Morphological Processing

Morphological Processing

- Analyze images
- Define structure
- Define regions and boundaries
- Based on set theory
- Unified theoretical approach

Morphology

The word morphology refers to the scientific branch that deals the forms and structures of animals/plants.

Morphology in image processing is a tool for extracting image components that are useful in the representation and description of region shape, such as **boundaries** and **skeletons**.

Furthermore, the morphological operations can be used for **filtering, thinning and pruning.**

The language of the Morphology comes from the set theory, where image objects can be represented by sets. For example an image object containing black pixels can be considered a set of black pixels in 2D space of Z^2

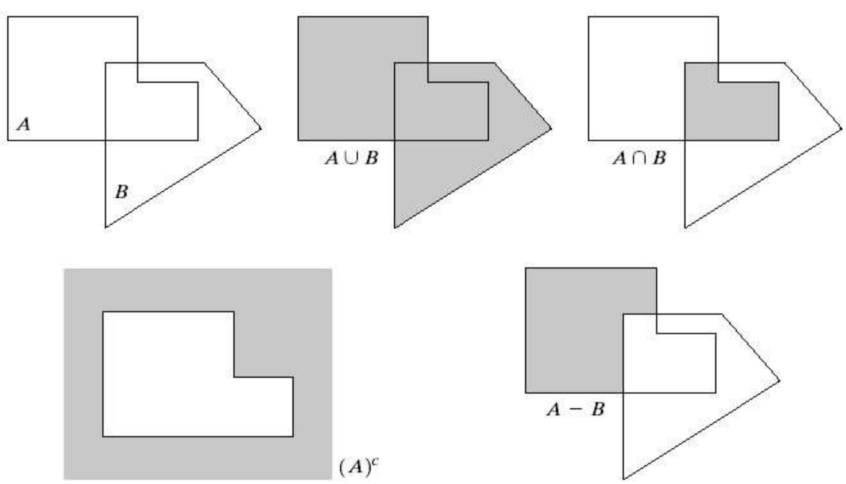
Morphological Operations

- Dilation
  - Fill in gaps
- Erosion
  - Delete unneeded detail (noise)
- Opening
  - Smooth inner contours
- Closing
  - Smooth outer contours
- Hit or Miss Transformation
  - Detect shapes

Set Theory Summary

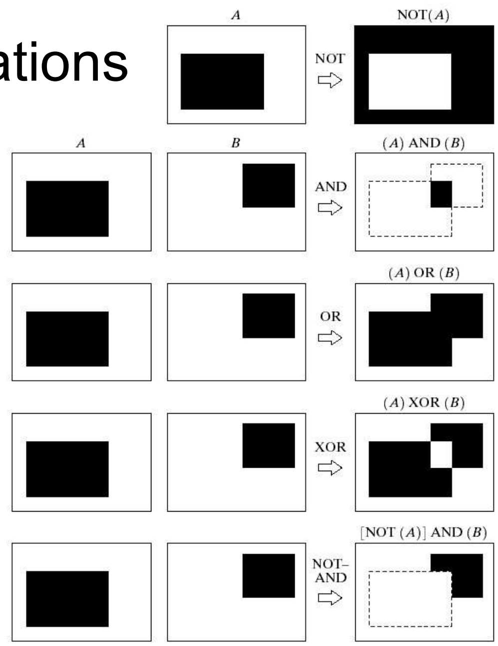
- *a* =(x, y) is an element of A — $a \in A$
- elements of images
  - pixel coordinates
- subset — $A \subseteq B$
- union — $A \bigcup B$
- intersection — $A \bigcap B$
- complement — $A^C = \{w \mid w \notin A\}$
- difference — $A - B = A \bigcap B^C$

