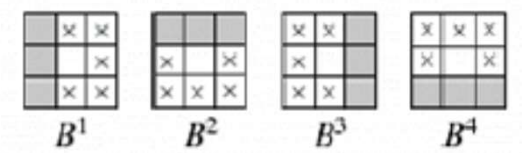
Convex Hull

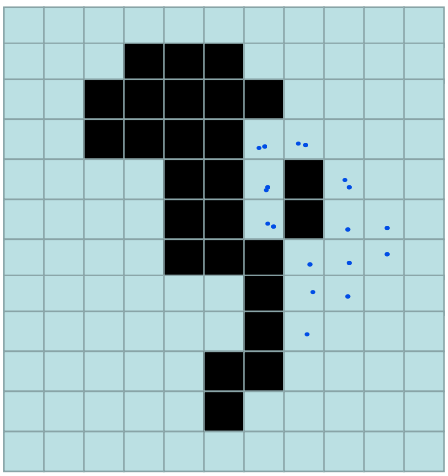
$$X_k^i = (X_{k-1} \circledast B^i) \cup A, \; i = 1, 2, 3, 4, \; k = 1, 2, \ldots, \; X_0^i = A$$

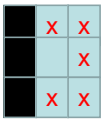
Now let $D^i = X_{\text{conv}}^i$, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. Then the convex hull of A is

