AI VIETNAM
All-in-One Course

Multi-layer Perception

Activation and Initialization

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Ph.D. in Computer Science

Year 2023

Outline

- Pipeline Recommendation
- Data Normalization
- Activation Functions
- MLP Examples
- Initialization Methods

To-do List for Training

Data Preparation

Data Normalization

Model (Network) Construction

Parameter Initialization

Optimizer Selection

Metric Selection

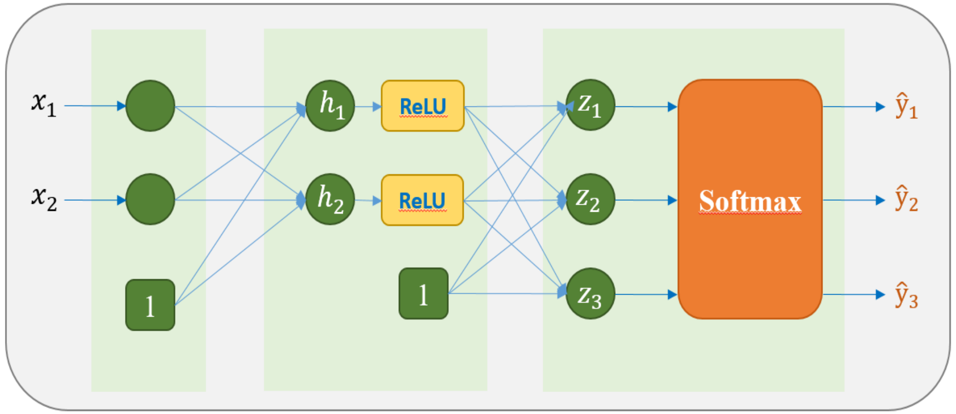
Loss function Selection

Data Preparation

Training Data

Used to train model
(Teach the model by examples)

Model

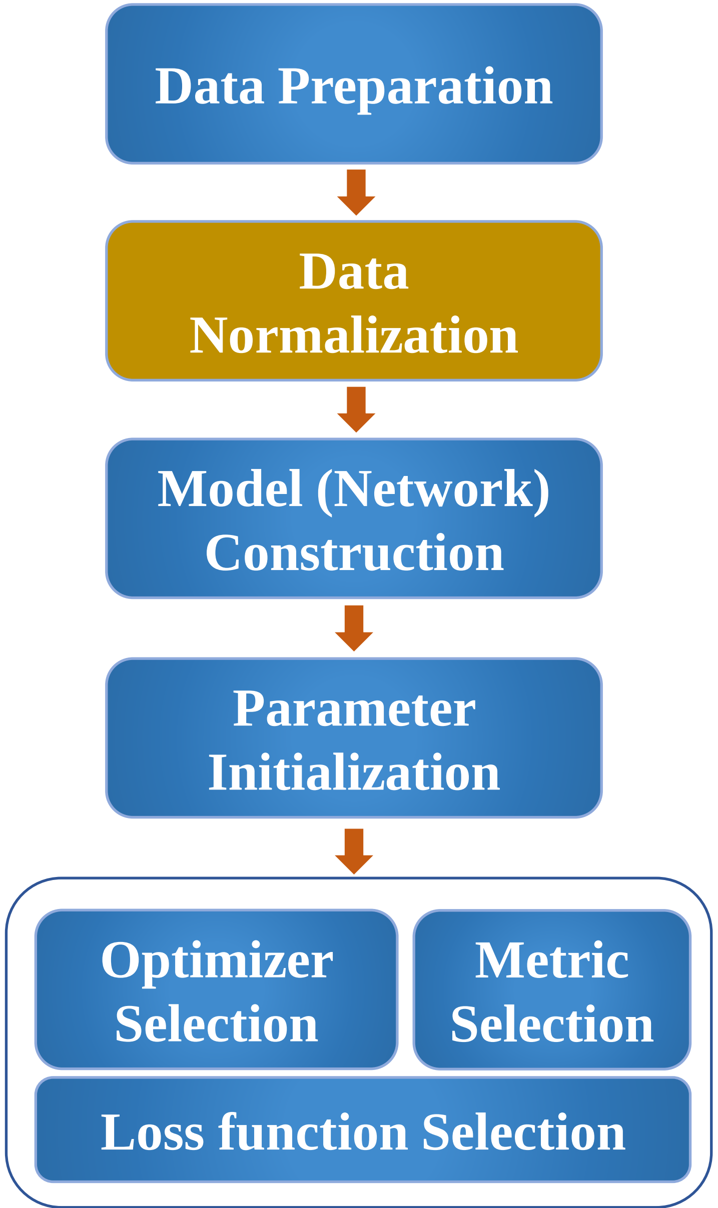


$\neq$

Testing Data

Used to validate model
(Check how good the model is)

Data Normalization



In Theory

$$X \in [0, 255]$$

Convert to the range [0,1]

$$\text{Image} = \frac{\text{Image}}{255}$$

Convert to the range [-1,1]

$$\text{Image} = \frac{\text{Image}}{127.5} - 1$$

Z-score normalization

$$\text{Image} = \frac{\text{Image} - \mu}{\sigma}$$

In Pytorch

$$X \in [0, 1]$$

Normalize(*mean*, std)

$$\text{Image} = \frac{\text{Image} - mean}{\text{std}}$$

[0,1]	mean = 0 ; std = 1
[-1,1]	mean = 0.5; std = 0.5

Compute mean and std from data

1

```python
transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize((0.5,), (0.5,))])

trainset = torchvision.datasets.FashionMNIST(root='data', train=True, download=True, transform=transform)
trainloader = torch.utils.data.DataLoader(trainset, batch_size=1024, num_workers=10, shuffle=True)

testset = torchvision.datasets.FashionMNIST(root='data', train=False, download=True, transform=transform)
testloader = torch.utils.data.DataLoader(testset, batch_size=1024, num_workers=10, shuffle=False)
```

2

```python
transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize((0,), (1.0,))])

trainset = torchvision.datasets.FashionMNIST(root='data', train=True, download=True, transform=transform)
trainloader = torch.utils.data.DataLoader(trainset, batch_size=1024, num_workers=10, shuffle=True)

testset = torchvision.datasets.FashionMNIST(root='data', train=False, download=True, transform=transform)
testloader = torch.utils.data.DataLoader(testset, batch_size=1024, num_workers=10, shuffle=False)
```

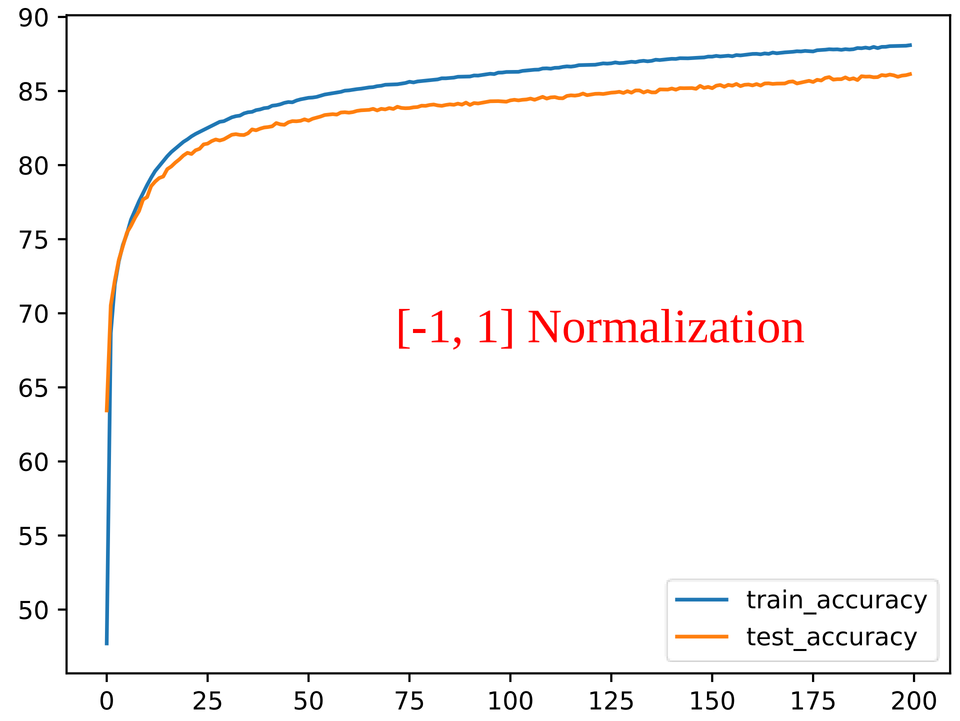
3

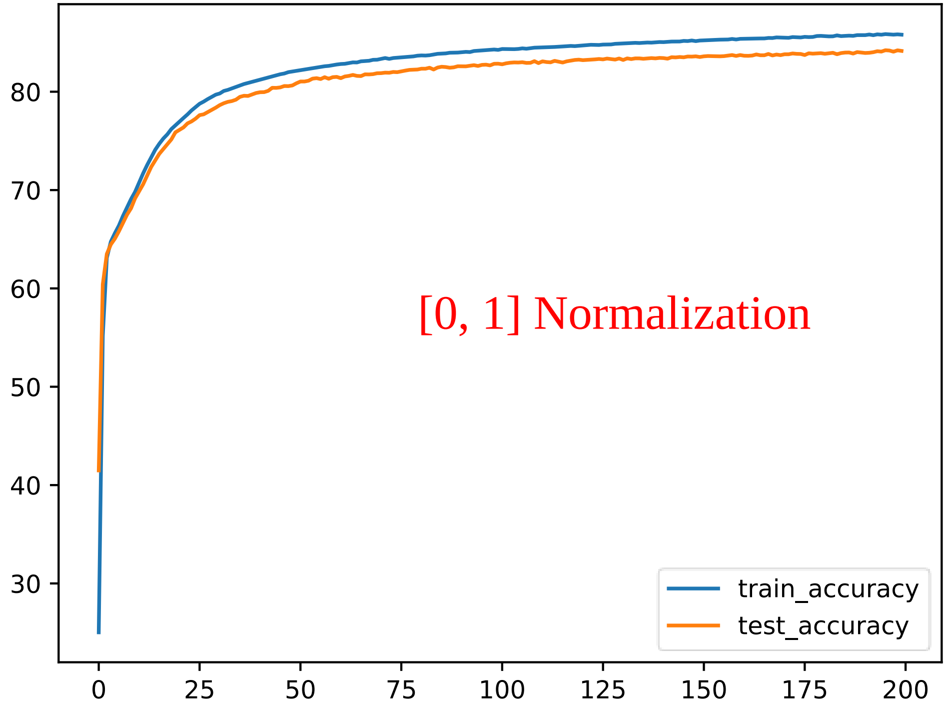
```python
# computed mean and std in advance
transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize((mean,), (std,))])

trainset = torchvision.datasets.FashionMNIST(root='data', train=True, download=True, transform=transform)
trainloader = torch.utils.data.DataLoader(trainset, batch_size=1024, num_workers=10, shuffle=True)

testset = torchvision.datasets.FashionMNIST(root='data', train=False, download=True, transform=transform)
testloader = torch.utils.data.DataLoader(testset, batch_size=1024, num_workers=10, shuffle=False)
```

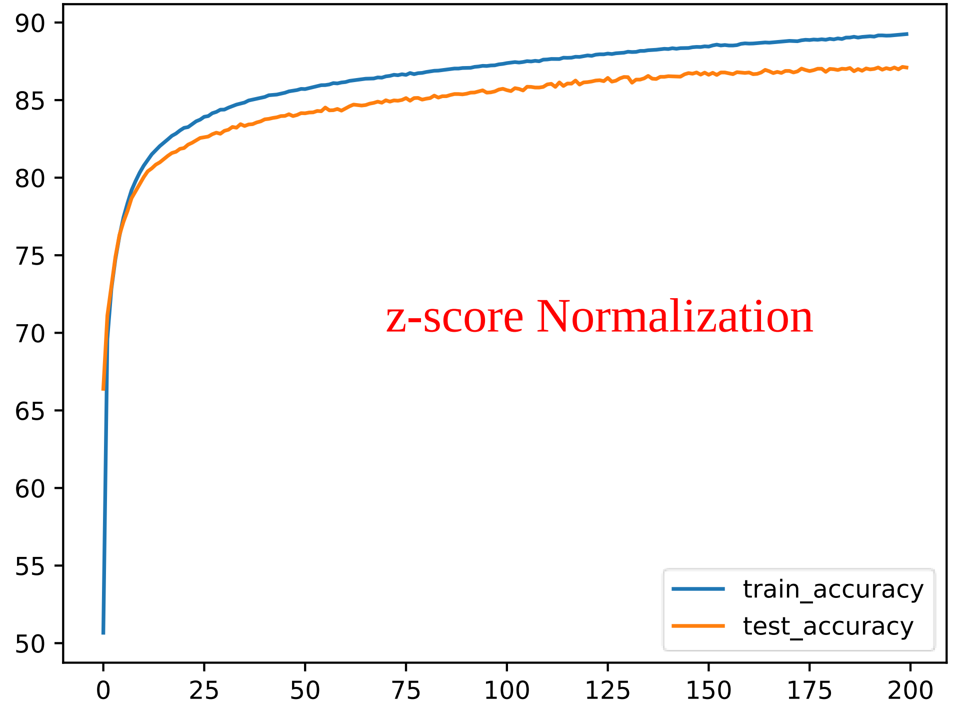
(a) [0, 1] Normalization (b) [-1, 1] Normalization (c) z-score Normalization

Data Normalization

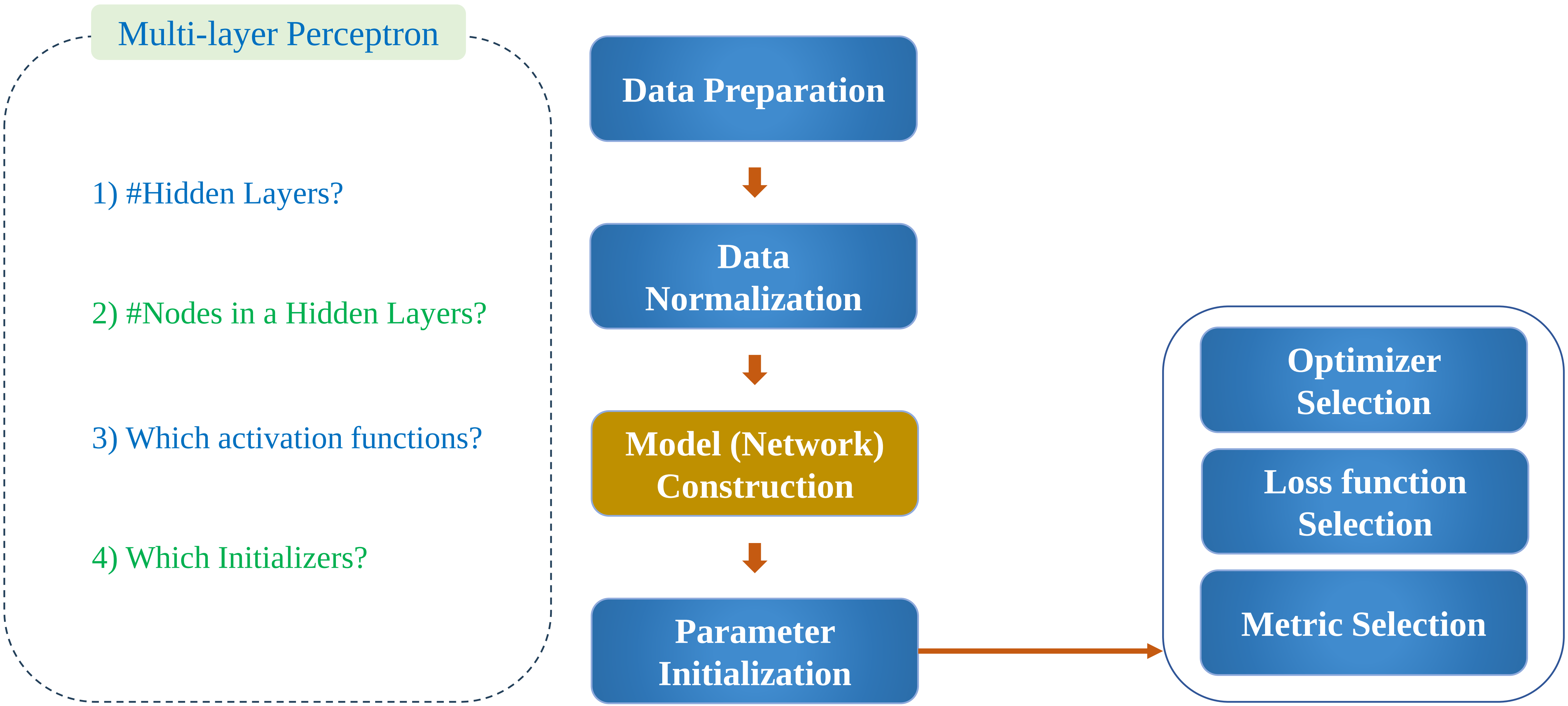
```python
model = nn.Sequential(
    nn.Flatten(), nn.Linear(784, 256),
    nn.ReLU(), nn.Linear(256, 10)
)
criterion = nn.CrossEntropyLoss()
optimizer = optim.SGD(model.parameters(),
                      lr=0.01)
```





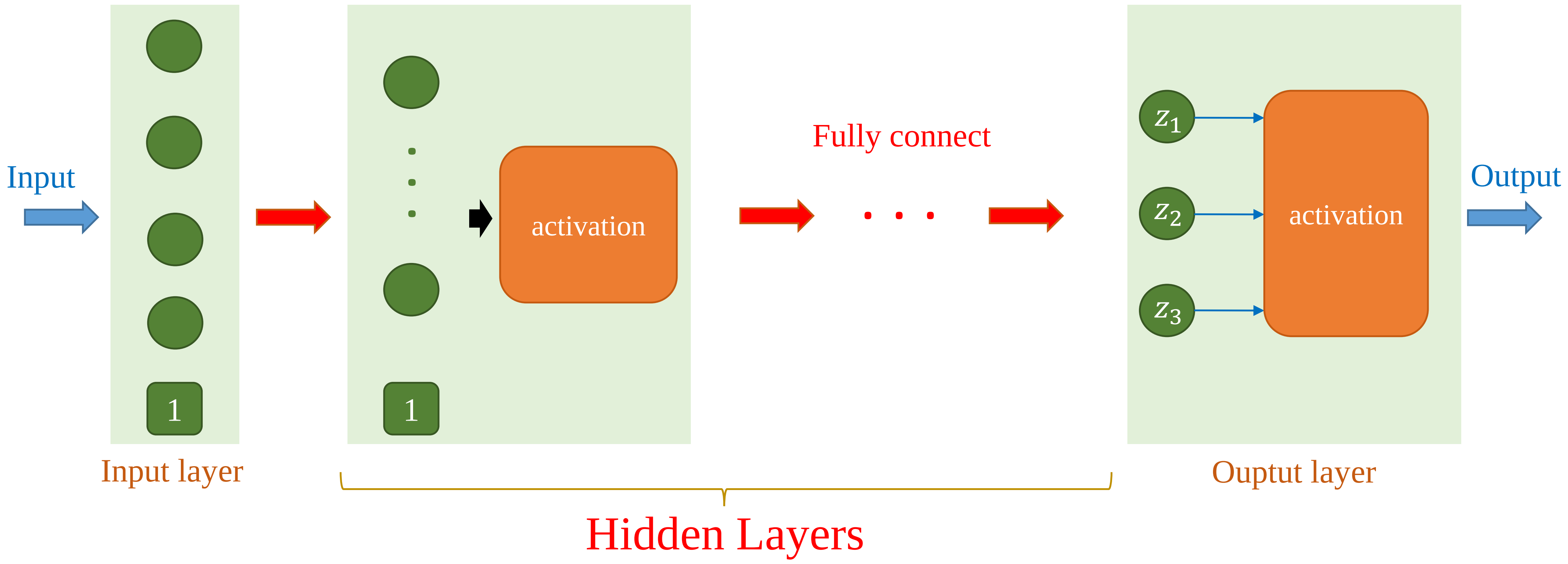


Training Pipeline



Training Pipeline

Model (Network) Construction

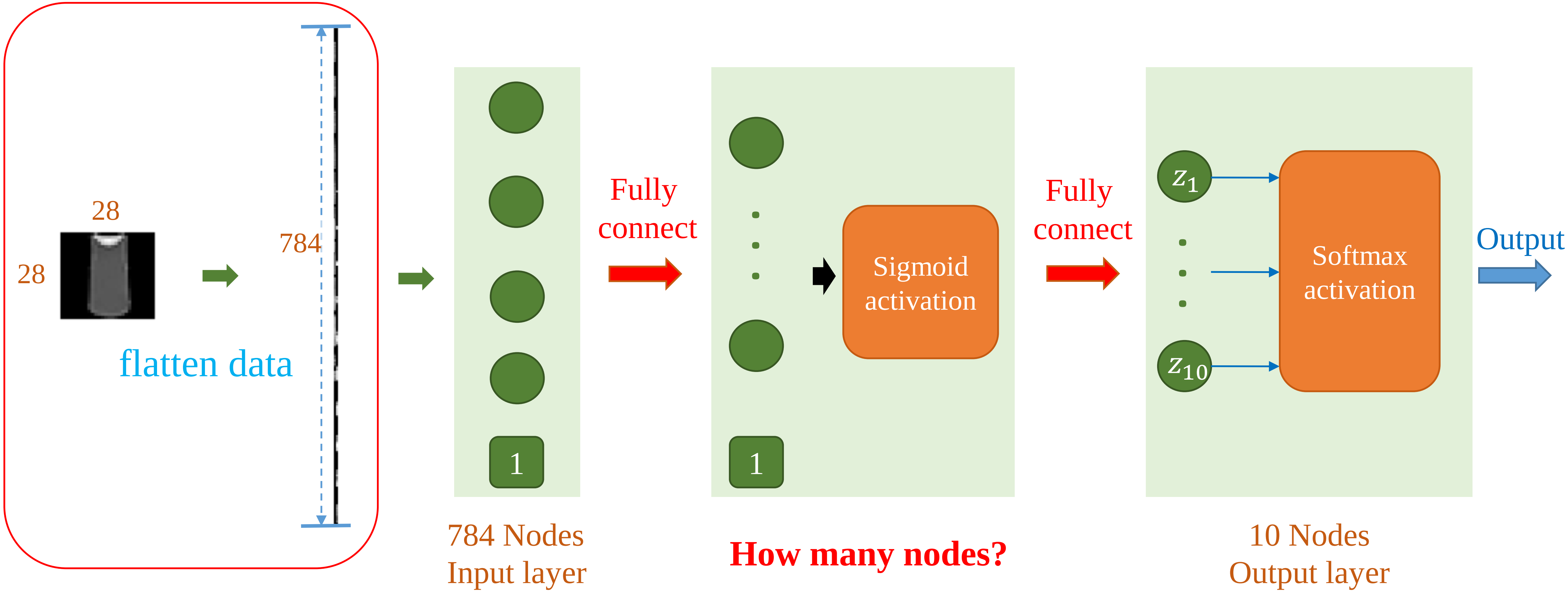


How many hidden layers?
How many nodes in a hidden layer?

Which activation function?
Which network components?

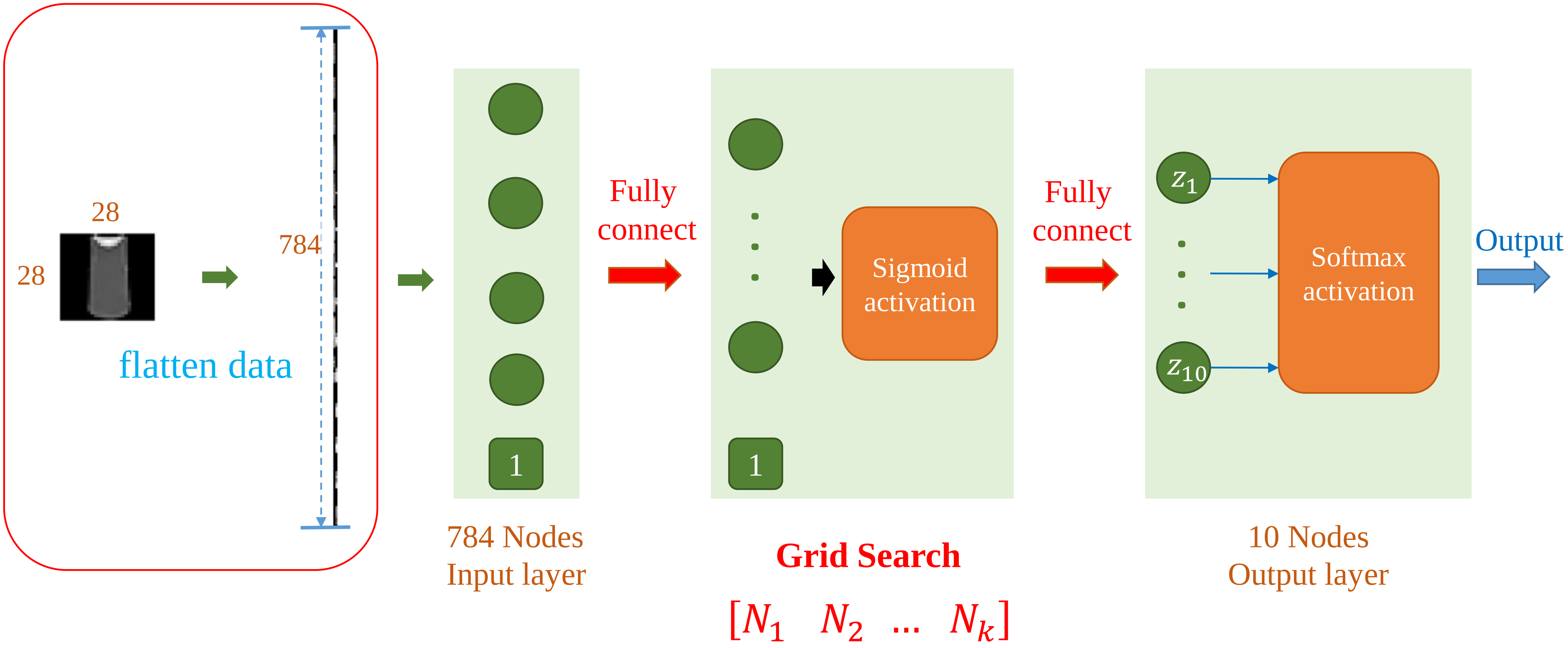
How many nodes?

Model (Network) Construction



How many nodes?

Model (Network) Construction

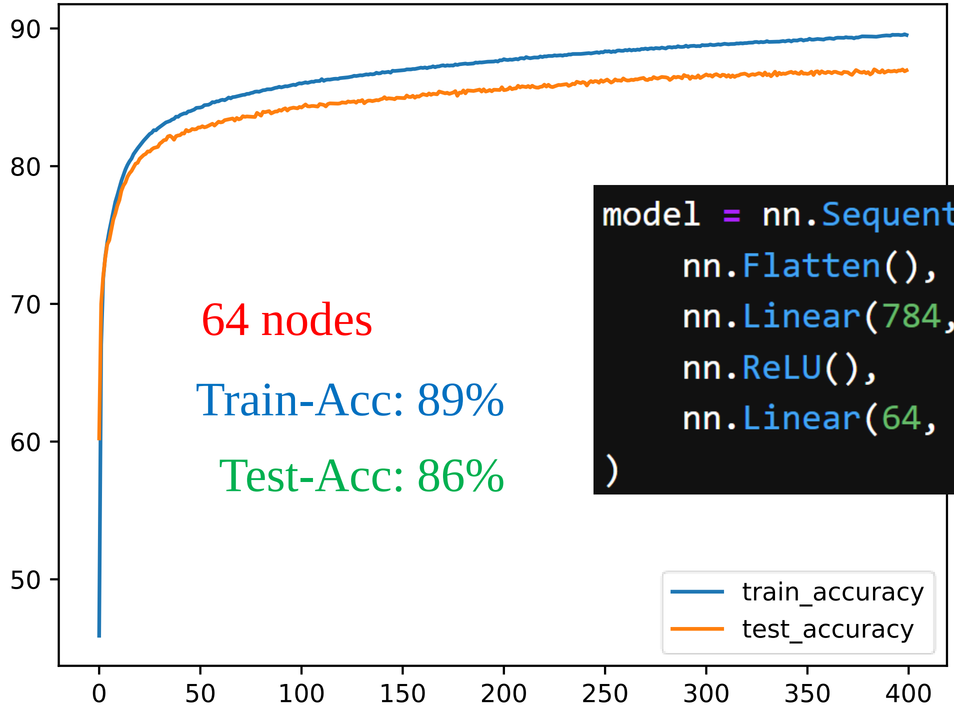


How many nodes?