Set Theory



Logic Operations

- Black = 1

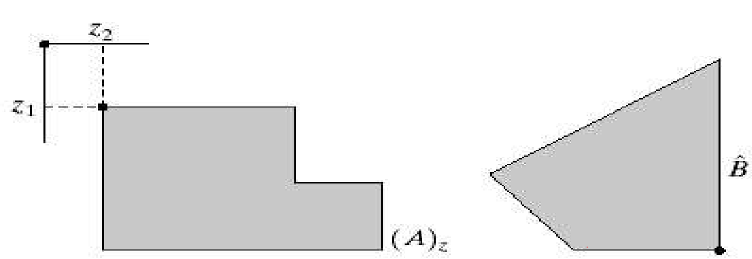


Morphological Operations

- Reflection
- Translation

$$\hat{B} = \{ w \mid w = -b \ \ for\ b \in B \}$$

$$(A)_Z = \{ c \mid c = a + z \ \ for\ a \in A \}$$



Dilation and Erosion

Dilation and erosion are the two fundamental operations used in morphological image processing. Almost all morphological algorithms depend on these two operations:

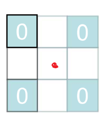
Dilation: Given A and B sets in Z2 , the dilation of A by B, is defined by:

$$A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\}$$

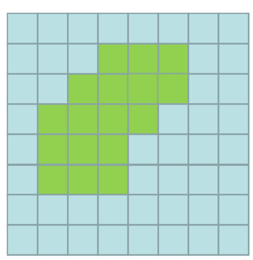
The dilation of A and B is a set of all displacements, z , such that B and A overlap by ***at least*** one element.

Set B is referred to as the **structuring element** and used in dilation as well as in other morphological operations. **Dilation expands/dilutes a given image.**

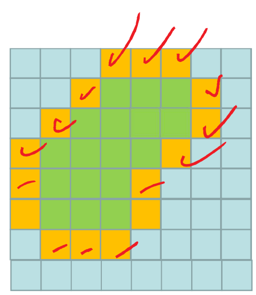
Dilation Operation



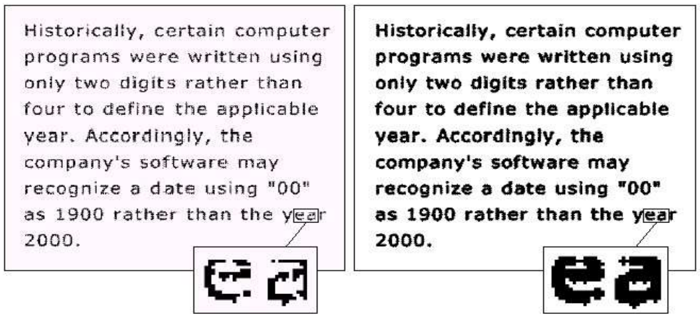
$B = \hat{B}$



A



Dilation Operation



0	1	0
1	1	1
0	1	0

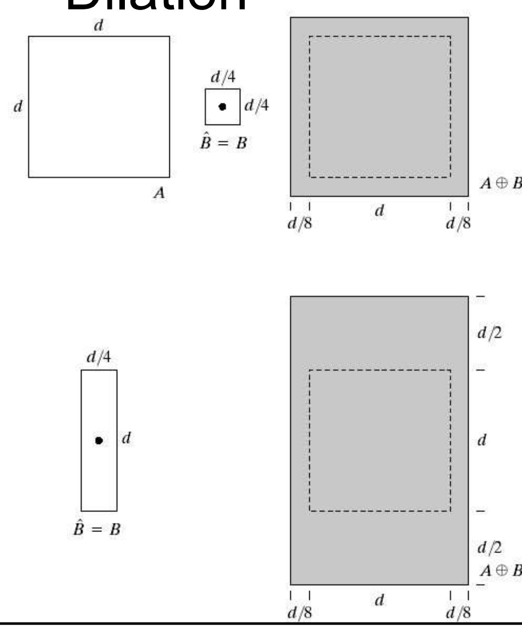
a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Dilation

