has label F)	F	* id + id $	reduce by T → F (r4)
4	0 2 (0 → 2 has label T)	T	* id + id $	Shift (s7)
5	0 2 7	T *	id + id $	Shift (s5)
6	0 2 7 5	T * id	+ id $	reduce by F → id (r6)
7	0 2 7 10 (7 → 10 has label F)	T * F	+ id $	reduce by T → T * F (r3)
8	0 2 (0 → 2 has label T)	T	+ id $	reduce by E → T (r2)
9	0 1 (0 → 1 has label E)	E	+ id $	Shift (s6)
10	0 1 6	E +	id $	Shift (s5)
11	0 1 6 5	E + id	$	reduce by F → id (r6)
12	0 1 6 3 (6 → 3 has label F)	E + F	$	reduce by T → F (r4)
13	0 1 6 9 (6 → 9 has label T)	E + T	$	reduce by E → E + T (r1)
14	0 1 (0 → 1 has label E)	E	$	accept

Every SLR(1) grammar is unambiguous, but there are many unambiguous grammars that are not SLR(1).

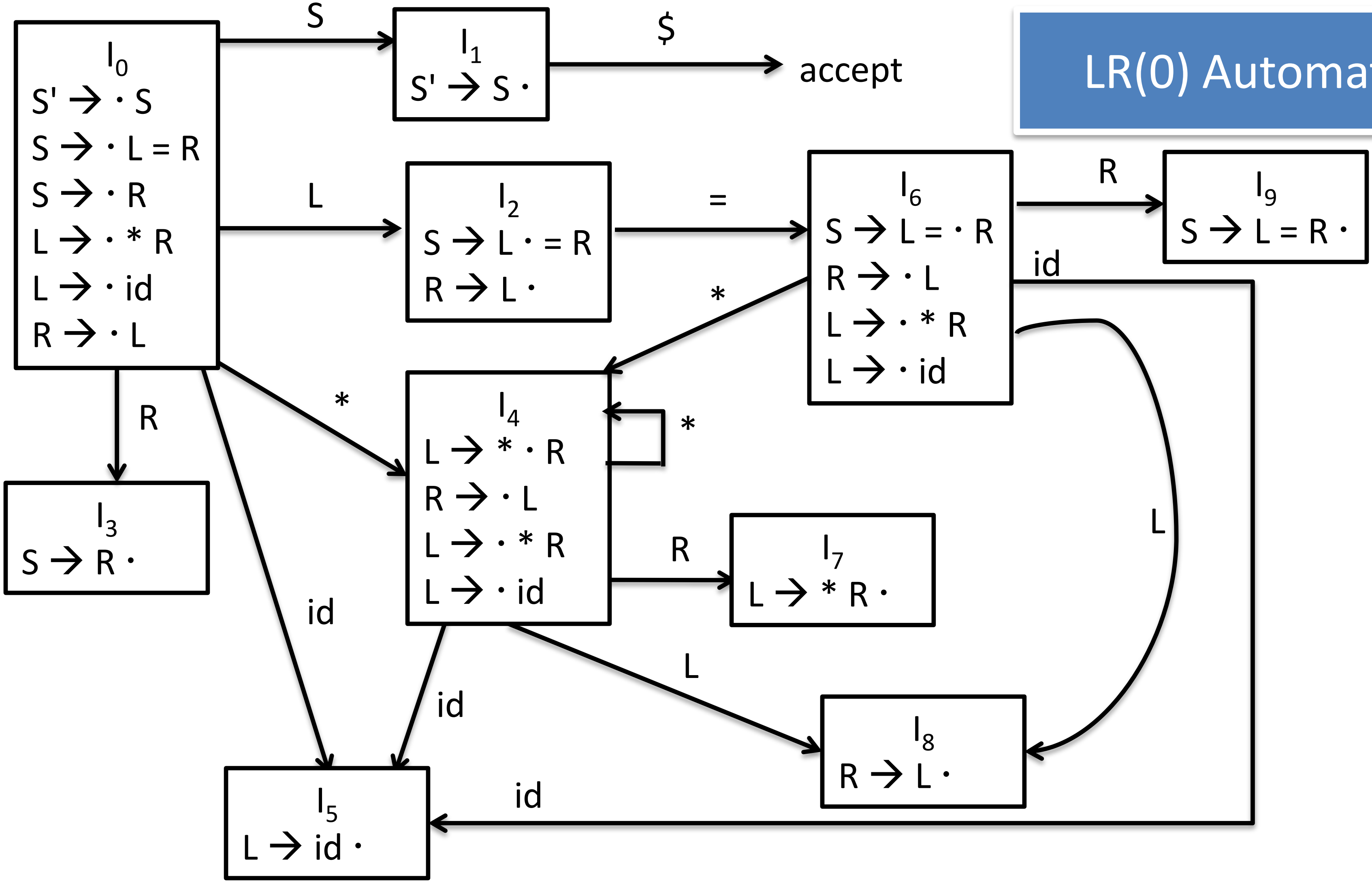
- Consider the grammar with productions

  S → L = R | R

  L → * R | id

  R → L
- Find the Canonical LR(0) collection for grammar.
- Draw LR(0) automation
- Draw parse table with ACTION and GOTO

LR(0) Automaton



I_0
S' → · S
S → · L = R
S → · R
L → · * R
L → · id
R → · L

I_1
S' → S ·

I_2
S → L · = R
R → L ·

I_3
S → R ·

I_4
L → * · R
R → · L
L → · * R
L → · id

I_5
L → id ·

I_6
S → L = · R
R → · L
L → · * R
L → · id

I_7
L → * R ·

I_8
R → L ·

I_9
S → L = R ·

Parse Table

STATE	ACTION				GOTO		
	id	=	*	$	S	L	R
0							
1							
2							
3							
4							
5							
6							
7							
8							
9							

Parse Table (filling GOTO and shift)

STATE	ACTION				GOTO		
	id	=	*	$	S	L	R
0	s5		s4		1	2	3
1				acc			
2		s6					
3							
4	s5		s4			8	7
5							
6	s5		s4			8	9
7							
8							
9							

