Histogram Processing Numerical

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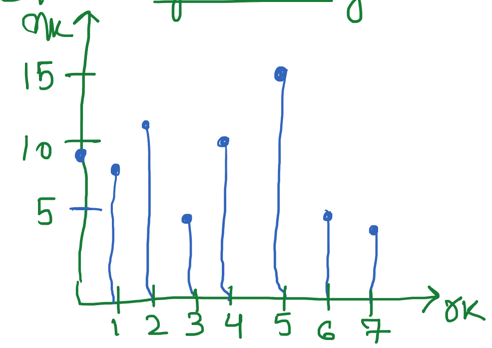
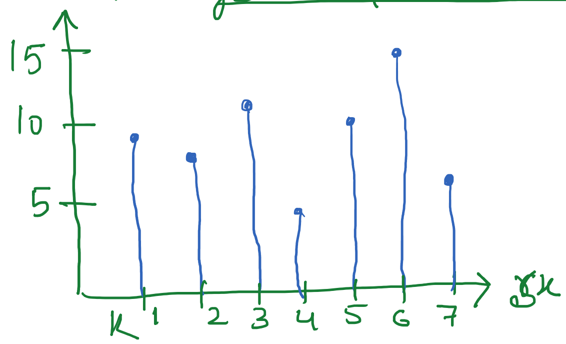
→ Perform the histogram equalisation for an 8×8 image shown below

Gray level	0	1	2	3	4	5	6	7
no. of pixels	9	8	11	4	10	15	4	3

Sol. original image



histogram equalised image



Gray levels	n_k	PDF $P_r(r_k) = n_k/MN$	CDF $= \sum_{j=0}^{k} P_r(r_j)$	$S_k = (L-1) \times CDF$	Round off
$r_0 = 0$	9	$P_r(r_0) = 0.141$	0.141	0.987	1
$r_1 = 1$	8	$P_r(r_1) = 0.125$	0.266	1.862	2
$r_2 = 2$	11	$P_r(r_2) = 0.172$	0.438	3.066	3
$r_3 = 3$	4	$P_r(r_3) = 0.062$	0.5	3.5	4
$r_4 = 4$	10	$P_r(r_4) = 0.156$	0.656	4.592	5
$r_5 = 5$	15	$P_r(r_5) = 0.234$	0.89	6.23	6
$r_6 = 6$	4	$P_r(r_6) = 0.062$	0.952	6.664	7
$r_7 = 7$	3	$P_r(r_7) = 0.047$	0.999	6.993	7

O/P image

Gray levels	1	2	3	4	5	6	7
no. of pixels	9	8	11	4	10	15	7

Q. For the given i/p image perform histogram equalisation & give the o/p image.

Sol. i/p image

$$\begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 3 & 4 & 5 & 4 & 3 \\ 3 & 5 & 5 & 5 & 3 \\ 3 & 4 & 5 & 4 & 3 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

$M \times N = 5 \times 5 = 25$ = Total no. of pixels

max intensity in image = 5

∴ no. of bits = 3

∴ max. gray levels = $2^3 - 1 = 7$

Gray levels	n_k	PDF $P_r(r_k) = n_k/MN$	CDF $= \sum_{j=0}^{k} P_r(r_j)$	$S_k = (L-1) \times CDF$	Round off
			0	0	0

Gray levels	n_k	PDF $P_r(r_k) = n_k/MN$	CDF $= \sum_{j=0} P_r(r_j)$	$s_k = (L-1) \times CDF$	Round off
$r_0 = 0$	0	0	0	0	0
$r_1 = 1$	0	0	0	0	0
$r_2 = 2$	0	0	0	0	0
$r_3 = 3$	6	0.24	0.24	1.68	2
$r_4 = 4$	14	0.56	0.80	5.6	6
$r_5 = 5$	5	0.2	1.00	7	7
$r_6 = 6$	0	0	1.00	7	7
$r_7 = 7$	0	0	1.00	7	7

O/P image

$$\begin{vmatrix} 6 & 6 & 6 & 6 & 6 \\ 2 & 6 & 7 & 6 & 2 \\ 2 & 7 & 7 & 7 & 2 \\ 2 & 6 & 7 & 6 & 2 \\ 6 & 6 & 6 & 6 & 6 \end{vmatrix}$$

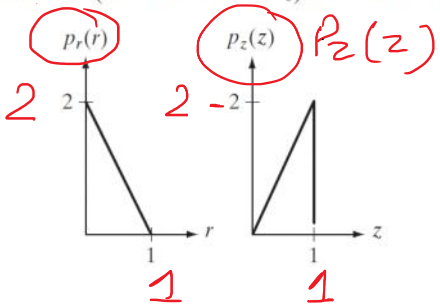
Q. Perform histogram equalisation for the following image

$$f(x,y) = \begin{vmatrix} 1 & 2 & 1 & 1 & 1 \\ 2 & 5 & 3 & 5 & 2 \\ 2 & 5 & 5 & 5 & 2 \\ 2 & 5 & 3 & 5 & 2 \\ 1 & 1 & 1 & 2 & 1 \end{vmatrix}$$

Try yourself!

$$s = T(r) = (L-1) \underbrace{\sum_{j=0}^{K} P_r(r_j)}_{CDF}$$

Q An image with intensities in the range [0, 1] has the PDF $p_r(r)$ shown in the following diagram. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.

$P_r(r)$  $P_z(z)$

$P_r(r) = -2r + 2$

$P_z(z) = 2z$

$$S = T(r) = (L-1)\int_0^r P_r(w)\,dw = \int_0^r (-2w+2)\,dw = -r^2 + 2r$$

$$G(z) = \int_0^z P_z(w)\,dw = \int_0^z 2z\,dw = z^2$$

$G(z) = S$

$z^2 = -r^2 + 2r$

$z = G^{-1}(s)$

$z = \sqrt{-r^2+2r}$

$z = \pm\sqrt{-r^2+2r}$

First, we obtain the histogram equalization transformation:

$$s = T(r) = \int_0^r p_r(w)\,dw = \int_0^r (-2w+2)\,dw = -r^2 + 2r.$$

Next we find

$$v = G(z) = \int_0^z p_z(w)\,dw = \int_0^z 2w\,dw = z^2.$$

Finally,

$$z = G^{-1}(v) = \pm\sqrt{v}.$$

But only positive intensity levels are allowed, so $z = \sqrt{v}$. Then, we replace v with s, which in turn is $-r^2 + 2r$, and we have

$$z = \sqrt{-r^2 + 2r}.$$