[-1, 1] Normalization

Cross-entropy Loss

SGD with lr=0.01

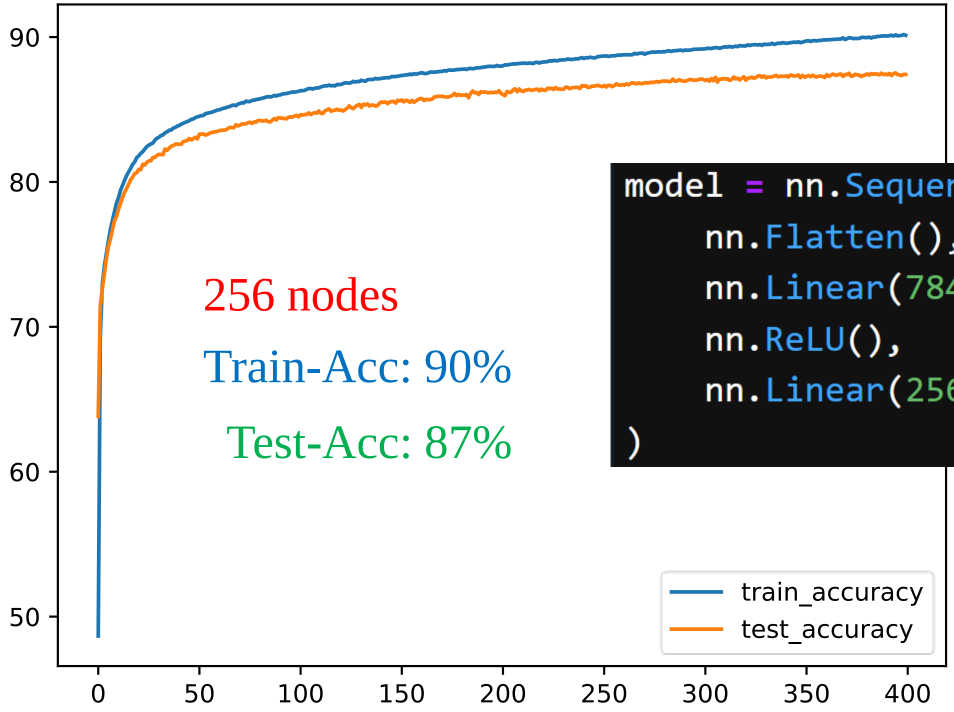


64 nodes

Train-Acc: 89%

Test-Acc: 86%

```
model = nn.Sequential(
    nn.Flatten(),
    nn.Linear(784, 64),
    nn.ReLU(),
    nn.Linear(64, 10)
)
```

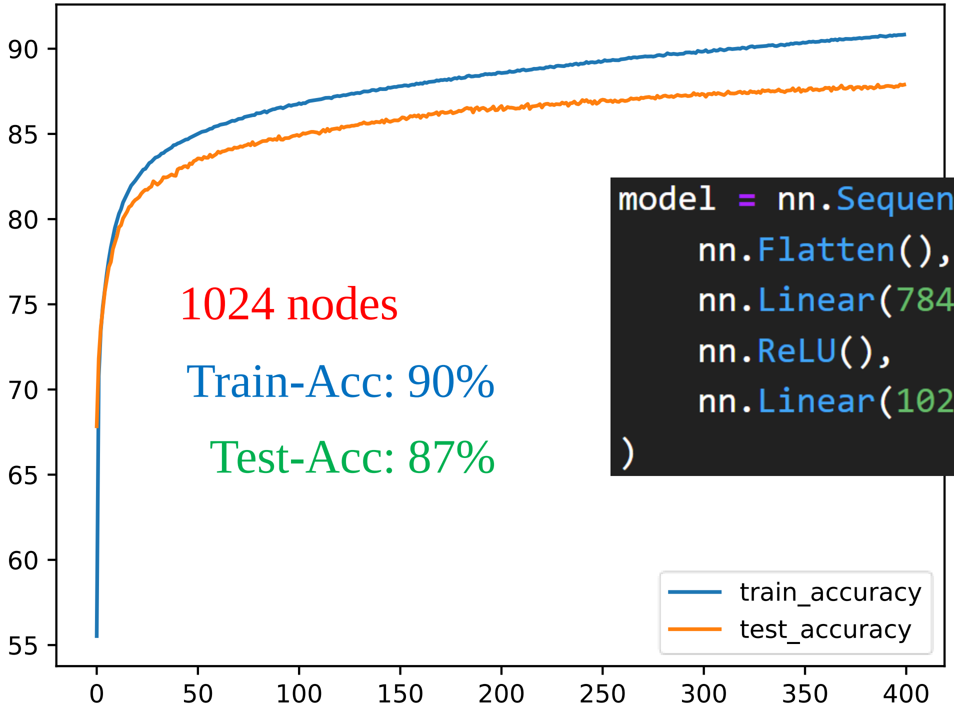


256 nodes

Train-Acc: 90%

Test-Acc: 87%

```
model = nn.Sequential(
    nn.Flatten(),
    nn.Linear(784, 256),
    nn.ReLU(),
    nn.Linear(256, 10)
)
```

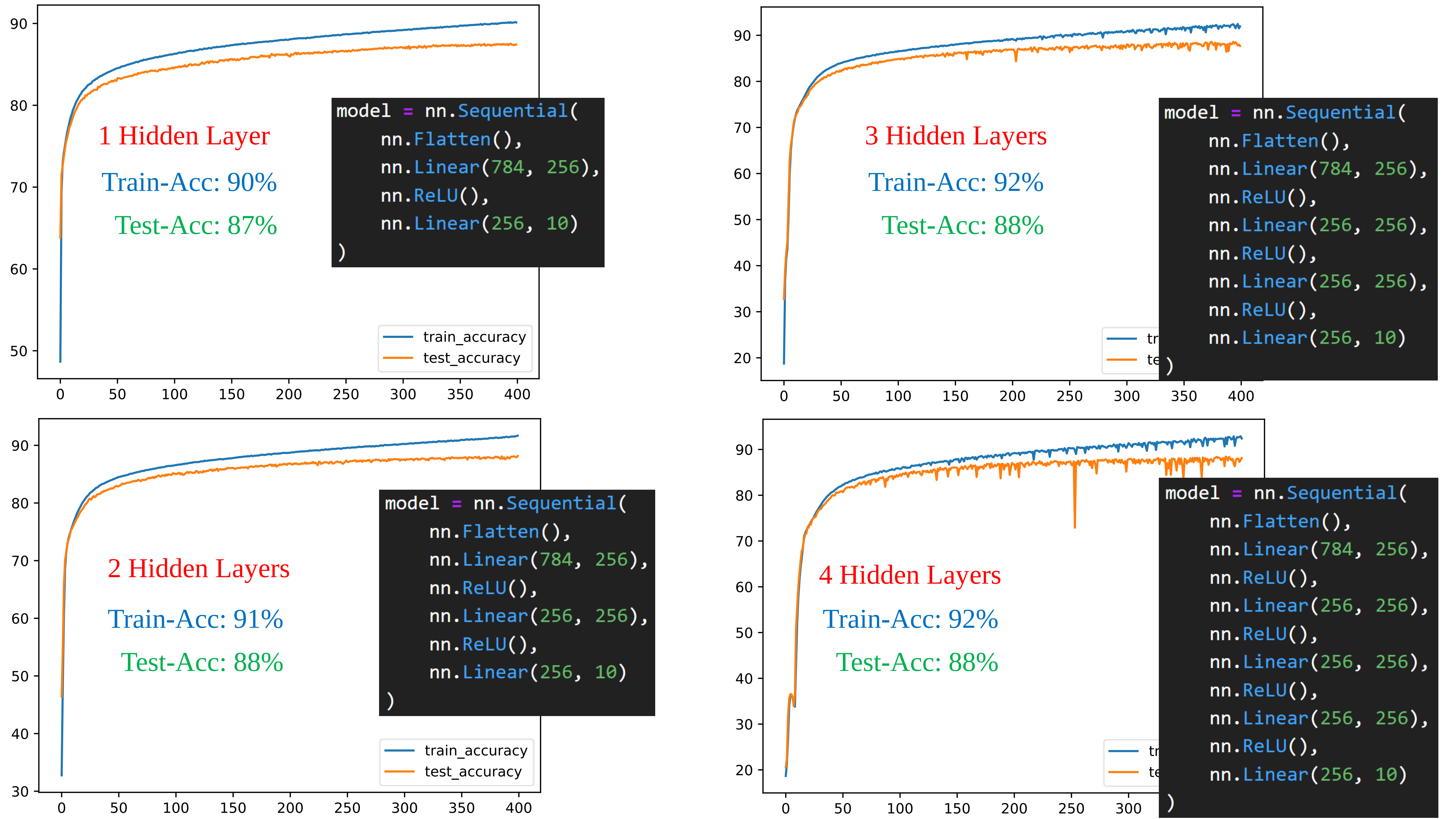


1024 nodes

Train-Acc: 90%

Test-Acc: 87%

```
model = nn.Sequential(
    nn.Flatten(),
    nn.Linear(784, 1024),
    nn.ReLU(),
    nn.Linear(1024, 10)
)
```


1 Hidden Layer
Train-Acc: 90%
Test-Acc: 87%
model = nn.Sequential(
nn.Flatten(),
nn.Linear(784, 256),
nn.ReLU(),
nn.Linear(256, 10)
)
train_accuracy
test_accuracy
3 Hidden Layers
Train-Acc: 92%
Test-Acc: 88%
model = nn.Sequential(
nn.Flatten(),
nn.Linear(784, 256),
nn.ReLU(),
nn.Linear(256, 256),
nn.ReLU(),
nn.Linear(256, 256),
nn.ReLU(),
nn.Linear(256, 10)
)
2 Hidden Layers
Train-Acc: 91%
Test-Acc: 88%
model = nn.Sequential(
nn.Flatten(),
nn.Linear(784, 256),
nn.ReLU(),
nn.Linear(256, 256),
nn.ReLU(),
nn.Linear(256, 10)
)
train_accuracy
test_accuracy
4 Hidden Layers
Train-Acc: 92%
Test-Acc: 88%
model = nn.Sequential(
nn.Flatten(),
nn.Linear(784, 256),
nn.ReLU(),
nn.Linear(256, 256),
nn.ReLU(),
nn.Linear(256, 256),
nn.ReLU(),
nn.Linear(256, 256),
nn.ReLU(),
nn.Linear(256, 10)
)

Activation Functions

Model (Network) Construction

Which activation function?

$$\text{sigmoid}(x) = \frac{1}{1+e^{-x}}$$

2010

$$\text{ReLU}(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

2017

$$SELU(x) = \begin{cases} \lambda x & \text{if } x \geq 0 \\ \lambda\alpha(e^x - 1) & \text{if } x < 0 \end{cases}$$

$$\lambda \approx 1.0507$$

$$\alpha \approx 1.6733$$

$$\tanh(x) = \frac{2}{1+e^{-2x}} - 1$$

2015

$$\text{ELU}(x) = \begin{cases} \alpha(e^x - 1) & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

2001

$$\text{softplus}(x) = \log(1 + e^x)$$

2015

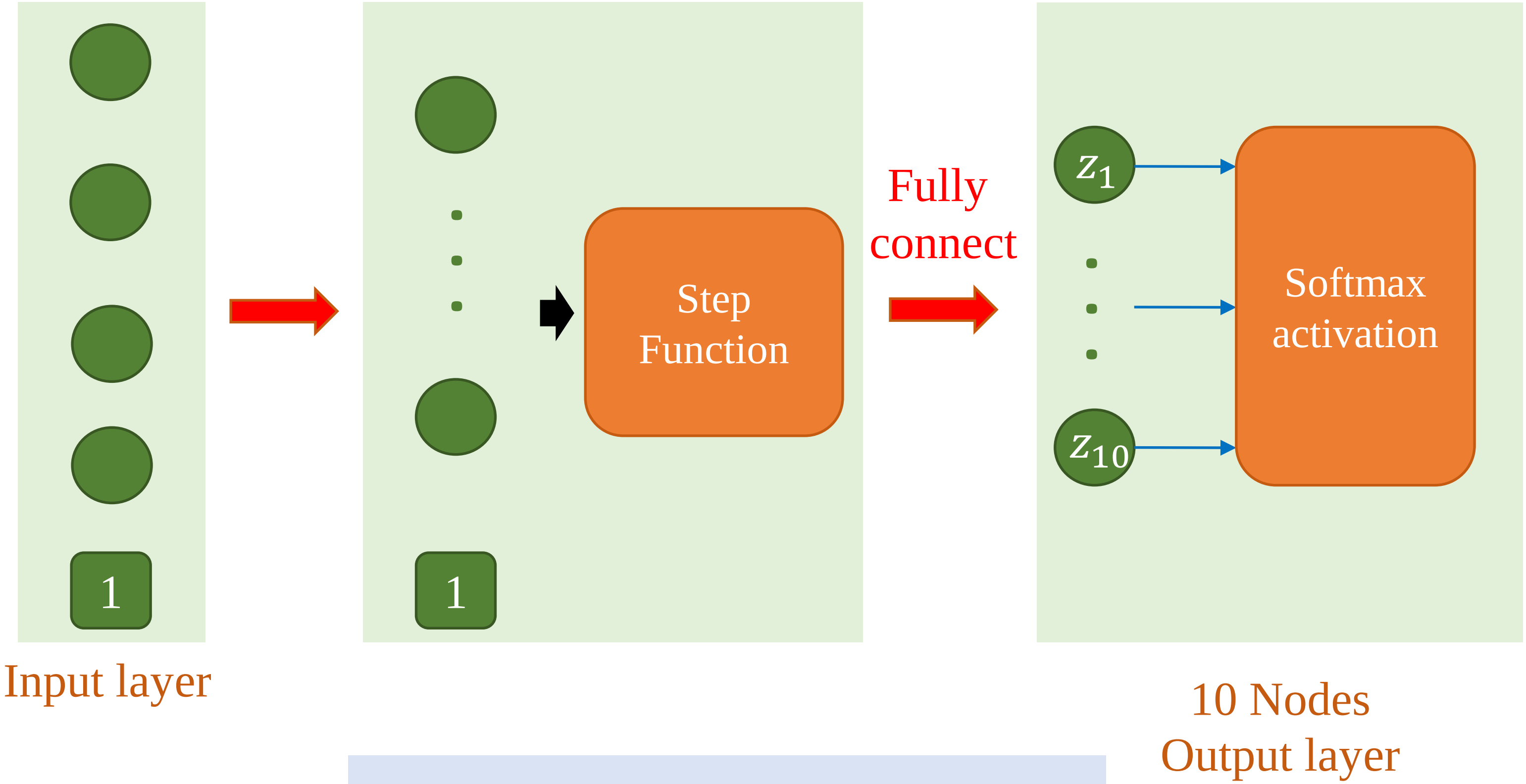
$$\text{PReLU}(x) = \begin{cases} \alpha x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

2017

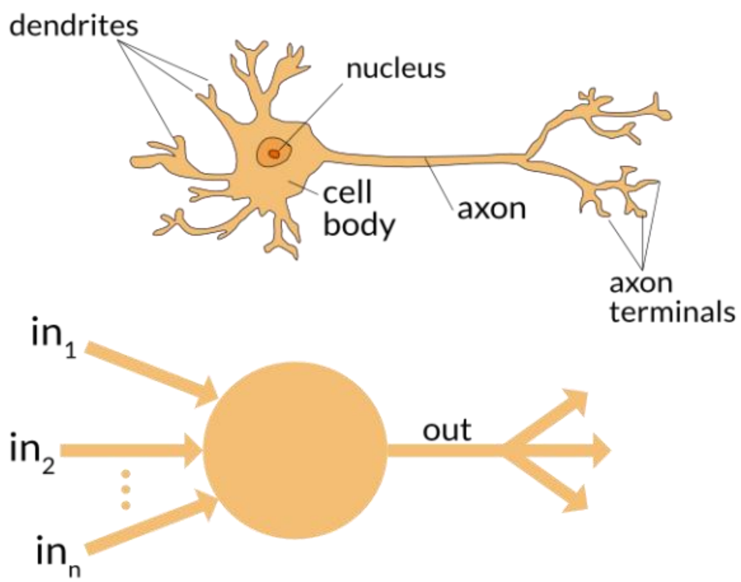
$$swish(x) = x * \frac{1}{1+e^{-x}}$$

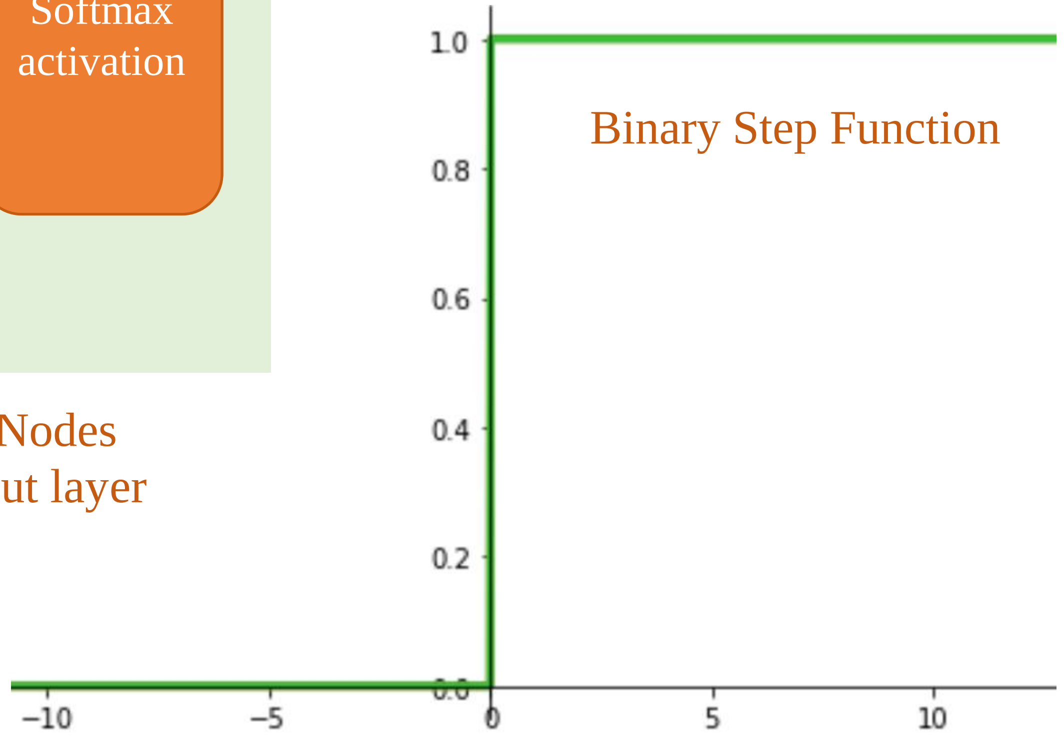
Activation Functions

❖ Step function



$$f(x) = \begin{cases} 0 & if \;\; x < 0 \\ 1 & if \;\; x \geq 0 \end{cases}$$





https://www.analyticsvidhya.com/blog/2020/01/fundamentals-deep-learning-activation-functions-when-to-use-them/

Activation Functions

❖ Sigmoid function

$$\text{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

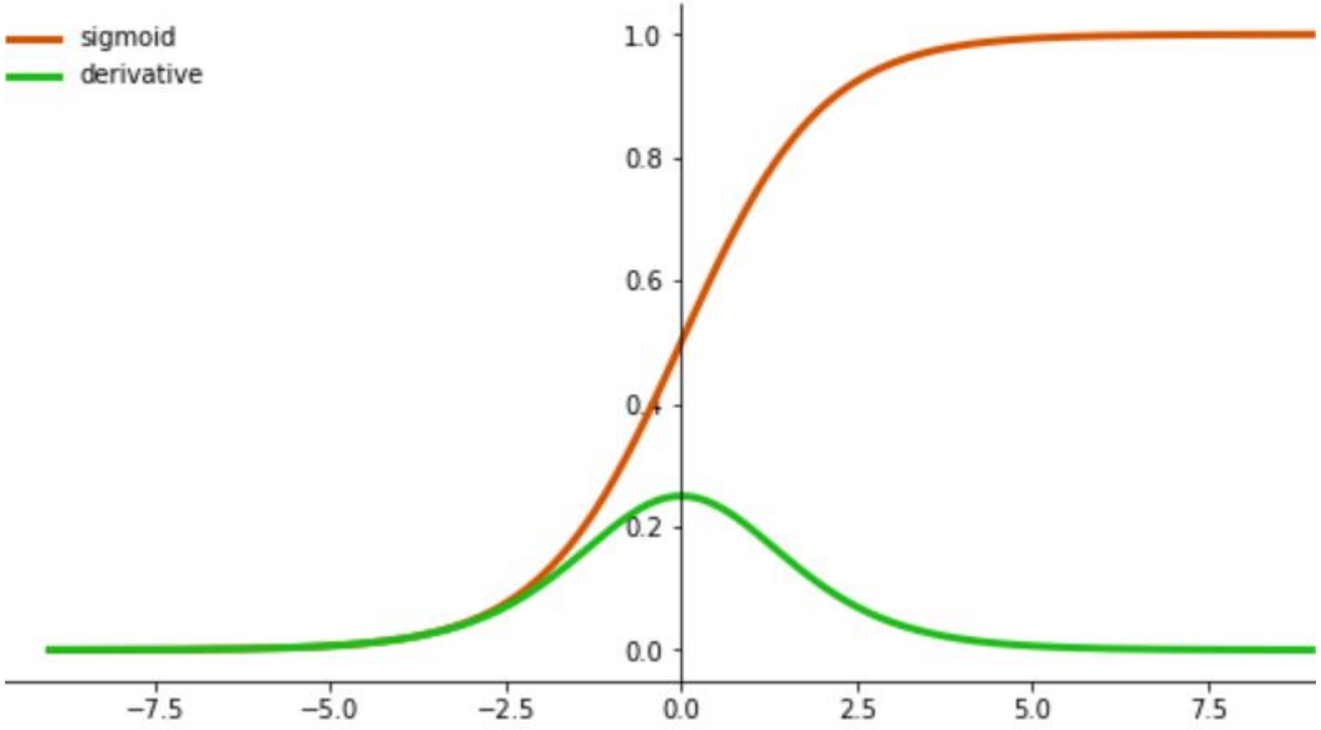
data =

1	5	-4	3	-2

data_a = sigmoid(data)

data_a =

0.731	0.993	0.017	0.95	0.119



$$\text{sigmoid}'(x) = \text{sigmoid}(x)\,(1 - \text{sigmoid}(x))$$

Activation Functions

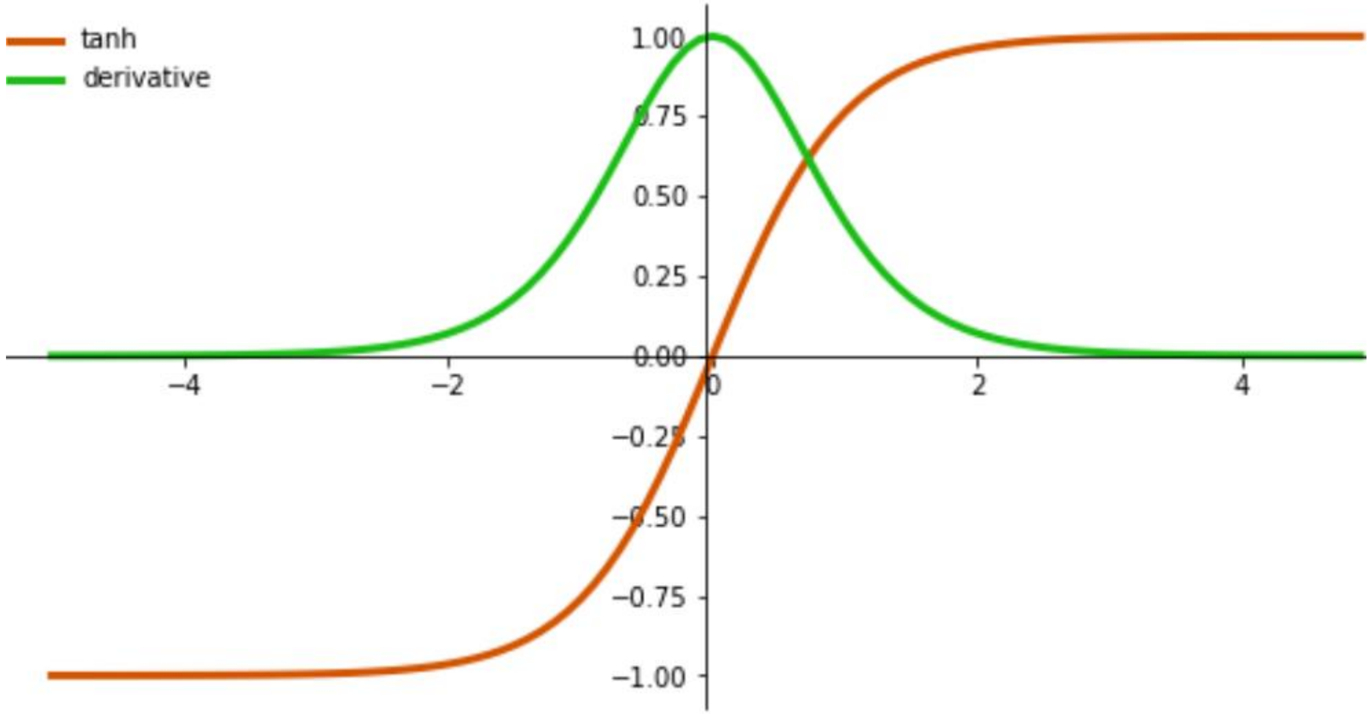
$$\text{sigmoid}(x) = \frac{1}{1+e^{-x}}$$

$$\begin{aligned}
\text{sigmoid}'(x) &= \left(\frac{1}{1+e^{-x}}\right)' = \frac{-1}{(1+e^{-x})^2}(-e^{-x}) \\
&= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{e^{-x}+1-1}{(1+e^{-x})^2} \\
&= \frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^2} \\
&= \frac{1}{1+e^{-x}}\left(1 - \frac{1}{1+e^{-x}}\right) \\
&= \text{sigmoid}(x)\,(1-\text{sigmoid}(x))
\end{aligned}$$

Activation Functions

❖ Tanh function

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
$$= \frac{2}{1 + e^{-2x}} - 1$$
$$= 1 - \frac{2}{e^{2x} + 1}$$



data =

1	5	-4	3	-2

data_a = tanh(data)

data_a =

0.761	0.999	-0.999	0.995	-0.964

$$tanh'(x) = 1 - tanh^2(x)$$

Activation Functions

$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = 1 - \frac{2}{e^{2x} + 1} = \frac{2}{e^{-2x} + 1} - 1$$

$$\tanh'(x) = \left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right)' = \frac{(e^{x} + e^{-x})(e^{x} + e^{-x}) - (e^{x} - e^{-x})(e^{x} - e^{-x})}{(e^{x} + e^{-x})^2}$$

$$= \frac{(e^{x} + e^{-x})^2 - (e^{x} - e^{-x})^2}{(e^{x} + e^{-x})^2}$$

$$= 1 - \left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right)^2 = 1 - tanh^2(x)$$

Activation Functions

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = 1 - \frac{2}{e^{2x}+1} = \frac{2}{e^{-2x}+1} - 1$$

$$tanh'(x) = \left(\frac{2}{e^{-2x}+1} - 1\right)' = \frac{4e^{-2x}}{(e^{-2x}+1)^2} = 4\left(\frac{e^{-2x}+1-1}{(e^{-2x}+1)^2}\right)$$

$$= 4\left(\frac{1}{e^{-2x}+1} - \frac{1}{(e^{-2x}+1)^2}\right) = -\left(\frac{4}{(e^{-2x}+1)^2} - \frac{4}{e^{-2x}+1}\right)$$

$$= -\left(\frac{4}{(e^{-2x}+1)^2} - \frac{4}{e^{-2x}+1} + 1 - 1\right) = 1 - \left(\frac{2}{e^{-2x}+1} - 1\right)^2 = 1 - tanh^2(x)$$

Activation Functions

❖ Softplus function

$$\text{softplus}(x) = \log(1 + e^x)$$

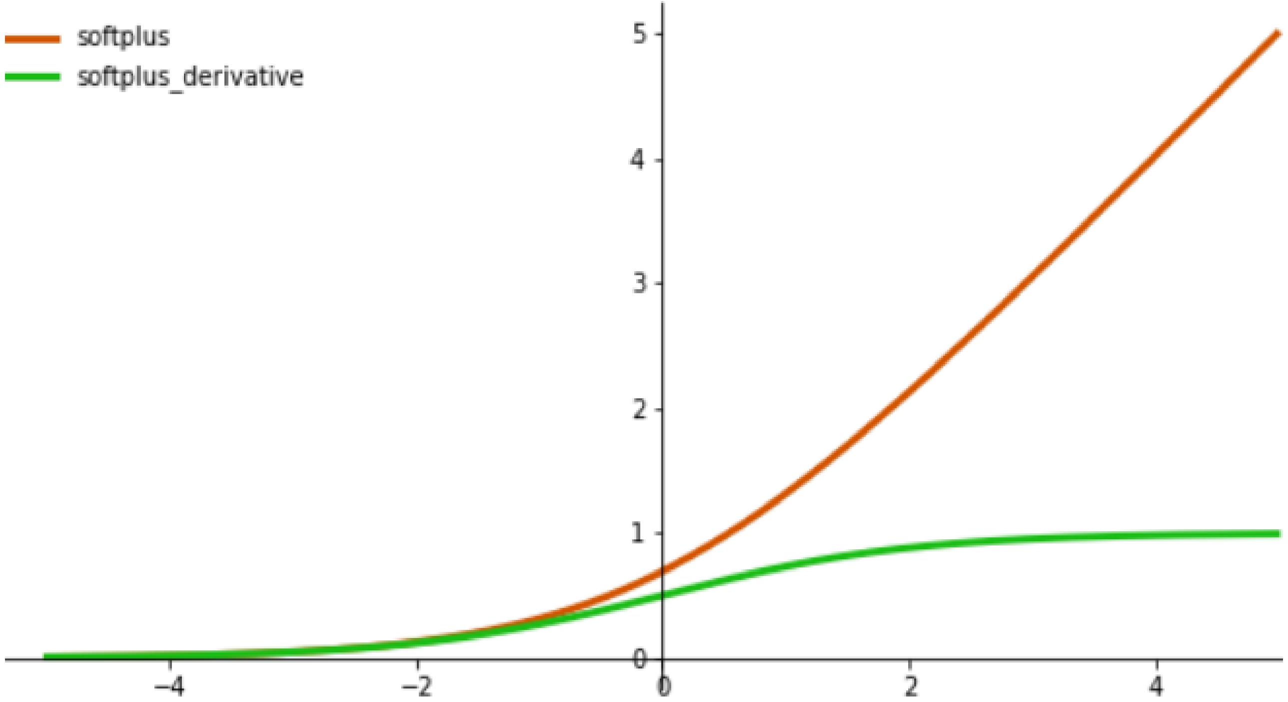
data =

1	5	-4	3	-2

data_a = softplus(data)

data_a =

1.313	5.006	0.018	3.048	0.126



$$\text{softplus}'(x) = \frac{1}{1 + e^{-x}}$$

Activation Functions

❖ ReLU function

$$\mathrm{ReLU}(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

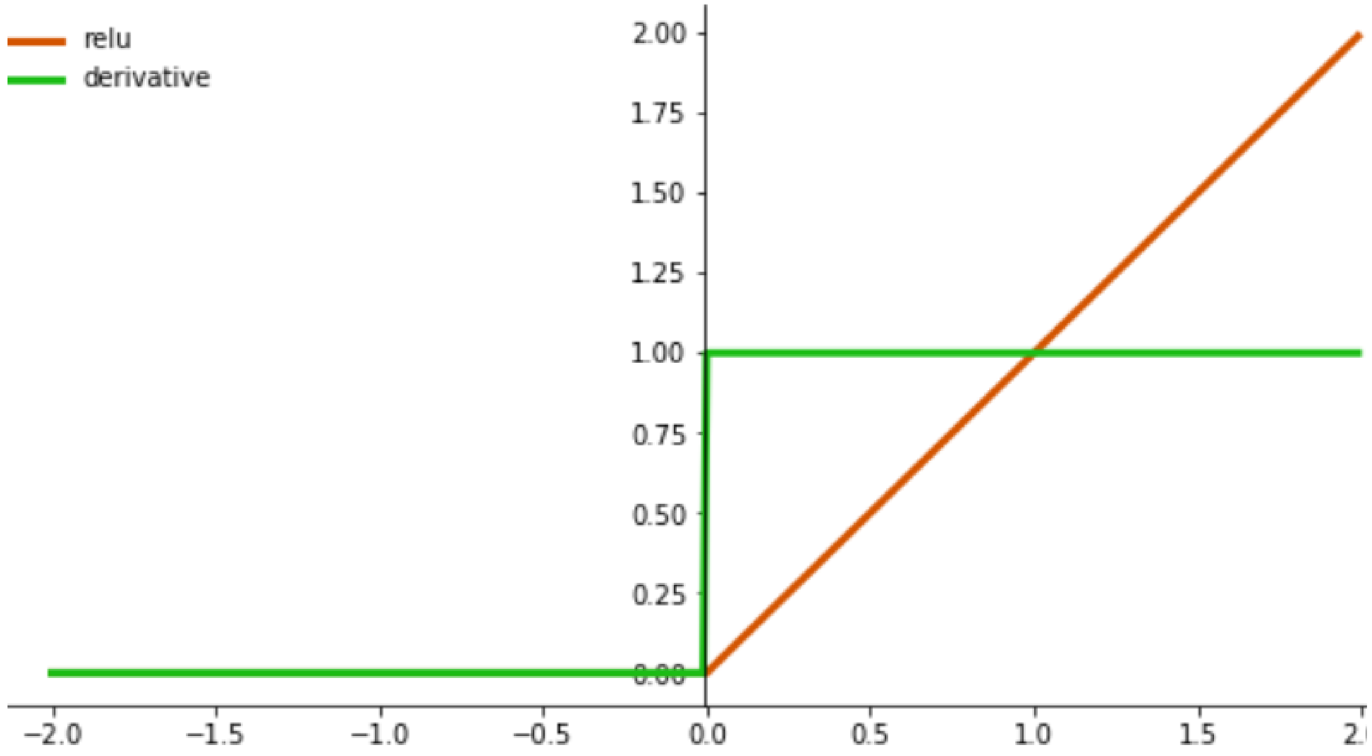
data =

1	5	-4	3	-2

data_a = ReLU(data)

data_a =

1	5	0	3	0



$$\mathrm{ReLU}'(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Activation Functions