a b c
d e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



Erosion

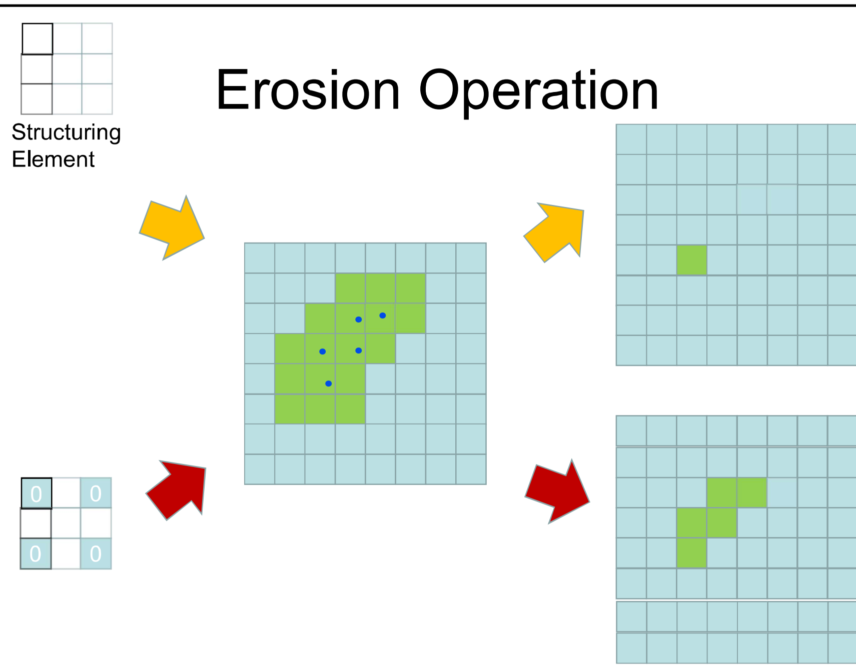
Erosion: Given A and B sets in Z^2 , the erosion of A by structuring element B, is defined by:

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

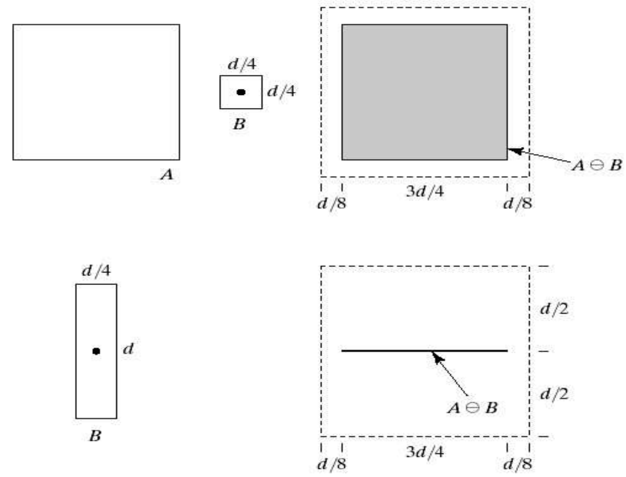
The erosion of A by structuring element B is the **set of all points z, such that B, translated by z, is contained in A.**

Note that in erosion the structuring element B erodes the input image A at its boundaries. Erosion **shrinks** a given image.

Erosion Operation

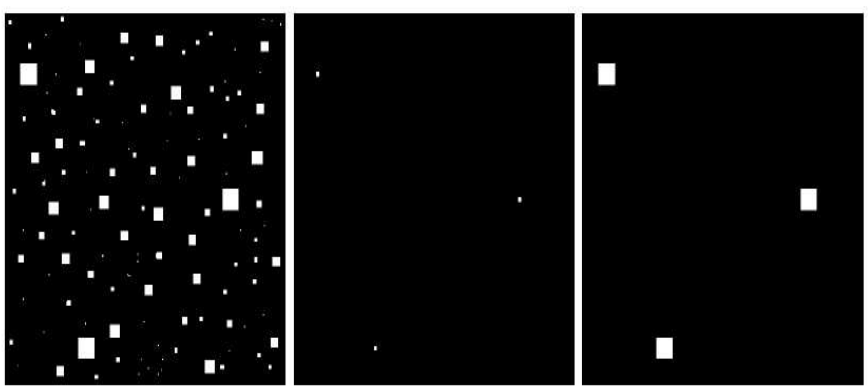


Erosion



a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening Operation

Opening: The process of erosion followed by dilation is called opening.

