Are Transformers Effective for Time Series Forecasting?

PDF: https://arxiv.org/pdf/2205.13504v3.pdf
Github: https://github.com/cure-lab/LTSF-Linear

Problem with Transformer in Time-Series Forecasting

- In time series modeling: extract the temporal relations (mối quan hệ thời gian) in an ordered set of continuous points ⇒ the order itself plays the most crucial role
- In Transformer:
  - positional encoding and using tokens to embed sub-series facilitate preserving some ordering information
  - the nature of the permutation-invariant self-attention mechanism inevitably results in temporal information loss
  - This is usually not a serious concern for semantic-rich applications such as NLP e.g., the semantic meaning of a sentence is largely preserved even if we reorder some words in it.

Motivation

- ❖ Not all time series are predictable
- ❖ Long-term forecasting is only feasible for those time series with a relatively clear trend and periodicity
- ❖ Linear models can already extract such information ⇒ introduce a set of simple models:
  - ➢ Linear (new baseline for comparison)
  - ➢ NLinear (Normalization Linear)
  - ➢ DLinear (Decomposition Linear)

Linear

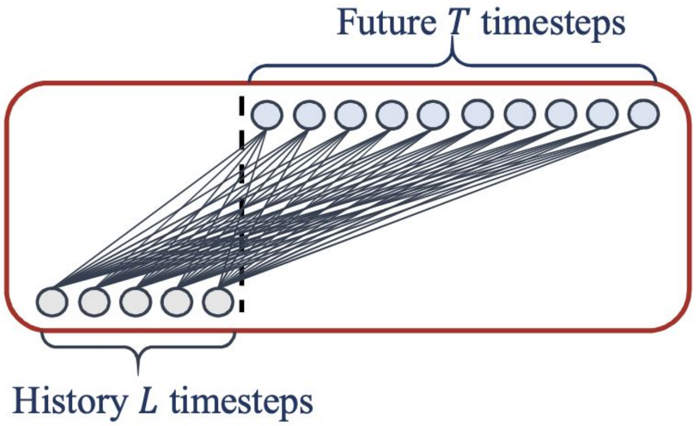


Figure 2: Illustration of the basic linear model.

Algorithm 1 Linear Model

procedure LINEAR(x)

$result \leftarrow Linear(x)$

return $result$

end procedure

- ❖ An O(1) maximum signal traversing path length ⇒ capable of capturing both short-range and long-range temporal relations
- ❖ High-efficiency: costs much lower memory and fewer parameters and has a faster inference speed than existing Transformers
- ❖ Interpretability: After training, can visualize weights from the seasonality and trend branches ⇒ have some insights on the predicted values
- ❖ Easy-to-use: without tuning model hyper-parameters

Algorithm 2 NLinear Model

procedure NLINEAR(x)