FOLLOW(S) = {$}
FOLLOW(L) = {=, $}
FOLLOW(R) = {=, $}

Parse Table (filling reduce)

STATE	ACTION				GOTO		
	id	=	*	$	S	L	R
0	s5		s4		1	2	3
1				acc			
2		s6 / r5					
3				r2			
4	s5		s4			8	7
5		r4		r4			
6	s5		s4			8	9
7		r3		r3			
8		r5		r5			
9				r1			

1.S → L = R
2.S → R
3.L → * R
4.L → id
5.R → L

Parse Table

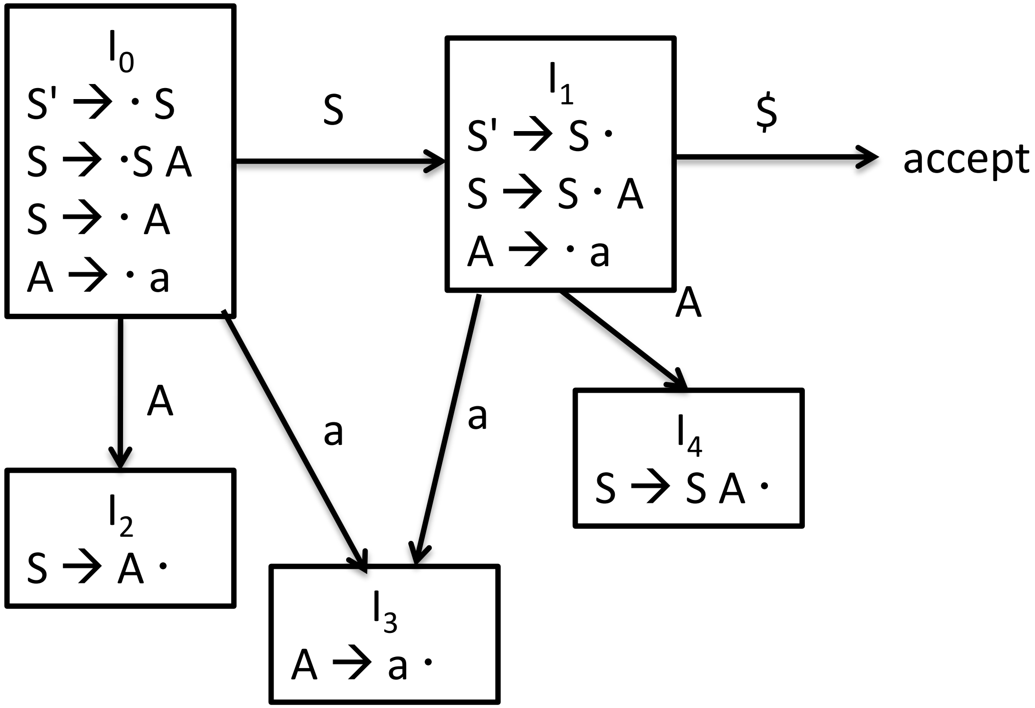
The grammar is not SLR(1) grammar.

STATE	ACTION				GOTO		
	id	=	*	$	S	L	R
0	s5		s4		1	2	3
1				acc			
2		s6 / r5					
3				r2			
4	s5		s4			8	7
5		r4		r4			
6	s5		s4			8	9
7		r3		r3			
8		r5		r5			
9				r1			

Show that the given grammar is SLR(1) but not LL(1)
S → SA | A
A → a

- For a grammar to be LL(1), it must not be left recursive or ambiguous.
  - But the given grammar is left recursive so not LL(1). S → SA
- For a grammar to be SLR(1), construct LR(0) automaton and parsing table. And there must be no conflict in any entry.