❖ LeakyReLU function

$$\text{LeakyReLU}(x) = \begin{cases} 0.01x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

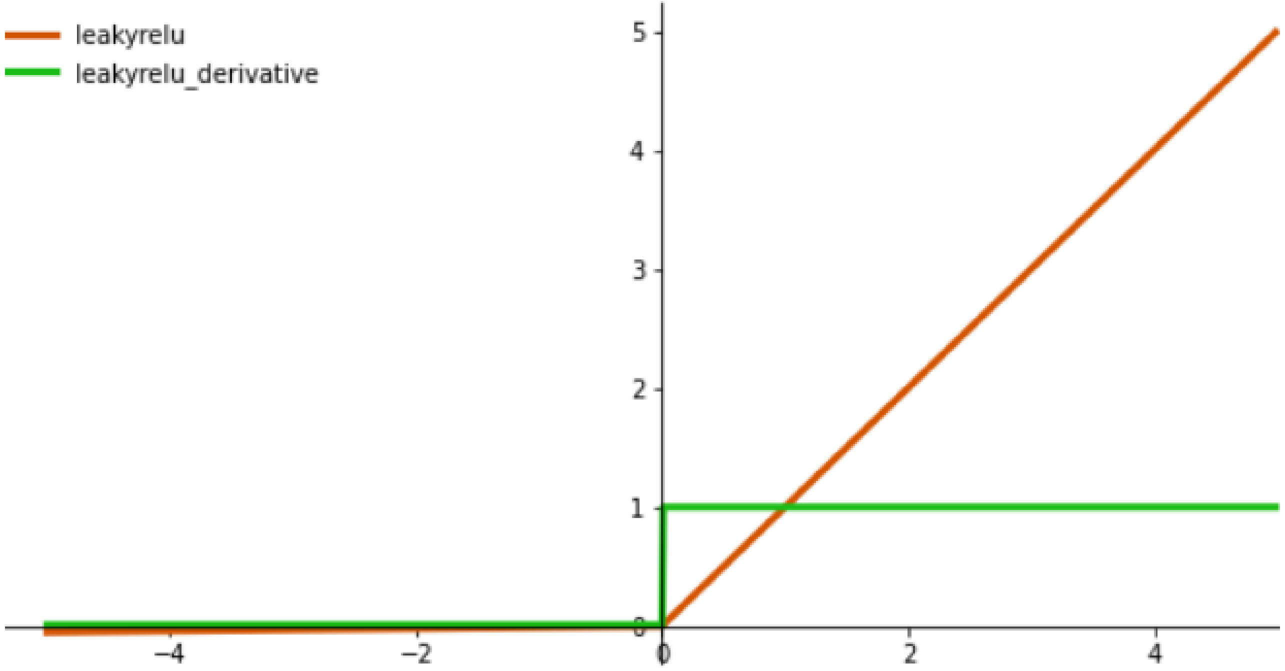
data =

1	5	-4	3	-2

data_a = leakyrelu(data)

data_a =

1	5	-0.04	3	-0.02



$$\text{LeakyReLU}'(x) = \begin{cases} 0.01 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Activation Functions

❖ ELU function

$$\text{ELU}(x) = \begin{cases} \alpha(e^x - 1) & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

$$\alpha = 0.1$$

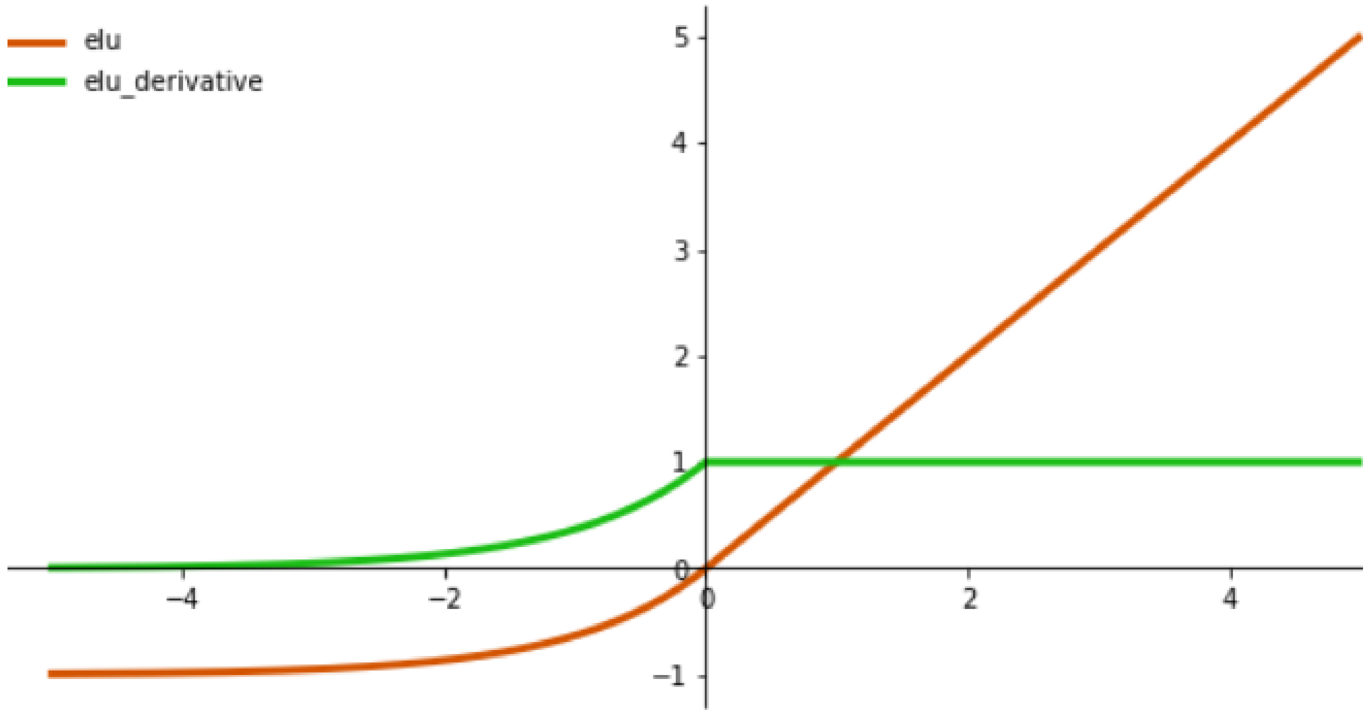
data =

1	5	-4	3	-2

data_a = ELU(data)

data_a =

1	5	-0.098	3	-0.086



$$\text{ELU}'(x) = \begin{cases} \alpha e^x & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Activation Functions

❖ PReLU function

$$\text{PReLU}(x) = \begin{cases} \alpha x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

$$\alpha = 0.1$$

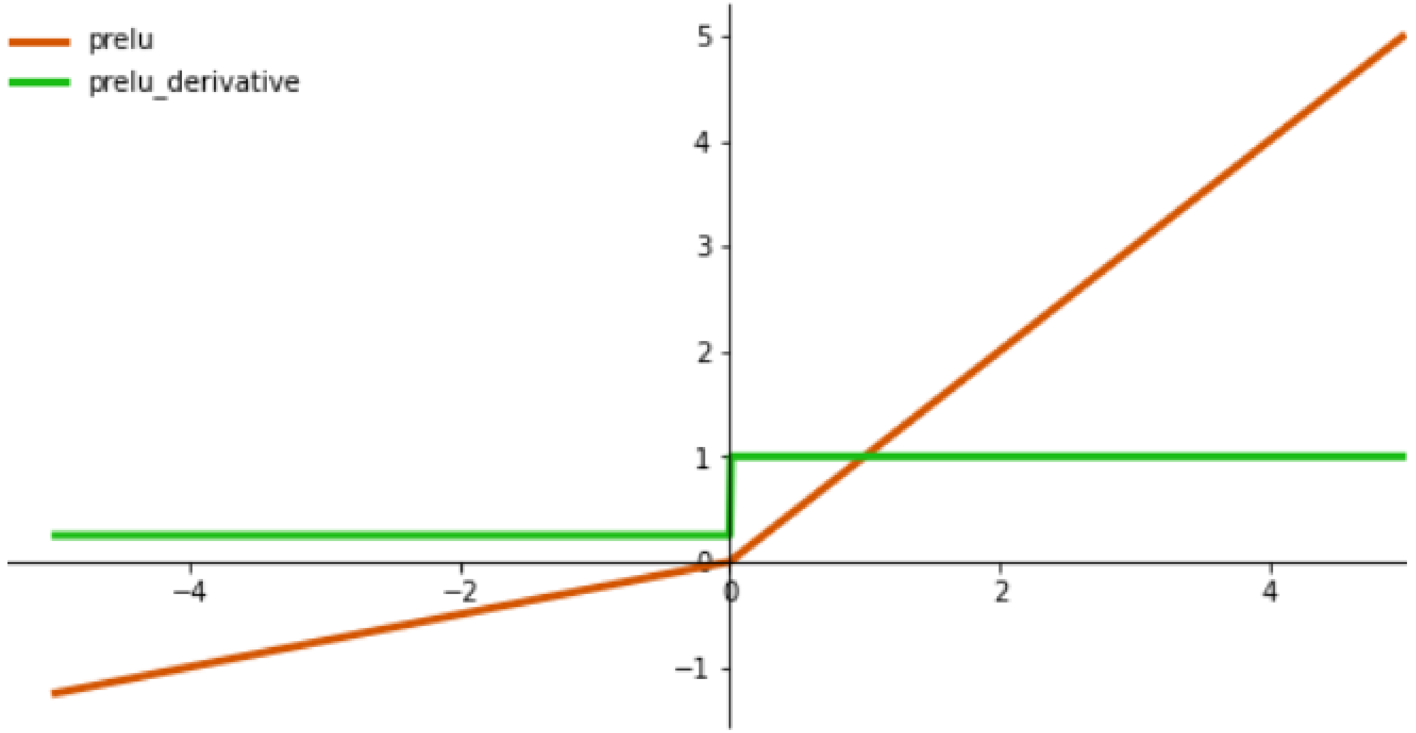
data =

1	5	-4	3	-2

data_a = PRELU(data)

data_a =

1	5	-0.4	3	-0.2



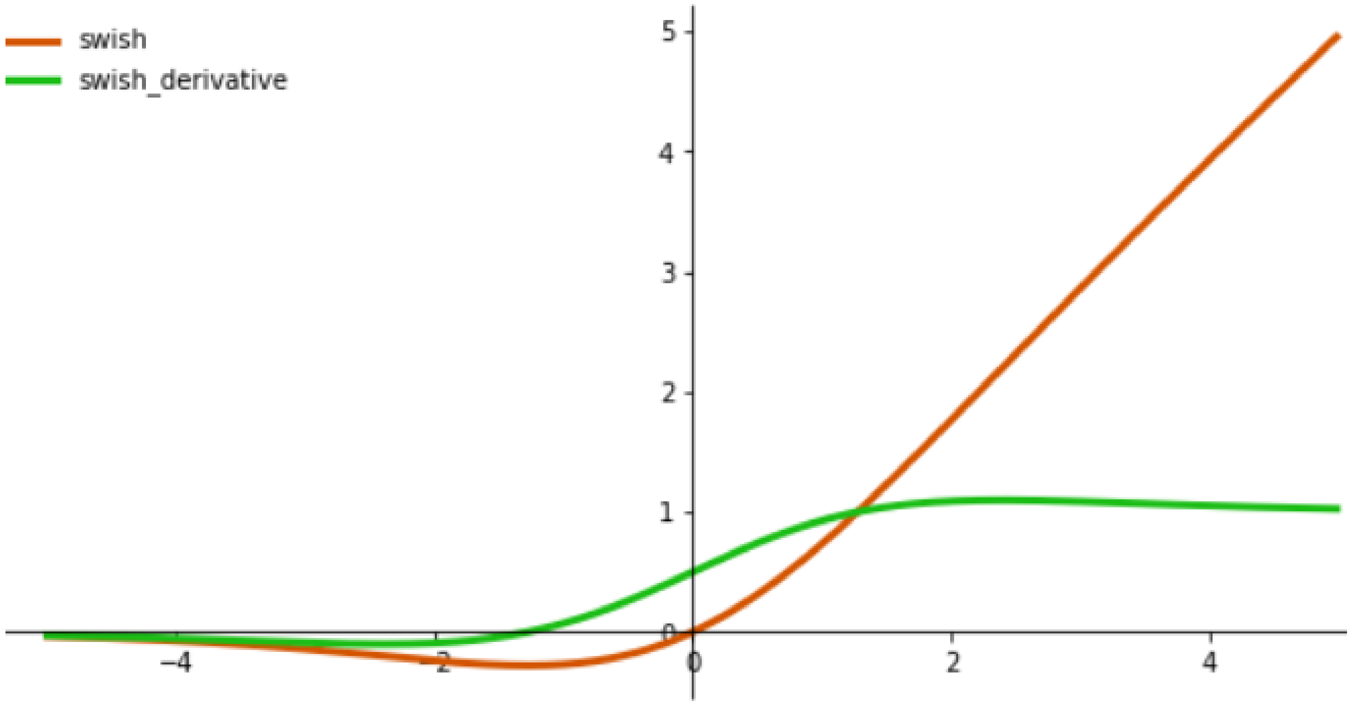
$$\text{PReLU}'(x) = \begin{cases} \alpha & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Activation Functions

❖ Swish function

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$swish(x) = \frac{x}{1+e^{-x}} = x\,\sigma(x)$$



data =

1	5	-4	3	-2

data_a = swish(data)

data_a =

0.731	4.966	-0.071	2.857	-0.238

$$swish'(x) = swish(x) + \sigma(x)\,(1 - swish(x))$$

Activation Functions

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$swish(x) = \frac{x}{1+e^{-x}} = x\,\sigma(x)$$

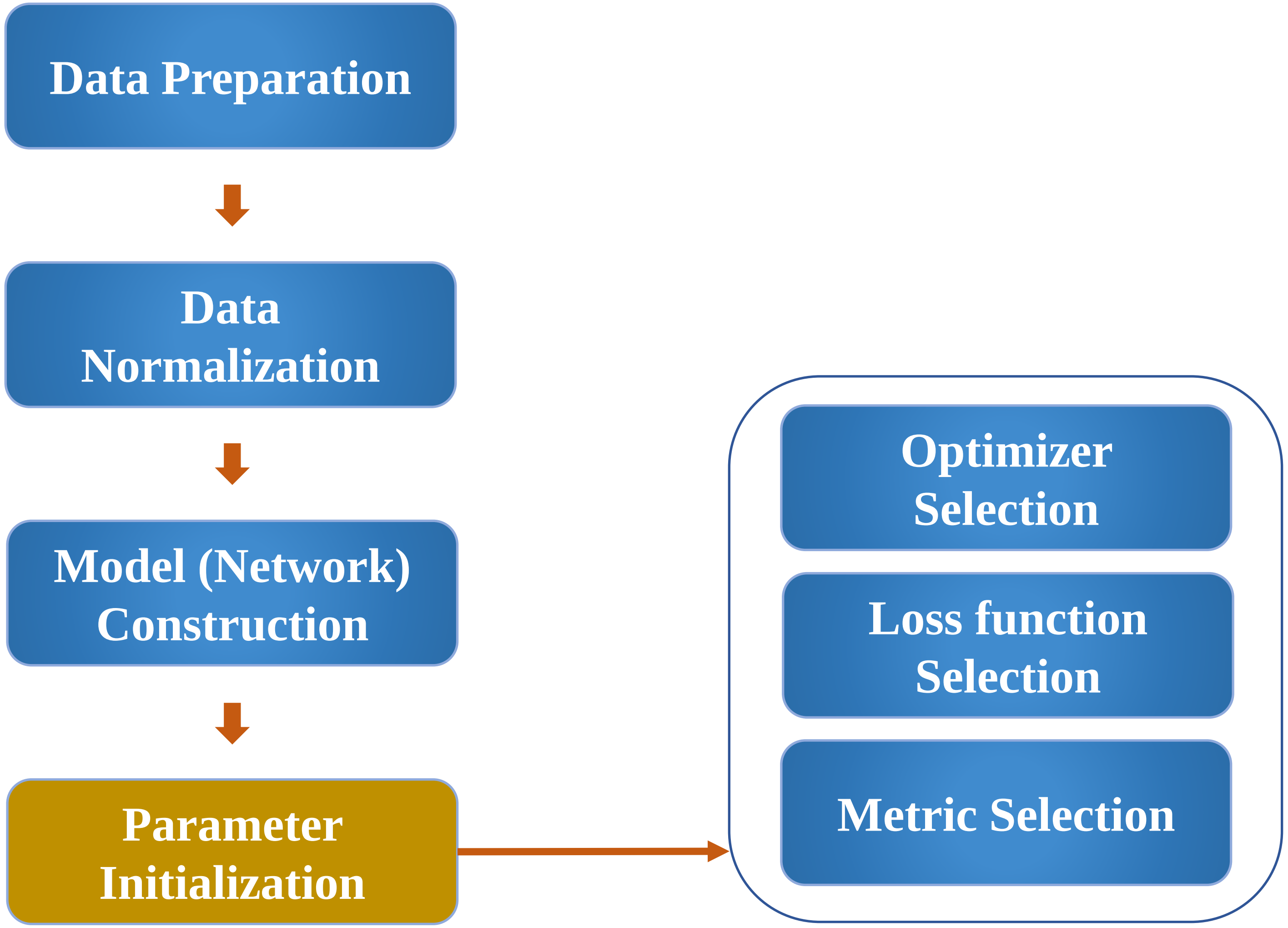
$$\begin{aligned}
swish'(x) &= (x\,\sigma(x))' = (x)'\,\sigma(x) + x(\sigma(x))' \\
&= \sigma(x) + x\,\sigma(x)\,(1-\sigma(x)) \\
&= \sigma(x) + x\,\sigma(x) - x\,\sigma(x)^2 \\
&= x\,\sigma(x) + \sigma(x)(1 - x\,\sigma(x)) \\
&= swish(x) + \sigma(x)\,(1 - swish(x))
\end{aligned}$$

Outline

- Pipeline Recommendation
- Data Normalization
- Activation Functions
- MLP Examples
- Initialization Methods

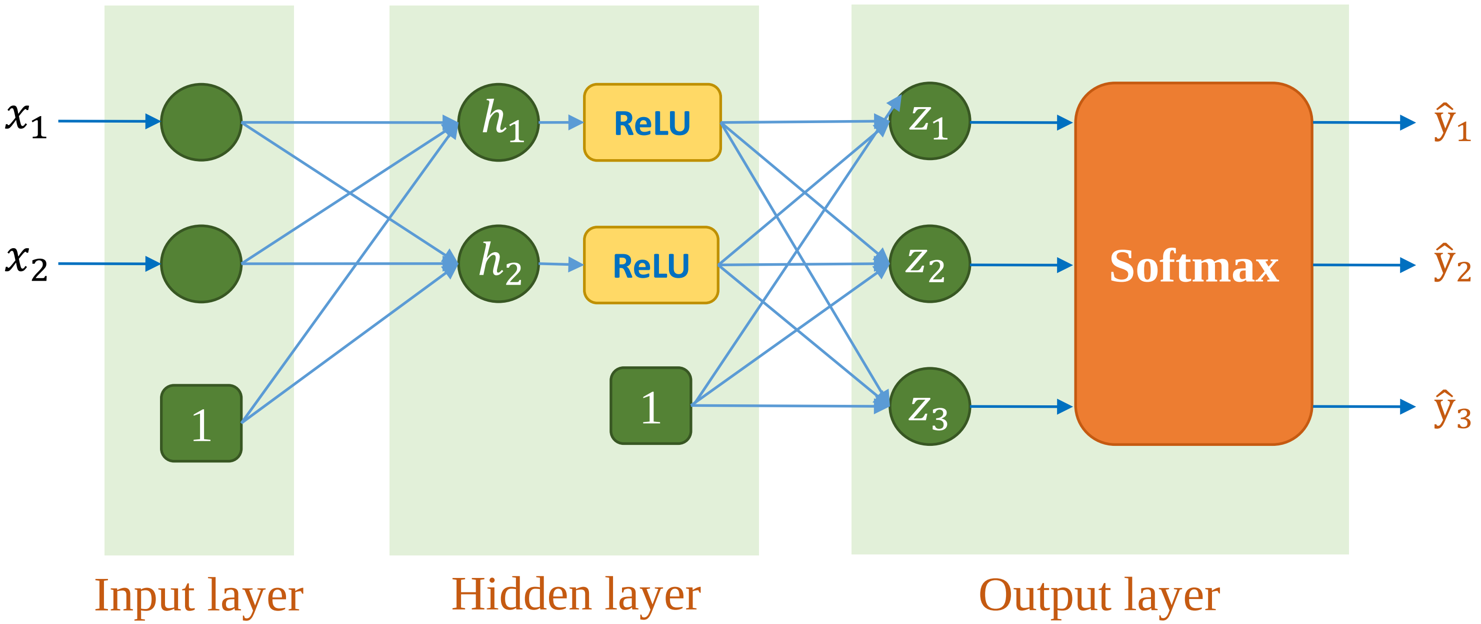
To-do List for Training

Train a model



MLP Example 1

Feature		Label
Petal_Length	Petal_Width	Label
1.5	0.2	0
1.4	0.2	0
1.6	0.2	0
4.7	1.6	1
3.3	1.1	1
4.6	1.3	1
5.6	2.2	2
5.1	1.5	2
5.6	1.4	2



$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \boldsymbol{x}^{(3)} \end{bmatrix} = \begin{bmatrix} 1.5 & 0.2 \\ 4.7 & 1.6 \\ 5.6 & 2.2 \end{bmatrix}$$

$$\boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

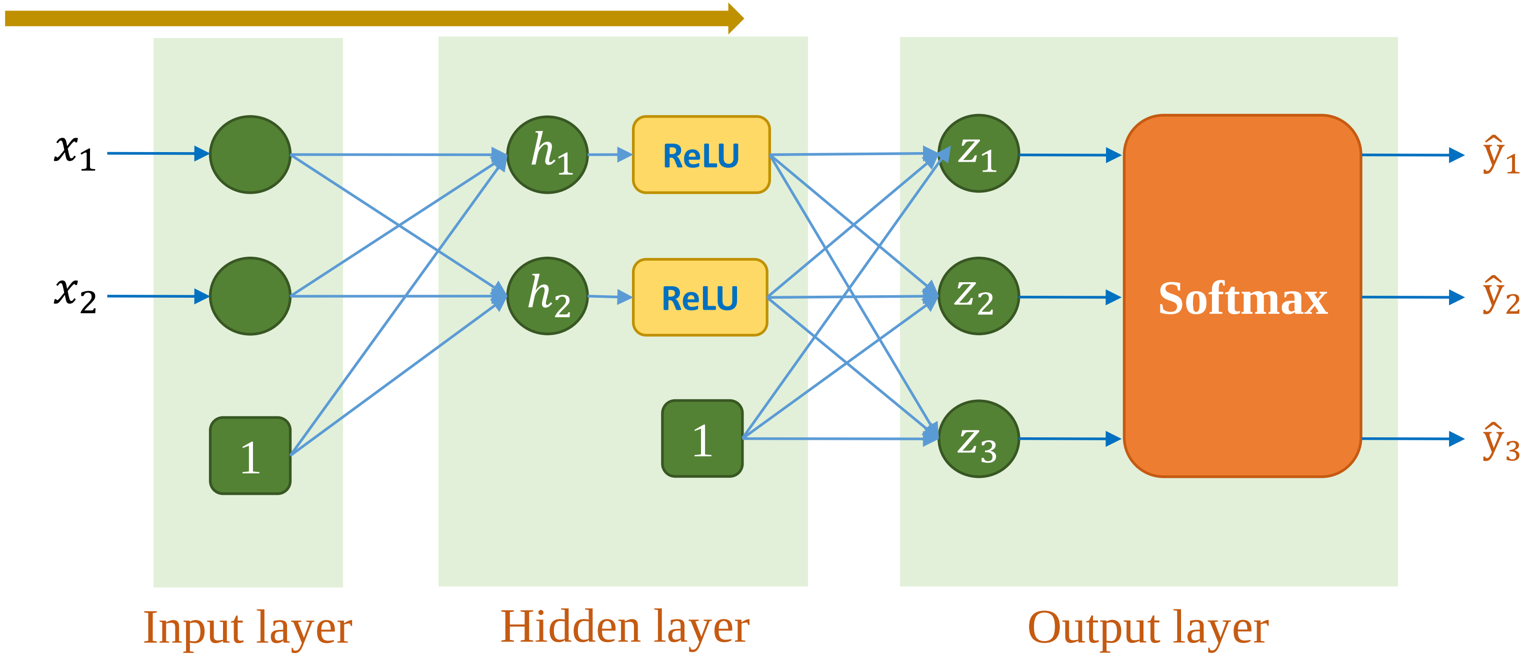
$$\boldsymbol{W}_h = [\boldsymbol{W}_{h1} \quad \boldsymbol{W}_{h2}] = \begin{bmatrix} 0.0 & 0.0 \\ 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix}$$

$$\boldsymbol{W}_z = [\boldsymbol{W}_{z1} \quad \boldsymbol{W}_{z2} \quad \boldsymbol{W}_{z3}] = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix}$$

$$\boldsymbol{h} = \boldsymbol{x}\boldsymbol{W}_h = \begin{bmatrix} 1 & 1.5 & 0.2 \\ 1 & 4.7 & 1.6 \\ 1 & 5.6 & 2.2 \end{bmatrix} \begin{bmatrix} 0.0 & 0.0 \\ 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix} = \begin{bmatrix} 1.373 & -1.696 \\ 4.708 & -5.951 \\ 5.731 & -7.281 \end{bmatrix}$$

$$\mathrm{ReLU}(\boldsymbol{h}) = \begin{bmatrix} 1.373 & 0 \\ 4.708 & 0 \\ 5.731 & 0 \end{bmatrix}$$

Feature		Label
Petal_Length	Petal_Width	Label
1.5	0.2	0
1.4	0.2	0
1.6	0.2	0
4.7	1.6	1
3.3	1.1	1
4.6	1.3	1
5.6	2.2	2
5.1	1.5	2
5.6	1.4	2



$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \boldsymbol{x}^{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 1.5 & 0.2 \\ 1 & 4.7 & 1.6 \\ 1 & 5.6 & 2.2 \end{bmatrix} \quad \boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\boldsymbol{W}_h = [\boldsymbol{W}_{h1} \quad \boldsymbol{W}_{h2}] = \begin{bmatrix} 0.0 & 0.0 \\ 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix}$$

$$\boldsymbol{W}_z = [\boldsymbol{W}_{z1} \quad \boldsymbol{W}_{z2} \quad \boldsymbol{W}_{z3}] = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix}$$

$$\mathrm{ReLU}(\boldsymbol{h}) = \begin{bmatrix} 1.373 & 0 \\ 4.708 & 0 \\ 5.731 & 0 \end{bmatrix}$$