$$C(A) = \cup_{i=1}^{4} D^i$$

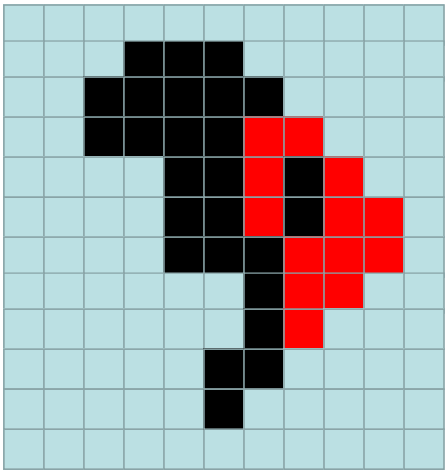


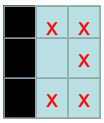
Convex Hull



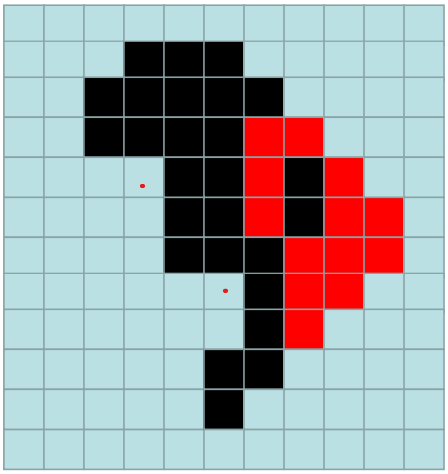


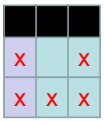
Convex Hull



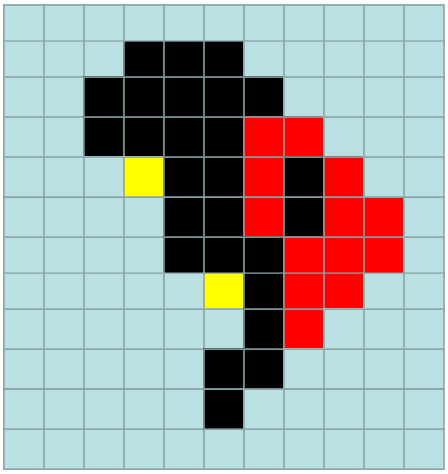


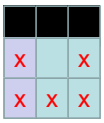
Convex Hull



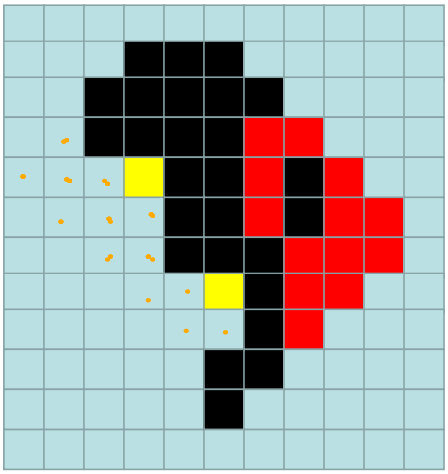


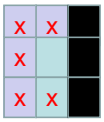
Convex Hull



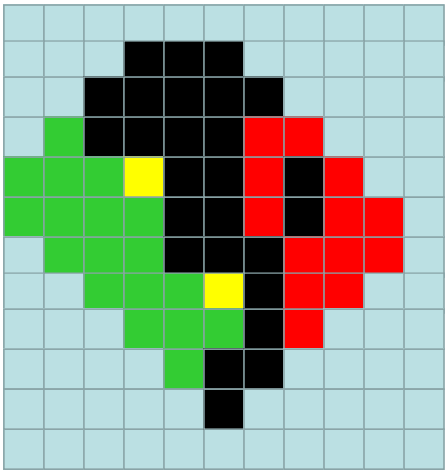


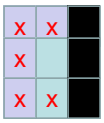
Convex Hull



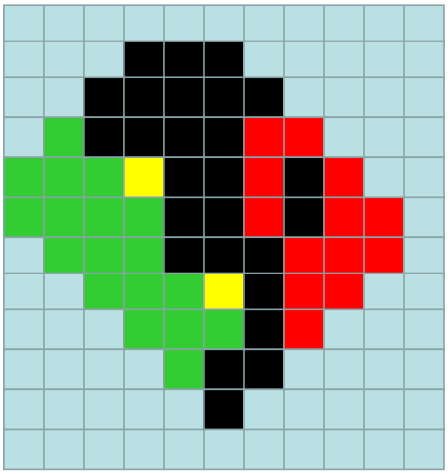


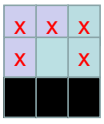
Convex Hull



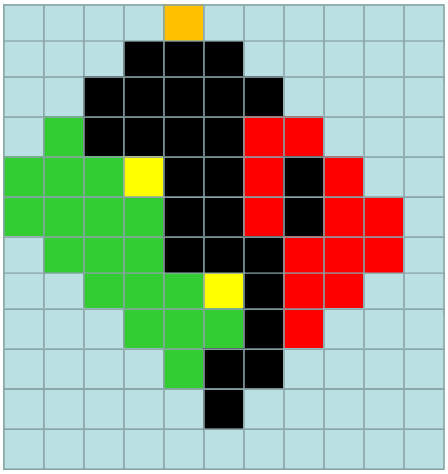


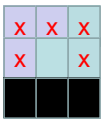
Convex Hull



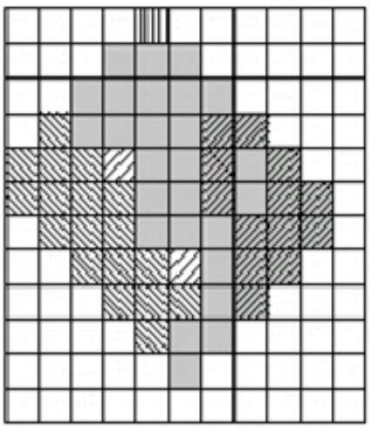


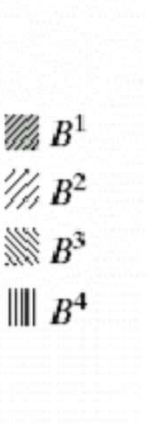
Convex Hull

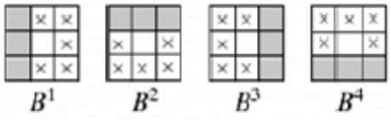




Convex Hull





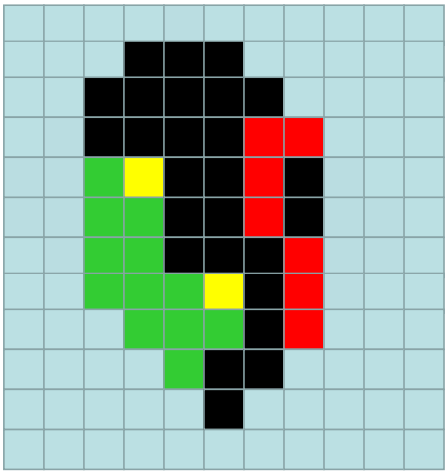


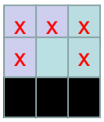
Convex Hull

Shortcoming of above algorithm: convex hull can grow beyond the minimum dimensions required to guarantee convexity

Possible solution: Limit growth so that it does not extend past the vertical and horizontal dimensions of the original set of points

