PRACTICAL No. 1

LIMITS & CONTINUITY

Q.1) 1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$= \lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$= \lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$= \frac{1}{3} \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$= \frac{1}{3} \times \frac{\overset{2}{\cancel{4}}\sqrt{a}}{\cancel{2}\sqrt{3a}}$

$= \frac{2}{3\sqrt{3}}$

2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$= \lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$= \lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$= \frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})}$

$= \frac{1}{\sqrt{a}(2\sqrt{a})}$

$= \frac{1}{2a}$

3) $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \pi/6 = h$

$x = h + \pi/6$

where $h \to 0$

$\therefore \lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3}\sin(h + \pi/6)}{\pi - 6(h + \pi/6)}$

$\therefore \lim_{h \to 0} \frac{\cos h \cos \pi/6 - \sin h \sin \pi/6 - \sqrt{3}(\sin h \cos \pi/6 + \cos h \sin \pi/6)}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$

$\therefore \lim_{h \to 0} \frac{\cos h \cdot \sqrt{3}/2 - \sin h \cdot 1/2 - \sqrt{3}(\sin h \cdot \sqrt{3}/2 + \cos h \cdot 1/2)}{\pi - 6h + \pi}$

$\therefore \lim_{h \to 0} \frac{\cos h \cdot \sqrt{3}/2\, h - \sin h/2 - \sin 3h/2 - \cos \frac{\sqrt{3}}{2} h}{-6h}$

$$\therefore \lim_{h\to 0} \frac{-\cancel{5}\sin 4h/3}{\cancel{4}\,6h}$$

$$\therefore \lim_{h\to 0} \frac{\sin 4h}{\frac{3}{4}h}$$

$$\therefore \frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h}$$

$$\therefore \frac{1}{3}(1)$$

$$\therefore \frac{1}{3}$$

4) $$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$$

By rationalizing numerator and denominator both

$$\therefore \lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\therefore \lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\therefore \lim_{x\to\infty}\frac{\cancel{8}^{4}(\sqrt{x^2+3}+\sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$\therefore 4\lim_{x\to\infty}\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}$$

$\therefore$ After applying limit, we get

$= 4$

$f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$
$\quad = \dfrac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$ } at $x = \pi/2$

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos(\pi/2)}}$ $\therefore f(\pi/2) = 0$

at $x = \pi/2$ is define

$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting method

$x - \pi/2 = h$

$x = \pi/2 + h$

where $h \to 0$

$\therefore \lim_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$

$\therefore \lim_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$

$\therefore \lim_{h \to 0} \dfrac{\cos(h + \pi/2)}{-2h}$

$\therefore \lim_{h \to 0} \dfrac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$

$\therefore \lim_{h \to 0} \dfrac{\cos h \cdot \cancel{0} - \sin h \cdot}{-2h}$

$\therefore \lim_{h \to 0} \dfrac{\cancel{-}\sin h}{\cancel{-}2h}$

$\therefore 1/2$

$\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\therefore \lim_{x \to \pi/2} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$\therefore \lim_{x \to \pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2}\ \cancel{\sin x}}$

$\therefore \lim_{x \to \pi/2^-} \frac{2\cos x}{\sqrt{2}}$

$\therefore \frac{2}{\sqrt{2}} \lim_{x \to \pi/2} \cos x$

L.H.L $\neq$ R.H.L

$\therefore$ f is not continous at $x = \pi/2$

5)
4) ii) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $6 \le x < 9$

at $x = 3$ & $x = 6$

at $x=3$

i) $f(3) = \frac{x^2-9}{x-3} = 6$

f at $x=3$ define

ii) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$F(3) = 3+3 = 6$

f is define at $x = 3$

$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ L.H.L = R.H.L

f is continuous at $x = 3$

For $x = 6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = 3$

$\lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$\therefore \lim_{x \to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\therefore 6-3$

$\therefore 3$

$\lim_{x \to 6^-} f(x) = \lim_{x \to 6^-} x+3$

$= 3+6$

$= 9$

L.H.L $\neq$ R.H.L

Function is not continuous

6)) $f(x) = \frac{1-\cos 4x}{x^2}$, $x < 0$
$= k$ $x = 0$
} at $x = 0$

Sol^n f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

18.

$\therefore \lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$

$\therefore \lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$

$\therefore 2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$

$\therefore 2 \lim_{x \to 0} \frac{\sin^2 2x}{2x^2} = k$

$\therefore 2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$\therefore 2(2)^2 = k$

$\therefore k = 8$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$
$= k$ $x = 0$ at $x = 0$

Sol^n $f(x) = (\sec^2 x)^{\cot^2 x}$

$\therefore k = (1 + \tan^2 x)^{1/\tan^2 x}$

we know that

$\lim_{x \to 0} (1+px)^{1/px} = e$

$= e$

$\therefore k = e$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \pi/3$; $= k$ $x = \pi/3$ } at $x = \pi/3$.

$x - \pi/3 = h$

$x = h + \pi/3$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(h + \pi/3)}{\pi - 3(\pi/3 + h)}$$

$$\therefore \lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\therefore \lim_{h \to 0} \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 - \tan h}}{\pi - \pi - 3h}$$

$$\therefore \lim_{h \to 0} \frac{\sqrt{3}(1 - \tan \pi/3 \cdot \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 \cdot \tan h} \times (-3h)$$

$$\therefore \lim_{h \to 0} \frac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3}) + \tan h)}{1 - \tan \pi/3 \cdot \tan h} \times (-3h)$$

$$\therefore \lim_{h \to 0} \frac{(\sqrt{3} - \cancel{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \tan h} \cdot (-3h)$$

$$\therefore \lim_{h \to 0} \frac{\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h}{1 - \sqrt{3} \tan h} (-3h)$$

$$\therefore \lim_{h \to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$\therefore \lim_{h \to 0} \frac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$\therefore 4/3 \lim_{h \to 0} \frac{\tan h}{h} \cdot \lim_{h \to 0} \frac{1}{1 - \sqrt{3}\tan h}$$

$$= \frac{4}{3} \frac{1}{1 - \sqrt{3}(0)} = 4/3 (1/1) = 4/3$$

7) i) $f(x) = \frac{1-\cos 3x}{x \tan x}$ $\quad x \neq 0$
$\quad = 9 \quad x = 0$ $\Big\}$ at $x = 0$