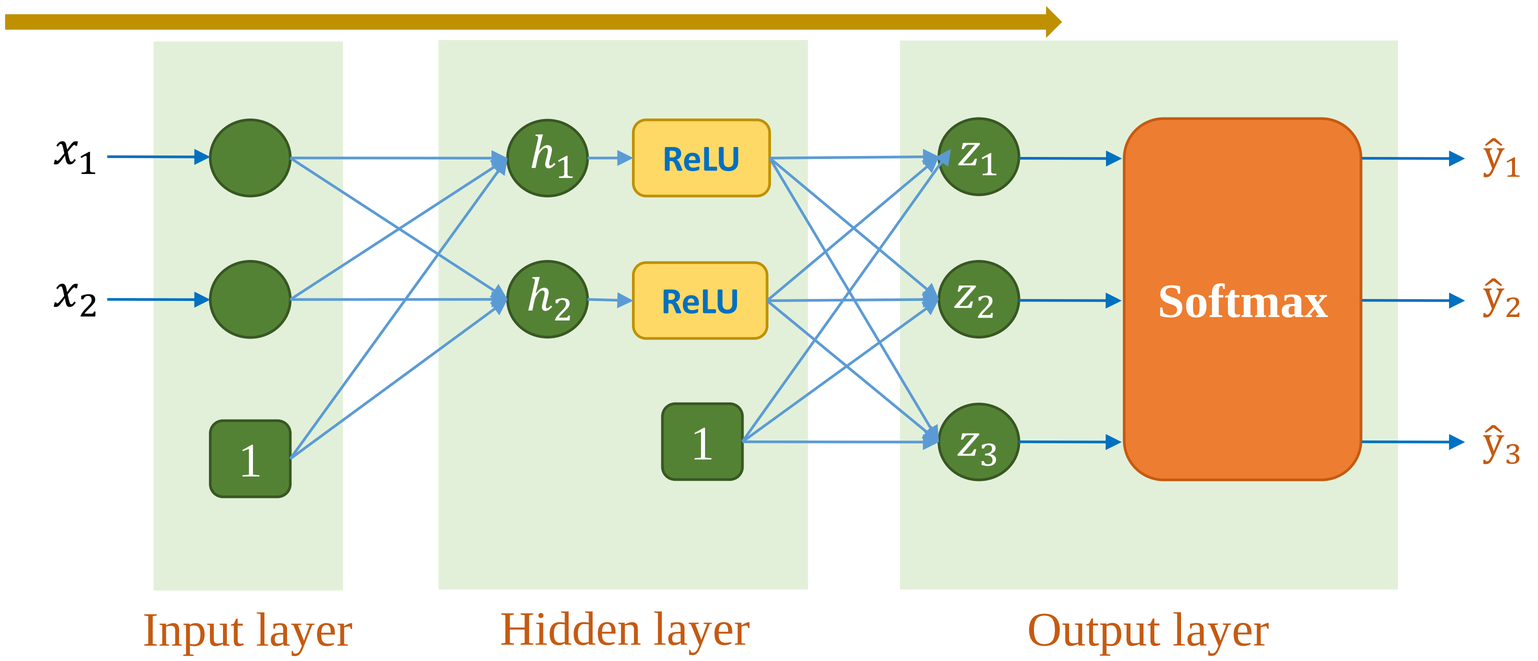
$$[\mathbf{1} \quad \mathrm{ReLU}(\boldsymbol{h})] = \begin{bmatrix} 1 & 1.373 & 0 \\ 1 & 4.708 & 0 \\ 1 & 5.731 & 0 \end{bmatrix}$$

$$\boldsymbol{z} = [\mathbf{1} \quad \mathrm{ReLU}(\boldsymbol{h})]\boldsymbol{W}_z = \begin{bmatrix} 1 & 1.373 & 0 \\ 1 & 4.708 & 0 \\ 1 & 5.731 & 0 \end{bmatrix} \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix}$$

$$= \begin{bmatrix} 0.439 & 0.356 & 0.195 \\ 1.507 & 1.220 & 0.670 \\ 1.835 & 1.485 & 0.816 \end{bmatrix}$$

Feature **Label**

Petal_Length	Petal_Width	Label
1.5	0.2	0
1.4	0.2	0
1.6	0.2	0
4.7	1.6	1
3.3	1.1	1
4.6	1.3	1
5.6	2.2	2
5.1	1.5	2
5.6	1.4	2



$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \boldsymbol{x}^{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 1.5 & 0.2 \\ 1 & 4.7 & 1.6 \\ 1 & 5.6 & 2.2 \end{bmatrix} \quad \boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

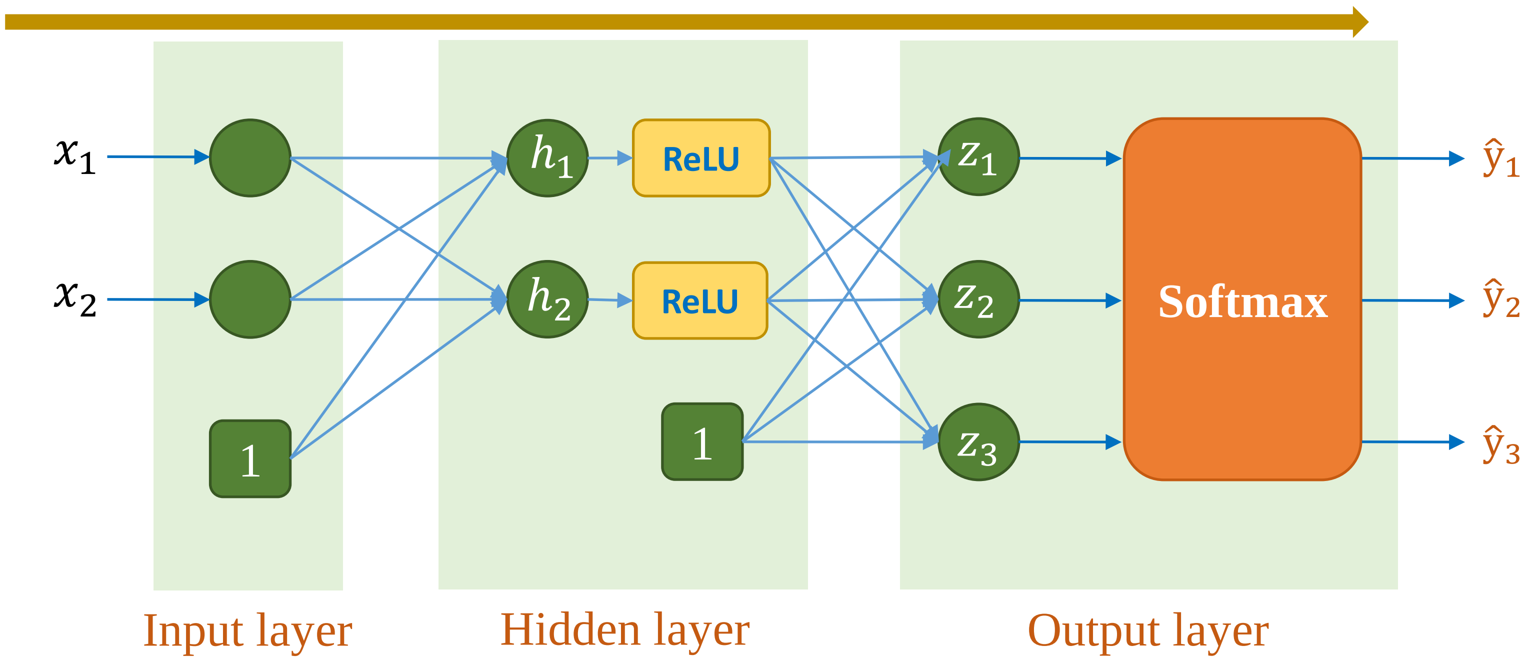
$$\boldsymbol{W}_h = [\boldsymbol{W}_{h1} \quad \boldsymbol{W}_{h2}] = \begin{bmatrix} 0.0 & 0.0 \\ 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix}$$

$$\boldsymbol{W}_z = [\boldsymbol{W}_{z1} \quad \boldsymbol{W}_{z2} \quad \boldsymbol{W}_{z3}] = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix}$$

$$\boldsymbol{z} = \begin{bmatrix} 0.439 & 0.356 & 0.195 \\ 1.507 & 1.220 & 0.670 \\ 1.835 & 1.485 & 0.816 \end{bmatrix} \qquad \hat{\boldsymbol{y}} = \text{softmax}(\boldsymbol{z}) = \begin{bmatrix} \hat{\boldsymbol{y}}^{(1)} \\ \hat{\boldsymbol{y}}^{(2)} \\ \hat{\boldsymbol{y}}^{(3)} \end{bmatrix} = \begin{bmatrix} 0.369 & 0.340 & 0.289 \\ 0.458 & 0.343 & 0.198 \\ 0.484 & 0.341 & 0.174 \end{bmatrix}$$

$$\text{loss} = 1.269$$

Feature		Label
Petal_Length	Petal_Width	Label
1.5	0.2	0
1.4	0.2	0
1.6	0.2	0
4.7	1.6	1
3.3	1.1	1
4.6	1.3	1
5.6	2.2	2
5.1	1.5	2
5.6	1.4	2



$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \boldsymbol{x}^{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 1.5 & 0.2 \\ 1 & 4.7 & 1.6 \\ 1 & 5.6 & 2.2 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

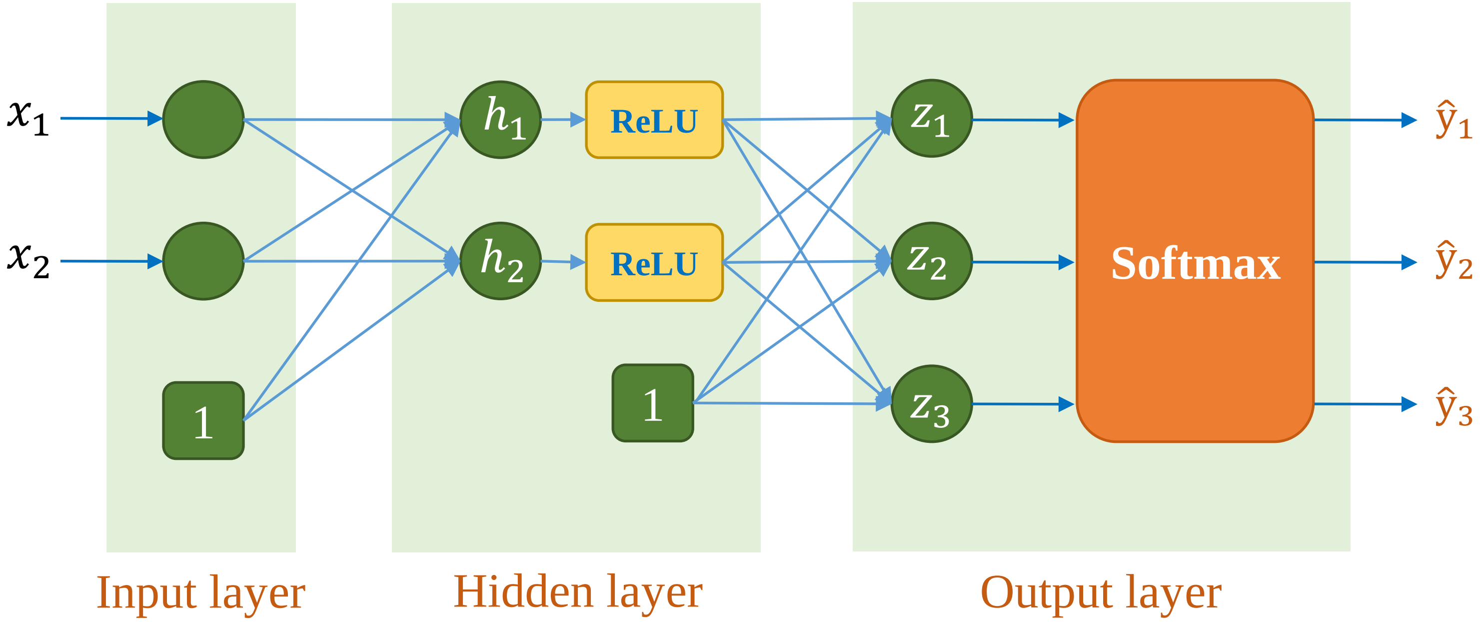
$$\boldsymbol{W}_h = [\boldsymbol{W}_{h1} \quad \boldsymbol{W}_{h2}] = \begin{bmatrix} 0.0 & 0.0 \\ 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix} \qquad \boldsymbol{W}_z = [\boldsymbol{W}_{z1} \quad \boldsymbol{W}_{z2} \quad \boldsymbol{W}_{z3}] = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix}$$

Example 2 - Dying ReLU

Petal Length	Petal Width	Label
1.5	0.2	0
1.4	0.2	0
1.6	0.2	0
4.7	1.6	1
3.3	1.1	1
4.6	1.3	1
5.6	2.2	2
5.1	1.5	2
5.6	1.4	2

Feature | Label

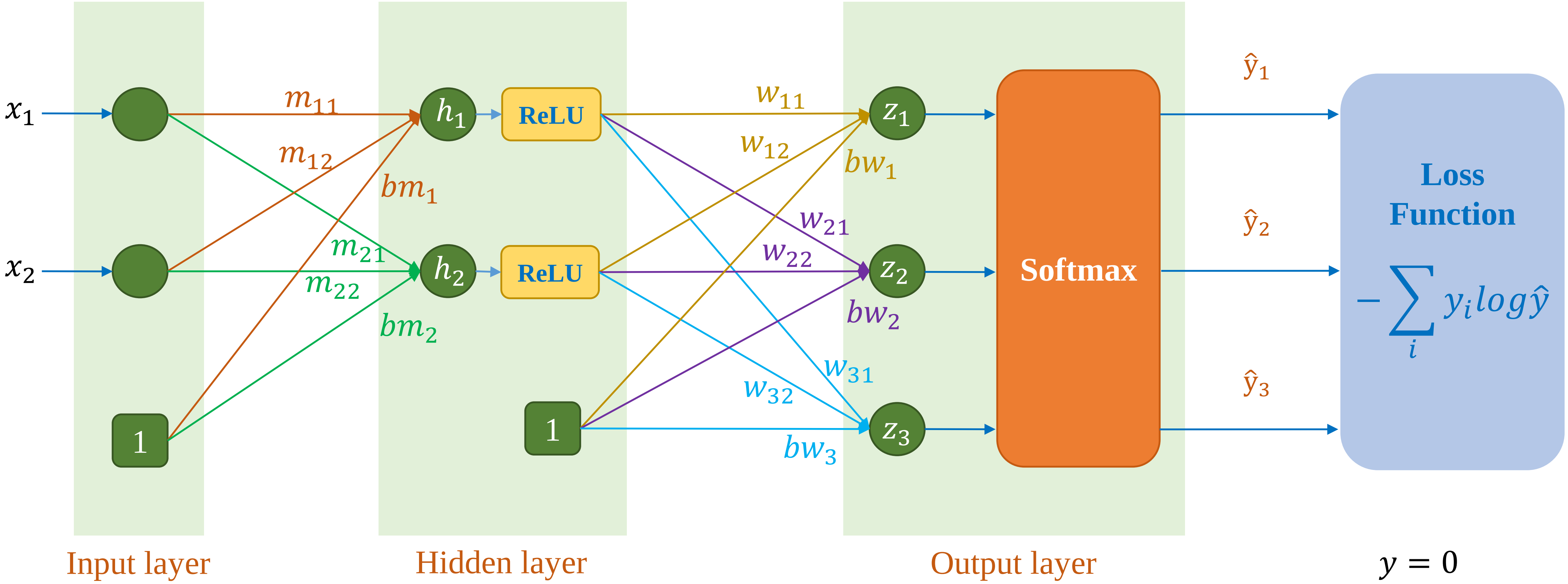
$$\boldsymbol{x} = \begin{bmatrix} 1.5 \\ 0.2 \end{bmatrix} \qquad y = 0$$



$$\boldsymbol{m} = [\boldsymbol{m}_1 \quad \boldsymbol{m}_2] = \begin{bmatrix} 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix}$$

$$\boldsymbol{w} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \boldsymbol{w}_3] = \begin{bmatrix} 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix}$$

$$\boldsymbol{bm} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix} \qquad \boldsymbol{bw} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$



$$\boldsymbol{x} = \begin{bmatrix} 1.5 \\ 0.2 \end{bmatrix}$$

$$\rightarrow \boldsymbol{y} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{m} = [\boldsymbol{m_1} \quad \boldsymbol{m}_2] = \begin{bmatrix} 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix}$$

$$\boldsymbol{w} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \boldsymbol{w}_3] = \begin{bmatrix} 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix}$$

$$\boldsymbol{bm} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

$$\boldsymbol{bw} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

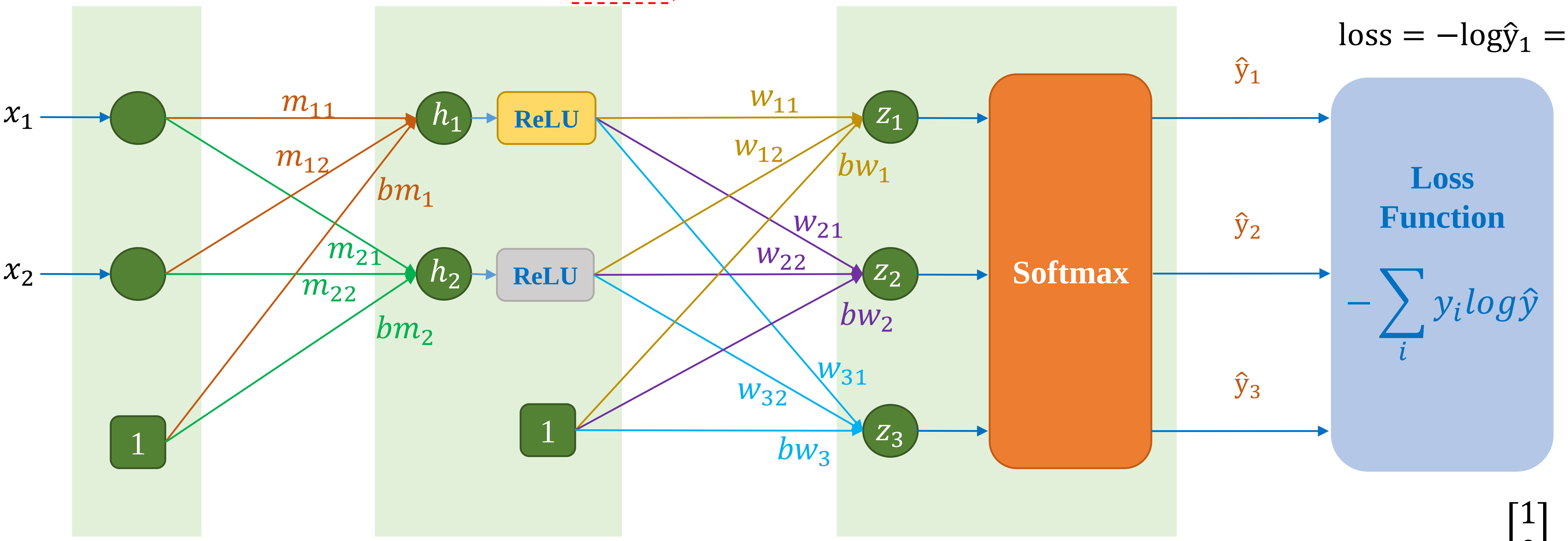
Forward pass

$$\boldsymbol{h} = \begin{bmatrix} 1.372 \\ -1.68 \end{bmatrix}$$

zero value

$$\boldsymbol{x} = \begin{bmatrix} 1.5 \\ 0.2 \end{bmatrix} \qquad \mathbf{ReLU} = \begin{bmatrix} 1.372 \\ \boxed{0.0} \end{bmatrix} \qquad \boldsymbol{z} = \begin{bmatrix} 0.439 \\ 0.343 \\ 0.192 \end{bmatrix} \qquad \hat{\boldsymbol{y}} = \begin{bmatrix} 0.372 \\ 0.338 \\ 0.290 \end{bmatrix}$$

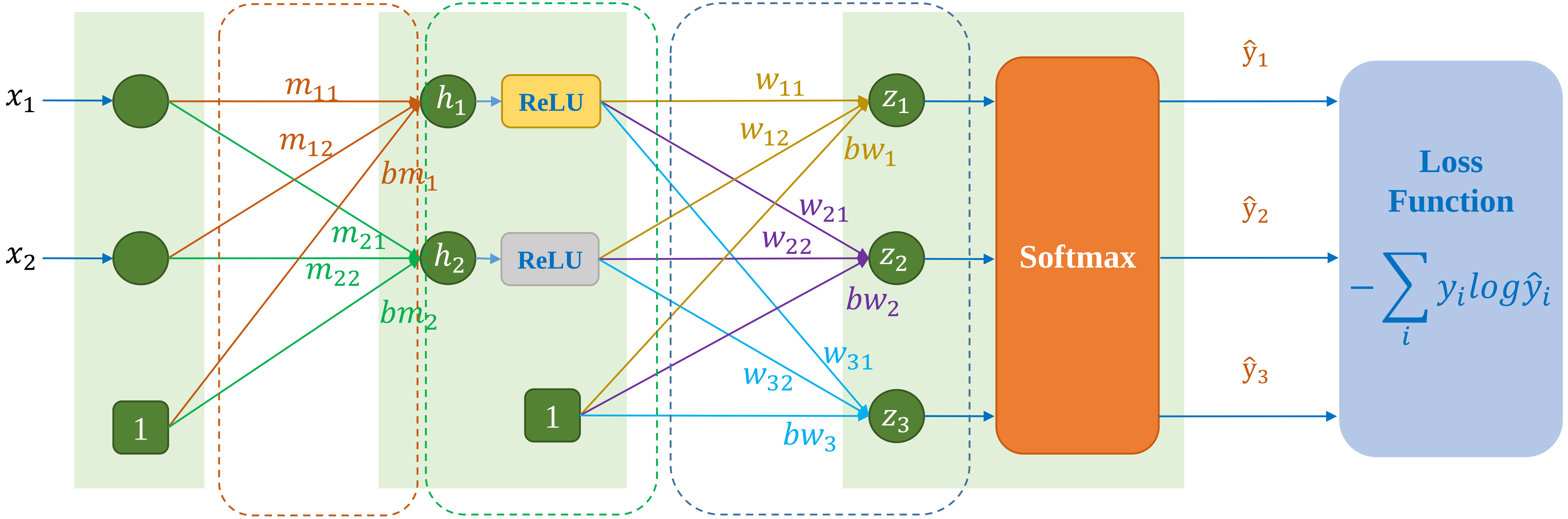
$$\text{loss} = -\log\hat{y}_1 = 0.989$$



$$\boldsymbol{y} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{m} = [\boldsymbol{m_1} \quad \boldsymbol{m}_2] = \begin{bmatrix} 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix} \qquad \boldsymbol{bm} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

$$\boldsymbol{w} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \boldsymbol{w}_3] = \begin{bmatrix} 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix} \qquad \boldsymbol{bw} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$



$$\frac{\partial L}{\partial m_{jk}} = x_k \frac{\partial L}{\partial h_j}$$

$$\frac{\partial L}{\partial bm_j} = \frac{\partial L}{\partial h_j}$$

$$\frac{\partial L}{\partial relu_j} = \sum_i w_{ij} \frac{\partial L}{\partial z_i}$$

$$\text{ReLU}'(h_j) = \begin{cases} 0 & \text{if } \ h_j \le 0 \\ 1 & \text{if } \ h_j > 0 \end{cases}$$

$$\frac{\partial L}{\partial h_j} = \begin{cases} 0 & \text{if } \ h_j \le 0 \\ \dfrac{\partial L}{\partial relu_j} & \text{if } \ h_j > 0 \end{cases}$$

$$\frac{\partial L}{\partial z_i} = \hat{y}_i - y_i$$

$$\frac{\partial L}{\partial w_{ij}} = \text{ReLU}_j \frac{\partial L}{\partial z_i}$$

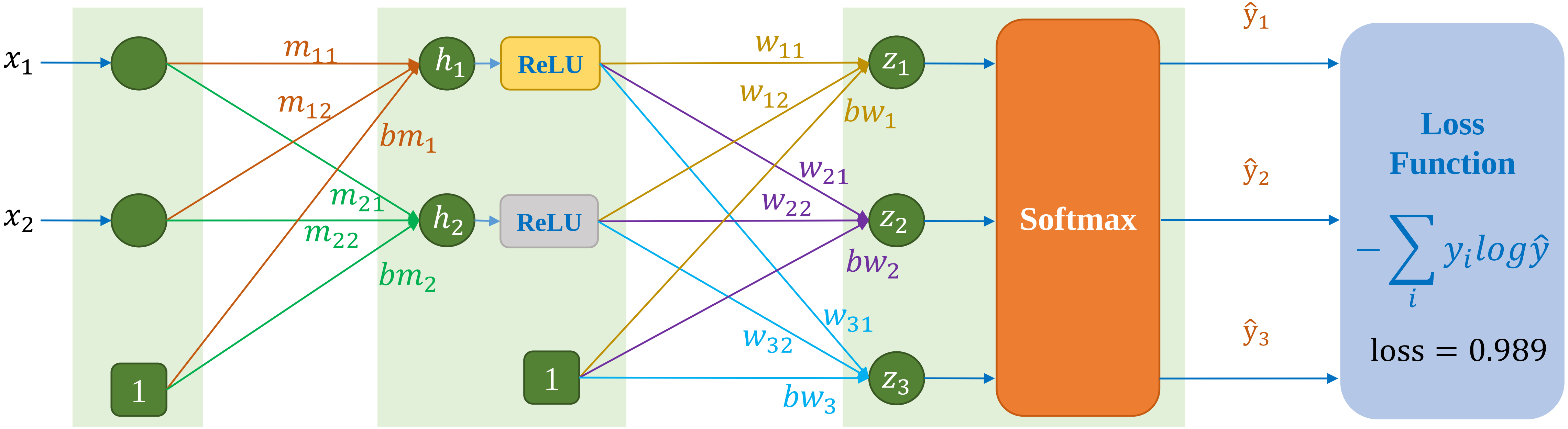
$$\frac{\partial L}{\partial bw_i} = \frac{\partial L}{\partial z_i}$$

Backward pass

Backward pass

$$\boldsymbol{x} = \begin{bmatrix} 1.5 \\ 0.2 \end{bmatrix} \quad \boldsymbol{h} = \begin{bmatrix} 1.372 \\ -1.68 \end{bmatrix} \quad \mathbf{ReLU} = \begin{bmatrix} 1.372 \\ 0.0 \end{bmatrix} \quad \boldsymbol{z} = \begin{bmatrix} 0.439 \\ 0.343 \\ 0.192 \end{bmatrix} \quad \hat{\boldsymbol{y}} = \begin{bmatrix} 0.372 \\ 0.338 \\ 0.290 \end{bmatrix}$$

$$\boldsymbol{m} = \begin{bmatrix} 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix} \quad \boldsymbol{bm} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix} \quad \boldsymbol{w} = \begin{bmatrix} 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix} \quad \boldsymbol{bw} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$



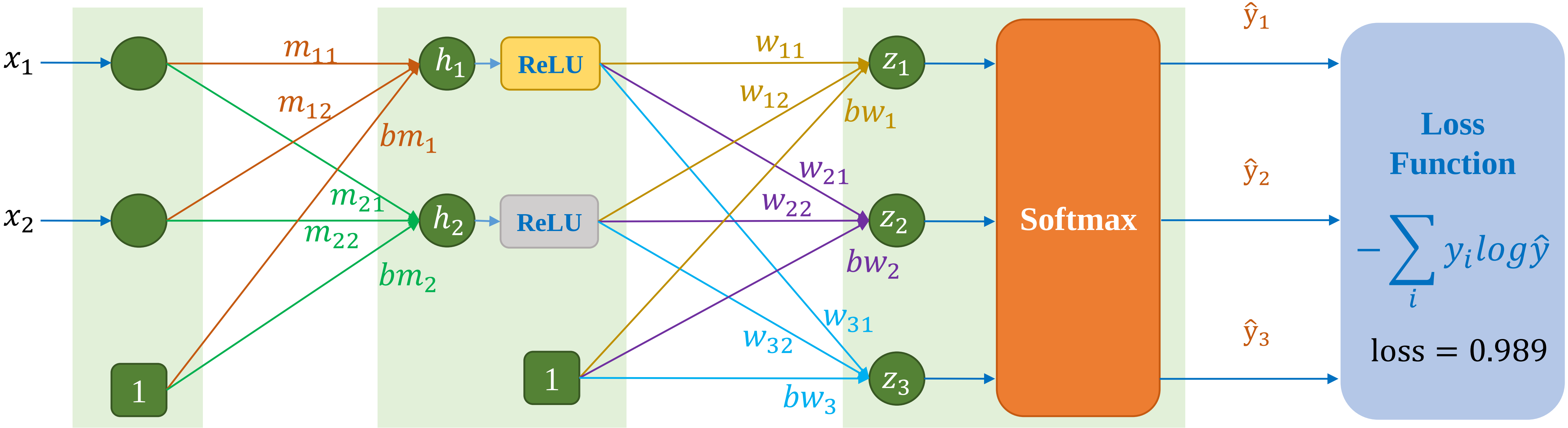
$$\frac{\partial L}{\partial relu_j} = \sum_i w_{ij} \frac{\partial L}{\partial z_i} \qquad \frac{\partial L}{\partial w_{ij}} = \mathrm{ReLU}_j \frac{\partial L}{\partial z_i} \qquad \frac{\partial L}{\partial bw_i} = \frac{\partial L}{\partial z_i} \qquad \frac{\partial L}{\partial z_i} = \hat{y}_i - y_i$$

$$\boldsymbol{\nabla}_{\mathbf{ReLU}} L = \begin{bmatrix} -0.0759 \\ -0.0445 \end{bmatrix} \quad \boldsymbol{\nabla}_{\boldsymbol{w}} L = \begin{bmatrix} -0.861 & 0.463 & 0.398 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \quad \boldsymbol{\nabla}_{\boldsymbol{bw}} L = \begin{bmatrix} -0.628 \\ 0.338 \\ 0.290 \end{bmatrix} \quad \boldsymbol{\nabla}_{\boldsymbol{z}} L = \begin{bmatrix} -0.628 \\ 0.338 \\ 0.290 \end{bmatrix}$$

Backward pass

$$\boldsymbol{x} = \begin{bmatrix} 1.5 \\ 0.2 \end{bmatrix} \quad \boldsymbol{h} = \begin{bmatrix} 1.372 \\ -1.68 \end{bmatrix} \quad \textbf{ReLU} = \begin{bmatrix} 1.372 \\ 0.0 \end{bmatrix} \quad \boldsymbol{z} = \begin{bmatrix} 0.439 \\ 0.343 \\ 0.192 \end{bmatrix} \quad \hat{\boldsymbol{y}} = \begin{bmatrix} 0.372 \\ 0.338 \\ 0.290 \end{bmatrix}$$

$$\boldsymbol{m} = \begin{bmatrix} 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix} \quad \boldsymbol{bm} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix} \quad \boldsymbol{w} = \begin{bmatrix} 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix} \quad \boldsymbol{bw} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$



$$\frac{\partial L}{\partial m_{jk}} = x_k \frac{\partial L}{\partial h_j}$$

$$\boldsymbol{\nabla}_{\boldsymbol{m}} L = \begin{bmatrix} -0.114 & 0.0 \\ -0.015 & 0.0 \end{bmatrix}$$

$$\frac{\partial L}{\partial bm_j} = \frac{\partial L}{\partial h_j}$$

$$\boldsymbol{\nabla}_{\boldsymbol{bm}} L = \begin{bmatrix} -0.0759 \\ 0.0 \end{bmatrix}$$

$$\frac{\partial L}{\partial h_j} = \begin{cases} 0 & \text{if } \ h_j \leq 0 \\ \dfrac{\partial L}{\partial relu_j} & \text{if } \ h_j > 0 \end{cases}$$