It has the effect of eliminating small and thin objects, breaking the objects at thin points and smoothing the boundaries/contours of the objects.

Given set A and the structuring element B. Opening of A by structuring element B is defined by:

$$A \circ B = (A \ominus B) \oplus B$$

The opening of A by the structuring element B is obtained by taking the union of all translates of B that fit into A.

$$A \circ B = \bigcup \{ B_z \mid (B_z) \subseteq A \}$$

Opening Operation

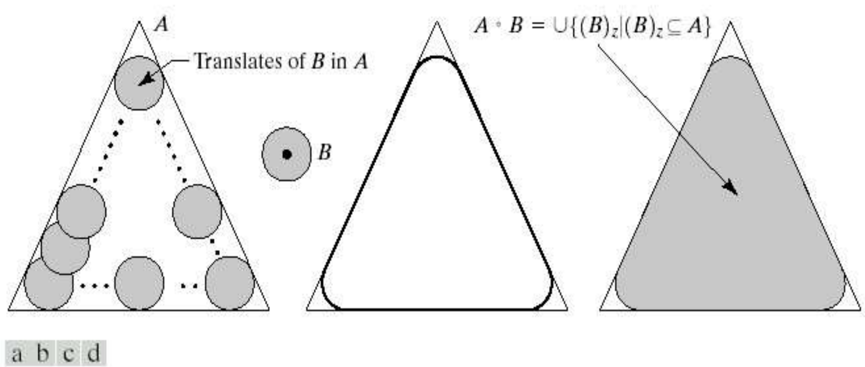
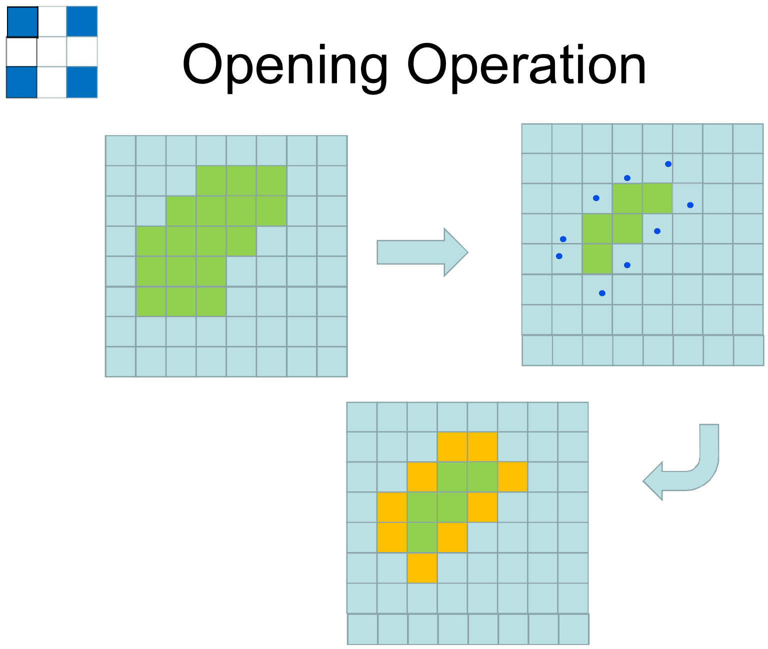
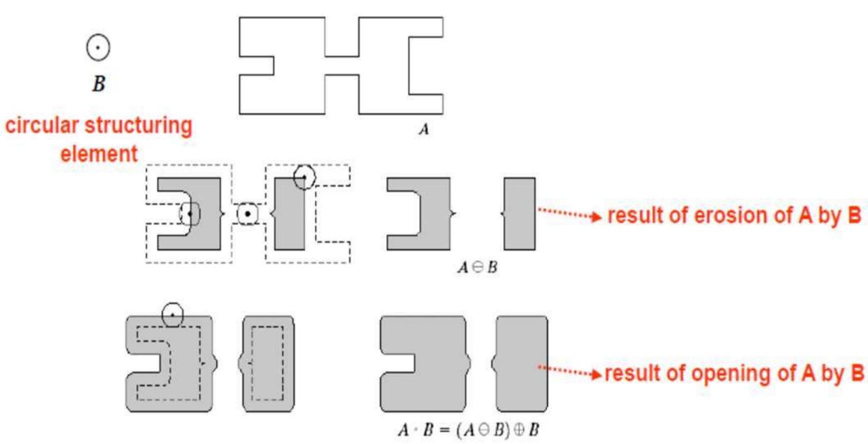