$x.shape = [batch\_size, input\_length, num\_features]$

$seq\_last \leftarrow x[:, -1:, :]$

$x \leftarrow x - seq\_last$

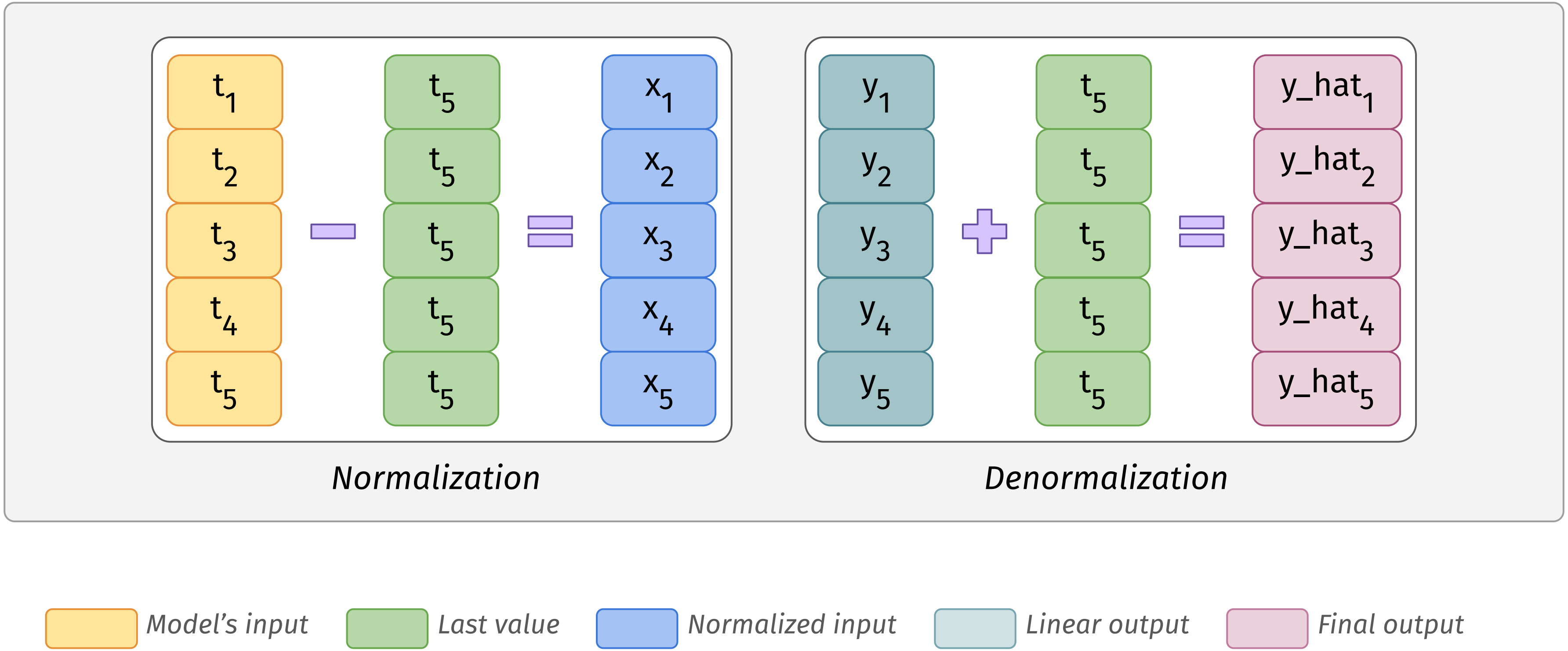
$result \leftarrow Linear(x)$

$result \leftarrow x + seq\_last$

return $result$

end procedure

Normalization in NLinear



Decomposition Linear (DLinear)

Algorithm 3 DLinear Model

```
procedure MovingAverage(x)
    avg ← AveragePooling1D(pool_size = kernel_size, strides = 1, padding = 'valid')
    front ← tile(x[:, 0 : 1, :], multiples = [1, (kernel_size − 1)//2, 1])
    end ← tile(x[:, −1 :, :], multiples = [1, (kernel_size − 1)//2, 1])
    x ← concatenate([front, x, end], axis = 1)
    x ← avg(x)
    return x
end procedure
procedure SeriesDecomposition(x)
    trend_init ← MovingAverage(x)
    seasonal_init ← x − trend_init
    return seasonal_init, trend_init
end procedure
procedure DLinear(x)
    seasonal_init, trend_init ← SeriesDecomposition(x)
    seasonal_output ← Linear_Seasonal(seasonal_init)
    trend_output ← Linear_Trend(trend_init)
    result ← seasonal_output + trend_output
    return result
end procedure
```