Q.1) i

$f(x) = \frac{1-\cos 3x}{x \tan x}$

$= \lim_{x \to 0} \frac{2\sin^2 3/2 x}{x \tan x}$

$= \lim_{x \to 0} \frac{\frac{2\sin^2 3x/2}{x^2} \times x^2}{\frac{x \cdot \tan x}{x^2} \times x^2}$

$= 2 \cdot \lim_{x \to 0} \frac{(3/2)^2}{1}$

$= \cancel{2} \times \frac{9}{\cancel{4}\, 2}$

$= 9/2$

$\lim_{x \to 0} f(x) = 9/2 \quad 9 = f(0)$

$\therefore f$ is not continuous at $x = 0$

Redefine function

$f(x) = \begin{cases} \frac{1-\cos 3x}{x \tan x} & x \neq 0 \\ 9/2 & x = 0 \end{cases}$

Now, $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$

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7) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$, $x \neq 0$

$= \pi/6$, $x = 0$

} at $x = 0$

$\therefore \lim_{x\to 0} \frac{(e^{3x}-1)\sin(\frac{\pi x}{180})}{x^2}$

$\therefore \lim_{x\to 0} \frac{e^{3x}-1}{x} \cdot \lim_{x\to 0} \frac{\sin(\frac{\pi x}{180})}{x}$

$\therefore \lim_{x\to 0} \frac{3.e^{3x}-1}{3x} \cdot \lim_{x\to 0} \frac{\sin(\frac{\pi x}{180})}{x}$

$\therefore 3.\lim_{x\to 0} \frac{e^{3x}-1}{3x} \cdot \lim_{x\to 0} \frac{\sin(\frac{\pi x}{180})}{x}$

$\therefore 3.\log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

f is continous at $x = 0$

8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x = 0$

$\therefore$ Given,

f is continous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

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$\therefore \lim_{x\to 0} \frac{(e^{x^2}-1)+(1-\cos x)}{x^2}$

$\therefore \lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$

$\therefore \log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$

$\therefore \log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$

Multiply by 2 on Num & Demo

$\therefore 1 + 2\times \frac{1}{4} = \frac{3}{2} = f(0)$

3) $f(x) = \frac{\sqrt{2}-\sqrt{1-\sin x}}{\cos^2 x}, \quad x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$\therefore \lim_{x\to \pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$

$\therefore \lim_{x\to \pi/2} \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$\therefore \lim_{x\to \pi/2} \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$\therefore \lim_{x\to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2}+\sqrt{1+\sin x})}$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}}$$

PRACTICAL No. 2

Topic: Derivative

Q.1) Show that the following function defined from R to R are differentiable.

Q.1) i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x - a) \tan x \tan a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(a) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a + h - a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan(a - a - h) - (1 + \tan a . \tan(a+h))}{h \times \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{-\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h) \tan a}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = -\frac{1}{\cancel{\cos^2 a}} \times \frac{\cos^2 a}{\sin^2 a} = -\operatorname{cosec}^2 a$$

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$\therefore DF(a) = -\cos^2 a$

$\therefore$ f is differentiable $\forall a \in R$.

iii) cosec x

$f(x) = \operatorname{cosec} x$

$$DF(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x - a)\sin a \sin x}$$

Put $x - a = h$, $x = a + h$

as $x \to a$, $h \to 0$

$$DF(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a . \sin(a+h)}$$

$$= \lim_{h \to 0} - \cancel{\frac{\sin h/2}{h/2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= -\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

Q.1)

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2 \sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2 \sin\left(\frac{2a+h}{2}\right) \sin -h/2}{\cos a \cos(a+h) \times -h/2} \times -1/2$$

$$= -1/2 \times \frac{-2 \sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= -1/2 \times -2 \frac{\sin a}{\cos a \times \cos a}$$

$= \tan a \sec a$

Q.2)

Q.2) If $f(x) = 4x + 1 \quad , x \le 2$
$= x^2 + 5 \quad , x > 0$, at $x = 2$, then find fun(t)ion is differentiable or not

Soln:

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - (4 \times 2 + 1)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - 9}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x - 8}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 4$$

$Df(2^-) = 4$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 2 + 2$$

$$= 4$$

LHD = RHD

f is diff at $x = 2$

Q.3) If $f(x) = 4x+7$, $x<3$
$= x^2+3x+1$, $x \geq 3$, at $x=3$, then
find f is differentiable or not?

Sol^n

R.H.D:

$$Df(3^+) = \lim_{x\to3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to3^+} \frac{x^2+3x+1-(3^2+3\cdot3+1)}{x-3}$$

$$= \lim_{x\to3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$= 3+6 = 9$

$Df(3^+) = 9$

LHD $= Df(3^-)$

$$= \lim_{x\to3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$Df(3^+) = 4$

RHD $\neq$ LHD

f is not differentiable at $x = 3$

(Q.4) If $f(x) = 8x-5$, $x \leq 2$

$= 3x^2 - 4x + 7$, $x > 2$ at $x = 2$, then

find f is differentiable or not

Sol^n:

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 3 \times 2 + 2$

$= 8$

$Df(2^+) = 8$

L.H.D :

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 8$$

$Df(2^-) = 8$

L.H.D = R.H.D

~~D.~~

f is differentiable at $x = 3$

Siz

2nd ambiguous

Practical 3.