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Opening Operation



Opening Operation



Closing Operation

Closing: The process of dilation followed by erosion is called closing.

It has the effect of filling small and thin holes, connecting nearby objects and smoothing the boundaries/contours of the objects.

Given set A and the structuring element B. Closing of A by structuring element B is defined by:

$$A \bullet B = (A \oplus B) \ominus B$$

The closing has a similar geometric interpretation except that we roll B on the outside of the boundary.

$$A \bullet B = \bigcup \{ (B_z) \mid (B_z) \cap A \neq \varnothing \}$$

Closing Operation

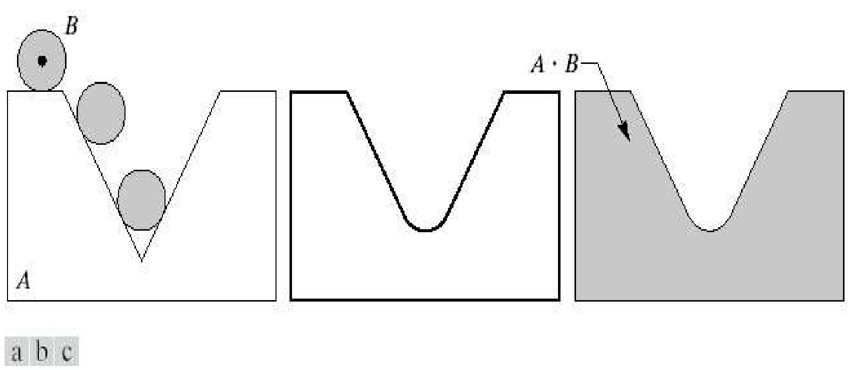
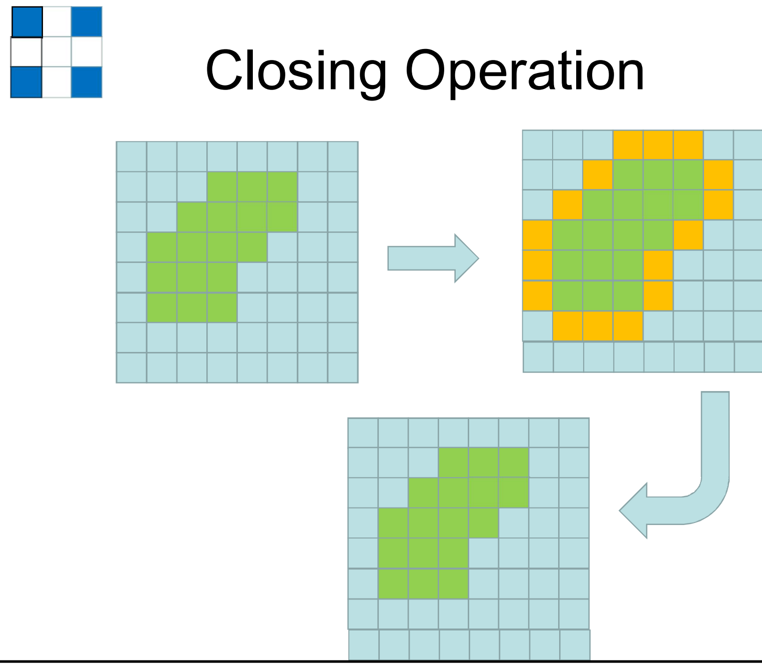
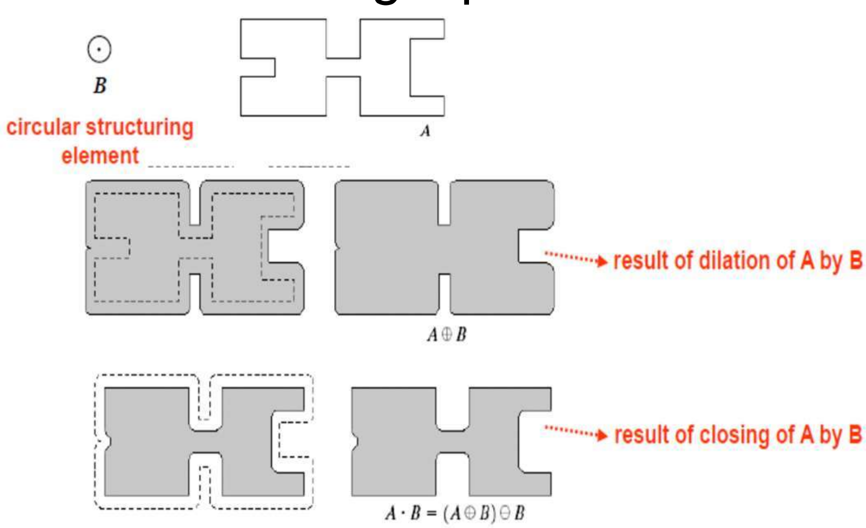