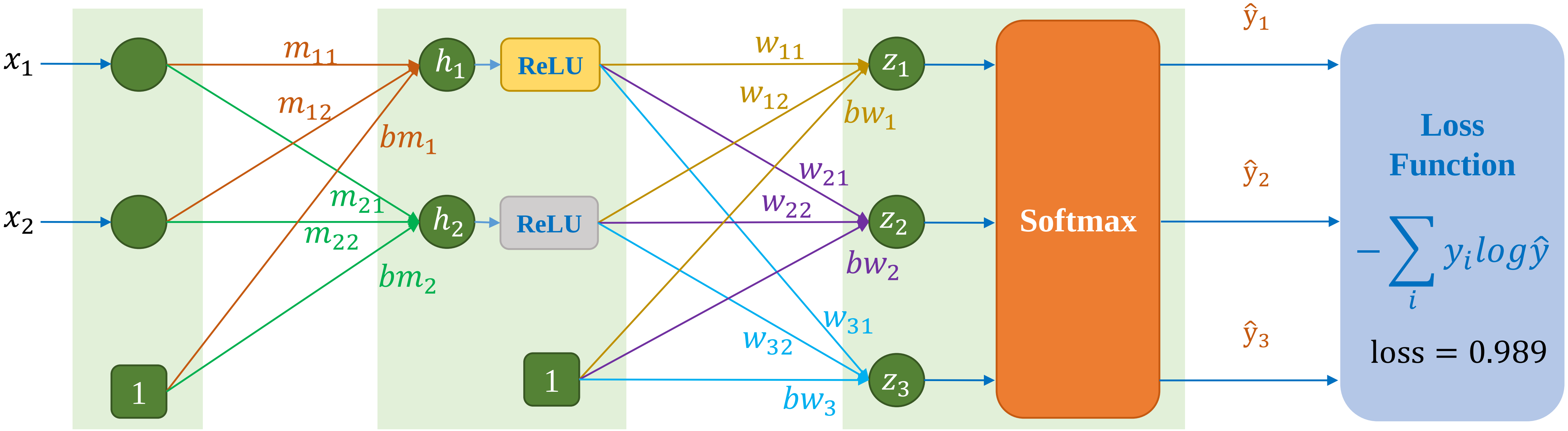
$$\boldsymbol{\nabla}_{\mathbf{h}} L = \begin{bmatrix} -0.0759 \\ 0.0 \end{bmatrix}$$

$$\frac{\partial L}{\partial relu_j} = \sum_i w_{ij} \frac{\partial L}{\partial z_i}$$

$$\boldsymbol{\nabla}_{\mathbf{ReLU}} L = \begin{bmatrix} -0.0759 \\ -0.0445 \end{bmatrix}$$

$$\boldsymbol{m} = \begin{bmatrix} 0.86 & -1.04 \\ 0.41 & -0.65 \end{bmatrix} \quad \boldsymbol{bm} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix} \quad \boldsymbol{w} = \begin{bmatrix} 0.32 & 0.25 & 0.14 \\ -0.47 & -1.06 & 0.063 \end{bmatrix} \quad \boldsymbol{bw} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$\nabla_{\boldsymbol{m}} L = \begin{bmatrix} -0.114 & 0.0 \\ -0.015 & 0.0 \end{bmatrix} \quad \nabla_{\boldsymbol{bm}} L = \begin{bmatrix} -0.0759 \\ 0.0 \end{bmatrix} \quad \nabla_{\boldsymbol{w}} L = \begin{bmatrix} -0.628 & 0.338 & 0.29 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \quad \nabla_{\boldsymbol{bw}} L = \begin{bmatrix} -0.628 \\ 0.338 \\ 0.290 \end{bmatrix}$$



Update the parameters with $\eta = 0.01$

$$\boldsymbol{m} = \begin{bmatrix} 0.861 & -1.04 \\ 0.4105 & -0.65 \end{bmatrix} \quad \boldsymbol{bm} = \begin{bmatrix} 0.000759 \\ 0.0 \end{bmatrix} \quad \boldsymbol{w} = \begin{bmatrix} 0.328 & 0.245 & 0.136 \\ -0.47 & -1.06 & 0.063 \end{bmatrix} \quad \boldsymbol{bw} = \begin{bmatrix} 0.0062 \\ -0.0033 \\ -0.0029 \end{bmatrix}$$

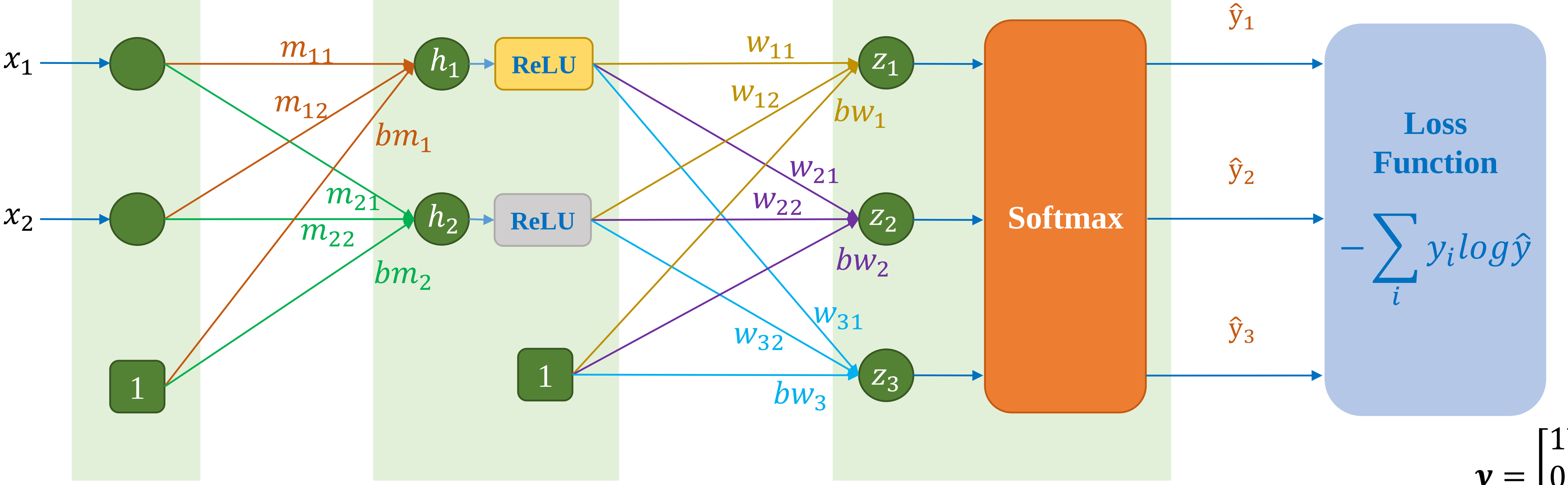
Forward pass again

$$\boldsymbol{h} = \begin{bmatrix} 1.374 \\ -1.68 \end{bmatrix}$$

still zero value

$$\boldsymbol{x} = \begin{bmatrix} 1.5 \\ 0.2 \end{bmatrix} \qquad \textbf{ReLU} = \begin{bmatrix} 1.374 \\ 0.0 \end{bmatrix} \qquad \boldsymbol{z} = \begin{bmatrix} 0.458 \\ 0.334 \\ 0.184 \end{bmatrix} \qquad \hat{\boldsymbol{y}} = \begin{bmatrix} 0.378 \\ 0.334 \\ 0.287 \end{bmatrix}$$

$$\text{loss} = -\log\hat{y}_1 = 0.972$$



$$\boldsymbol{y} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

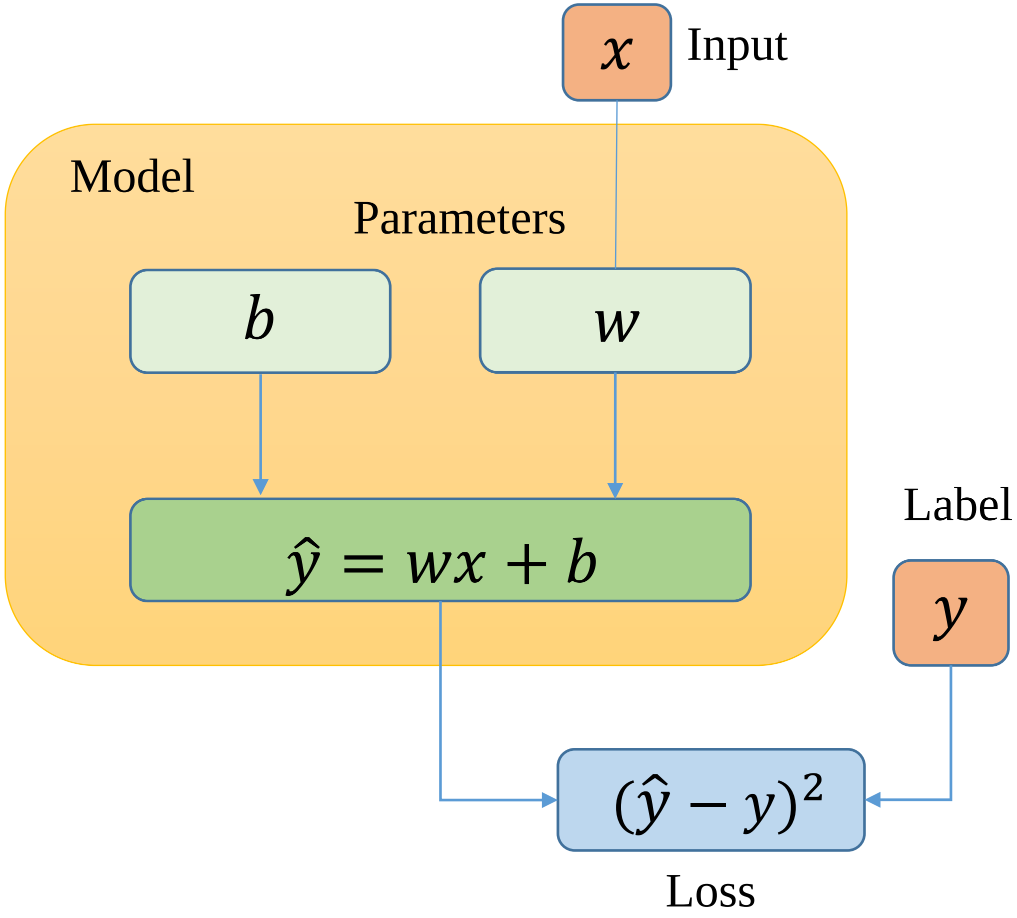
$$\boldsymbol{m} = [\boldsymbol{m_1} \quad \boldsymbol{m}_2] = \begin{bmatrix} 0.861 & -1.04 \\ 0.4105 & -0.65 \end{bmatrix} \qquad \boldsymbol{bm} = \begin{bmatrix} 0.000759 \\ 0.0 \end{bmatrix}$$

$$\boldsymbol{w} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \boldsymbol{w}_3] = \begin{bmatrix} 0.328 & 0.245 & 0.136 \\ -0.47 & -1.06 & 0.063 \end{bmatrix} \qquad \boldsymbol{bw} = \begin{bmatrix} 0.0062 \\ -0.0033 \\ -0.0029 \end{bmatrix}$$

Example 3 - Zero Initialization

❖ Linear regression

Diagram



Cheat sheet

Compute the output $\hat{y}$

$$\hat{y} = wx + b$$

Compute the loss

$$L = (\hat{y} - y)^2$$

Compute derivative

$$L'_w = 2x(\hat{y} - y)$$

$$L'_b = 2(\hat{y} - y)$$

Update parameters

$$w = w - \eta L'_w$$

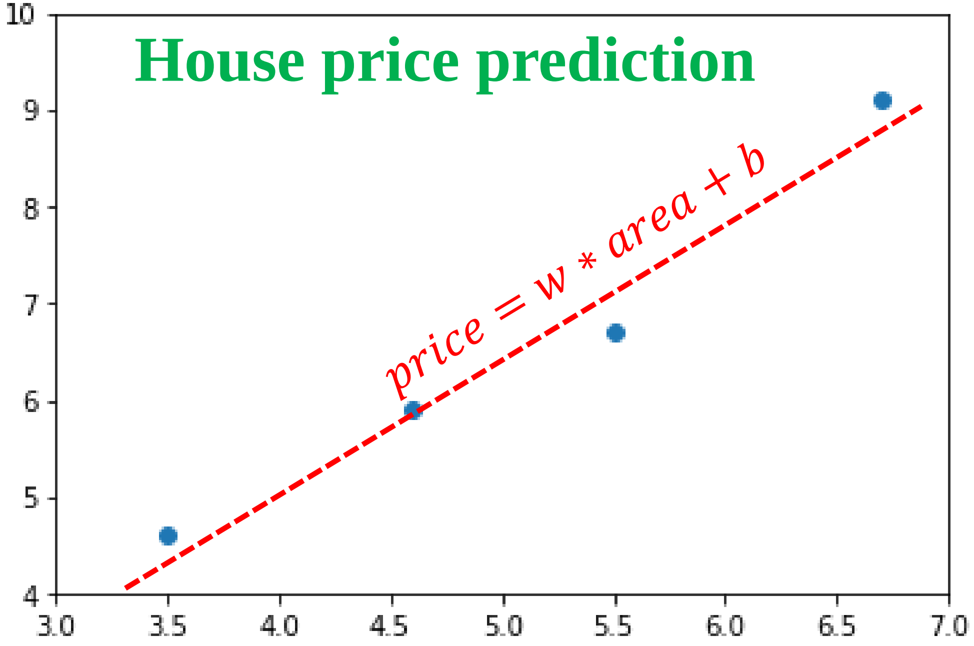
$$b = b - \eta L'_b$$

Example 3 - Zero Initialization

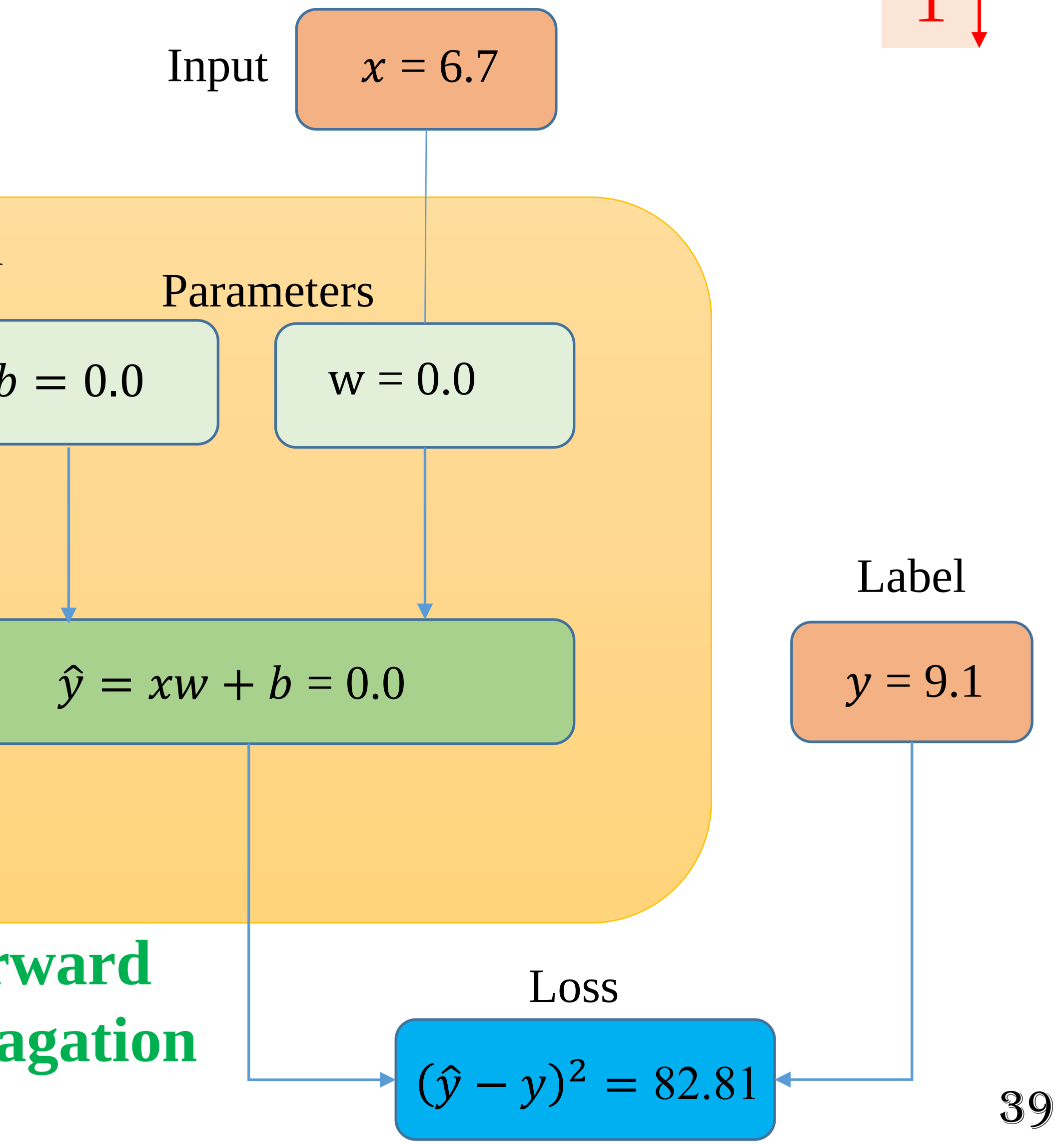
1

Given sample data

	Feature	Label
	area	price
	6.7	9.1
	4.6	5.9
	3.5	4.6
	5.5	6.7

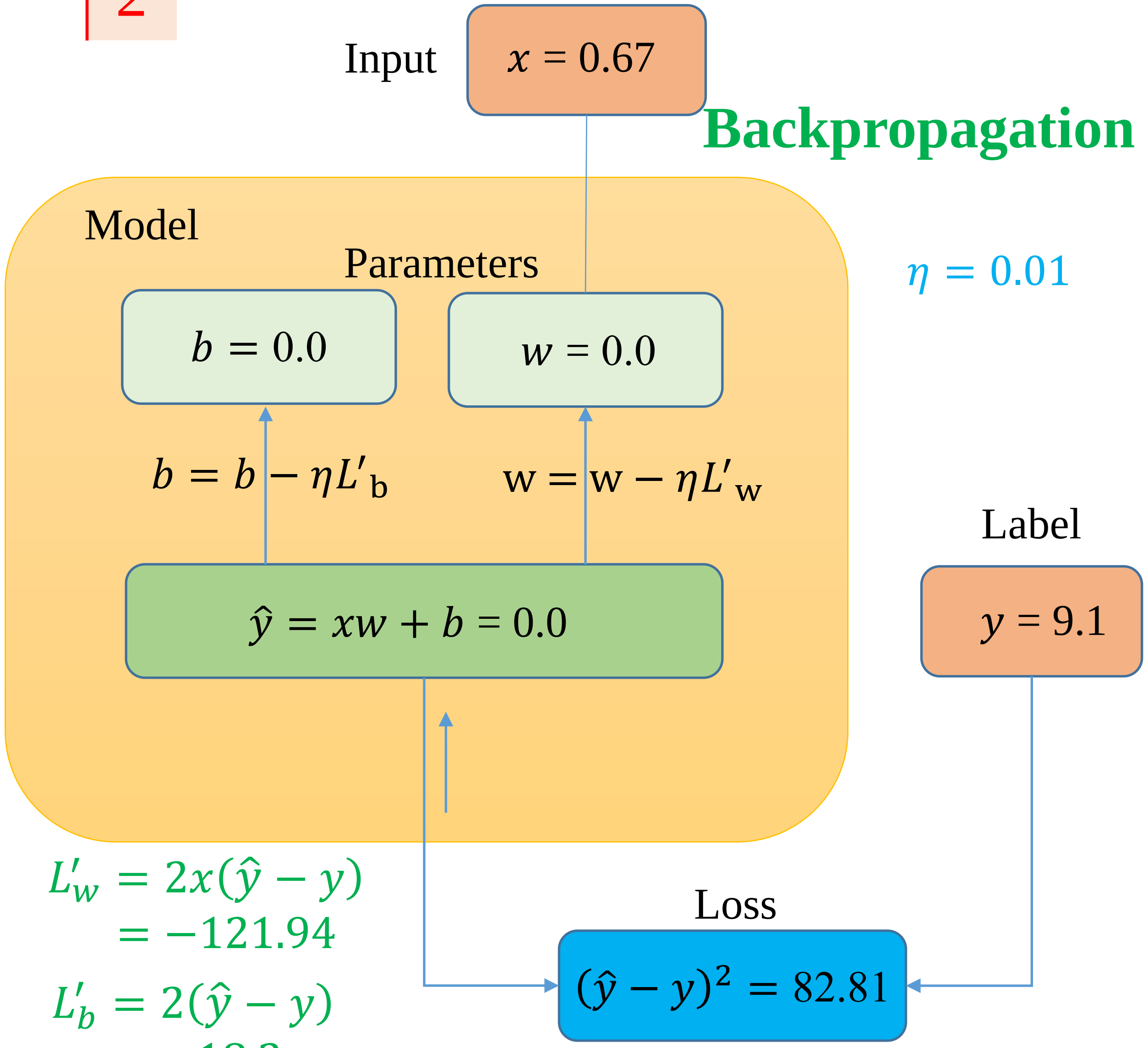


Initialize
b=0.0 and
w=0.0



2

Backpropagation



$\eta = 0.01$

$$L'_w = 2x(\hat{y} - y) = -121.94$$

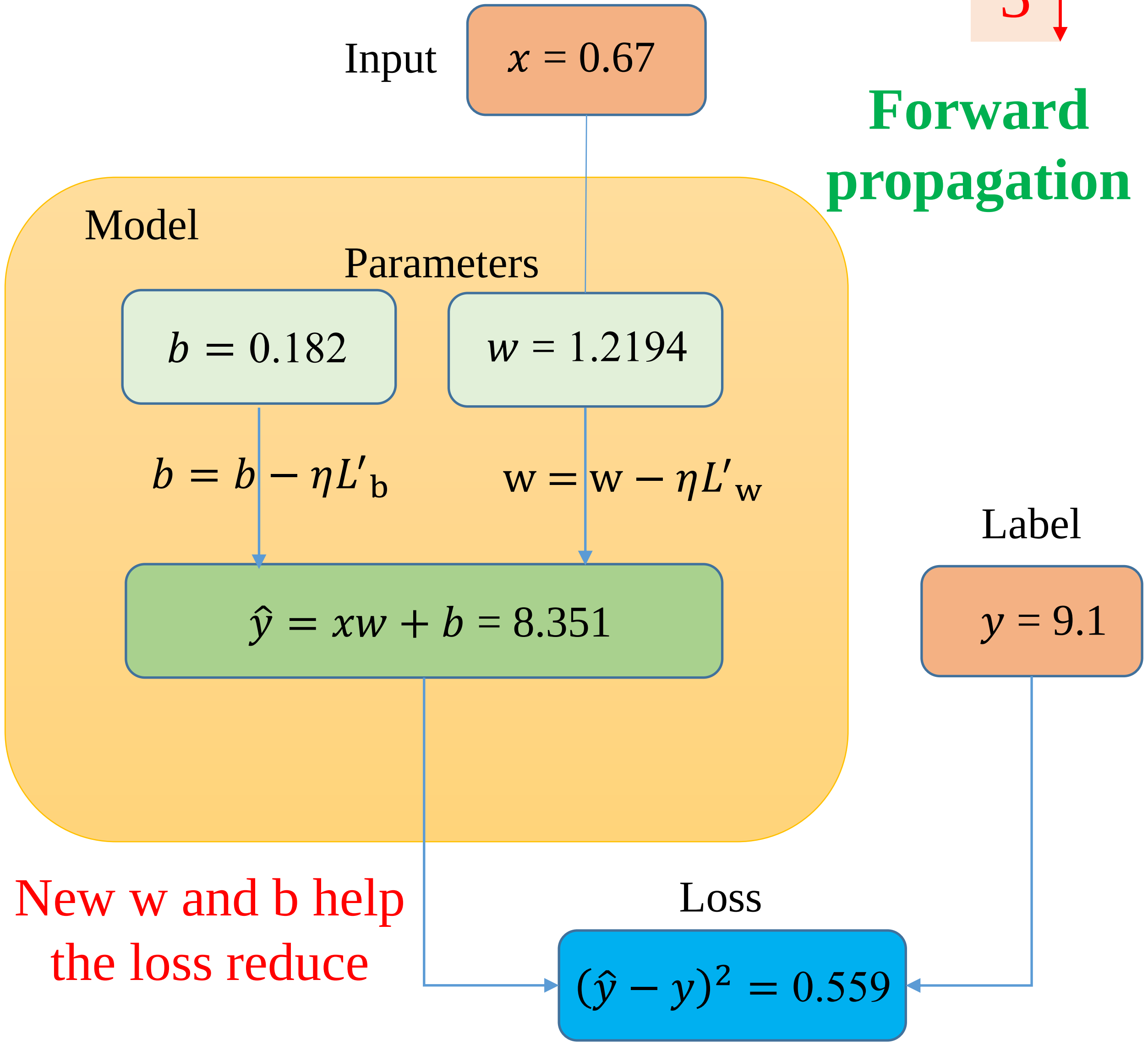
$$L'_b = 2(\hat{y} - y) = -18.2$$

$$b = b - \eta L'_{\mathrm{b}} = 0.182$$

$$w = w - \eta L'_{\mathrm{w}} = 1.2194$$

3

Forward propagation



New w and b help the loss reduce

Example 4 - Zero Initialization

❖ Logistic regression

$$\boldsymbol{\theta}^T = [b \quad w_1 \quad w_2]$$

$$\boldsymbol{x}^T = [1 \quad x_1 \quad x_2]$$

1) Pick a sample (x, y) from training data

2) Compute output $\hat{y}$

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

3) Compute loss

$$L(\boldsymbol{\theta}) = (-\mathrm{y}\log\hat{\mathrm{y}} - (1-\mathrm{y})\log(1-\hat{\mathrm{y}}))$$