Topic : Application of Derivative.

Q1) Find the intervals in which function is increasing or decreasing.

i) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > 5/3$

$x > \pm\sqrt{5/3}$

$$\begin{array}{ccc} + & - & + \\ \hline -\sqrt{5/3} & & \sqrt{5/3} \end{array}$$

$x \in (\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < 5/3$

$x < \pm\sqrt{5/3}$

$$\begin{array}{ccc} - + & - & + - \\ \hline -\sqrt{5/3} & & \sqrt{5/3} \end{array}$$

$x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{5/3}\right)$

Q.1)

ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f$ is increasing iff $f'(x) > 0$

$2x - 4 > 0$

$2x > 4$

$x > 2$

$\therefore x \in (2, \infty)$

$\therefore f$ is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x < 2$

$x \in (\infty, 2)$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$= 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0 \quad \therefore x = -2, \ x = 10/6$

$- \quad + \quad - \quad +$

$-2 \qquad 10/6$

$x \in (-\infty, -2) \cup (10/6, \infty)$

f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x+2) < 0$

$(x+2)(6x-10) < 0$

$+ \quad - \quad + \quad -$

$-2 \quad 10/6$

$x \in (-2, 10/6)$

iv) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$- \quad + \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-\infty, -3)(3, \infty)$

f is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad + \quad -$

$-3 \quad 3$

$x \in (-3, 3)$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24x - 18x + 6x^2$

i.e. $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

$\therefore$ f is increasing iff $f'(x) > 0$

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$\therefore 6(x^2-3x-4)>0$

$x^2-3x-4>0$

$x^2-4x-x-4>0$

$x(x-4)+1(x-4)>0$

$(x-4)(x+1)>0$

Q.i

$- \quad + \quad - \quad +$

$-1 \quad +4$

$x \in (-\infty,-1) \cup (4,\infty)$

f is decreasing iff $f'(x)<0$

$\therefore 6(x^2-3x-4)<0$

$x^2-3x-4<0$

$x^2-4x+x-4<0$

$x(x-4)+1(x-4)<0$

$(x+1)(x-4)<0$

$+ \quad - \quad + \quad -$

$-1 \quad 4$

$\therefore x \in (1,4)$

Q.2) find the intervals in which function is concave upwards are concave downwards

i) $y=3x^2-2x^3$

Let,

$f'(x)=y=3x^2-2x^3$

$f'(x)=6x-6x^2$

$f''(x)=6-12x$

$=6(1-2x)$

$f''(x)$ is concave upwards iff

$f''(x)>0$

$6(1-2x)>0$

$1-2x > 0$

$-2x > -1$

$2x < -1$

$x < 1/2$

$x \in (-\infty, 1/2)$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$6(1-2x) < 0$

$1-2x < 0$

$-2x < -1$

$x > 1/2$

$x \in (1/2, \infty)$

ii) $y = x^4 - 6x^3 + 12x^2 + 15x + 7$

det,

$f(x) = y = x^4 - 6x^3 + 12x^2 + 5x + 17$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$f''(x)$ is concave upwords iff

$f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$x^2 - x - 2x + 2 > 0$

$x(x-1) - 2(x-1) > 0$

$(x-1)(x-2) > 0$

$+ \quad - \quad - \quad +$

$1 \quad 2$

$x \in (-\infty, 1) \cup (2, \infty)$

Q.1) ii)

$f''(x)$ is concave downwards iff.

$$f''(x) = 0$$
$$12(x^2 - 3x + 2) < 0$$
$$x^2 - 3x + 2 < 0$$
$$x^2 - x - 2x + 2 < 0$$
$$x(x-1) - 2(x-1) < 0$$
$$(x-2)(x-1) < 0$$

+ - + -

1 2

$$\therefore x \in (1, 2)$$

iii) $y = x^3 - 27x + 5$

Let

$$F(x) = y = x^3 - 27x + 5$$
$$f'(x) = 3x^2 - 27$$
$$f''(x) = 6x$$

$f''(x)$ is concave iff upwards iff

$$f''(x) > 0$$
$$6x > 0$$
$$x > 0$$
$$x \in (0, \infty)$$

$f''(x)$ is concave downwards iff

$$f''(x) < 0$$
$$6x < 0$$
$$x < 0$$
$$x \in (-\infty, 0)$$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

Let

$f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$f''(x)$ is concave upward iff.

$f''(x) > 0$

$-18 + 12x > 0$

$12x > 18$

$x > 18/12$

$x \in (3/2, \infty)$

$f''(x)$ is concave downwards iff.

$f''(x) < 0$

$-18 + 12x < 0$

$12x < 18$

$x < 18/12$

$\therefore x \in (\infty, 3/2)$

v) $y = 2x^3 + x^2 - 20x + 4$

Let,

$f(x) = y$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

$= 2(6x + 1)$

$\therefore f''(x)$ is concave upwards iff.

$f''(x) > 0$

$2(6x+1) > 0$

$6x + 1 > 0$

$6x > -1$

$x = -1/6$

$x \in (-1/6, \infty)$

$\therefore f''(x)$ is concave downwards iff

$f''(x) < 0$

$2(6x+1) < 0$

$6x + 1 < 0$

$x < -1/6$

$\therefore x \in (-\infty, -1/6)$

AV
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Practical No. 4

Topic: Application of Derivative & Newton's Method

1) Find maximum and minimum value of following function

i) $f(x) = x^3 + 16/x^2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ is $[-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 + 12x + 1$ in $[-2, 3]$

Q.2) Find the root of following equation by newton's method (Take 4 iteration only). (correct upto 4 decimal

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ [taking $x_0 = 0$]

2) $f(x) = x^3 - 4x - 9$ in $[2,3]$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Q1) i) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider,

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + \frac{96}{(-2)^4}$

Q.