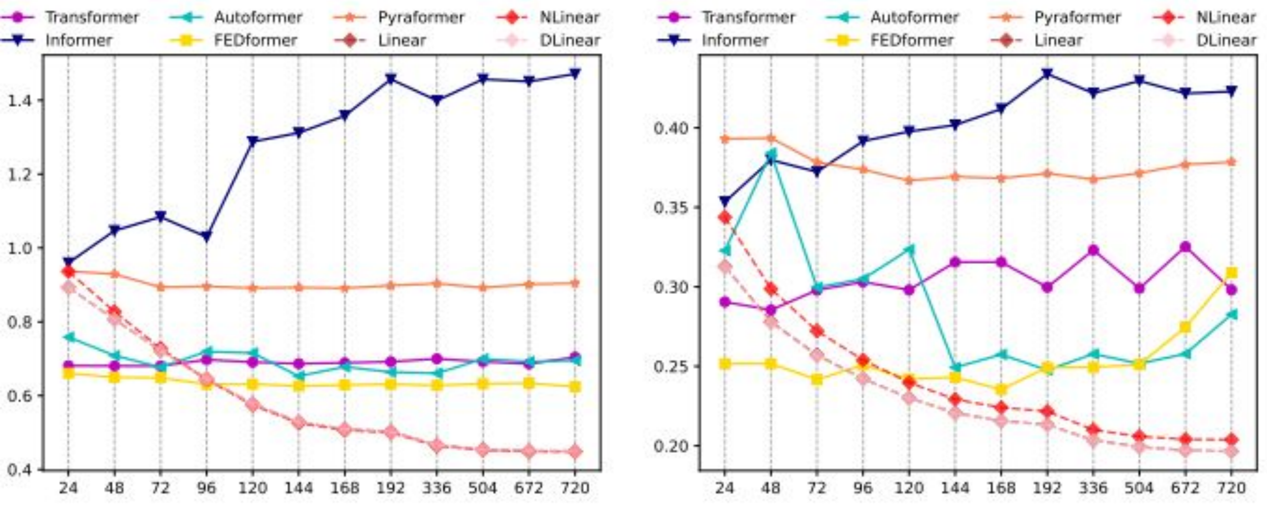
Results

Methods		IMP.	Linear*		NLinear*		DLinear*		FEDformer		Autoformer		Informer		Pyraformer*		LogTrans		*Repeat**	
Metric		MSE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Electricity	96	27.40%	**0.140**	**0.237**	0.141	**0.237**	**0.140**	**0.237**	0.193	0.308	0.201	0.317	0.274	0.368	0.386	0.449	0.258	0.357	1.588	0.946
	192	23.88%	**0.153**	0.250	0.154	**0.248**	**0.153**	0.249	0.201	0.315	0.222	0.334	0.296	0.386	0.386	0.443	0.266	0.368	1.595	0.950
	336	21.02%	**0.169**	0.268	0.171	**0.265**	**0.169**	0.267	0.214	0.329	0.231	0.338	0.300	0.394	0.378	0.443	0.280	0.380	1.617	0.961
	720	17.47%	**0.203**	0.301	0.210	**0.297**	**0.203**	0.301	0.246	0.355	0.254	0.361	0.373	0.439	0.376	0.445	0.283	0.376	1.647	0.975
Exchange	96	45.27%	0.082	0.207	0.089	0.208	**0.081**	0.203	0.148	0.278	0.197	0.323	0.847	0.752	0.376	1.105	0.968	0.812	**0.081**	**0.196**
	192	42.06%	0.167	0.304	0.180	0.300	**0.157**	0.293	0.271	0.380	0.300	0.369	1.204	0.895	1.748	1.151	1.040	0.851	0.167	**0.289**
	336	33.69%	0.328	0.432	0.331	0.415	**0.305**	0.414	0.460	0.500	0.509	0.524	1.672	1.036	1.874	1.172	1.659	1.081	**0.305**	**0.396**
	720	46.19%	0.964	0.750	1.033	0.780	**0.643**	**0.601**	1.195	0.841	1.447	0.941	2.478	1.310	1.943	1.206	1.941	1.127	0.823	0.681
Traffic	96	30.15%	**0.410**	0.282	**0.410**	**0.279**	**0.410**	0.282	0.587	0.366	0.613	0.388	0.719	0.391	2.085	0.468	0.684	0.384	2.723	1.079
	192	29.96%	**0.423**	0.287	**0.423**	**0.284**	**0.423**	0.287	0.604	0.373	0.616	0.382	0.696	0.379	0.867	0.467	0.685	0.390	2.756	1.087
	336	29.95%	0.436	0.295	**0.435**	**0.290**	0.436	0.296	0.621	0.383	0.622	0.337	0.777	0.420	0.869	0.469	0.734	0.408	2.791	1.095
	720	25.87%	0.466	0.315	**0.464**	**0.307**	0.466	0.315	0.626	0.382	0.660	0.408	0.864	0.472	0.881	0.473	0.717	0.396	2.811	1.097
Weather	96	18.89%	**0.176**	0.236	0.182	**0.232**	**0.176**	0.237	0.217	0.296	0.266	0.336	0.300	0.384	0.896	0.556	0.458	0.490	0.259	0.254
	192	21.01%	**0.218**	0.276	0.225	**0.269**	0.220	0.282	0.276	0.336	0.307	0.367	0.598	0.544	0.622	0.624	0.658	0.589	0.309	0.292
	336	22.71%	**0.262**	0.312	0.271	**0.301**	0.265	0.319	0.339	0.380	0.359	0.395	0.578	0.523	0.739	0.753	0.797	0.652	0.377	0.338
	720	19.85%	0.326	0.365	0.338	**0.348**	**0.323**	0.362	0.403	0.428	0.419	0.428	1.059	0.741	1.004	0.934	0.869	0.675	0.465	0.394
ILI	24	47.86%	1.947	0.985	**1.683**	**0.858**	2.215	1.081	3.228	1.260	3.483	1.287	5.764	1.677	1.420	2.012	4.480	1.444	6.587	1.701