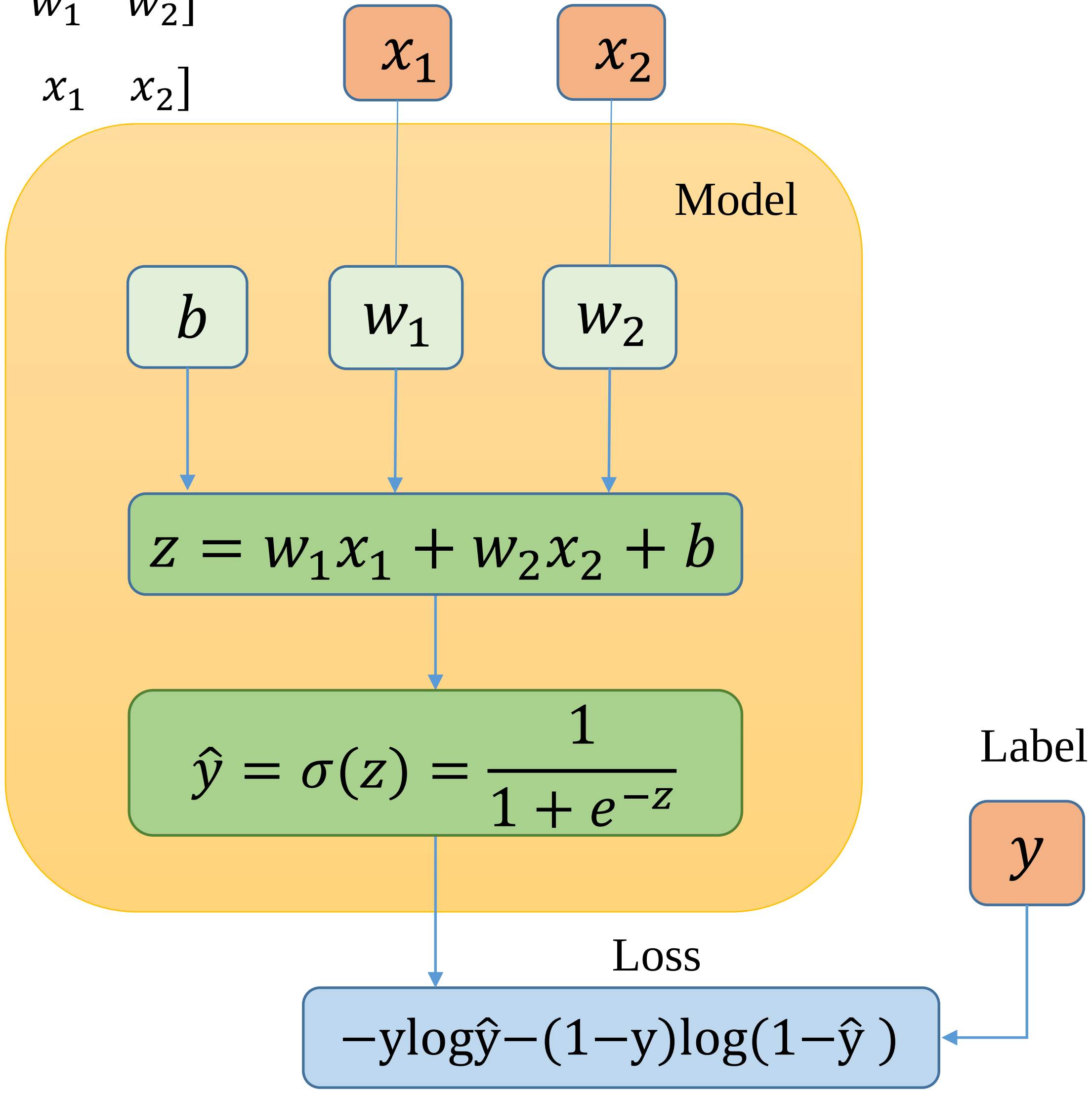
4) Compute derivative

$$\nabla_{\boldsymbol{\theta}} L = \mathbf{x}(\hat{\mathrm{y}} - y)$$

5) Update parameters

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

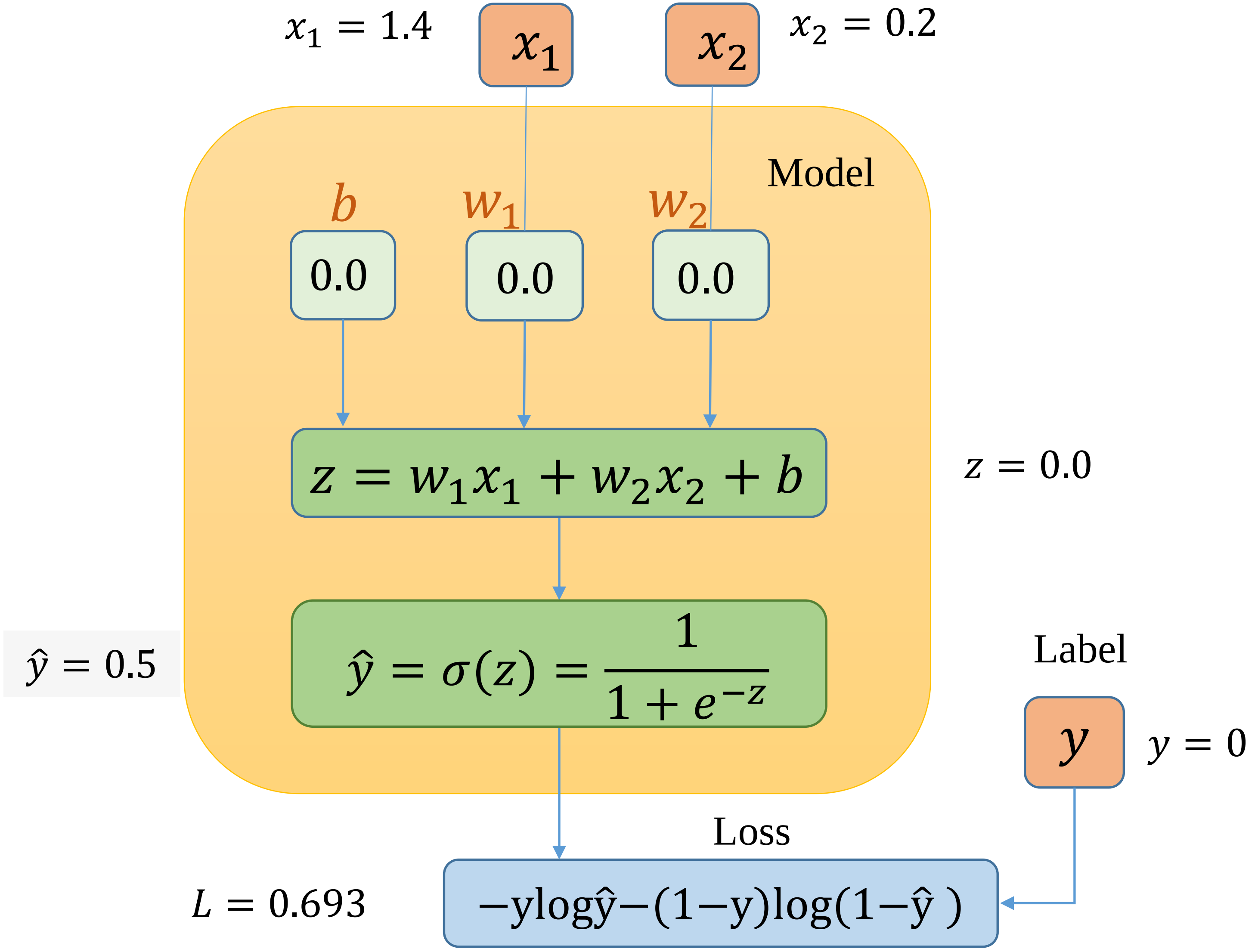


Example 4 - Zero Initialization

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} \qquad \boldsymbol{y} = [0]$$



Example 4 - Zero Initialization

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} \qquad \boldsymbol{y} = [0]$$

$\eta = 0.01$

$b = 0.005$

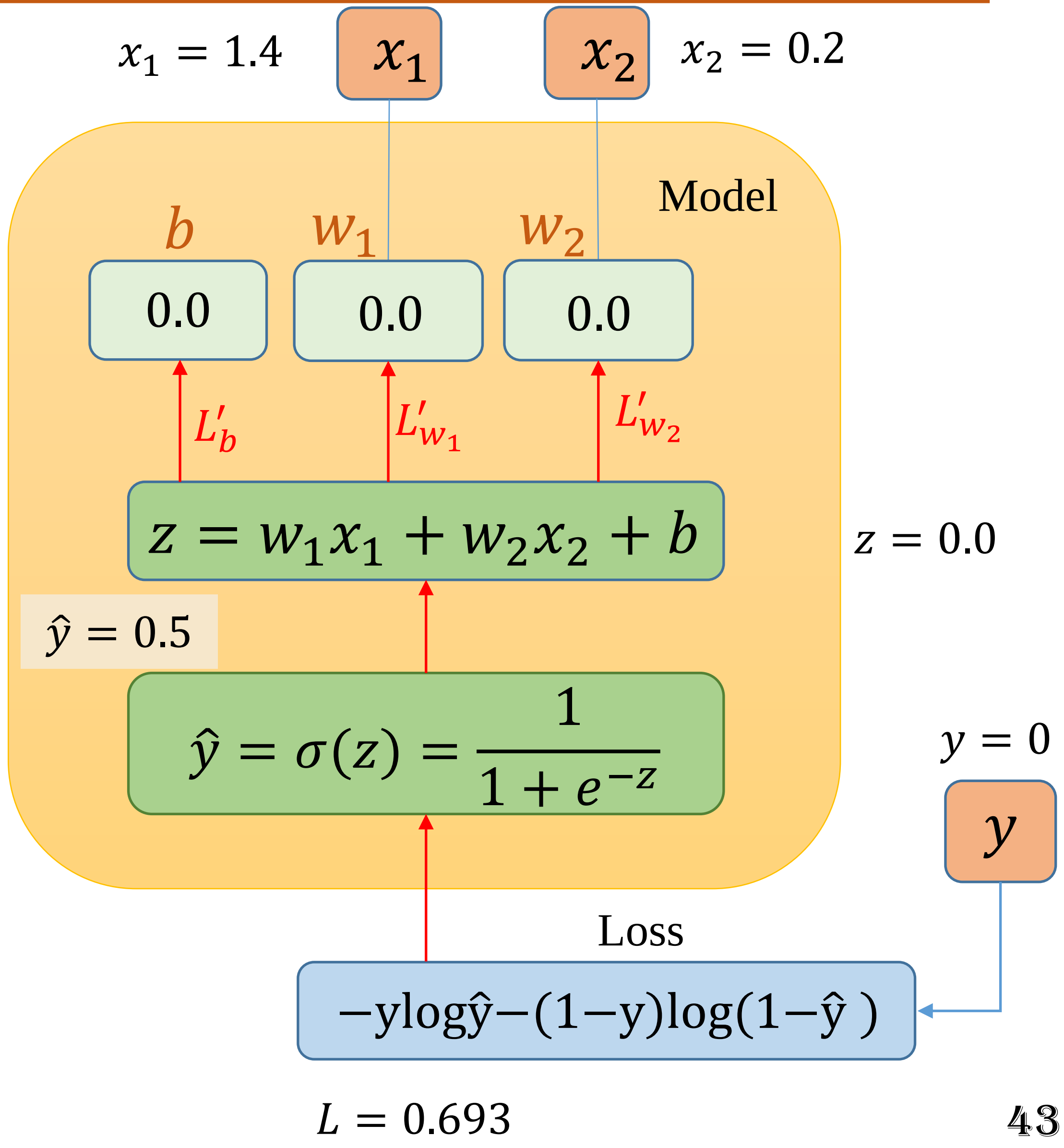
$w_1 = 0.007$

$w_2 = 0.001$

$$L'_{\boldsymbol{\theta}} = \mathbf{x}(\hat{y} - y)$$

$$= \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} [0.5]$$

$$= \begin{bmatrix} 0.5 \\ 0.7 \\ 0.1 \end{bmatrix} = \begin{bmatrix} L'_b \\ L'_{w_1} \\ L'_{w_2} \end{bmatrix}$$



Example 4 - Zero Initialization

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} \qquad \boldsymbol{y} = [0]$$

$\eta = 0.01$

$b = -0.005$

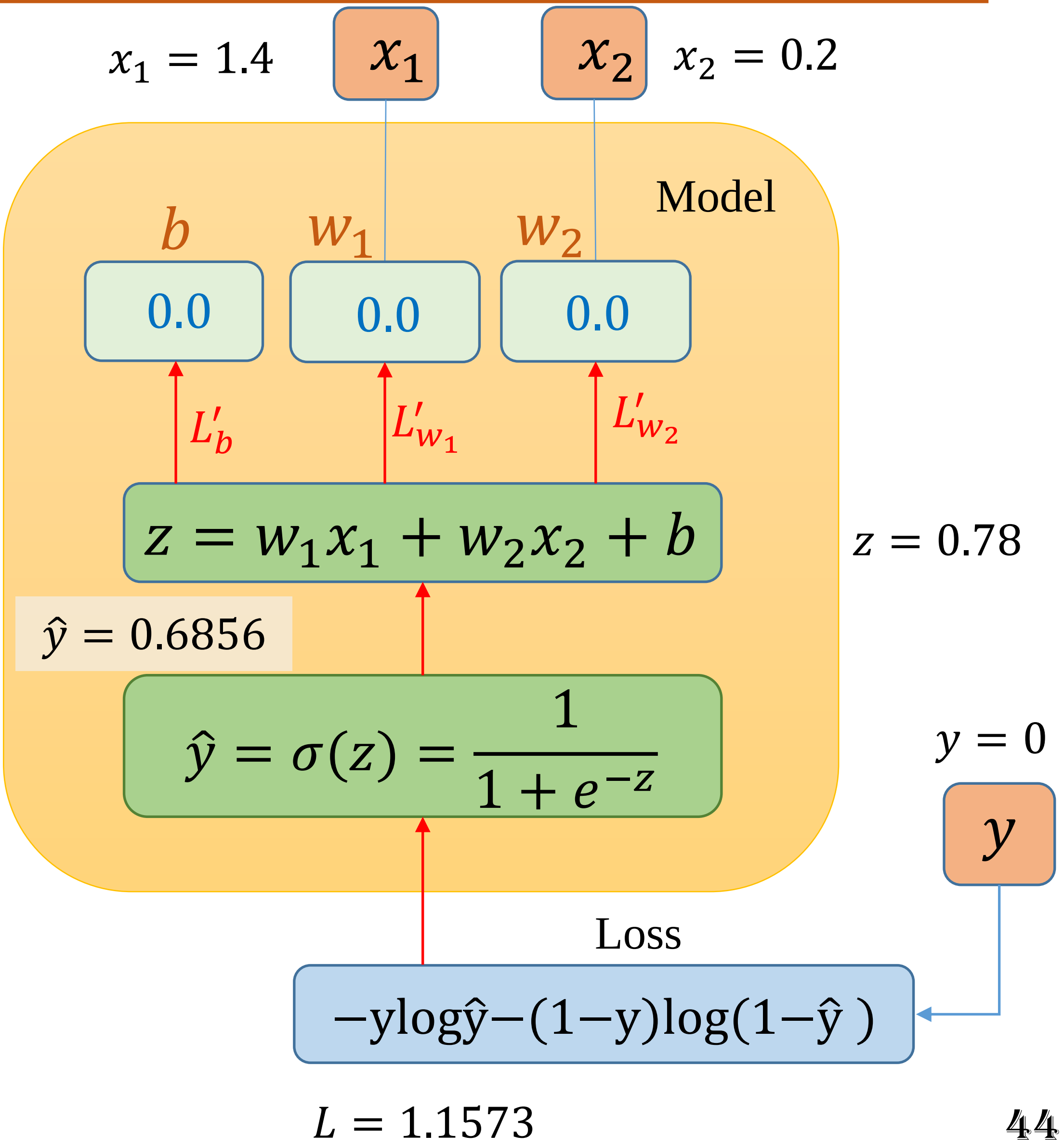
$w_1 = -0.007$

$w_2 = -0.001$

$$L'_{\boldsymbol{\theta}} = \mathbf{x}(\hat{y} - y)$$

$$= \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} [0.5]$$

$$= \begin{bmatrix} 0.5 \\ 0.7 \\ 0.1 \end{bmatrix} = \begin{bmatrix} L'_b \\ L'_{w_1} \\ L'_{w_2} \end{bmatrix}$$

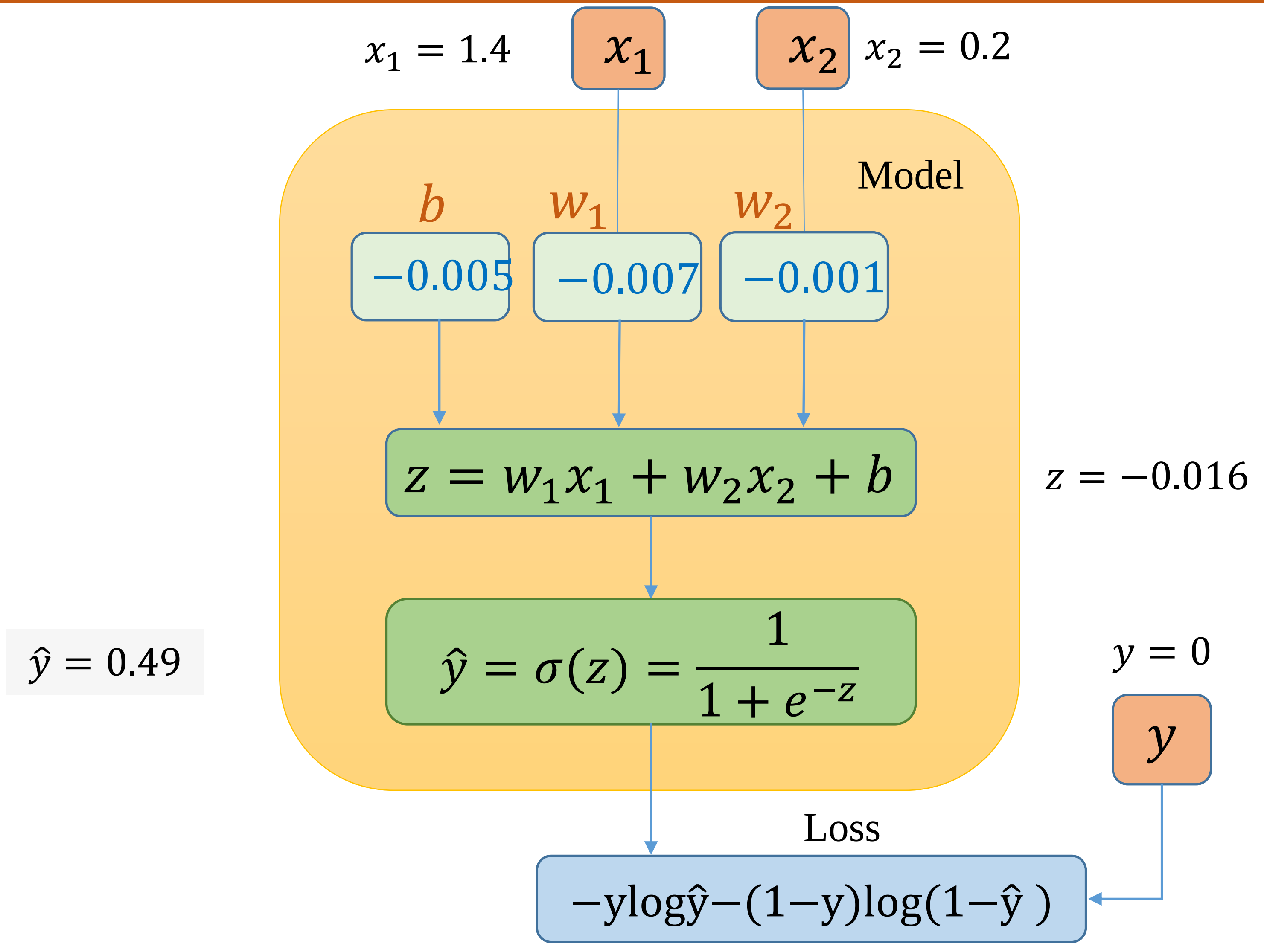


Example 4 - Zero Initialization

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} \qquad \boldsymbol{y} = [0]$$



previous $\boldsymbol{L} = 1.1573$ $\qquad L = 0.68$

Example 5 - Zero Initialization

❖ Softmax regression

Training data

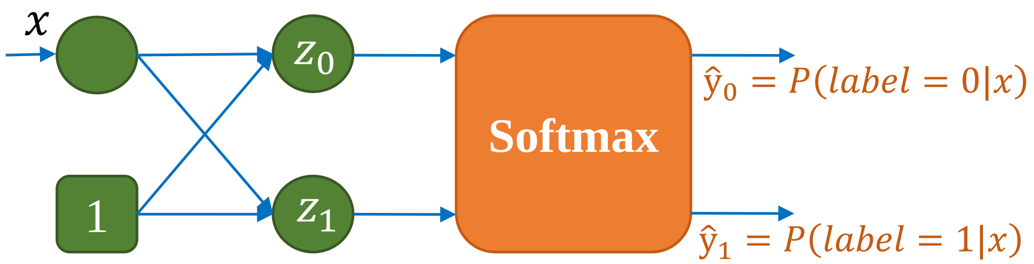
Feature	Label	
Petal_Length	Label	
1.4	0	Category A
1.3	0	Category A
1.5	0	Category A
4.5	1	Category B
4.1	1	Category B
4.6	1	Category B

One-hot encoding for labels

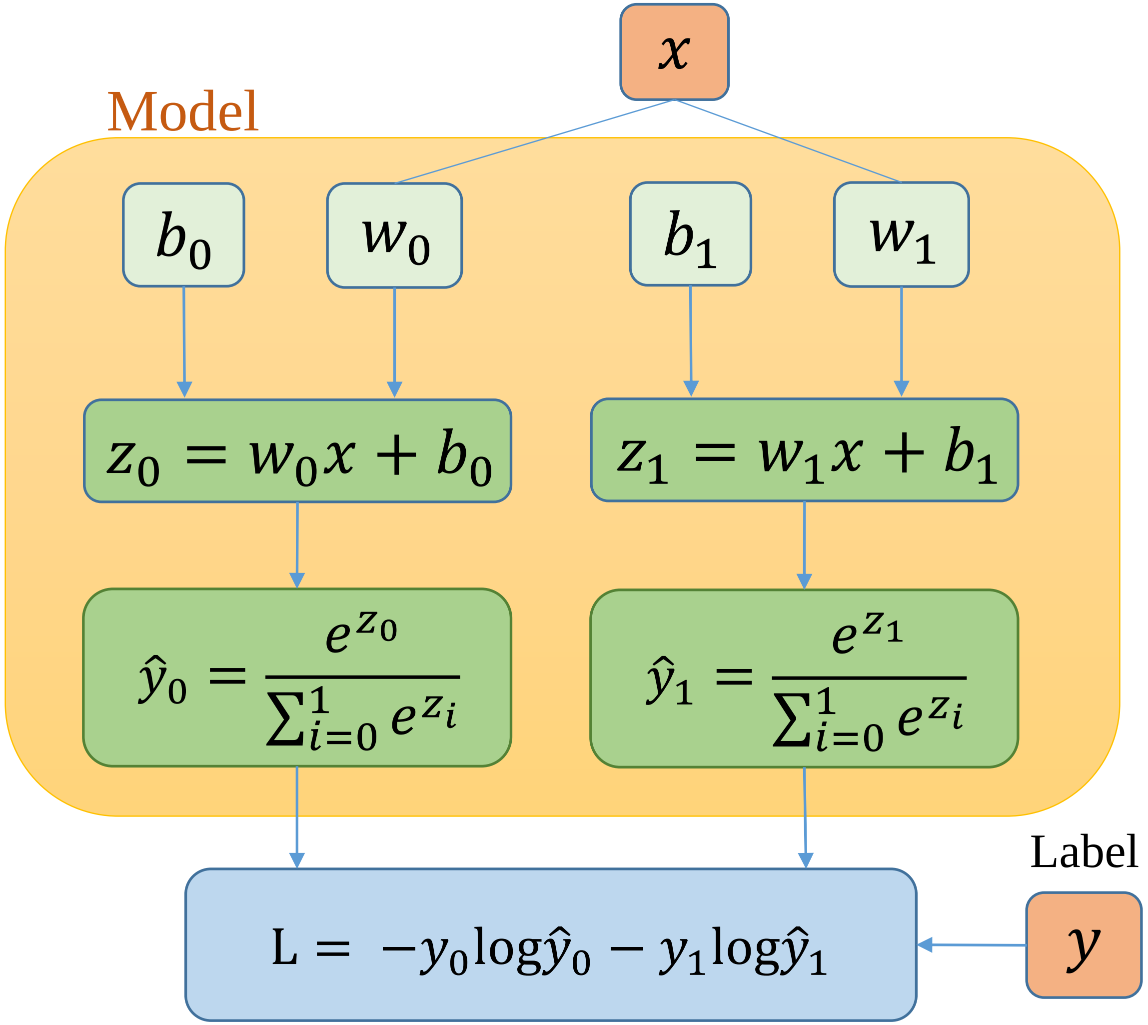
index 0 1

$$y = 0 \rightarrow \boldsymbol{y}^T = [1, 0]$$

$$y = 1 \rightarrow \boldsymbol{y}^T = [0, 1]$$



Model



Example 5 - Zero Initialization

Training data

Petal_Length	Label
1.4	0
1.3	0
1.5	0
4.5	1
4.1	1
4.6	1

Feature: Petal_Length; Label: Label

#class=2

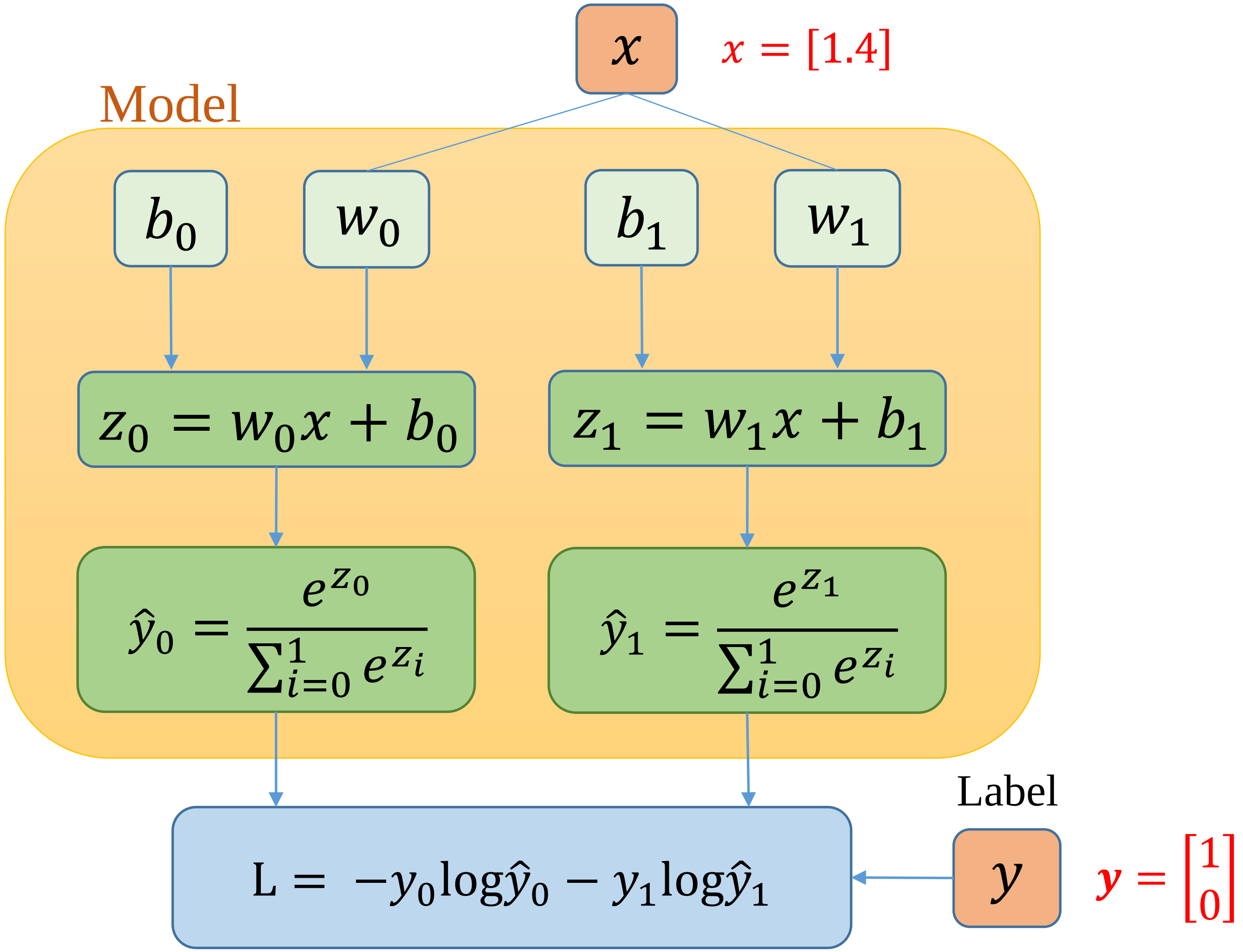
#feature=1

One-hot encoding for label

$$\begin{array}{ccc} & & y_0 \quad y_1 \\ y = 0 \rightarrow \boldsymbol{y}^T & = & [1 \quad 0] \\ y = 1 \rightarrow \boldsymbol{y}^T & = & [0 \quad 1] \end{array}$$

Training example

$$(x, y) = (1.4, 0)$$



Example 5 - Zero Initialization

Training data

Feature: Petal_Length	Label: Label
1.4	0
1.3	0
1.5	0
4.5	1
4.1	1
4.6	1

#class=2

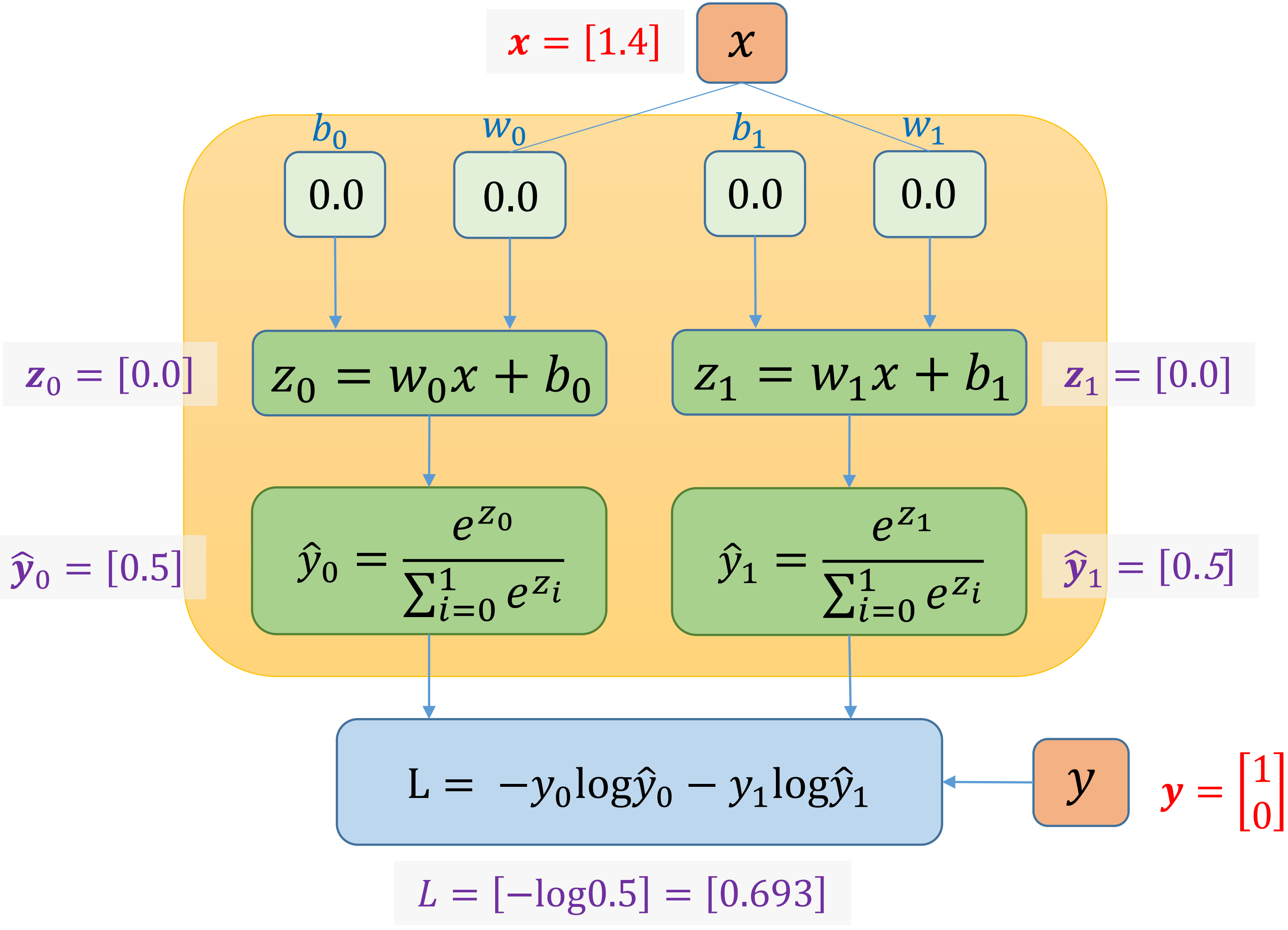
#feature=1

One-hot encoding for label

$$\begin{array}{cccc} & & y_0 & y_1 \\ y = 0 \rightarrow \boldsymbol{y}^T = & [& 1 & 0 \,] \\ y = 1 \rightarrow \boldsymbol{y}^T = & [& 0 & 1 \,] \end{array}$$