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$ & $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider,

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = \cancel{-30 + 60}$

$= \cancel{30} > 0$

$\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$
$= 30 - 60$
$= -30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$
$= 3 + 5 - 3$
$= 5$

$\therefore$ f has the maximom value 5 at $x = -1$ and has the mininum value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$
$f'(x) = 3x^2 - 6x$

consider ?
$f'(x) = 0$
$\therefore 3x^2 - 6x = 0$
$3x(x-2) = 0$
$3x = 0$ or $x - 2 = 0$
$x = 0$ or $x = 2$

$f''(x) = 6x - 6$
$f''(0) = 6(0) - 6$
$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$
$= 1$

$\therefore$ f has minimum value at $x = 2$
$f(2) = (2)^3 - 3(2)^2 + 1$
$= 8 - 3(4) + 1$
$= 9 - 12 = -3$

$\therefore$ f has maximum vale 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ OR $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has maximum value 8 at $x = -1$ and

f has minimum value at $x = 2$

Q.2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $x_0 \to 0$ $\to$ Given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = 0 + \frac{9.5}{55}$$

$$x_1 = 0.1727$$

$$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$
$$= 0.0051 - 0.0895 - 9.4985 + 9.5$$
$$= -0.0829$$

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$
$$= 0.0895 - 1.0362 - 55$$
$$= -55.9467$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)}$$

$$= 0.1727 - \frac{0.0829}{55.9467}$$

$$= 0.1712$$

Q.1)

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.1712 + \frac{0.0011}{55.9393}$

$= 0.1712$

The root of the equation is 0.1712.

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9 = -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation.

$\therefore$ By Newton's method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{6}{23}$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.7392 - \frac{0.596}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 19.8386 - 10.8284 - 9$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.6056$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
Q.1) $= 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943}$
$= 2.7015 + 0.0050$
$= 2.7005$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$
$f'(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= 1 - 1.8 - 10 + 17$
$= 6.2$
$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17$
$= -2.2$

Let $x_0 = 2$ be initial approximation
By Newton's method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)}$
$= 2 - \frac{2.2}{5.2}$
$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 1.7$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$
$= 1.577 - \frac{0.6755}{8.2164}$
$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$
$= 1.6592 + \frac{0.0204}{7.7143}$
$= 1.6618$

x_4 $f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 16(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2847 - 5.9824 - 10$
$= -7.6977$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.6618 + \frac{0.0004}{7.6971}$$

$$= 1.6618$$

$\therefore$ The root of equation is 1.6618

Practical No. 5

Topic : Integration.

Q1] Solve the following integration

i) $\frac{dx}{\sqrt{x^2+2x-3}}$

$\rightarrow \int \frac{1}{\sqrt{x^2+2x+1-4}} dx$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx \rightarrow [a^2+2ab+b^2 = (a+b)^2$

Substitute $x-1=t$

$dx = \frac{1}{t} \times dt$ where $t = 1, t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

Using. $\int \frac{1}{\sqrt{x^2-(a)^2}} dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+C$

2) $\int(4e^{3x}+1)dx$

$= \int (4e^{3x})dx + \int 1 dx$

$= 4\int e^{3x}dx + \int 1 dx \quad \ldots \int e^{ax}dx = \frac{1}{a} \times e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

3) $\int(2x^2 - 3\sin x + 5\sqrt{x})dx$

$= \int 2x^2 - 3\sin x + 5x^{1/2}dx \qquad \sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2 dx - \int 3\sin(x)dx + \int 5x^{1/2}dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}dx$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}dx$

$= \int x^{3/2} + 3x^{1/2} + \frac{4}{x^{1/2}}dx$

$= \int x^{5/2}dx + \int 3x^{1/2}dx + \int \frac{4}{x^{1/2}}dx$

$\frac{x^{5/2+1}}{5/2+1}$

$\frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)dx$

Put $u = 2t^4$

$du = 2\times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}du$

$\int t^4 \sin(2t^4) \times \frac{1}{8}du$

$= \frac{t^4 \times \sin(2t^4)}{8}du$

Substitute t^4 with $u/2$

$= \int \frac{u/2 \times \sin(u)du}{8}$

$= \int \frac{u \times \sin(u)}{2}/8\, du$

$= \int \frac{u \times \sin(u)}{16}du$

$= \frac{1}{16}\int u \times \sin(u)du$

$\int u dv = uv - \int v du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1du$

$V = -\cos(u)$

$$= \frac{1}{16}\left(u \times (-\cos(u)) - \int -\cos(u)\, du\right)$$

$$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u)\, du$$

$\cos x\, dx = \sin x$

$$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$$

Return the substitution $u = 2+t$

$$= \frac{1}{16} \times (2t^4 \times (-\cos(2+u)) + \sin(2+u))$$

$$= -\frac{t^4 \times \cos(2+u)}{8} + \frac{\sin(2+u)}{16} + C$$

6) $\int \sqrt{x}\,(x^2-1)\,dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} x^2 - x^{1/2} dx$

$= \int x^{5/2} - x^{1/2} dx$

$= \int x^{5/2} dx - \int x^{1/2} dx$

$= I_1 - \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$

7) $\int \frac{\cos x}{3\sqrt{\sin^2 x}}\, dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$

Put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos x}{\sin(x)^{3/2}} \times \frac{1}{\cos(x) dt}$

$= \frac{1}{\sin x^{3/2}}\, dt$

$= \frac{1}{t^{2/3}}\, dt$

$I = \int \frac{1}{t^{2/3}}\, dt = -\frac{1}{(2/3-1)t^{2/3-1}}$

$= -\frac{1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

Return substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + C$

8) $\int \frac{x^2-2x}{x^3-3x^2+1}\, dx$

Put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2 - 3\times 2x}\, dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\, dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\, dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\, dt$