LR(0) Automaton



I_0

S' → · S

S → ·S A

S → · A

A → · a

I_1

S' → S ·

S → S · A

A → · a

I_2

S → A ·

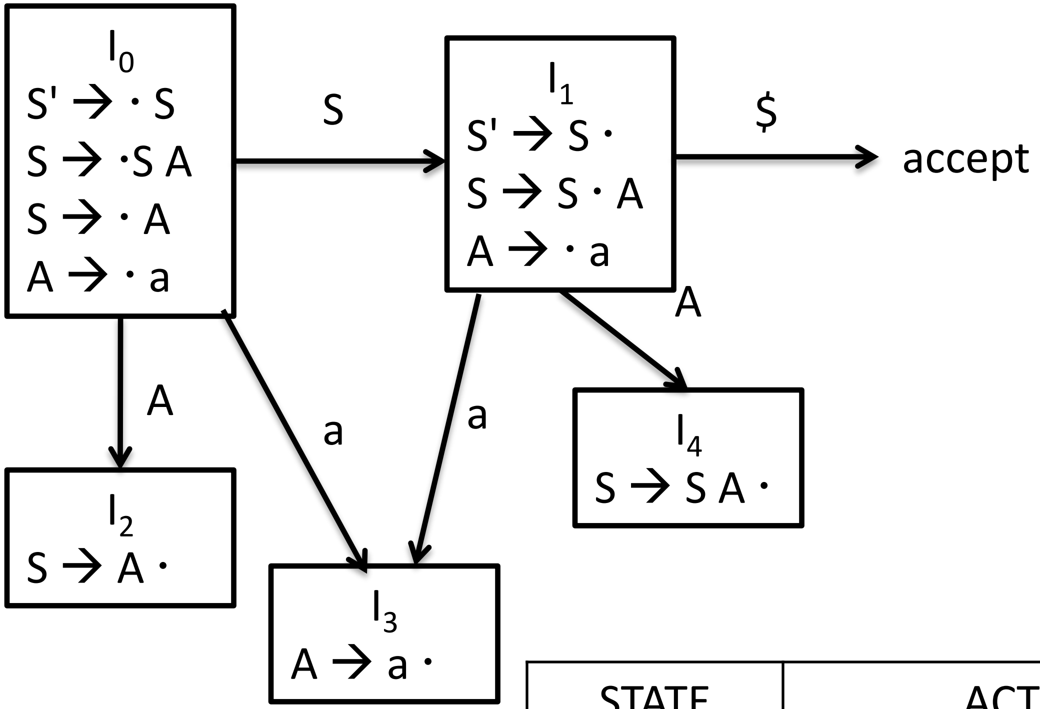
I_3

A → a ·

I_4

S → S A ·

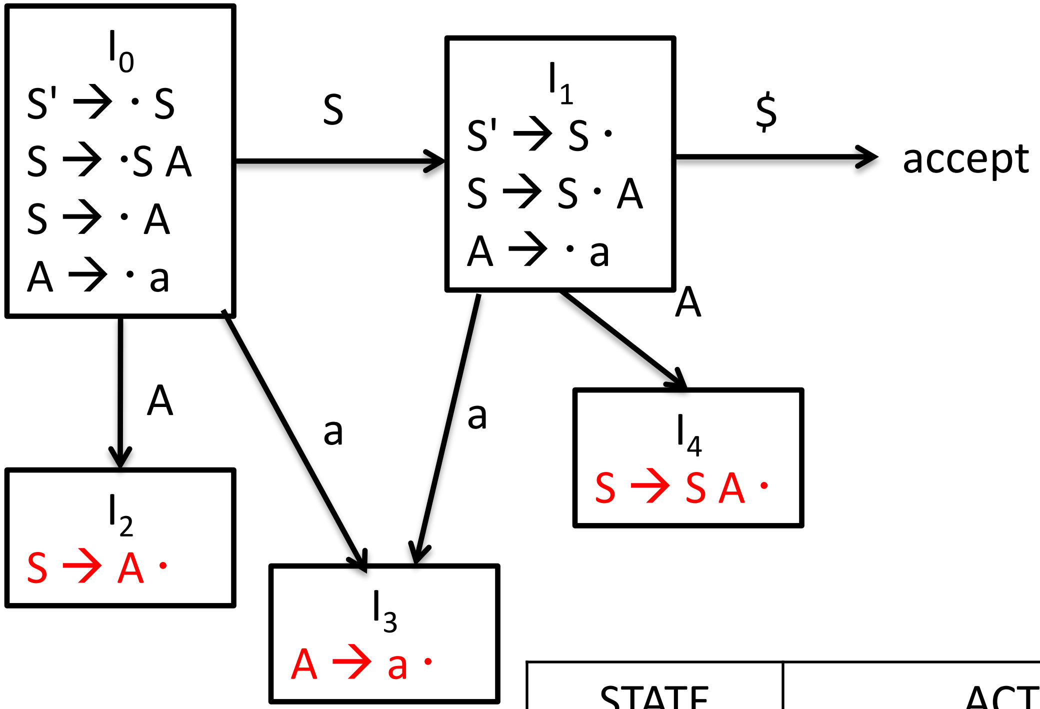
LR(0) Automaton



Filling shift and goto entries

STATE	ACTION		GOTO	
	a	$	S	A
0	s3		1	2
1	s3	acc		4
2				
3				
4				

LR(0) Automaton



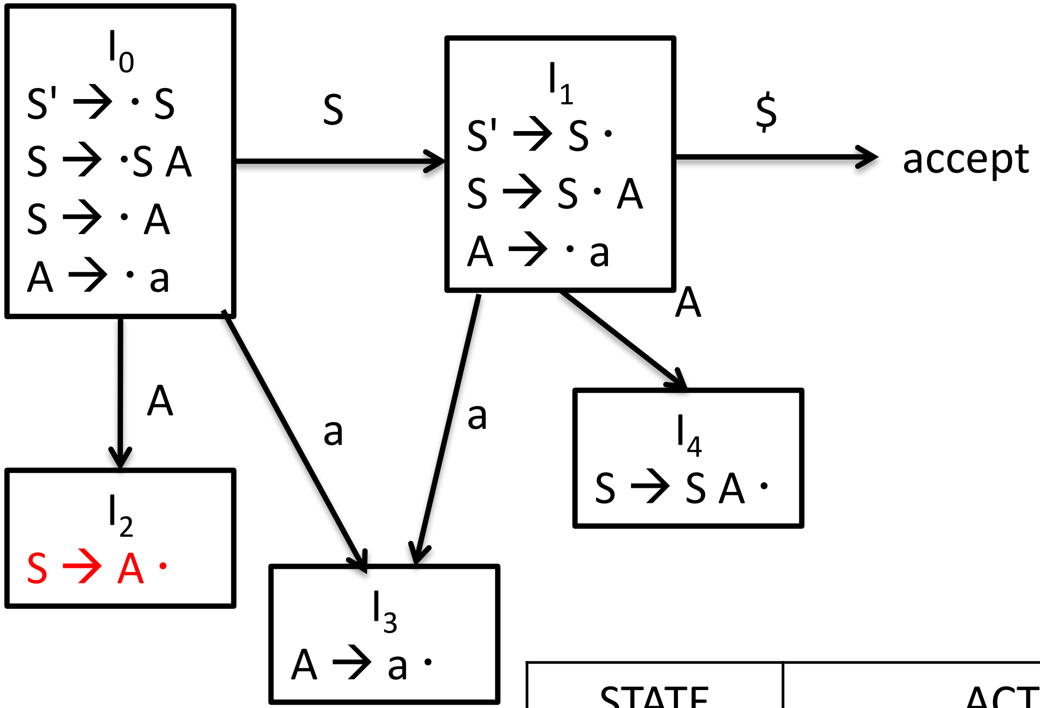
1. S → S A
2. S → A
3. A →a

FOLLOW(S) = {$} U FIRST(A) = {$, a}
FOLLOW(A) = FOLLOW(S) = {$, a}

Filling reduce entries

STATE	ACTION		GOTO	
	a	$	S	A