Training example

$$(x, y) = (1.4, 0)$$



Example 5 - Zero Initialization

Derivative

$$\frac{\partial L}{\partial z_i} = \hat{y}_i - y_i$$

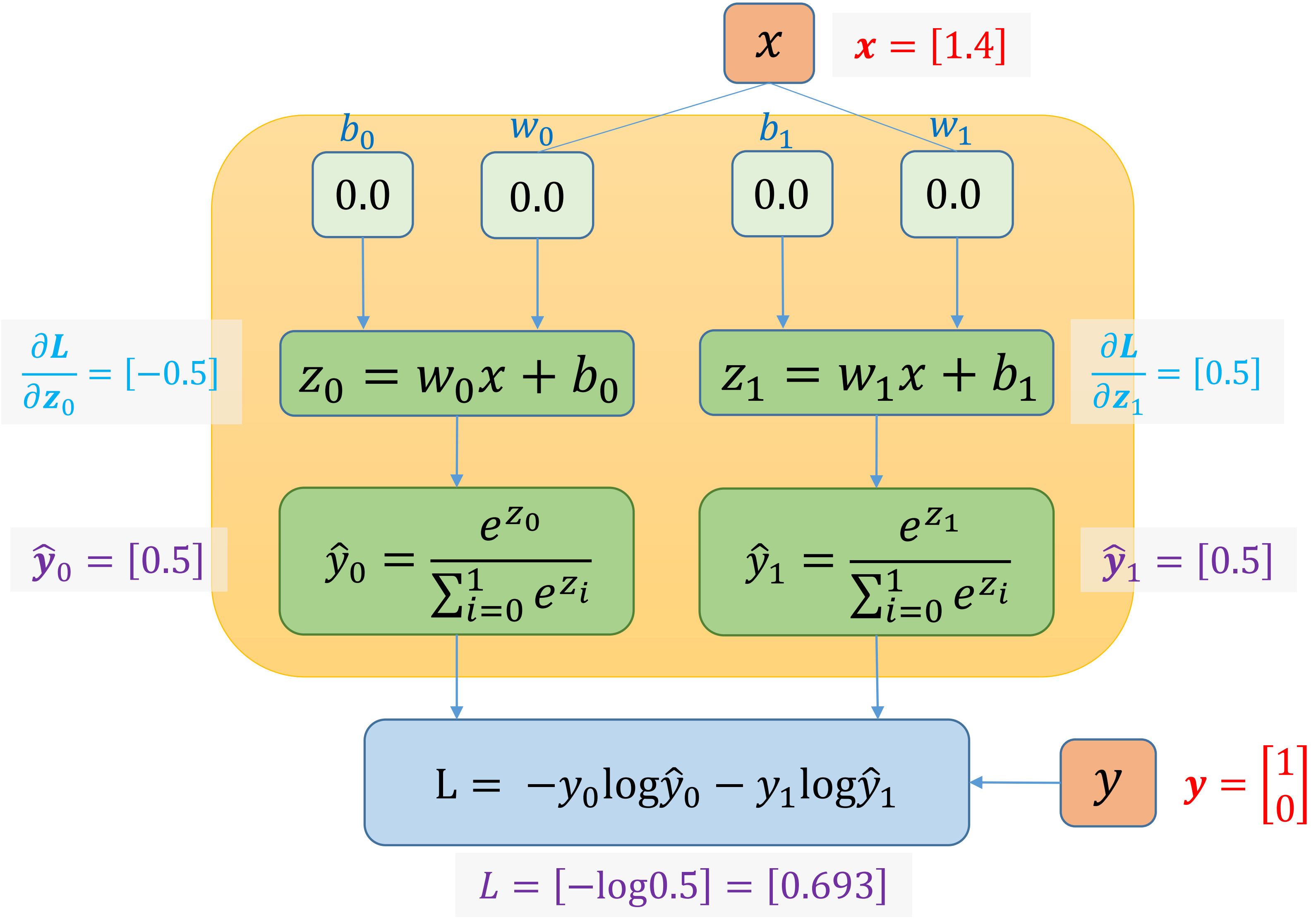
$$\frac{\partial L}{\partial w_i} = x(\hat{y}_i - y_i)$$

$$\frac{\partial L}{\partial b_i} = \hat{y}_i - y_i$$

$$\begin{array}{ll} & \begin{array}{cc} y_0 & y_1 \end{array} \\ y = 0 \rightarrow \boldsymbol{y}^T = & [1 \quad 0] \\ y = 1 \rightarrow \boldsymbol{y}^T = & [0 \quad 1] \end{array}$$

$$\frac{\partial L}{\partial \boldsymbol{z}_0} = \hat{y}_0 - 1$$
$$= 0.5 - 1 = -0.5$$

$$\frac{\partial L}{\partial \boldsymbol{z}_1} = \hat{y}_1 - 0 = 0.5$$



Example 5 - Zero Initialization

Derivative

$$\frac{\partial L}{\partial z_i} = \hat{y}_i - y_i$$

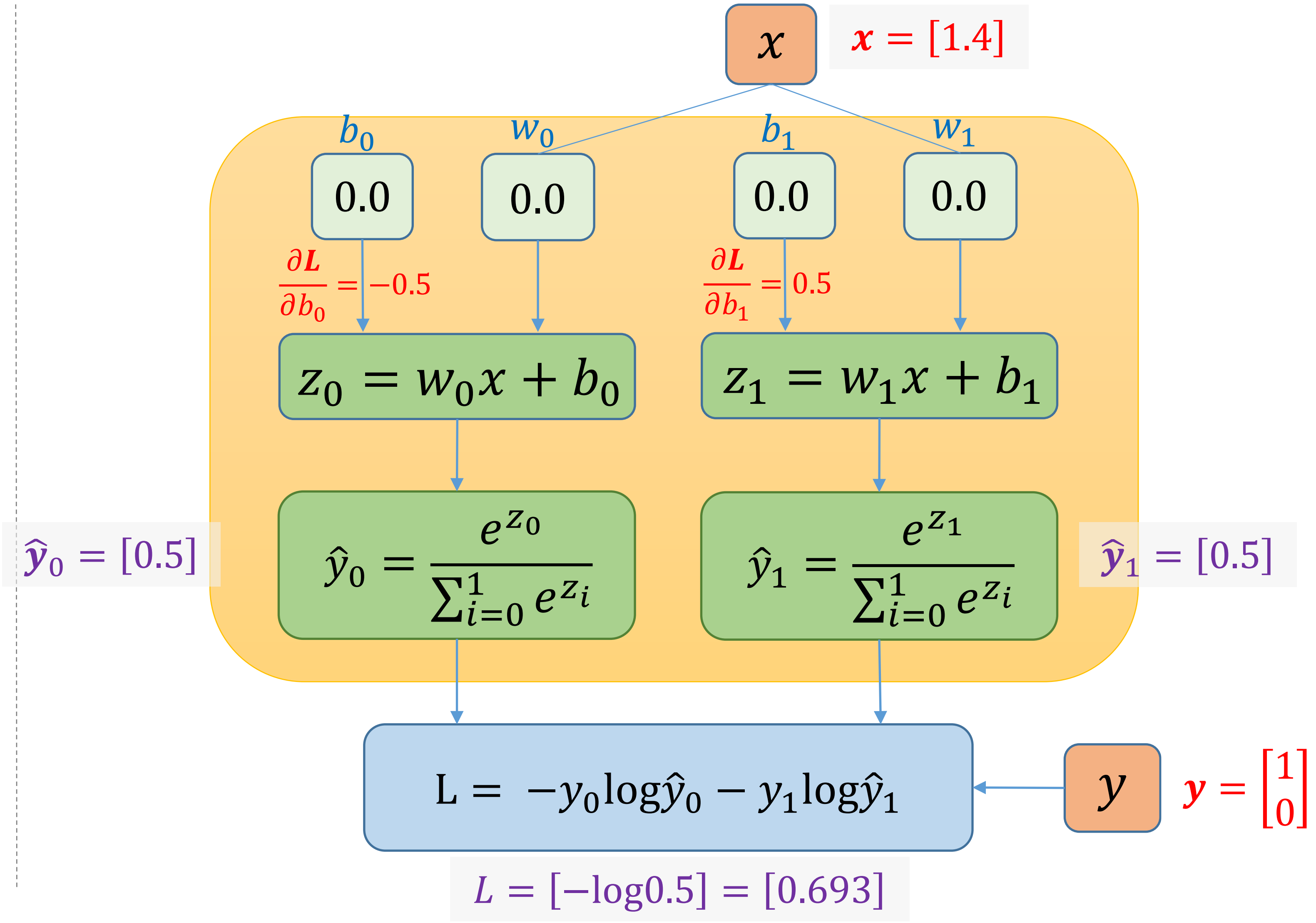
$$\frac{\partial L}{\partial w_i} = x(\hat{y}_i - y_i)$$

$$\frac{\partial L}{\partial b_i} = \hat{y}_i - y_i$$

$$\begin{array}{l} \phantom{y = 0 \rightarrow \boldsymbol{y}^T = [}\, y_0 \quad y_1 \\ y = 0 \rightarrow \boldsymbol{y}^T = [1 \quad 0] \\ y = 1 \rightarrow \boldsymbol{y}^T = [0 \quad 1] \end{array}$$

$$\frac{\partial \boldsymbol{L}}{\partial b_0} = (\hat{y}_0 - 1\,) = -0.5$$

$$\frac{\partial \boldsymbol{L}}{\partial b_1} = (\hat{y}_1 - 0\,) = 0.5$$



Example 5 - Zero Initialization

Derivative

$$\frac{\partial L}{\partial z_i} = \hat{y}_i - y_i$$

$$\frac{\partial L}{\partial w_i} = x(\hat{y}_i - y_i)$$

$$\frac{\partial L}{\partial b_i} = \hat{y}_i - y_i$$

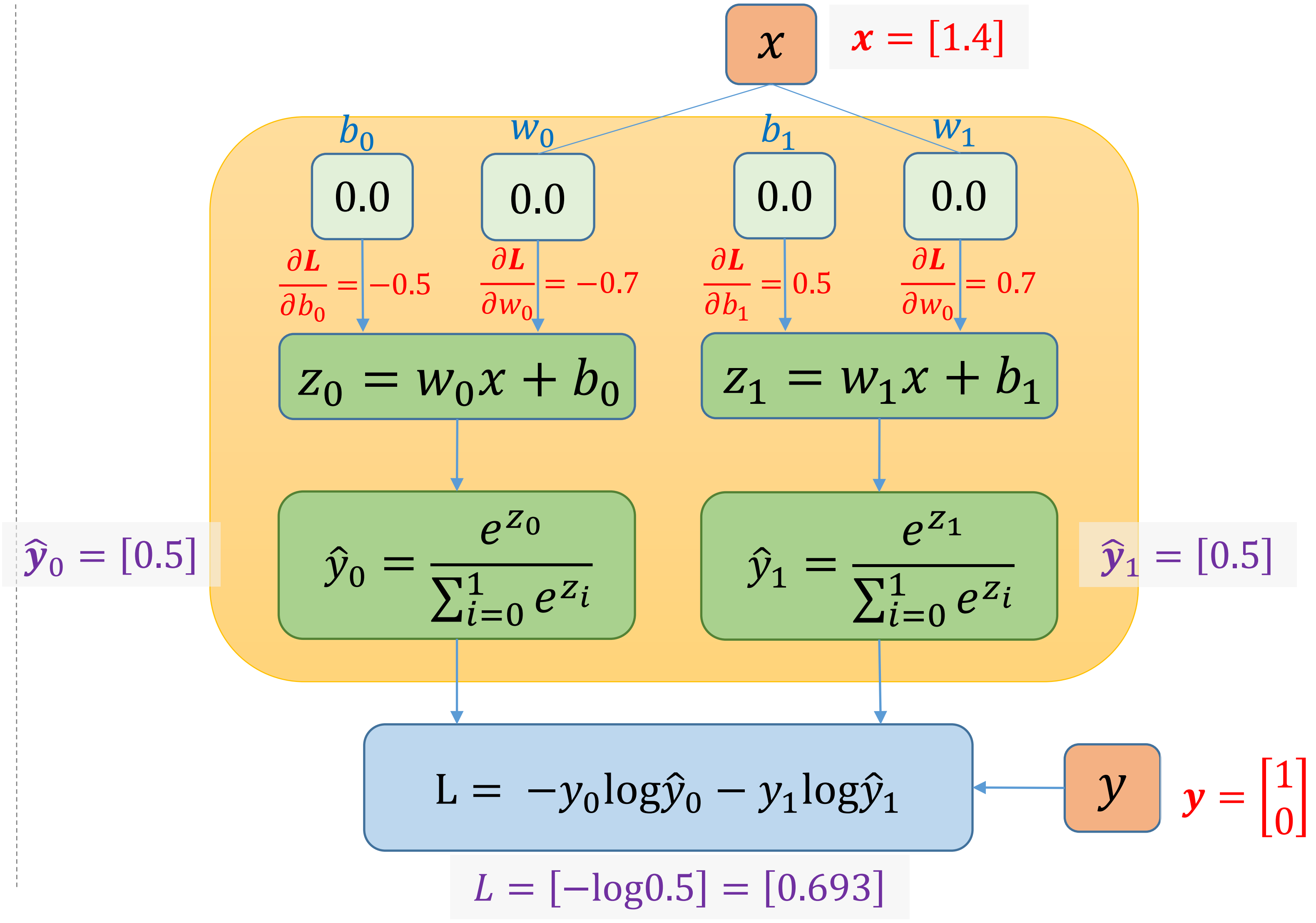
$$\begin{array}{ccc} & y_0 & y_1 \\ y = 0 \rightarrow \boldsymbol{y}^T = & [1 & 0] \\ y = 1 \rightarrow \boldsymbol{y}^T = & [0 & 1] \end{array}$$

$$\frac{\partial \boldsymbol{L}}{\partial w_0} = x(\hat{y}_0 - 1\ )$$

$$= -0.5 * 1.4 = -0.7$$

$$\frac{\partial \boldsymbol{L}}{\partial w_1} = x(\hat{y}_1 - 0\ )$$

$$= 0.5 * 1.4 = 0.7$$



Example 5 - Zero Initialization

Update parameters

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

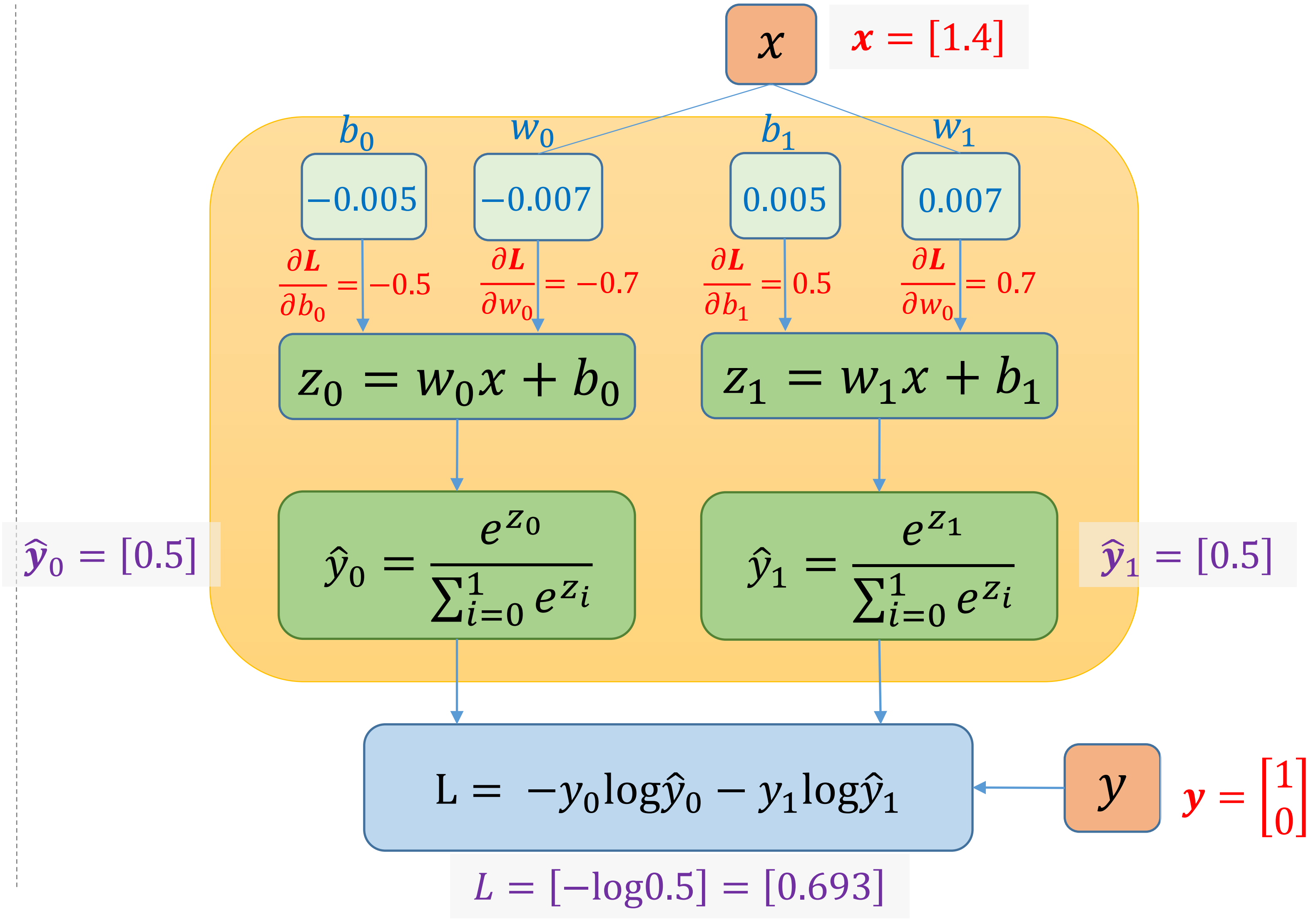
η is learning rate

$$\boldsymbol{\theta} = \begin{bmatrix} b_0 & b_1 \\ w_0 & w_1 \end{bmatrix} \qquad L'_{\boldsymbol{\theta}} = \begin{bmatrix} \frac{\partial L}{\partial b_0} & \frac{\partial L}{\partial b_1} \\ \frac{\partial L}{\partial w_0} & \frac{\partial L}{\partial w_1} \end{bmatrix}$$

$$\eta = 0.1$$

$$\boldsymbol{\theta} = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} - 0.01 \begin{bmatrix} -0.5 & 0.5 \\ -0.7 & 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} -0.005 & 0.005 \\ -0.007 & 0.007 \end{bmatrix}$$



Example 5 - Zero Initialization

Training data

Feature | **Label**

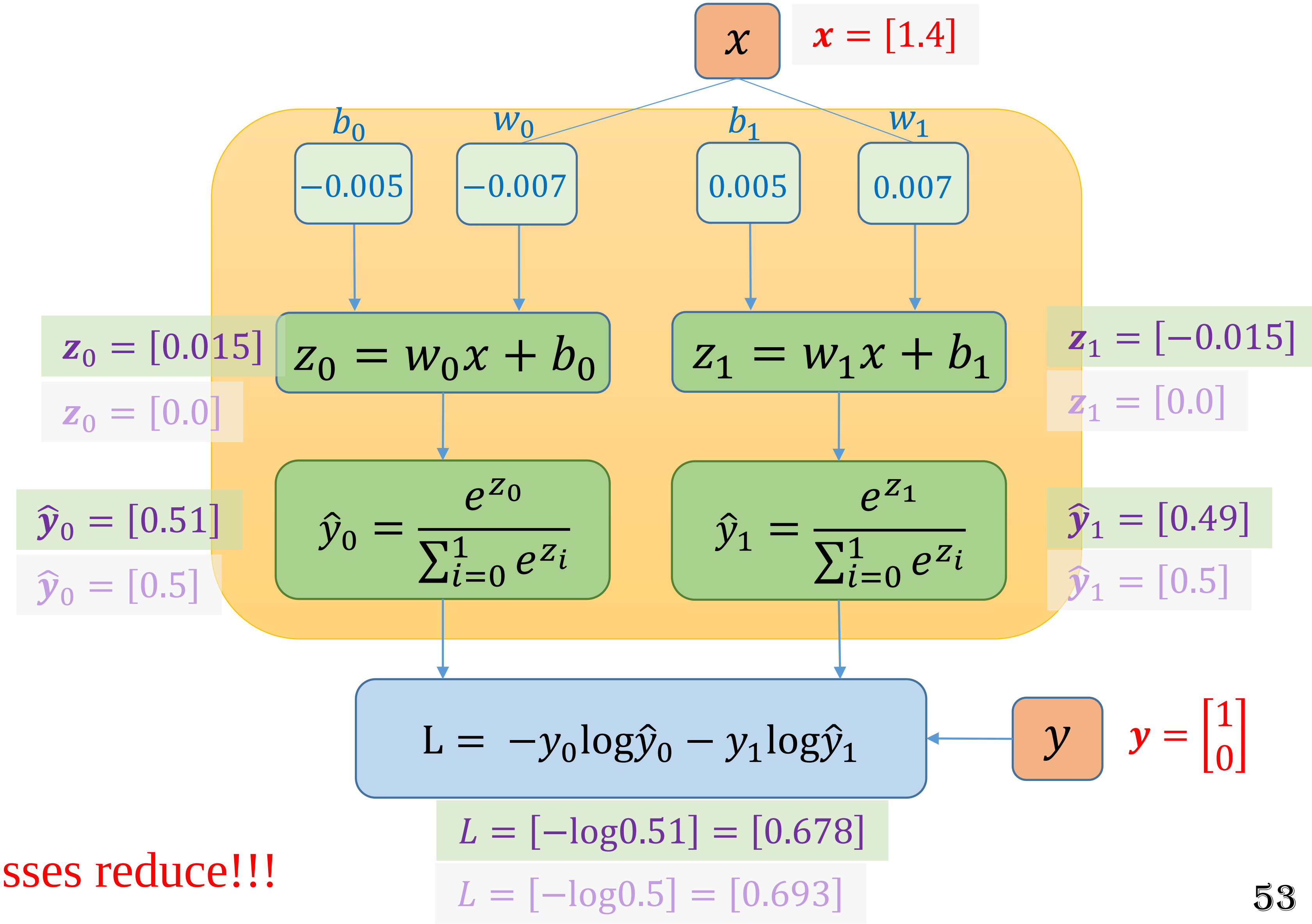
Petal_Length	Label
1.4	0
1.3	0
1.5	0
4.5	1
4.1	1
4.6	1

One-hot encoding for label

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad y_0 \quad y_1 \\ y = 0 \rightarrow \boldsymbol{y}^T = [1 \quad 0] \\ y = 1 \rightarrow \boldsymbol{y}^T = [0 \quad 1] \end{array}$$

Training example

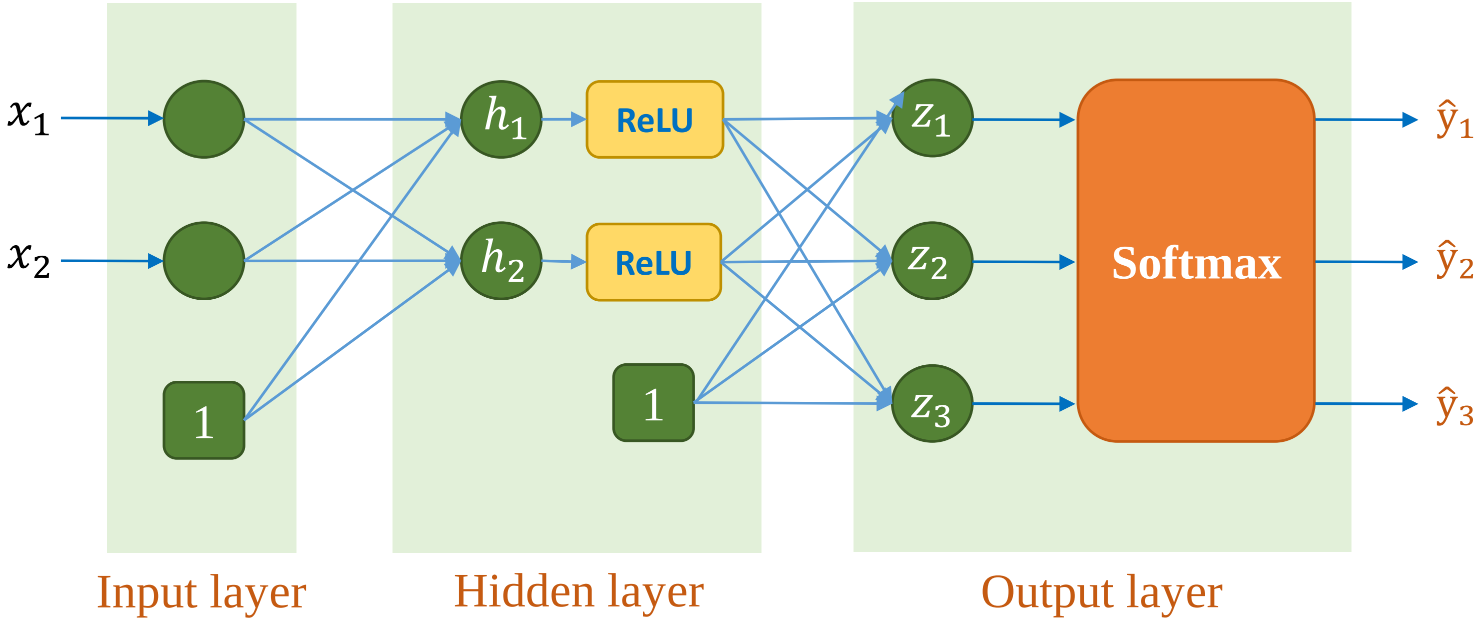
$$(x, y) = (1.4, 0)$$



losses reduce!!!

Example 6 - Zero Initialization

Feature		Label
Petal_Length	Petal_Width	Label
1.5	0.2	0
1.4	0.2	0
1.6	0.2	0
4.7	1.6	1
3.3	1.1	1
4.6	1.3	1
5.6	2.2	2
5.1	1.5	2
5.6	1.4	2



$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(2)} \\ \boldsymbol{x}^{(3)} \end{bmatrix} = \begin{bmatrix} 1.5 & 0.2 \\ 4.7 & 1.6 \\ 5.6 & 2.2 \end{bmatrix}$$

$$\boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\boldsymbol{h} = [\boldsymbol{h}_1 \quad \boldsymbol{h}_2] = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$\boldsymbol{w} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \boldsymbol{w}_3] = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\boldsymbol{b}_h = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

$$\boldsymbol{b}_w = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$\boldsymbol{h} = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

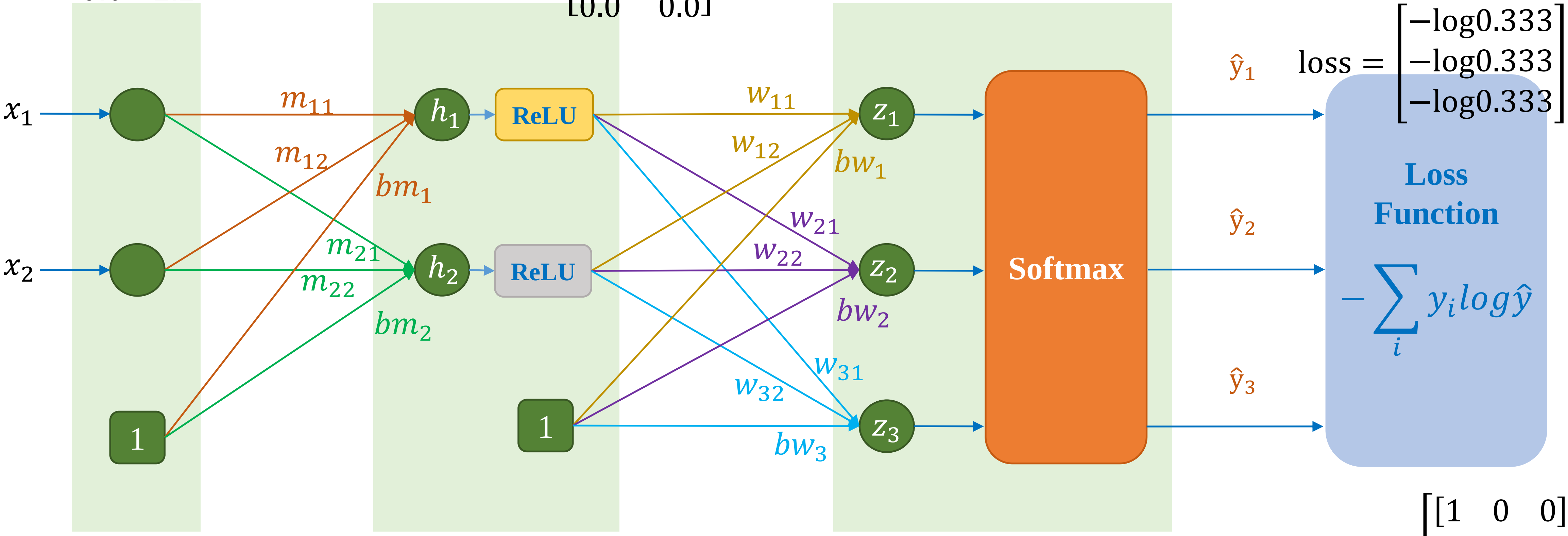
$$\boldsymbol{x} = \begin{bmatrix} 1.5 & 0.2 \\ 4.7 & 1.6 \\ 5.6 & 2.2 \end{bmatrix}$$

$$\textbf{ReLU} = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$\boldsymbol{z} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\hat{\boldsymbol{y}} = \begin{bmatrix} 0.333 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.333 \end{bmatrix}$$

$$\text{loss} = \begin{bmatrix} -\log 0.333 \\ -\log 0.333 \\ -\log 0.333 \end{bmatrix}$$



$$\boldsymbol{y} = \begin{bmatrix} [1 & 0 & 0] \\ [0 & 1 & 0] \\ [0 & 0 & 1] \end{bmatrix}$$

$$\boldsymbol{m} = [\boldsymbol{m}_1 \quad \boldsymbol{m}_2] = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$\boldsymbol{bm} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