Convex Hull



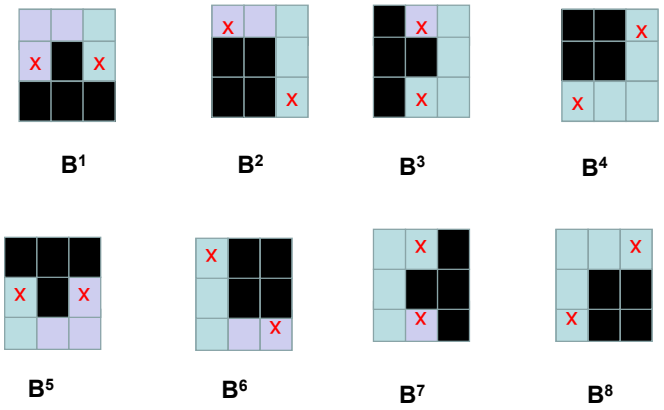


Thinning

- The thinning of a set A by a structuring element B, defined

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

Thinning



Thinning

$$A \otimes B = A - (A \circledast B)$$

hit-or-miss transform/template matching

- *Note that we are only interested in pattern matching of B in A, so no background operation is required of the hit-miss-transform.*

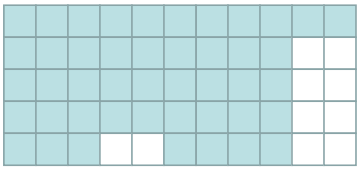
$$\{B\} = \{B^1, B^2, B^3, ..., B^n\}$$

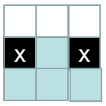
- *The structuring element B consists of a sequence of structuring elements, where B^i is the rotated version of B^{i-1}. Each structuring elements helps thinning in one direction. If there are 4 structuring elements thinning is performed from 4 directions separated by 90°. If 8 structuring elements are used the thinning is performed in 8 directions separated by 45°.*

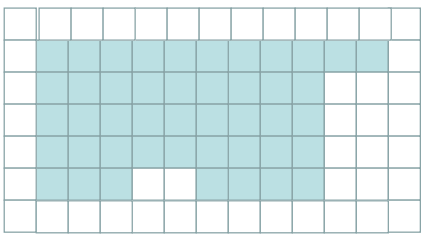
- *The process is to thin A by one pass with B^1, then the result with one pass of B^2, and continue until A is thinned with one pass of B^n.*

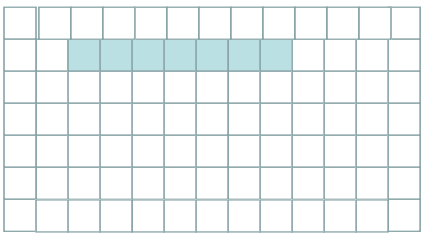
$$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$