0	s3		1	2
1	s3	acc		4
2				
3				
4				

LR(0) Automaton



I_0
S' → · S
S → ·S A
S → · A
A → · a

I_1
S' → S ·
S → S · A
A → · a

I_2
S → A ·

I_3
A → a ·

I_4
S → S A ·

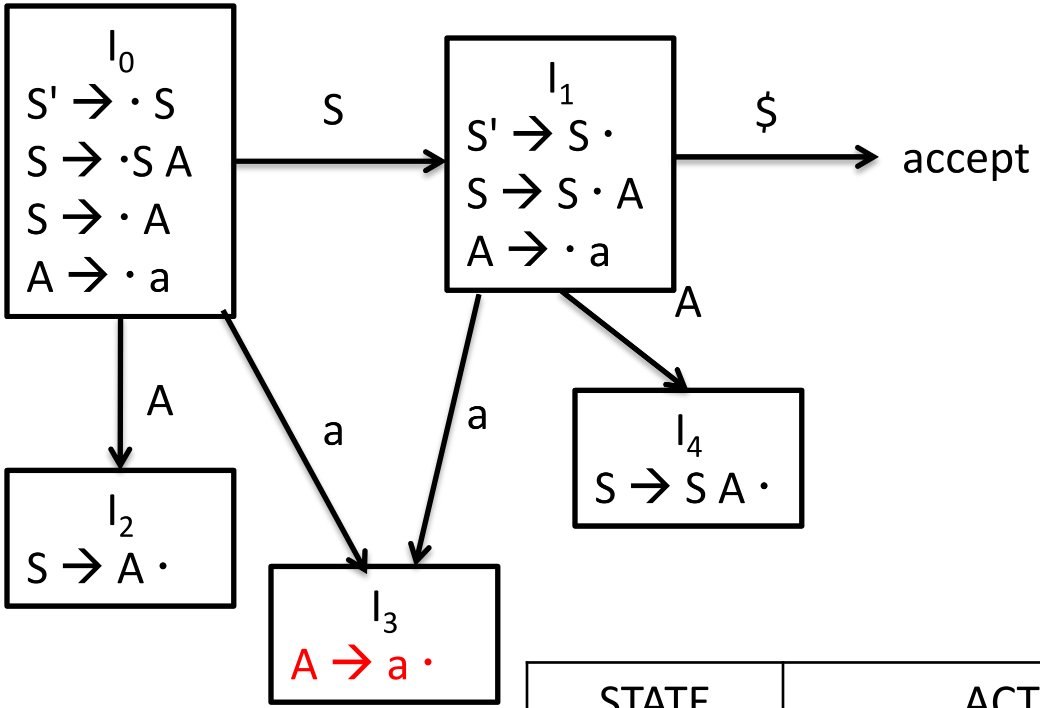
1. S → S A
2. S → A
3. A →a

FOLLOW(S) = {$} U FIRST(A) = {$, a}
FOLLOW(A) = FOLLOW(S) = {$, a}

Filling reduce entries

STATE	ACTION		GOTO	
	a	$	S	A
0	s3		1	2
1	s3	acc		4
2	r2	r2		
3				
4				

LR(0) Automaton



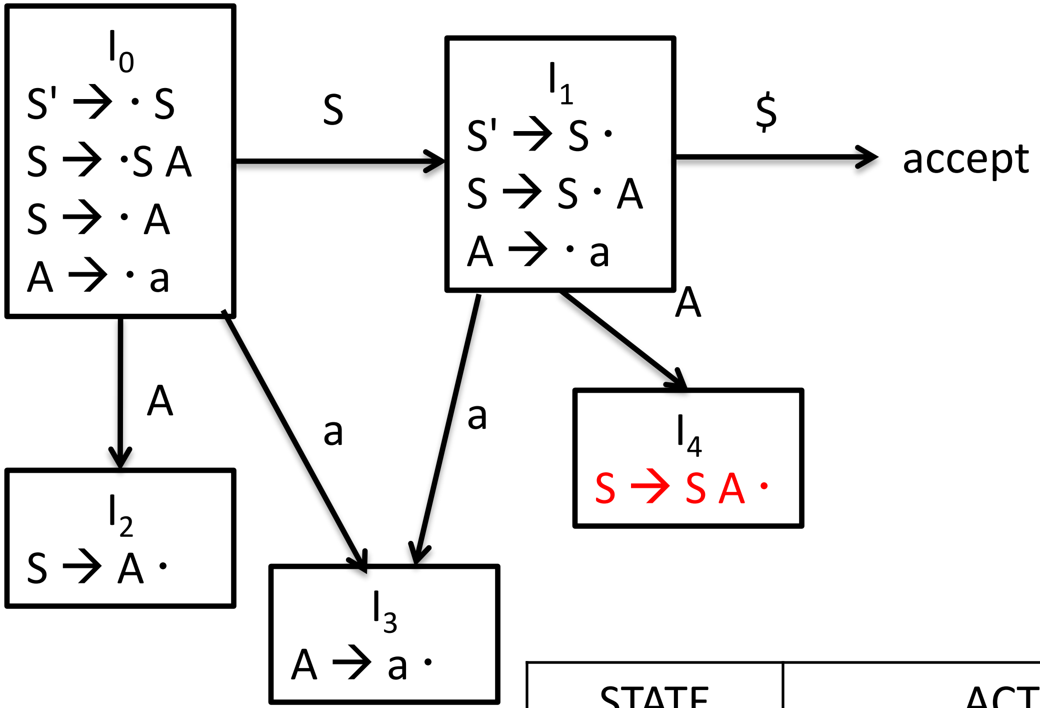
1. S → S A
2. S → A
3. A →a

FOLLOW(S) = {$} U FIRST(A) = {$, a}
FOLLOW(A) = FOLLOW(S) = {$, a}