	36	36.43%	2.182	1.036	**1.703**	**0.859**	1.963	0.963	2.679	1.080	3.103	1.148	4.755	1.467	7.394	2.031	4.799	1.467	7.130	1.884
	48	34.43%	2.256	1.060	**1.719**	**0.884**	2.130	1.024	2.622	1.078	2.669	1.085	4.763	1.469	7.551	2.057	4.800	1.468	6.575	1.798
	60	34.33%	2.390	1.104	**1.819**	**0.917**	2.368	1.096	2.857	1.157	2.770	1.125	5.264	1.564	7.662	2.100	5.278	1.560	5.893	1.677
ETTh1	96	0.80%	0.375	0.397	**0.374**	**0.394**	0.375	0.399	0.376	0.419	0.449	0.459	0.865	0.713	0.664	0.612	0.878	0.740	1.295	0.713
	192	3.57%	0.418	0.429	0.408	**0.415**	**0.405**	0.416	0.420	0.448	0.500	0.482	1.008	0.792	0.790	0.681	1.037	0.824	1.325	0.733
	336	6.54%	0.479	0.476	**0.429**	**0.427**	0.439	0.443	0.459	0.465	0.521	0.496	1.107	0.809	0.891	0.738	1.238	0.932	1.323	0.744
	720	13.04%	0.624	0.592	**0.440**	**0.453**	0.472	0.490	0.506	0.507	0.514	0.512	1.181	0.865	0.963	0.782	1.135	0.852	1.339	0.756
ETTh2	96	19.94%	0.288	0.352	**0.277**	**0.338**	0.289	0.353	0.346	0.388	0.358	0.397	3.755	1.525	0.645	0.597	2.116	1.197	0.432	0.422
	192	19.81%	0.377	0.413	**0.344**	**0.381**	0.383	0.418	0.429	0.439	0.456	0.452	5.602	1.931	0.788	0.683	4.315	1.635	0.534	0.473
	336	25.93%	0.452	0.461	**0.357**	**0.400**	0.448	0.465	0.496	0.487	0.482	0.486	4.721	1.835	0.907	0.747	1.124	1.604	0.591	0.508
	720	14.25%	0.698	0.595	**0.394**	**0.436**	0.605	0.551	0.463	0.474	0.515	0.511	3.647	1.625	0.963	0.783	3.188	1.540	0.588	0.517
ETTm1	96	21.10%	0.308	0.352	0.306	0.348	**0.299**	**0.343**	0.379	0.419	0.505	0.475	0.672	0.571	0.543	0.510	0.600	0.546	1.214	0.665
	192	21.36%	0.340	0.369	0.349	0.375	**0.335**	**0.365**	0.426	0.441	0.553	0.496	0.795	0.669	0.557	0.537	0.837	0.700	1.261	0.690
	336	17.07%	0.376	0.393	0.375	0.388	**0.369**	**0.386**	0.445	0.459	0.621	0.537	1.212	0.871	0.754	0.655	1.124	0.832	1.283	0.707
	720	21.73%	0.440	0.435	0.433	0.422	**0.425**	**0.421**	0.543	0.490	0.671	0.561	1.166	0.823	0.908	0.724	1.153	0.820	1.319	0.729
ETTm2	96	17.73%	0.168	0.262	**0.167**	**0.255**	**0.167**	0.260	0.203	0.287	0.255	0.339	0.365	0.453	0.435	0.507	0.768	0.642	0.266	0.328
	192	17.84%	0.232	0.308	**0.221**	**0.293**	0.224	0.303	0.269	0.328	0.281	0.340	0.533	0.563	0.730	0.673	0.989	0.757	0.340	0.371
	336	15.69%	0.320	0.373	**0.274**	**0.327**	0.281	0.342	0.325	0.366	0.339	0.372	1.363	0.887	1.201	0.845	1.334	0.872	0.412	0.410
	720	12.58%	0.413	0.435	**0.368**	**0.384**	0.397	0.421	0.421	0.415	0.433	0.432	3.379	1.338	3.625	1.451	3.048	1.328	0.521	0.465