Thinning

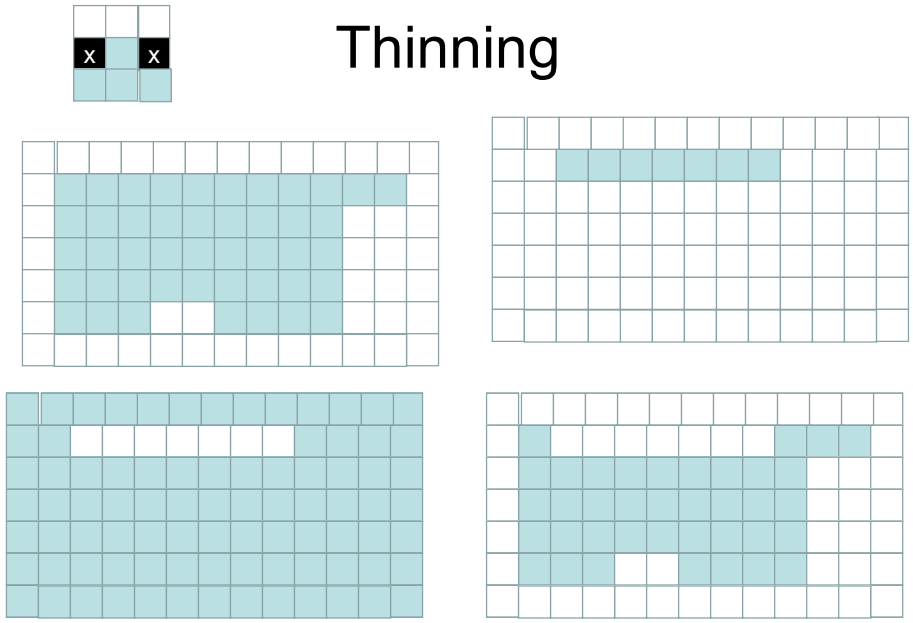




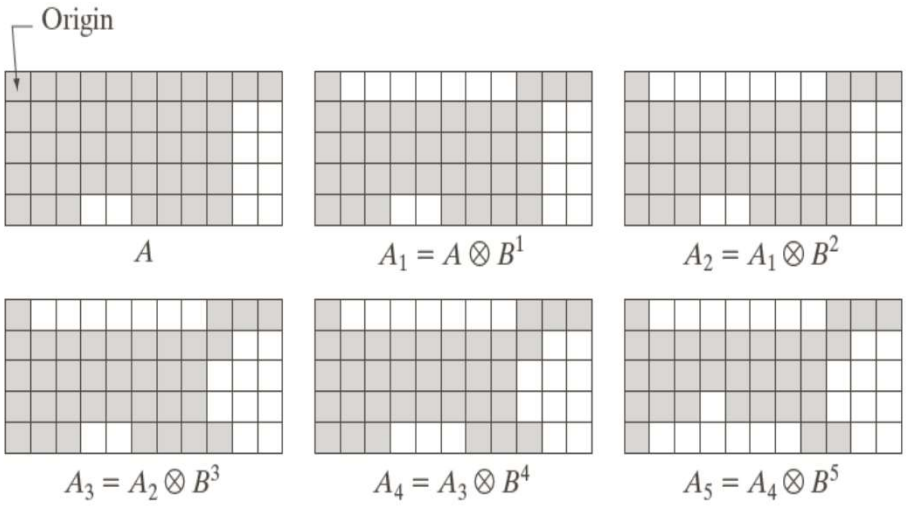




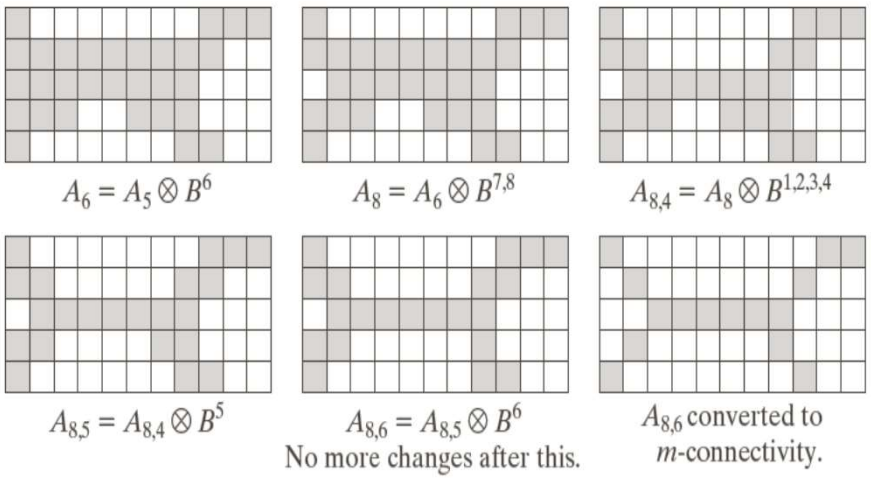
Thinning



Thinning



Thinning



Thickening

The thickening is defined by the expression

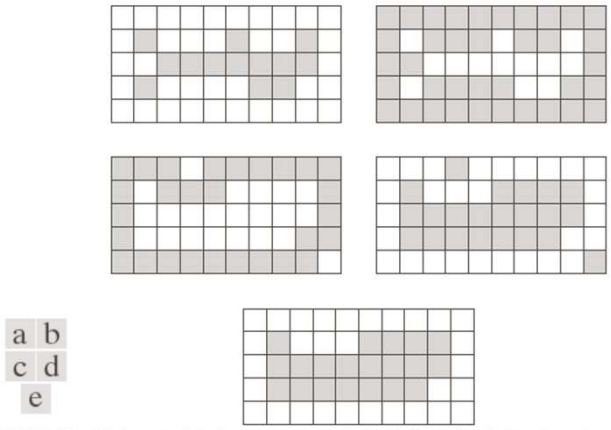
$$A \odot B = A \cup \left(A \circledast B\right)$$

The thickening of A by a sequence of structuring element $\{B\}$

$$A \odot \{B\} = ((...((A \odot B^1) \odot B^2)...) \odot B^n)$$

In practice, the usual procedure is to thin the background of the set and then complement the result.

Thickening



a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.