$= \int \frac{1}{3(x^3-3x^2+1)}\, dt = \int \frac{1}{3t}\, dt$

$= \frac{1}{3} \int \frac{1}{t} dt$

$= \frac{1}{3} \times \ln|t| + c$

$= \frac{1}{3} \times \ln(1-3x^2+1) + c$

9) $\int \frac{1}{x^3} \sin(1/x^2) dx$

$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

Let $\frac{1}{x^2} = t$

$x^{-2} = t$

$-\frac{2}{x^3} dx = dt$

$I = \frac{1}{-2} \int -\frac{2}{x^3} \sin(1/x^2) dx$

$= -\frac{1}{2} \int \sin t \, dt$

$= -\frac{1}{2} [-\cos t] + c$

$= \frac{1}{2} \cos t + c$

Resubstituting $t = 1/x^2$

$\therefore I = \frac{1}{2} \cos(1/x^3) + c$

(10) $\int e^{\cos 2x} \sin^2 x \, dx$

$I = \int e^{\cos 2x} \sin 2x \, dx$

Let $\cos^2 x = t$

$= -2\cos x \sin x \, dx = dt$

$= -2\sin 2x \, dx = dt$

$I = -\int -\sin 2x \cdot e^{\cos^2 x} dx$

$= e \int e^t dt$

$= -e^t + c$

AK 03/01/2020

Resubstituting $t = \cos^2 x$

$\therefore I = -e^{\cos^2 x} + c$

PRACTICAL No. 6

Aim: Application of integration & numerical integration

1) $y = \sqrt{4-x^2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$

$= -\frac{x}{\sqrt{4-x^2}}$

$I = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

$= \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}} dx$

$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}$

$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}} dx$

$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}} dx$

$= 2\left[\sin^{-1}(x/2)\right]^{2}$

$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$

Q1)

1) $x = t\sin t,\ y = 1-\cos t,\ t \in [0, 2\pi]$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t - \sin t \quad \therefore \frac{dx}{dt} = 1-\cos t$

$y = 1-\cos t \quad \therefore \frac{dy}{dt} = 0-(-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1-\cos t + \sin t)^2 + (\sin t)^2 dt}$

$= \int_0^{2\pi} \sqrt{1-2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{1-2\cos t \quad + 1}\quad dt$

$= \int_0^{2\pi} \sqrt{2-2\cos t}\, dt = \int_0^{2\pi} \sqrt{2(1-\cos t)} = \int_0^{2\pi} \sqrt{2 \times 2\sin^2 t/2}$

$= \cancel{\sqrt{4\sin^2 t/2}} \quad \int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt \quad \therefore \sin^2 \frac{t}{2} = \frac{1-\cos t}{2}$

$= \int_0^{2\pi} 2\sin \frac{t}{2}\, dt.$

$= \left(-4\cos(t/2)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0) = 4+4$
$= 8$

(2) $y = \sqrt{4-x^2} \quad x \in [-2, 2]$

$I = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$y = \sqrt{4-x^2} \quad \therefore \frac{dy}{dx} = 2\int_0^2 \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2} dx$

$= 2\int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\, dx$

$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$

$= 4\left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

(3) $y = x^{3/2}$ in $[0, 4]$

$f'(x) = 3/2\, x^{1/2}$

$[f'(x)]^2 = 9/4\, x$

$L = \int_a^b \sqrt{1+[f'(x)]^2}\, dx$

$= \int_0^b \sqrt{1+9/4\, x}\ dx$

$= \int_0^4 \sqrt{\frac{4+9x}{4}}\ dx$

$= 1/2 \int_0^4 \sqrt{4+9x}\, dx$

$= 1/2 \left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$

$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$

$= \frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right].$

$= \frac{1}{24}\left[(4)^{3/2} - (40)^{3/2}\right]$

Scanned by CamScanner

iv) $x = 3\sin t$, $y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -3\sin t$$

$$I = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\, dt$$

$$= \int_0^{2\pi} 3\, dt = 3\int_0^{2\pi} dt = 3[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ $\quad y = [1, 2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\cancel{\frac{(y^4+1)^2}{(2y)^2}}}\, dy$$

$$= \int_1^2 \frac{y^4 + 1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$$

$$= \frac{17}{12}$$

Q2) i) $\int_0^2 e^{x^2}\, dx$ with $n = 4$

$a = 0$, $b = 2$, $n = 4$

$$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182 + 54.5981)\right]$$

$$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 14.6326\right]$$

$$= 1.1779$$

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ii) $\int_0^4 x^2 dx$

$L = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

$= \int_0^4 x^2 dx = \frac{1}{3}[16+4(10)+8]$

$= 64/3$

$\int_0^4 x^2 dx = 21.533$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n=6$

$L = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.80687	0.8722	0.9907

$\int_0^{\pi/3} \sqrt{\sin(x)} = \pi/54 \times 12.1163$

$\cancel{= \int_0^{\pi/3} \sqrt{\sin x}\, dx}$

$= 0.7049$

Practical No. 7

Aim: Differential equation

Q1) 1) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$I(x) = 1/x \qquad Q(x) = \frac{e^x}{x}$

$I.F = e^{\int 1(x) dx}$

$= e^{\int 1/x\, dx}$

$= e^{\ln x}$

$I.F = x$

$y(I.F) = \int Q(x)\,(I.F)dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x dx + 1$

$xy = e^x + c$

$y(I.F) = \int Q(x)(I.F)dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3 dx + c$

$= \int \sin x\, dx + c$

$x^3 y = -\cos x + c$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$ $(\div \text{ by } e^x)$

$\frac{dy}{dx} + 2y = 1/e^x$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \quad Q(x) = e^{-x}$

$\int P(x)dx$

$IF = e^{\int 2dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)dx + c$

$ye^{2x} \int e^{-x} + 2x dx + c$

$= \int e^x dx + c$

$y.e^{2x} = e^x + c$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x.\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \quad Q(x) = \frac{\cos x}{x^2}$