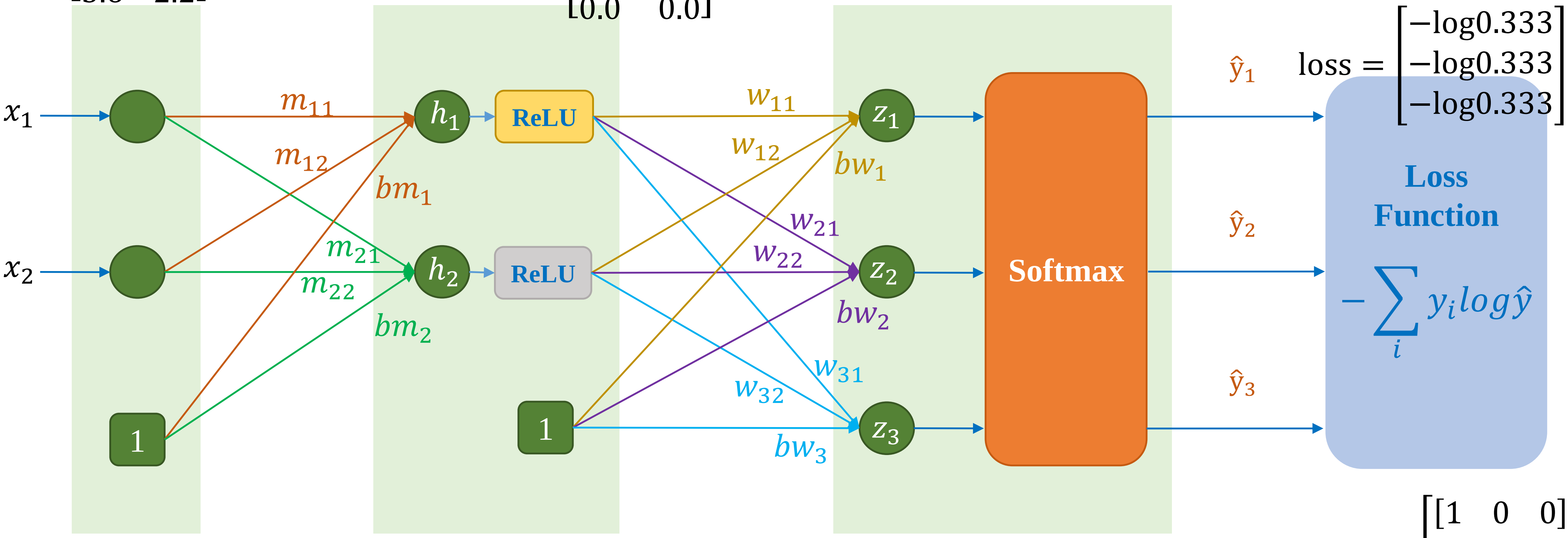
$$\boldsymbol{w} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \boldsymbol{w}_3] = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\boldsymbol{bw} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} 1.5 & 0.2 \\ 4.7 & 1.6 \\ 5.6 & 2.2 \end{bmatrix} \quad \boldsymbol{h} = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} \quad \textbf{ReLU} = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$\boldsymbol{z} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \quad \hat{\boldsymbol{y}} = \begin{bmatrix} 0.333 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.333 \end{bmatrix}$$

$$\text{loss} = \begin{bmatrix} -\log 0.333 \\ -\log 0.333 \\ -\log 0.333 \end{bmatrix}$$



$$\boldsymbol{y} = \begin{bmatrix} [1 & 0 & 0] \\ [0 & 1 & 0] \\ [0 & 0 & 1] \end{bmatrix}$$

$$\boldsymbol{m} = [\boldsymbol{m}_1 \quad \boldsymbol{m}_2] = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} \quad \boldsymbol{bm} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

$$\boldsymbol{w} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \boldsymbol{w}_3] = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \quad \boldsymbol{bw} = \begin{bmatrix} v \\ v \\ v \end{bmatrix}$$

Optimizers

Optimizer Selection

Data Preparation

Data Normalization

Model (Network) Construction

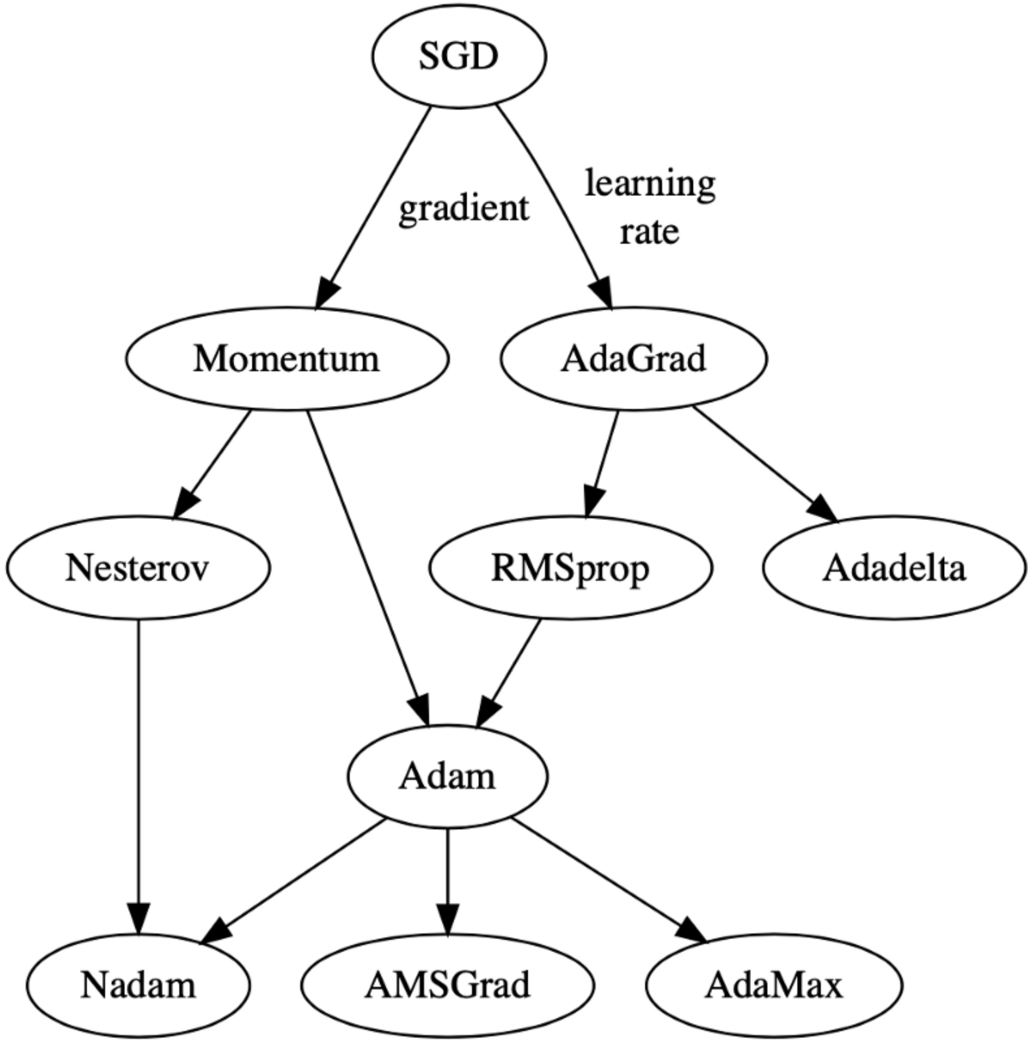
Parameter Initialization

Define a way to update parameters

Optimizer Selection

Loss function Selection

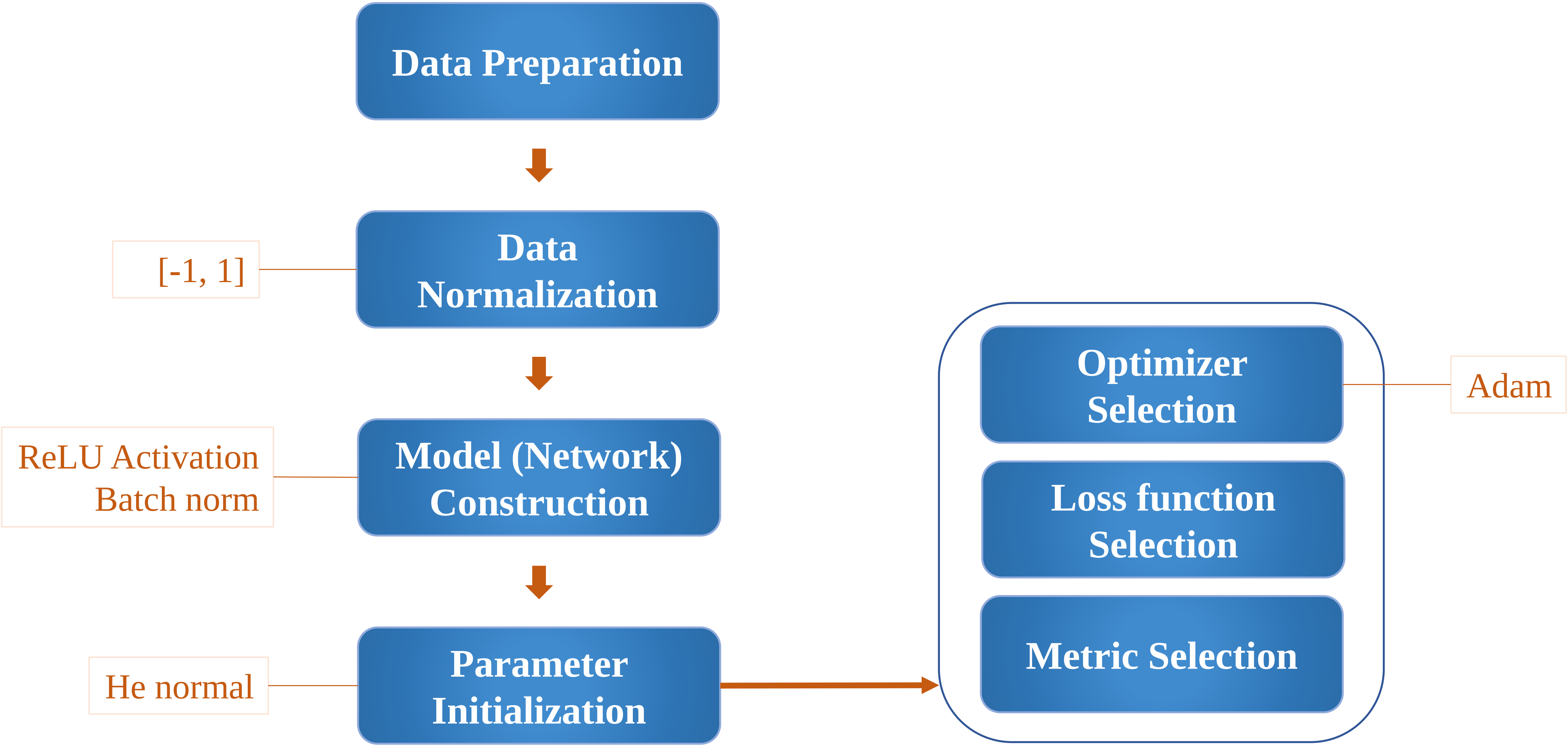
Metric Selection



https://www.kdnuggets.com/2019/06/gradient-descent-algorithms-cheat-sheet.html

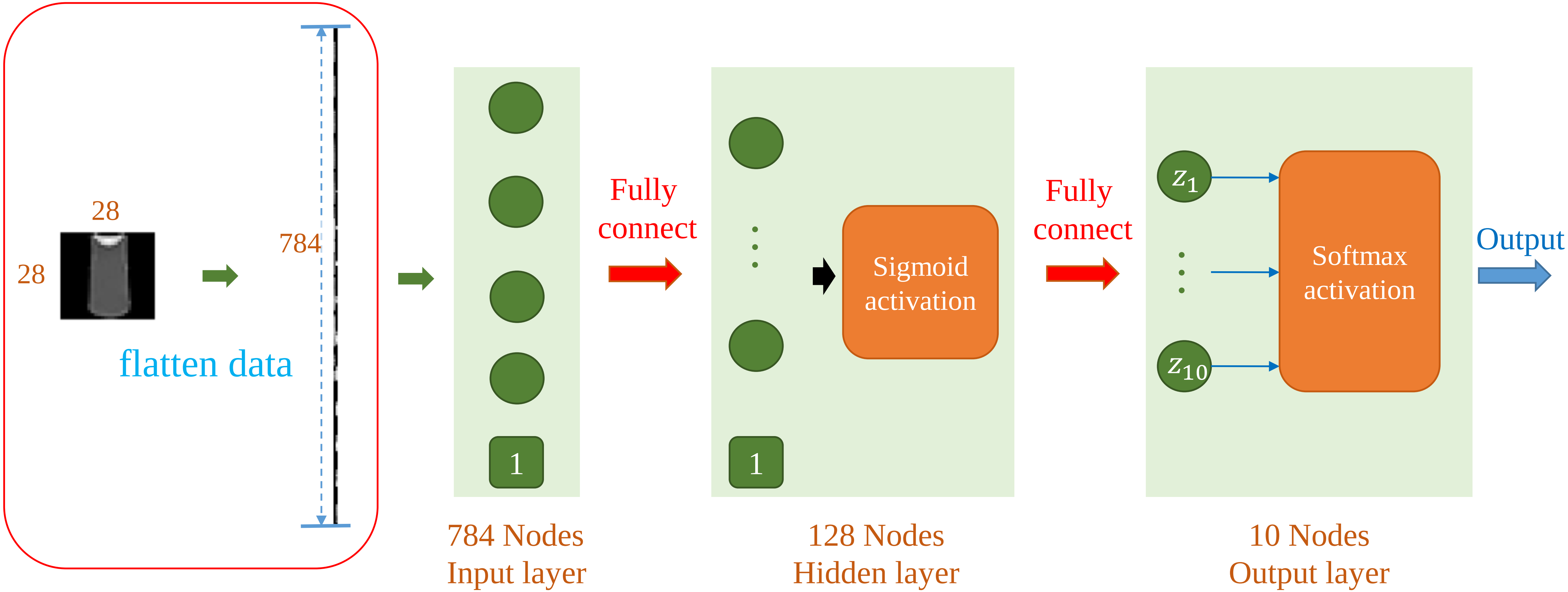
Summary

Recommendation



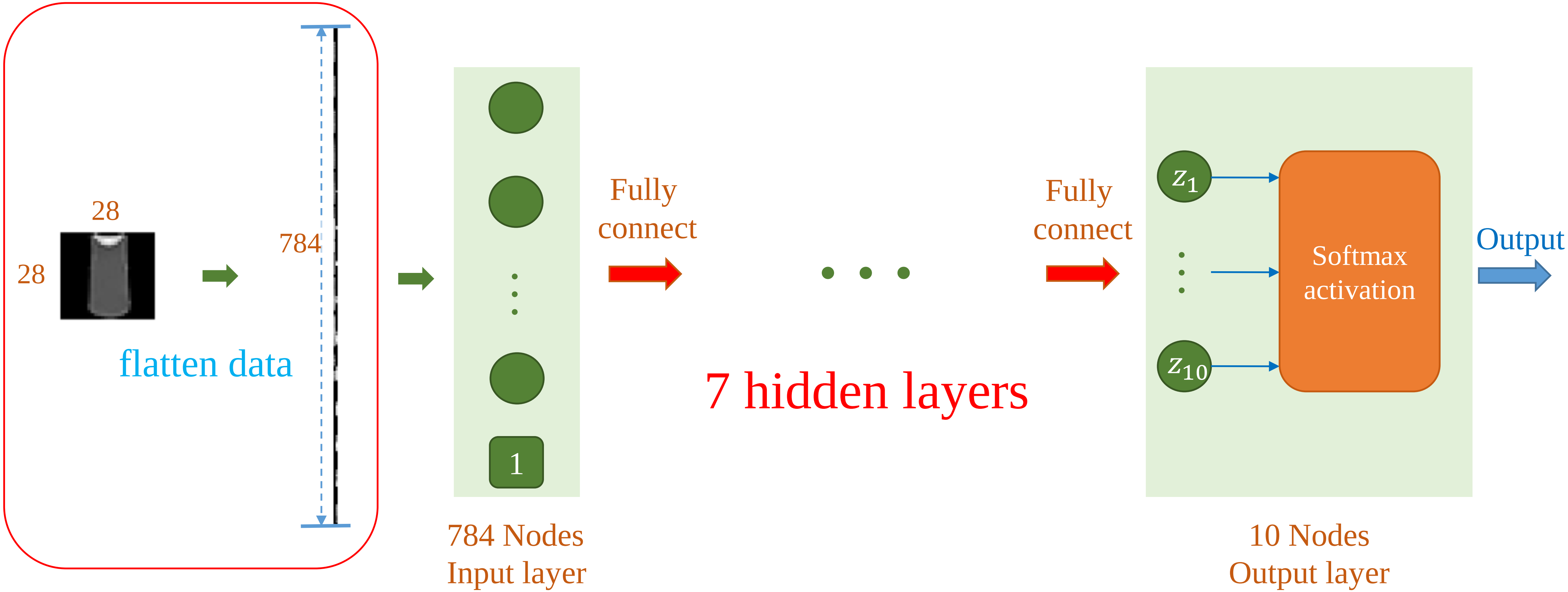
Discussion

- ❖ **Sigmoid and SGD**
- ❖ **W/o using normalization**



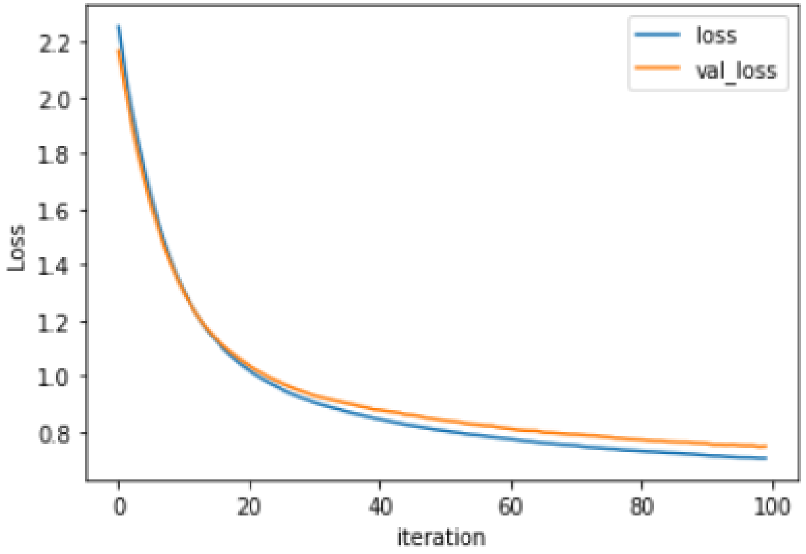
Discussion

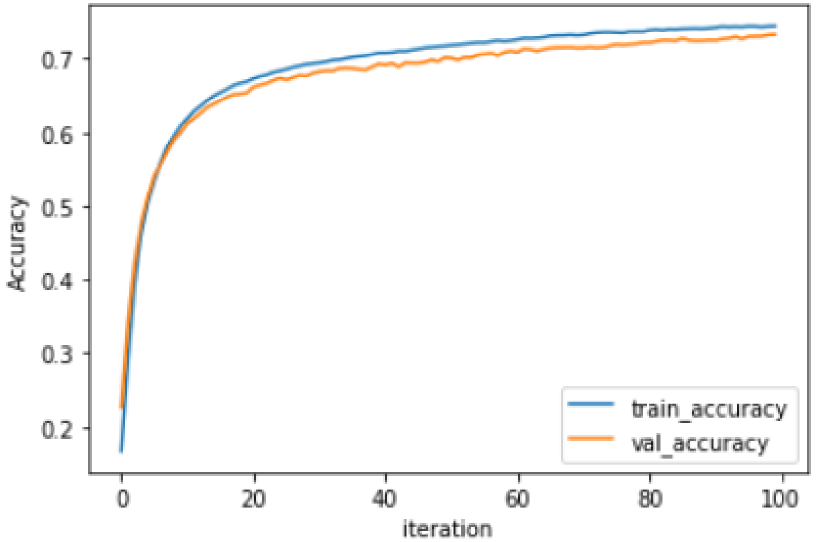
- ❖ **Sigmoid and SGD**
- ❖ **W/o using normalization**



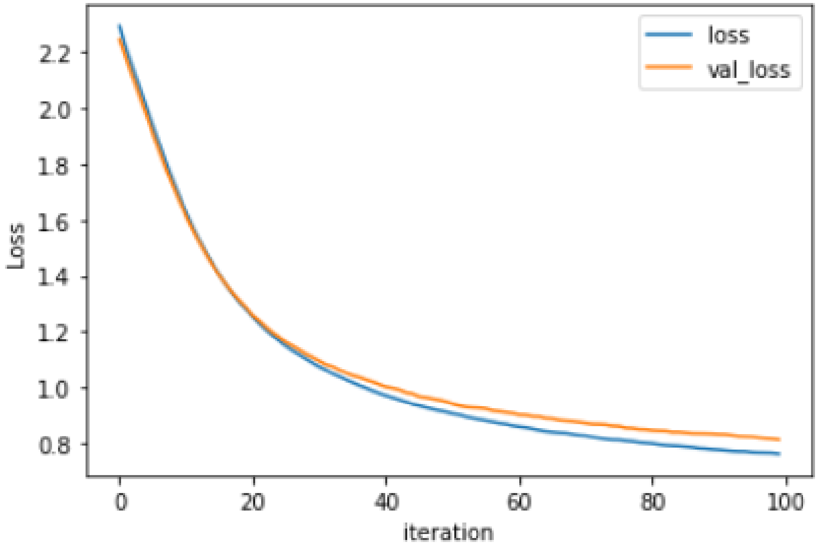
Discussion

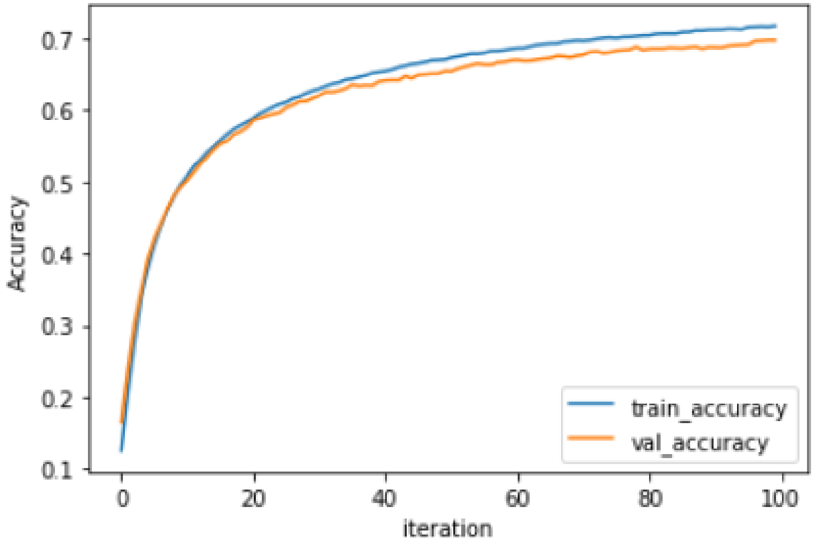
2 hidden layers



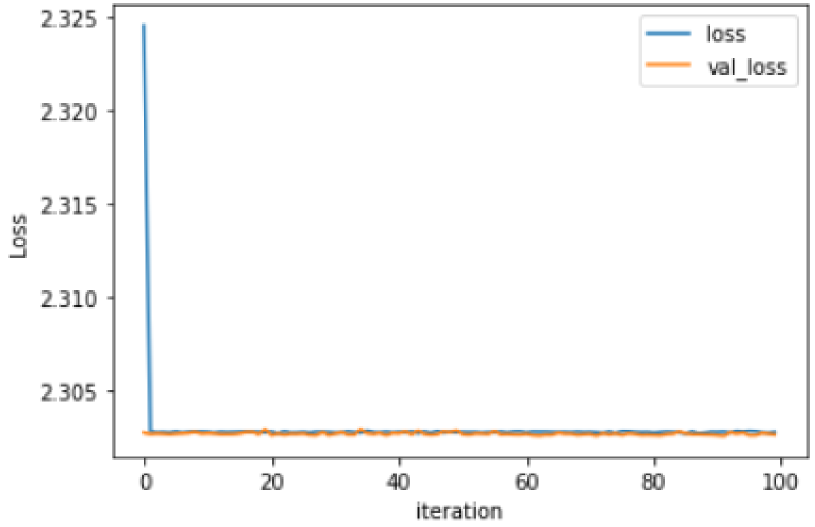


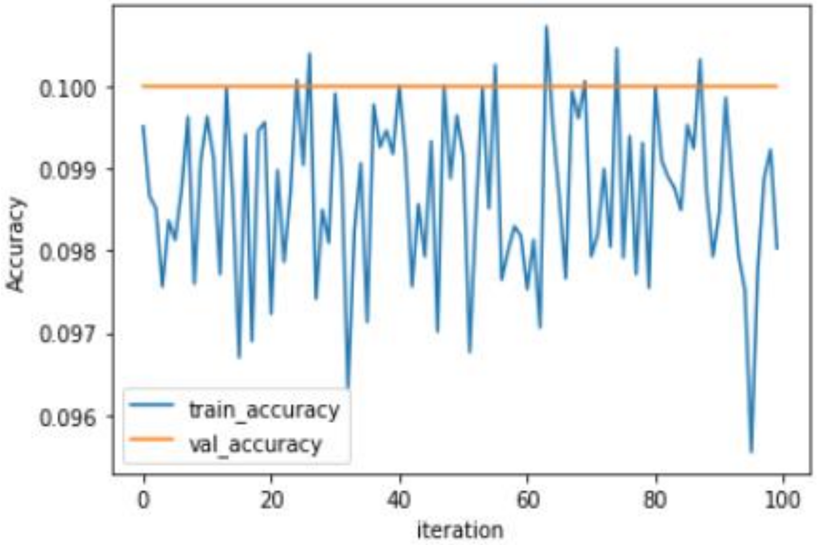
5 hidden layers (!)





7 hidden layers

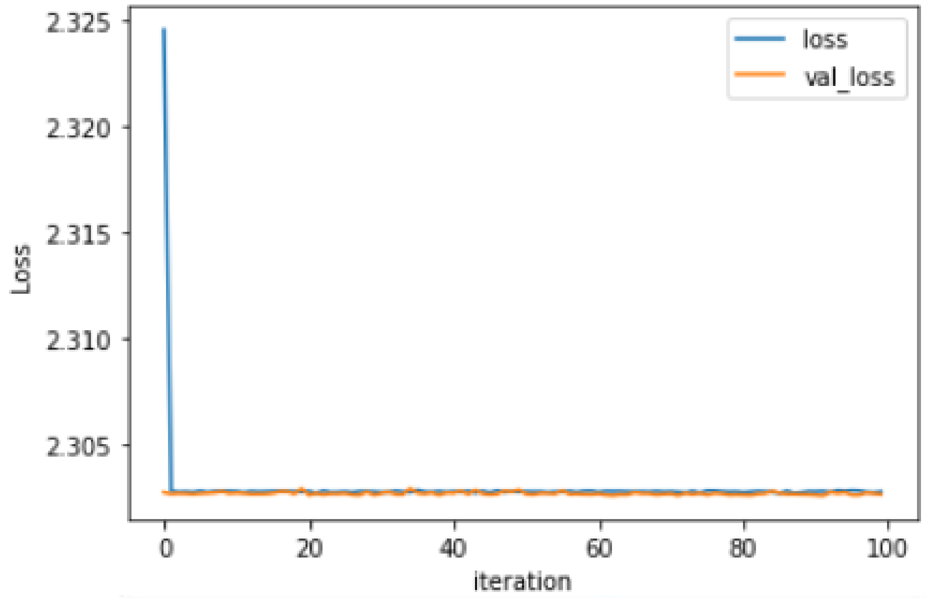


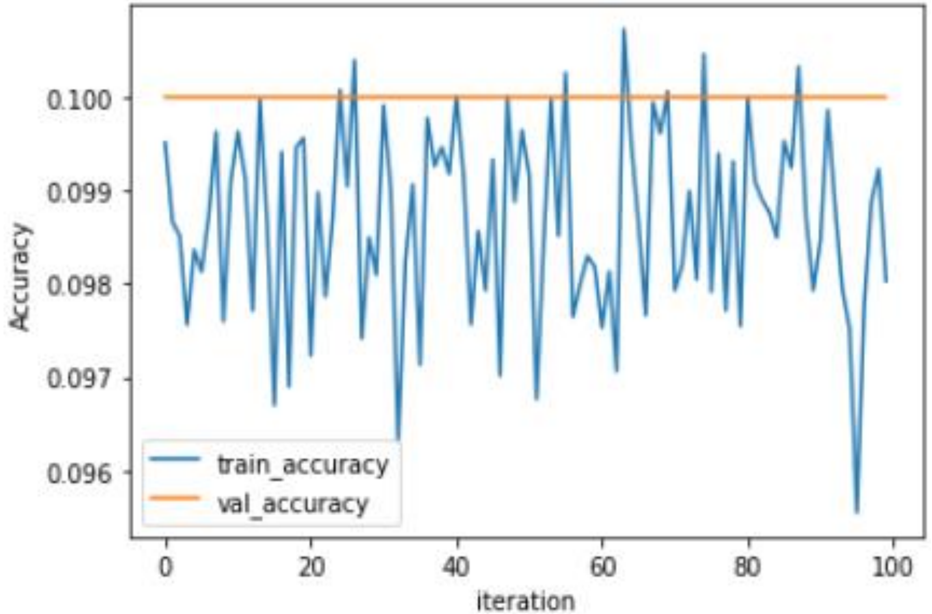


Discussion

Tensorflow

```python
import tensorflow as tf
import tensorflow.keras as keras

# set seed
tf.random.set_seed(1234)
initializer = tf.keras.initializers.RandomNormal()

# create model
model = keras.Sequential()
model.add(keras.Input(shape=(784,)))
model.add(keras.layers.Dense(128, activation='sigmoid',
                             kernel_initializer=initializer))
model.add(keras.layers.Dense(128, activation='sigmoid',
                             kernel_initializer=initializer))
model.add(keras.layers.Dense(128, activation='sigmoid',
                             kernel_initializer=initializer))
model.add(keras.layers.Dense(128, activation='sigmoid',
                             kernel_initializer=initializer))
model.add(keras.layers.Dense(128, activation='sigmoid',
                             kernel_initializer=initializer))
model.add(keras.layers.Dense(128, activation='sigmoid',
                             kernel_initializer=initializer))
model.add(keras.layers.Dense(10, activation='softmax'))
```





Further Reading

Dying ReLU

https://towardsdatascience.com/the-dying-relu-problem-clearly-explained-42d0c54e0d24

Initialization

https://www.deeplearning.ai/ai-notes/initialization/index.html