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

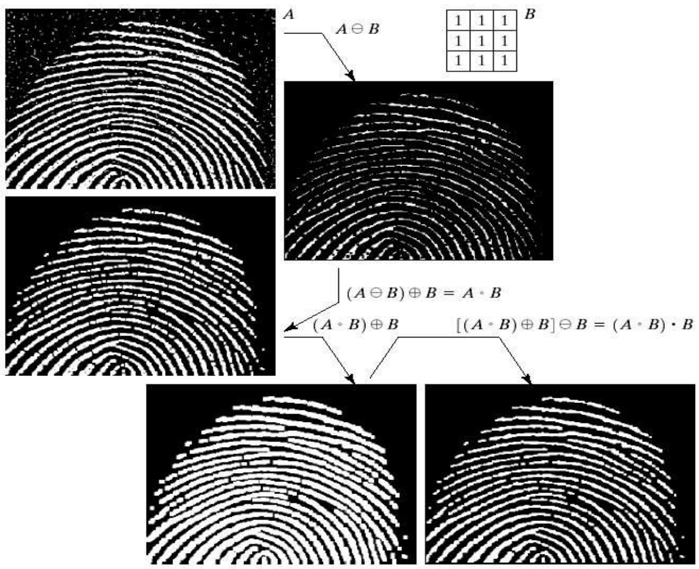
Closing Operation



Closing Operation



Mixed Example



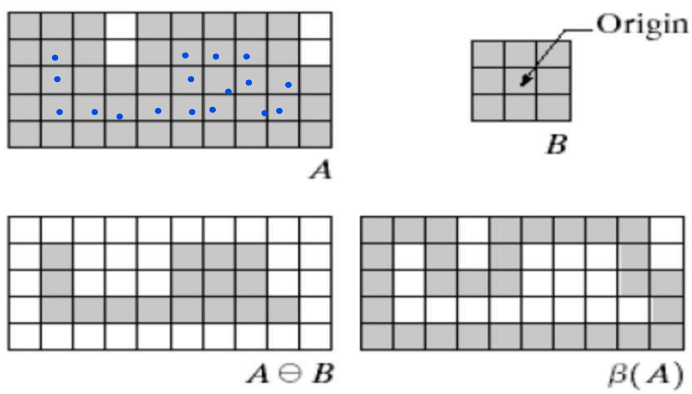
a b
d c
e f

FIGURE 9.11 (a) Noisy image. (c) Eroded image. (d) Opening of *A*. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Opening and Closing