$I.F = e^{\int P(x)dx}$

$= e^{\int 2/x \, dx}$

$= e^{2/x \, x}$

$= \ln x^2$

$I.F = x^2$

$y(I.F) = \int Q(x)(I.F)dx + c$

$= \int \frac{\cos x}{x^2} - x^2 dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e^{\int P(x)dx}$

$= e^{\int 3/x \, dx}$

$= e^{3/x \, \cancel{dx}}$

$= \cancel{e}^{\ln x^3}$

$I.F = x^3$

$y(I.F) = \int Q(x)(I.F)dx + c$

$= \int \frac{\sin x}{x^3} . x^3 . dx + c$

$= \int \sin x \, dx + c$

$x^3 y = -\cos x + c$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$IF = e^{\int P(x)dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)dx + c$

$= \int 2xe^{-2x}\, e^{2x}\, dx + c$

$= \int 2x\, dx + c$

$ye^{2x} = x^2 + c$

vi) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\sec^2 x \cdot \tan y\, dx = -\sec^2 y \cdot \tan x\, dy$

$\frac{\sec^2 x\, dx}{\tan x} = -\frac{\sec^2 y\, dy}{\tan y}$

$\int \frac{\sec^2 x\, dy}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\therefore \log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

Differentiating on both sides

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 x} = dx$

$\int \sec^2 v\, dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$

viii) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x + 3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\frac{dv}{dx} = \frac{v+1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$\therefore \int\left(\frac{v+2}{v+1}\right)dv = 3dx$

$\therefore \int\frac{v+1}{v}dx + \int\frac{1}{v+1}dv = 3x$

$\therefore v + \log|x| = 3x + c$

$\therefore 2x+3y+\log|2x+3y+1| = 3x+c$

$\therefore 3y = x - \log|2x+3y+1| + c$

10/01/2020

PRACTICAL No. 8

(I) $\frac{dy}{dx} = y + e^x - 2$

$F(x, y) = y + e^x - 2, \quad y_0 = 2, \quad x_0 = 0, \quad h = 0.5$

x	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.407	3.57435
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h\ F(x_n, y_n)$

x	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28505
4	2	9.2831		

By Euler's method.
$y(2) = 9.2831$

(II) $\frac{dy}{dx} = 1 + y^2$

$F(x, y) = 1 + y^2, \quad y_0 = 0, \quad x_0 = 0, \quad h = 0.2$

using Euler's formula

$y_{n+1} = y_n + h$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0		0.2	0	0.2
1	0.2	0.408	0.104	0.408
2	0.4	0.6413	1.665	0.6413
3	0.6	0.9236	1.4113	0.9236
4	0.8	1.2942	1.8530	1.2442
5	1			

$\therefore$ By Euler's formula
$y(1) = 1.2942$

(3) $\frac{dy}{dx} = \sqrt{x/y}$. $y(0) = 1$, $x_0 = 0$. $h = 0.2$

Using Euler's ~~for~~ iteration formula
$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	0	0
1	0.2			
2	0.4			
3	0.6			
4	0.8			
5	1			

(4) $dy/dx = 3x^2 + 1$. $y_0 = 2$, $x_0 = 1$, $h = 0.5$
For $h = 0.5$
Using Euler's itration formula
$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Euler's Formula
$y(2) = 28.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.3	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.90483
4	2	8.9048		

By Euler's formula.
$y(2) = 8.9048$

(5) $dy/dx = \sqrt{2y} + 2$. $y_0 = 1$, $x_0 = 1$, $h = 0.2$

Using Euler's iteration formula.

$y_{n+1} = y_n + hf$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

By Euler's formula
$y(1.2) = 1.6$

7/01/2020

Practical No. 9.

57 Limits & Partial order derivatives

Q1)

i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$

$= \frac{-64+3+1-1}{4+5}$

$= -\frac{61}{9}$

ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit.

$= \frac{(0+1)((2)^2+0-4(2))}{2+0}$

$= \frac{1(4+0-8)}{2}$

$= \frac{-4}{2}$

$= -2$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, Denominator $= 0$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{\cancel{(x-yz)}(x+yz)}{x^2\cancel{(x-yz)}}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

On applying limit

$= \frac{1+1(1)}{(1)^2}$

$= 2$

Q2)

i) $f(x,y) = xye^{x^2+y^2}$

$\therefore f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f_x = \frac{\delta}{\delta x}(f(x,y))$

$= \frac{\delta}{\delta x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{\delta}{\delta y}(f(x,y))$

$= \frac{\delta}{\delta y}(e^x \cos y)$

$\therefore f_y = -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\delta}{\delta x}(f(x,y))$

$= \frac{\delta}{\delta x}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \frac{\delta}{\delta y}(f(x,y))$

$= \frac{\delta}{\delta y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_y = \cancel{2}x^3y - 3x^2 + 3y^2$

a) i) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{\delta}{\delta x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\delta}{\delta x}(2x) - 2x \frac{\delta}{\delta x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2 - 0}{(1+y^2)^2}$

$= \frac{2\cancel{(1+y^2)}}{\cancel{(1+y^2)}(1+y)^2}$

$= \frac{2}{1+y^2}$

At $(0,0)$

$= \frac{2}{1+0}$

$= 2$

$f_y = \frac{\delta}{\delta y}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\delta}{\delta x}(2x) - 2x \frac{\delta}{\delta x}(1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At $(0,0)$

$= \frac{-4(0)(0)}{(1+0)^2}$

$= 0$

4)

1) $f(x,y) = \frac{y^2 - xy}{x^2}$

$f_x = \frac{x^2 \frac{\delta}{\delta x}(y^2 - xy) - (y^2 - xy)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$