Filling reduce entries

STATE	ACTION		GOTO	
	a	$	S	A
0	s3		1	2
1	s3	acc		4
2	r2	r2		
3	r3	r3		
4				

LR(0) Automaton



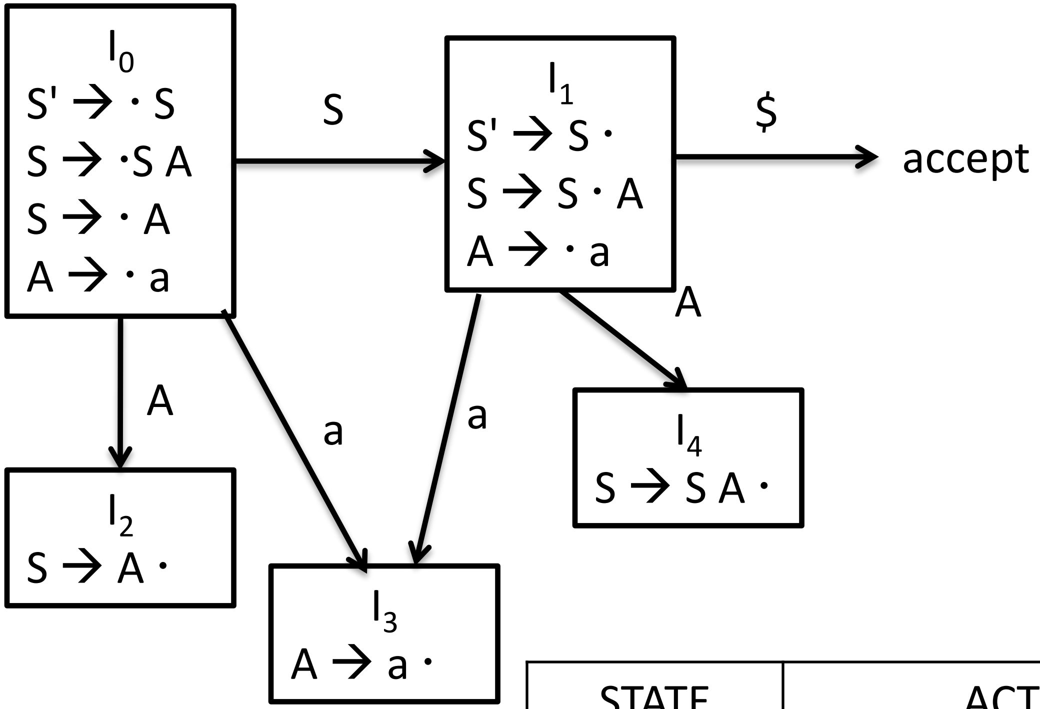
1. S → S A
2. S → A
3. A → a

FOLLOW(S) = {$} U FIRST(A) = {$, a}
FOLLOW(A) = FOLLOW(S) = {$, a}

Filling reduce entries

STATE	ACTION		GOTO	
	a	$	S	A
0	s3		1	2
1	s3	acc		4
2	r2	r2		
3	r3	r3		
4	r1	r1		

LR(0) Automaton



I_0
S' → · S
S → ·S A
S → · A
A → · a

I_1
S' → S ·
S → S · A
A → · a

I_2
S → A ·

I_3
A → a ·

I_4
S → S A ·

As there is no conflict so the grammar is SLR(1).