- Methods* are implemented by us; Other results are from FEDformer [31].

Table 2. Multivariate long-term forecasting errors in terms of MSE and MAE, the lower the better. Among them, ILI dataset is with forecasting horizon $T \in \{24, 36, 48, 60\}$. For the others, $T \in \{96, 192, 336, 720\}$. *Repeat* repeats the last value in the look-back window. The **best results** are highlighted in **bold** and the best results of Transformers are highlighted with a underline. Accordingly, IMP. is the best result of linear models compared to the results of Transformer-based solutions.

Can existing LTSF-Transformers extract temporal relations well from longer input sequences?



(a) **720** steps-**Traffic** (b) **720** steps-**Electricity**

Figure 4. The MSE results (Y-axis) of models with different look-back window sizes (X-axis) of long-term forecasting (T=**720**) on the Traffic and Electricity datasets.

- ❖ Existing Transformer-based models' performance deteriorates or stays stable when the look-back window size increases
- ❖ The performances of all LTSF-Linear are significantly boosted with the increase of look-back window size

Are the self-attention scheme effective for LTSF?

Methods		Informer	*Att.-Linear*	*Embed + Linear*	Linear
Exchange	96	0.847	1.003	0.173	0.084
	192	1.204	0.979	0.443	0.155
	336	1.672	1.498	1.288	0.301
	720	2.478	2.102	2.026	0.763
ETTh1	96	0.865	0.613	0.454	0.400
	192	1.008	0.759	0.686	0.438
	336	1.107	0.921	0.821	0.479
	720	1.181	0.902	1.051	0.515

Table 4. The MSE comparisons of gradually transforming Informer to a Linear from the left to right columns. *Att.-Linear* is a structure that replaces each attention layer with a linear layer. *Embed + Linear* is to drop other designs and only keeps embedding layers and a linear layer. The look-back window size is 96.

Can existing LTSF-Transformers preserve temporal order well?