$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$

$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$

$f_y = \frac{2y - x}{x^2}$

$f_{xx} = \frac{\delta}{\delta x}\left(\frac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$

$= \frac{x^4\left(\frac{\delta}{\delta x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{\delta}{\delta x}(x^4)}{(x^4)^2}$

$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6}$ — ①

$f_{yy} = \frac{\delta}{\delta y}\left(\frac{2y - x}{x^2}\right)$

$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$ — ②

$f_{xy} = \frac{\delta}{\delta y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$

$= \frac{-x^2 - 4xy + 2x^2}{x^4}$ — ③

$f_{yx} = \frac{\delta}{\delta x}\left(\frac{2y - x}{x^2}\right)$

$= \frac{x^2\frac{\delta}{\delta x}(2y - x) - (2y - x)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$

$= \frac{-x^2 - 4xy + 2x^2}{x^4}$ — ④

From ③ & ④;

$f_{xy} = f_{yx}$

2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\delta}{\delta x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{\delta}{\delta y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 0 + 6x^2y - 0$

$= 6x^2y$

$f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1\frac{\delta}{\delta x}(2x) - 2x\frac{\delta}{\delta x}(x^2+1)}{(x^2+1)^2}\right)$

$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$ — ①

$f_{yy} = \frac{\delta}{\delta y}(6x^2y)$

$= 6x^2$ — ②

$f_{xy} = \frac{\delta}{\delta y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy$ — ③

$f_{yx} = \frac{\delta}{\delta x}(6x^2y)$

$= 12xy$ — ④

From ③ & ④

$f_{xy} = f_{yx}$

3) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$\therefore f_{xx} = \frac{\delta}{\delta x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$ — 1

$f_{yy} = \frac{\delta}{\delta y}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ — 2

$f_{xy} = \frac{\delta}{\delta y}(y\cos(xy) + e^{x+y})$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — 3

$f_{yx} = \frac{\delta}{\delta x}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — 4

$\therefore$ From 3 & 4

$f_{xy} \neq f_{yx}$

Q.5)

1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

f_y at $(1,1) = 1/\sqrt{2}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \cancel{2/\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

2) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \pi/2 + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$

$f_y = 0 - 0 + \sin x$

f_x at $(\pi/2, 0) = -1 + 0 = -1$

f_y at $(\pi/2, 0) = \sin \pi/2 = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-0)$

$= 1 - \pi/2 - x + \pi/2 + y$

$= 1 - x + y$

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(3) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\quad f_y = 0 + 1/y$

f_x at $(1,1) = 1$ $\quad f_y$ at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

Practical No. 10

Directional derivative, Gradient Vector & maxima, minima Tangent & normal vector.

Q1)

1) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1)$, $u = 3i - j$

→ Here,

$u = 3i - j$ is not a unit vector

$\bar{u} = 3\bar{i} - \bar{j}$

$|u| = \sqrt{10}$

Unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3\bar{i} - \bar{j})$

$= \frac{1}{\sqrt{10}}(3, -1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

Now,

$f(a+hu) = f\left((1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$

$= f\left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$

$= 1 + \frac{3h}{\sqrt{10}} + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 - 2 - 3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$

$= -4 + \frac{h}{\sqrt{10}}$

$$\therefore D_uF(a) = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{-4+h/\sqrt{10}-(-4)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\sqrt{10}}$$

$$= 1/\sqrt{10}$$

ii) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

→ Here,

$u = i+5j$ is not a unit vector

$\bar{u} = \bar{i}+5\bar{j}$

$|\bar{u}| = \sqrt{26}$

∴ Unit Vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(\bar{i}+5\bar{j})$

$$= \frac{1}{\sqrt{26}}(1, 5)$$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

Now,

$$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$$

$$= f\left(3+\frac{h}{\sqrt{26}}, 4+\frac{5h}{\sqrt{26}}\right)$$

$$= \left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_uF(a) = \lim_{h\to 0} \frac{f(a+hu) + f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \lim_{h\to 0} \frac{25h^2}{26} + \frac{36h}{\sqrt{26}}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left(\frac{25h}{26} + \frac{36h}{\sqrt{26}}\right)}{\cancel{h}}$$

$$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

3) $f(x,y) = 2x+3y$ $a = (1,2)$, $u = 3i+4j$

Here,

$u = 3i+4j$ is not a unit vector

$\bar{u} = 3\bar{i}+4\bar{j}$

$|\bar{u}| = \sqrt{25} = 5$

∴ Unit vector along $u = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5}(3\bar{i}+4\bar{j})$

$$= \frac{1}{5}(3,4)$$

$$= \frac{3}{5}, \frac{4}{5}$$

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Now,

$f(a+hu) = f\left((1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4h}{5}\right)$

$= 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 2+\frac{6h}{5} + 6+\frac{12h}{5}$

$= 8 + \frac{18h}{5}$

$\therefore D_u f = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{\cancel{8} + \frac{18h}{5} - \cancel{8}}{h}$

$= \lim_{h\to 0} \frac{18\cancel{h}}{5h}$

$= \frac{18}{5}$

Q2)

i) $f(x,y) = x^y + y^x$ $\qquad a=(1,1)$

$f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + x^y \log x$

$\nabla f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^x \log y, x y^{x-1} + x^y \log x)$

$\nabla f(x,y)$ at $(1,1)$

$= (1(1)^0 + 1^1 \log(1), 1(1)^{(1-1)} + 1^1 \log 1)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$ $\qquad a = (1,-1)$

$f_x = y^2\left(\frac{1}{1+x^2}\right) = \frac{y^2}{1+x^2}$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$\nabla f(x,y)$ at $(1,-1)$

$= \left(\frac{(-1)^2}{1+(1)^2}, 2(-1)\tan^{-1}(1)\right)$

$= \left(\frac{1}{2}, \frac{-2\pi}{4}\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$ $\qquad a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0}, 1(0) - e^{1-1+0}, 1(-1) - e^{1-1+0})$