STATE	ACTION		GOTO	
	a	$	S	A
0	s3		1	2
1	s3	acc		4
2	r2	r2		
3	r3	r3		
4	r1	r1		

Show that the given grammar is LL(1) but not SLR(1)
S → AaAb | BbBa
A → ∈
B → ∈

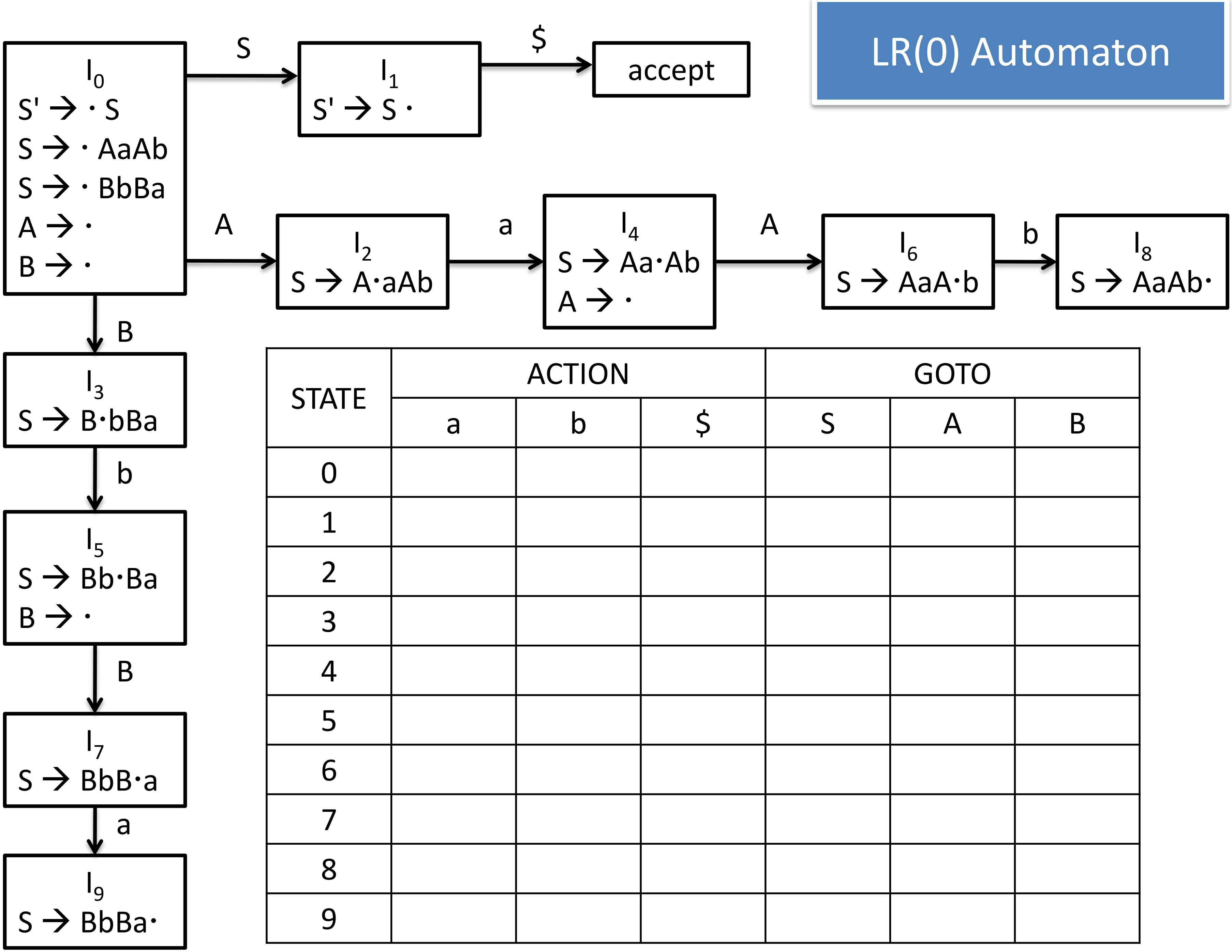
- To check if grammar is LL(1),

  FIRST(AaAb) ∩ FIRST(BbBa) = {a} ∩ {b} = ∅

- Hence, **it is LL(1) grammar**. (this is first condition and no need to check second condition).
- For a grammar to be SLR(1), construct LR(0) automaton and parsing table. And there must be no conflict in any entry.

In A → a | b
If FIRST(a) ∩ FIRST (b)!= ∅ : not LL(1)
If FIRST (a) ∩ FIRST(b) = ∅,
then FIRST (a) = ∈ **OR** FOLLOW(A) ∩ FIRST (b)== ∅ : then LL(1).