Methods		Linear			FEDformer			Autoformer			Informer		
Predict Length		*Ori.*	*Shuf.*	*Half-Ex.*	*Ori.*	*Shuf.*	*Half-Ex.*	*Ori.*	*Shuf.*	*Half-Ex.*	*Ori.*	*Shuf.*	*Half-Ex.*
Exchange	96	0.080	0.133	0.169	0.161	0.160	0.162	0.152	0.158	0.160	0.952	1.004	0.959
	192	0.162	0.208	0.243	0.274	0.275	0.275	0.278	0.271	0.277	1.012	1.023	1.014
	336	0.286	0.320	0.345	0.439	0.439	0.439	0.435	0.430	0.435	1.177	1.181	1.177
	720	0.806	0.819	0.836	1.122	1.122	1.122	1.113	1.113	1.113	1.198	1.210	1.196
	Average Drop	N/A	27.26%	46.81%	N/A	-0.09%	0.20%	N/A	0.09%	1.12%	N/A	-0.12%	-0.18%
ETTh1	96	0.395	0.824	0.431	0.376	0.753	0.405	0.455	0.838	0.458	0.974	0.971	0.971
	192	0.447	0.824	0.471	0.419	0.730	0.436	0.486	0.774	0.491	1.233	1.232	1.231
	336	0.490	0.825	0.505	0.447	0.736	0.453	0.496	0.752	0.497	1.693	1.693	1.691
	720	0.520	0.846	0.528	0.468	0.720	0.470	0.525	0.696	0.524	2.720	2.716	2.715
	Average Drop	N/A	81.06%	4.78%	N/A	73.28%	3.44%	N/A	56.91%	0.46%	N/A	1.98%	0.18%

Table 5. The MSE comparisons of models when shuffling the raw input sequence. *Shuf.* randomly shuffles the input sequence. *Half-EX.* randomly exchanges the first half of the input sequences with the second half. Average Drop is the average performance drop under all forecasting lengths after shuffling. All results are the average test MSE of five runs.

For the ETTh1 dataset, FEDformer and Autoformer introduce time series inductive bias into their models ⇒ can extract certain temporal information when the dataset has more clear temporal patterns ⇒ suffer when shuffle all the order information

Is training data size a limiting factor for existing LTSF-Transformers?

Methods	FEDformer		Autoformer	
Dataset	*Ori.*	*Short*	*Ori.*	*Short*
96	0.587	**0.568**	0.613	**0.594**
192	0.604	**0.584**	**0.616**	0.621
336	0.621	**0.601**	0.622	**0.621**
720	0.626	**0.608**	0.660	**0.650**

Table 7. The MSE comparison of two training data sizes.

Dataset: Traffic
Ori.: full dataset (17,544*0.7 hours)
Short: shortened dataset (8,760 hours)

Is efficiency really a top-level priority?

Method	MACs	Parameter	Time	Memory
DLinear	**0.04G**	**139.7K**	**0.4ms**	**687MiB**
Transformer×	4.03G	13.61M	26.8ms	6091MiB
Informer	3.93G	14.39M	49.3ms	3869MiB
Autoformer	4.41G	14.91M	164.1ms	7607MiB
Pyraformer	0.80G	241.4M*	3.4ms	7017MiB
FEDformer	4.41G	20.68M	40.5ms	4143MiB

- × is modified into the same one-step decoder, which is implemented in the source code from Autoformer.
- ∗ 236.7M parameters of Pyraformer come from its linear decoder.

Table 8. Comparison of practical efficiency of LTSF-Transformers under L=96 and T=720 on the Electricity. MACs are the number of multiply-accumulate operations. We use Dlinear for comparison since it has the double cost in *LTSF-Linear*. The inference time averages 5 runs.

Conclusion

- Most of the existing Transformer fail to extract temporal relations from long sequences, i.e., the forecasting errors are not reduced (sometimes even increased) with the increase of look-back window sizes
- The temporal modeling capabilities of Transformers for time series are exaggerated, at least for the existing LTSF benchmarks

Thank you for listening