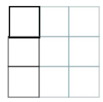
- Opening
  - smooth contours
  - break narrow isthmus
  - eliminate narrow protrusions
- Closing
  - smooth contours
  - fuse breaks
  - eliminate holes
  - fill in small gaps

Boundary Extraction

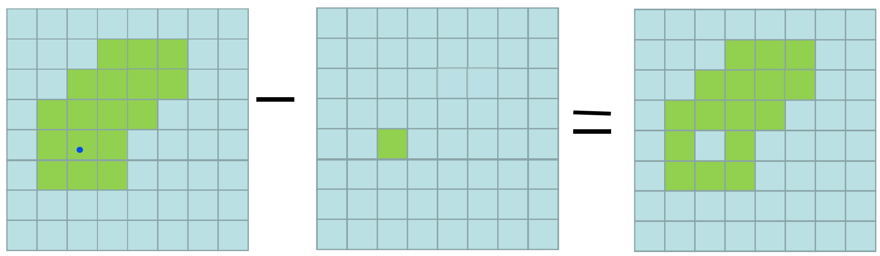


Boundary Extraction

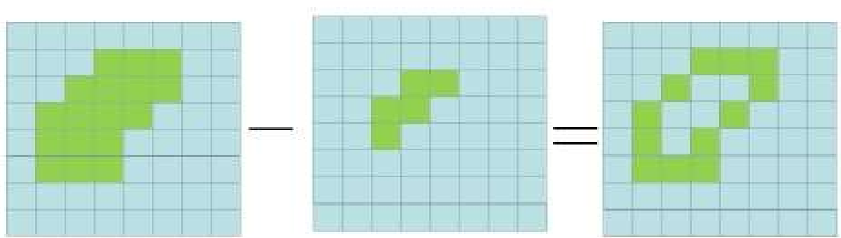
Note that thicker boundaries can be obtained by increasing the size of structuring element

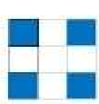


Boundary Extraction



Boundary Extraction





- Erode A by B
- Subtract from original

Region Filling

Region filling can be performed by using the following definition.

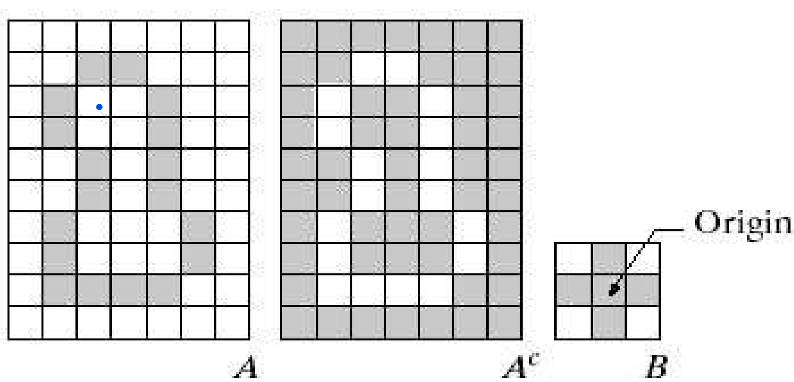
Given a symmetric structuring element B, one of the non-boundary pixels (Xk) is consecutively diluted and its intersection with the complement of A is taken as follows:

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad \begin{array}{r} k = 1,2,3,\ldots \\ terminates\ when\ X_k = X_{k-1} \\ X_0 = 1\ (inner\ pixel) \end{array}$$

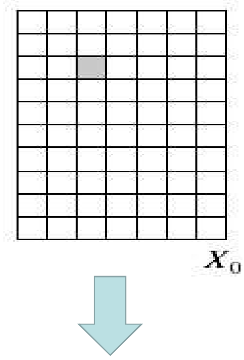
Following consecutive dilations and their intersection with the complement of A, finally resulting set is the filled inner boundary region and its union with A gives the filled region F(A)