LR(0) Automaton



I$_0$
S' → · S
S → · AaAb
S → · BbBa
A → ·
B → ·

S

I$_1$
S' → S ·

$

accept

A

I$_2$
S → A·aAb

a

I$_4$
S → Aa·Ab
A → ·

A

I$_6$
S → AaA·b

b

I$_8$
S → AaAb·

B

I$_3$
S → B·bBa

b

I$_5$
S → Bb·Ba
B → ·

B

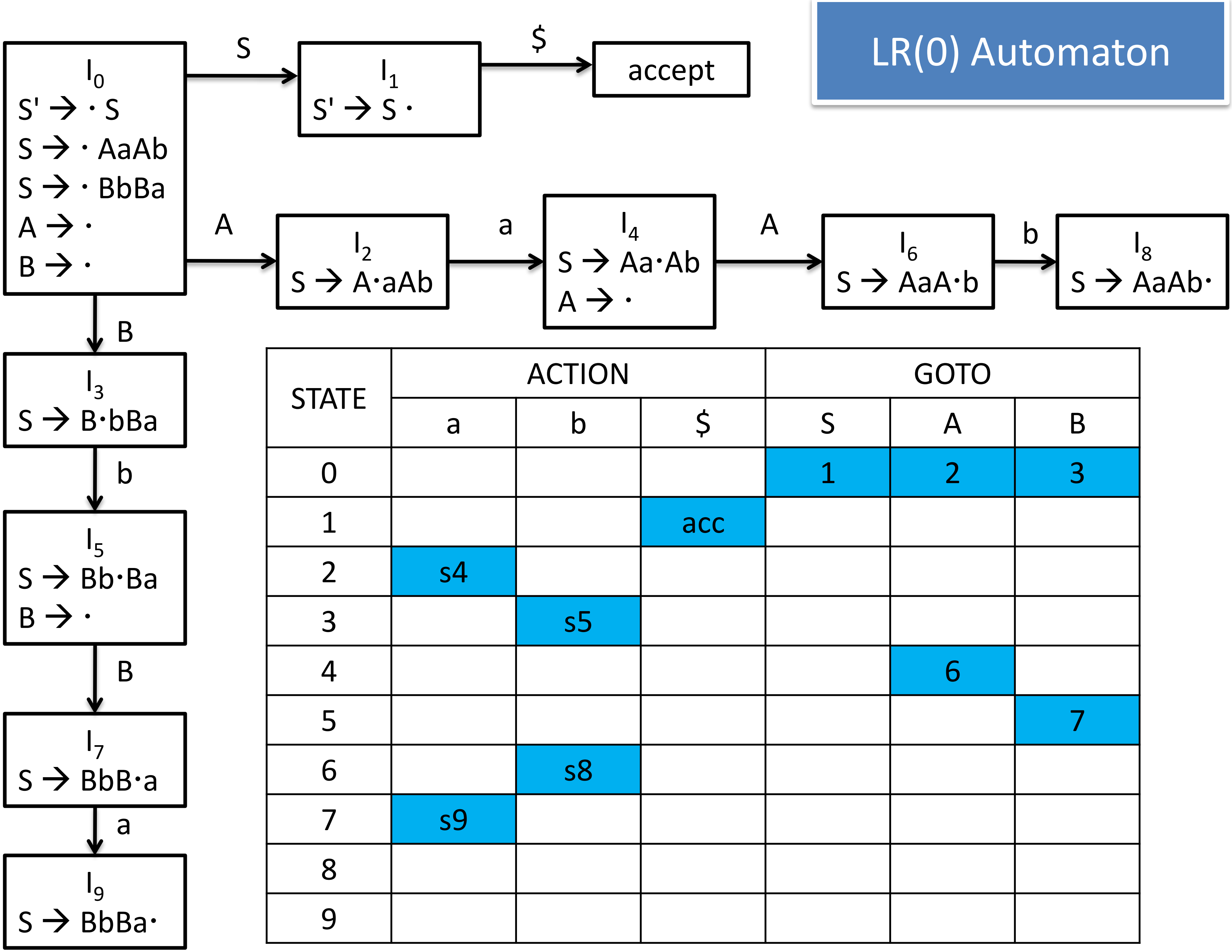
I$_7$
S → BbB·a

a

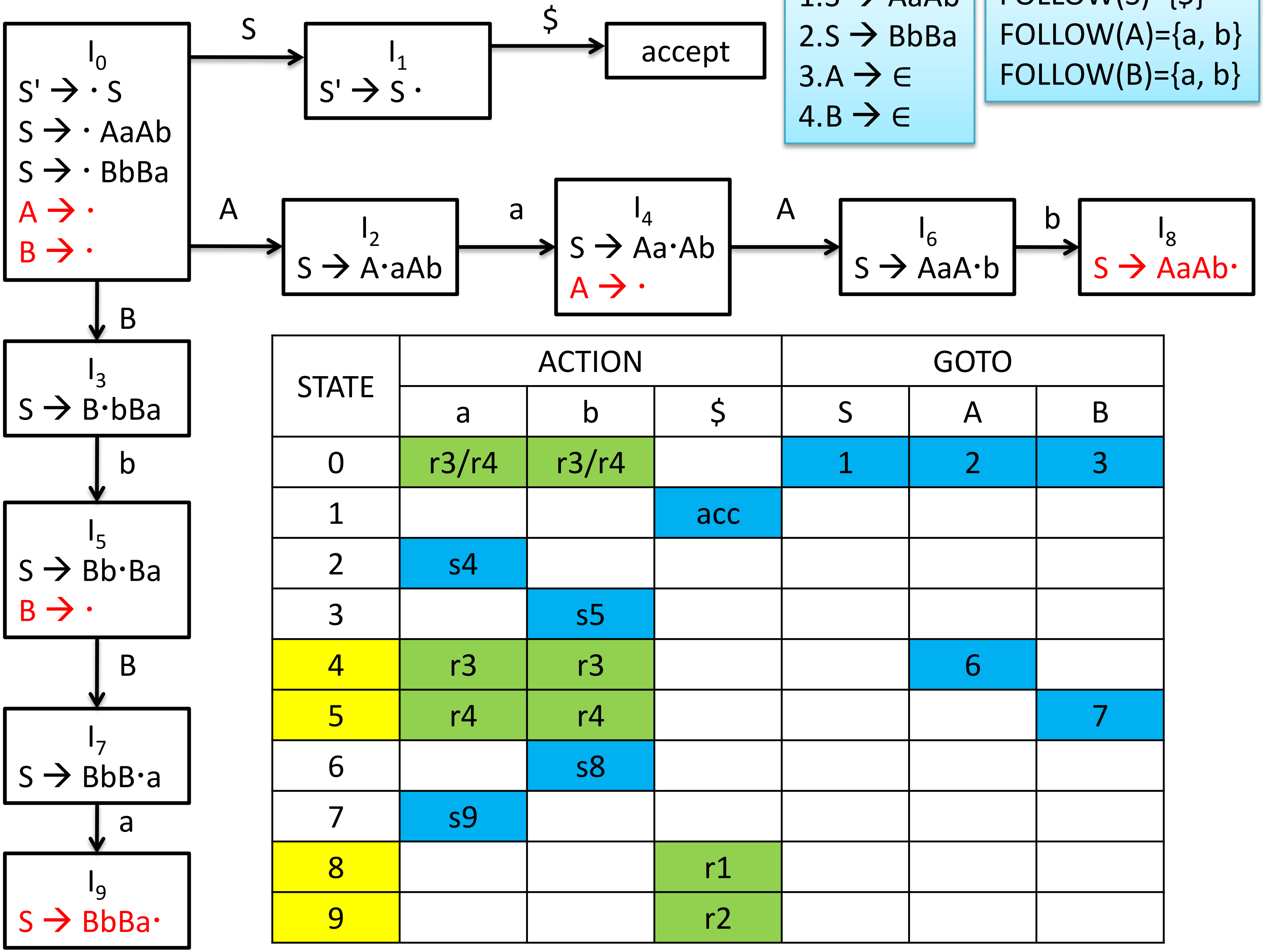
I$_9$
S → BbBa·

STATE	ACTION			GOTO		
	a	b	$	S	A	B
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						

LR(0) Automaton



STATE	ACTION			GOTO		
	a	b	$	S	A	B
0				1	2	3
1			acc			
2	s4					
3		s5				
4					6	
5						7
6		s8				
7	s9					
8						
9						