$= (0-1, 0-1, -1-1)$

$= (-1, -1, -2)$

Q3)

i) $x^2\cos y + e^{xy} = 2$ at $(1,0)$

$f(x,y) = x^2\cos y + e^{xy} - 2$

$f_x = 2x\cos y + ye^{xy}$

$f_y = -x^2\sin y + xe^{xy}$

$(x_0, y_0) = (1,0)$

f_x at $(1,0) = 2(1)\cos 0 + 0$

$= 2$

f_y at $(1,0) = -(1)^2\sin(0) + 1(e)^{1(0)}$

$= 1$

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ ... equ^n of tangent

Now,

For equation of normal

$bx + ay + d = 0$

$\therefore x + 2y + d = 0$

$\therefore (1) + 2(0) + d = 0$ at $(1,0)$

$1 + d = 0$

$d = -1$

$\therefore x + 2y - 1 = 0$ ... equ^n of normal

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2$

$f_x = 2x + 0 - 2 + 0 + 0$ $\therefore f_x$ at $(2,-2) = 2(2) - 2$

$= 2x - 2$ $= 2$

$f_y = 0 + 2y - 0 + 3 + 0$ $\therefore f_y$ at $(2,-2) = 2(-2) + 3$

$= 2y + 3$ $= -1$

$\therefore$ equation of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

$2x - 4 - y - 2 = 0$

$2x - y - 6 = 0$

$\therefore$ equation of normal;

$bx + ay + d = 0$

$-x + 2y + d = 0$

$-(2) + 2(-2) + d = 0$ at $(2,-2)$

$-2 - 4 + d = 0$

$d = 6$

$\therefore -x + 2y + 6 = 0$

(Q4)

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f(x,y,z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x - 0 + 0 + z - 0$
$= 2x + z$

f_x at $(2,1,0) = 2(2) + 0$
$= 4$

$f_y = -2z + 3 + 0 - 0$
$= -2z + 3$

f_y at $(2,1,0) = -2(0) + 3$
$= 3$

$f_z = 0 - 2y + 0 - x - 0$
$= -2y + x$

f_z at $(2,1,0) = -2(1) + 2$
$= 0$

→ equation of tangent;

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$4(x-2) + 3(y-1) + 0(z-0) = 0$

$\therefore 4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0$

→ equation of normal,

$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$

$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z-0}{0}$

ii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$f(x,y,z) = 3yz - x - y + z + 4$

$f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

f_x at $(1,-1,2) = 3(-1)(2) - 1$
$= -7$

$f_y = 3xz - 0 - 1 + 0 + 0$
$= 3xz - 1$

f_y at $(1,-1,2) = 3(1)(2) - 1$
$= 5$

$f_z = 3xy - 0 - 0 + 1 - 0$
$= 3xy + 1$

f_z at $(1,-1,2) = 3(1)(-1) + 1$
$= -2$

equation of tangent;

$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$

$-7(x-1) + 5(y+1) + (-2)(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

Equation of normal;

$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$

$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$

Q.5)

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$\therefore f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$ — (1)

$f_y = 2y - 3x + 0 - 4$
$= 2y - 3x - 4$ — (2)

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — (3)

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — (4)

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Multiplying ③ by ② & subtracting ④ from ③

$4x - \cancel{2y} = -4$

$-\cancel{2y} - 3x = 4$

$7x = 0$

$x = 0$

Substituting value of x in ③

$(0) - y = -2$

$-y = -2$

$y = 2$

∴ Critical points are (0, 2)

Now.

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 - 9$

$= 3 > 0$

Here. $r > 0$ and $rt - s^2 > 0$

∴ f has minimum at (0, 2)

∴ $f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

2) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy - 0$

$= 8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2y$

$= 3x^2 - 2y$

Now,

$f_x = 0$

$8x^3 + 6xy = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 6xy) = 0$

$4x^2 + 6y = 0$ - ①

$f_y = 0$

$3x^2 - 2y = 0$

$3x^2 - 2y = 0$

$3x^2 - 2y = 0$ - ②

Multiply in ① by 3 and ② by 4 and Subtracting ② from ①

$12x^2 + 18y = 0$

$-12x^2 - 8y = 0$

$24y = 0$

$y = 0$ - ③

Substituting ③ in ②

$3x^2 - 2(0) = 0$

$3x^2 = 0$

$x^2 = 0$

$x = 0$ - ④

Critical points are (0, 0)

Now,

$r = f_{xx} = 24x^2 + 6y$

$t = f_{yy} = -2$

$s = f_{xy} = 6x$

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$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$= -48x^2 - 12y - 36x^2$

$= -84x^2 - 12y$

At $(0,0)$

$r = 24(0)^2 + 6(0)$

$= 0$

$s = 6(0) = 0$

$rt - s^2 = -84(0)^2 - 12(0) = 0$

$r = 0$ and $rt - s^2 = 0$

$\therefore$ Nothing can besaid.

iii) $f(x,y) = x^2 - y^2 + 2x + 3y - 70$

$f_x = 2x - 0 + 2 + 0 - 0$ $\quad$ $f_y = -2y + 0 + 8 - 0$

$= 2x + 2$ $\quad$ $= -2y + 8$

$f_x = 0$

$2x + 2 = 0$

$2(x+1) = 0$

$x + 1) = 0$

$x = 1$

$f_y = 0$

$-2y + 8 = 0$

$-2(y - 4) = 0$

$y - 4 = 0$

$y = 4$

$\therefore$ The critical points are $(-1, 4)$

$\therefore r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$rt - s^2 = 2(-2) - 0^2$

$= -4 < 0$

Here $r > 0$ & $rt - s^2 < 0$

$\therefore$ Nothing can said.

Ap
07/02/2022