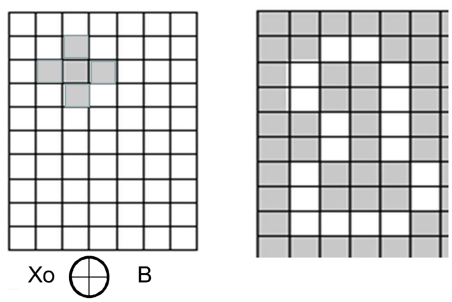
$$F(A) = X_k \cup A$$

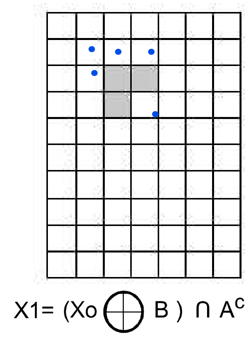
Region Filling



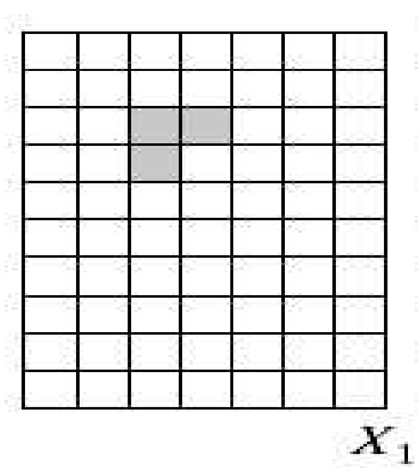
Region Filling

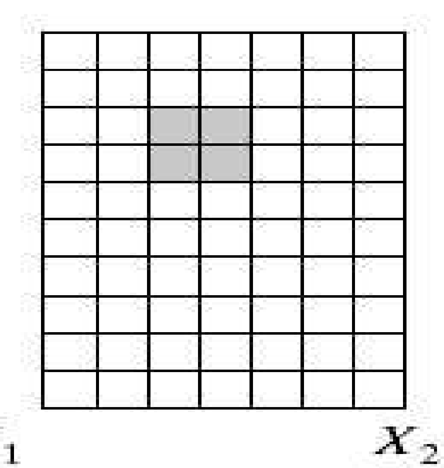




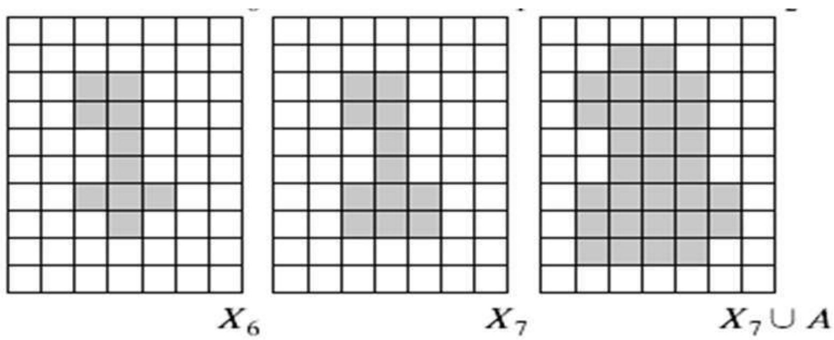


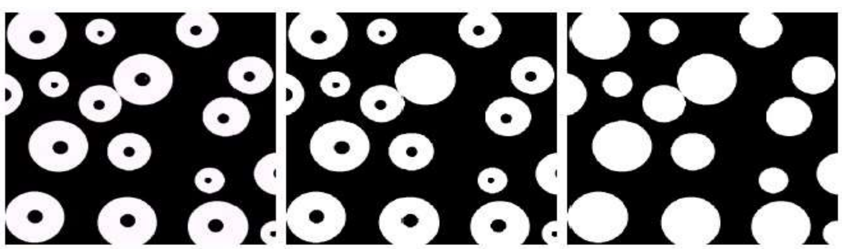
Region Filling





Region Filling





a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Finding the **starting** points is often done **manually**