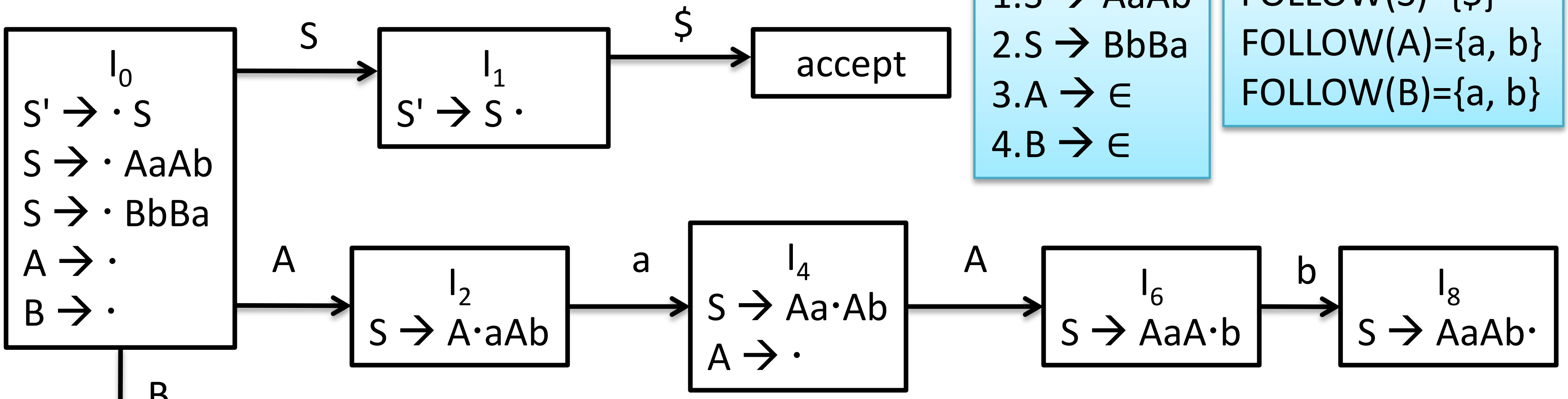
1.S → AaAb
2.S → BbBa
3.A → ∈
4.B → ∈

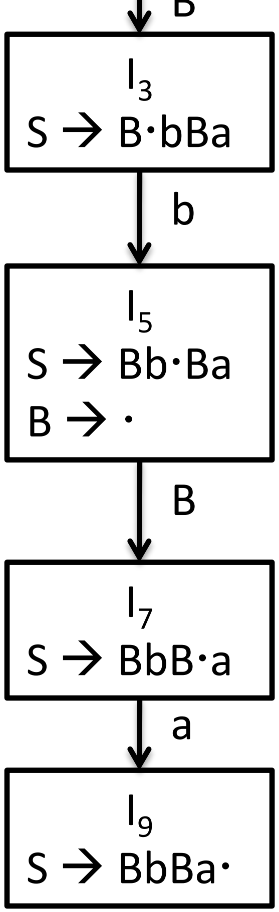
FOLLOW(S)={$}
FOLLOW(A)={a, b}
FOLLOW(B)={a, b}

STATE	ACTION			GOTO		
	a	b	$	S	A	B
0	r3/r4	r3/r4		1	2	3
1			acc			
2	s4					
3		s5				
4	r3	r3			6	
5	r4	r4				7
6		s8				
7	s9					
8			r1			
9			r2			

1.S → AaAb
2.S → BbBa
3.A → ∈
4.B → ∈

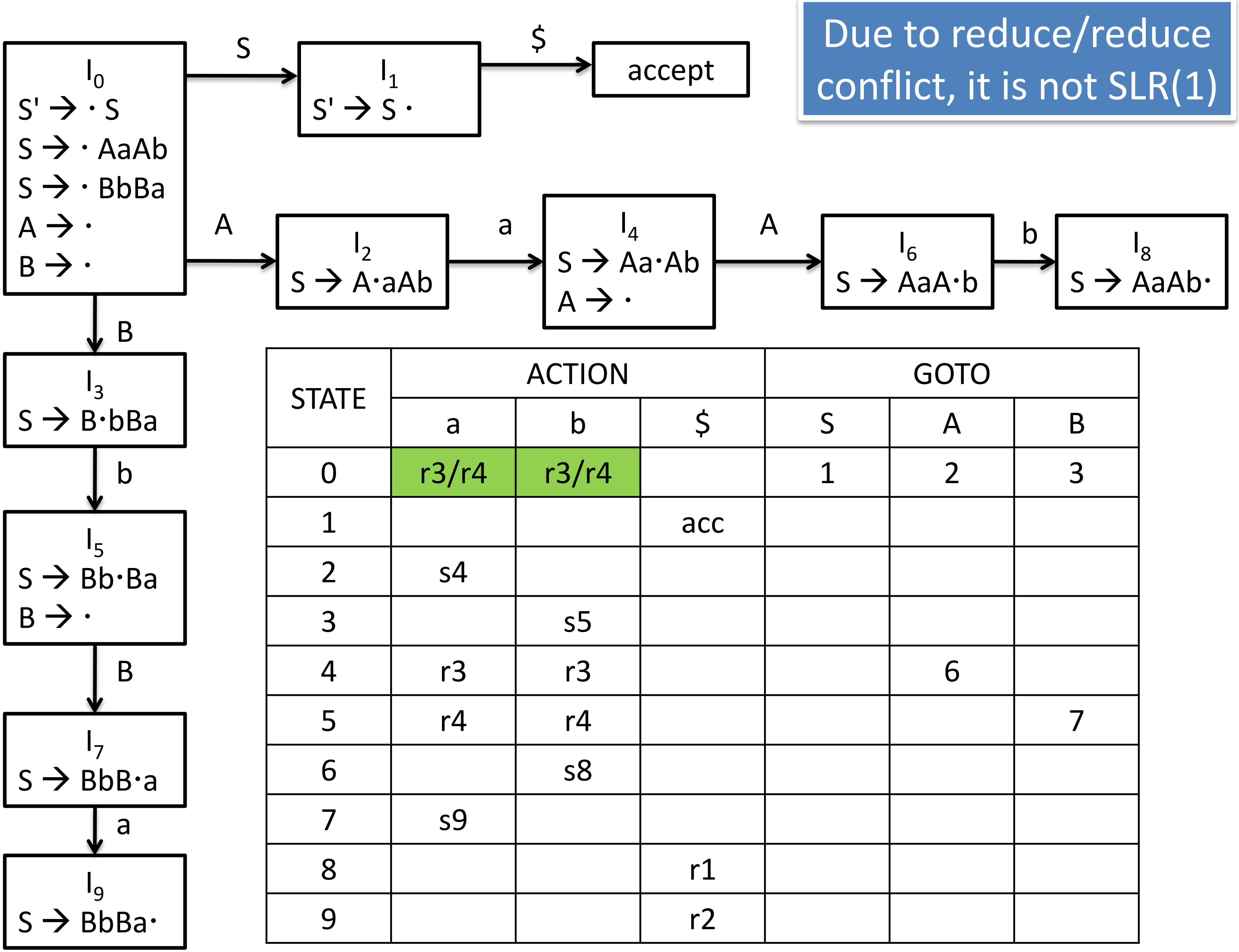
FOLLOW(S)={$}
FOLLOW(A)={a, b}
FOLLOW(B)={a, b}





STATE	ACTION			GOTO		
	a	b	$	S	A	B
0	r3/r4	r3/r4		1	2	3
1			acc			
2	s4					
3		s5				
4	r3	r3			6	
5	r4	r4				7
6		s8				
7	s9					
8			r1			
9			r2			

Due to reduce/reduce conflict, it is not SLR(1)



STATE	ACTION			GOTO		
	a	b	$	S	A	B
0	r3/r4	r3/r4		1	2	3
1			acc			
2	s4					
3		s5				
4	r3	r3			6	
5	r4	r4				7
6		s8				
7	s9					
8			r1			
9			r2			