ME 344L – Spring 2025

Laboratory 3 - Computational Tools for Transfer Functions

Thursday 10:05 am - 12:40 pm
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Due: February 17th, 2025

1 Introduction

This lab employed MATLAB for modeling, simulating, and analyzing dynamic systems through transfer functions. In the three experiments, we examined both mechanical and electrical systems to gain a deeper understanding of system response characteristics. In the electrical portion, we derived and simulated the transfer function for a bridged-T filter circuit. By using step and ramp inputs, we observed the transient and steady-state behavior of the circuit, including the effects of capacitor voltage continuity and time delays.

For the mechanical system, we simulated a vehicle suspension model using a quarter-car representation. This experiment involved generating the simulated response of the passenger compartment to a road bump, modeled as a step input. The simulation further allowed us to identify key parameters such as the damped natural frequency by analyzing the oscillatory response. We then compared the simulated data with experimental measurements recorded from the actual suspension setup, highlighting the similarities and discrepancies between the theoretical model and real-world behavior.

2 Assignment 1

2.1 Mechanical System

2.1.1 Motion of the Masses

Both masses act in similar fashion in response to the step input. Both have stable oscillations, due to the fact that they are under-damped. However, m_1 reacts more severely, due to the fact that the step is directly applied to it, whereas m_2 experiences the input damped through spring k_2. This applies both to the initial reactions and amplitudes of the oscillations. Because the oscillations are smaller for m_2, it will reach steady-state faster.

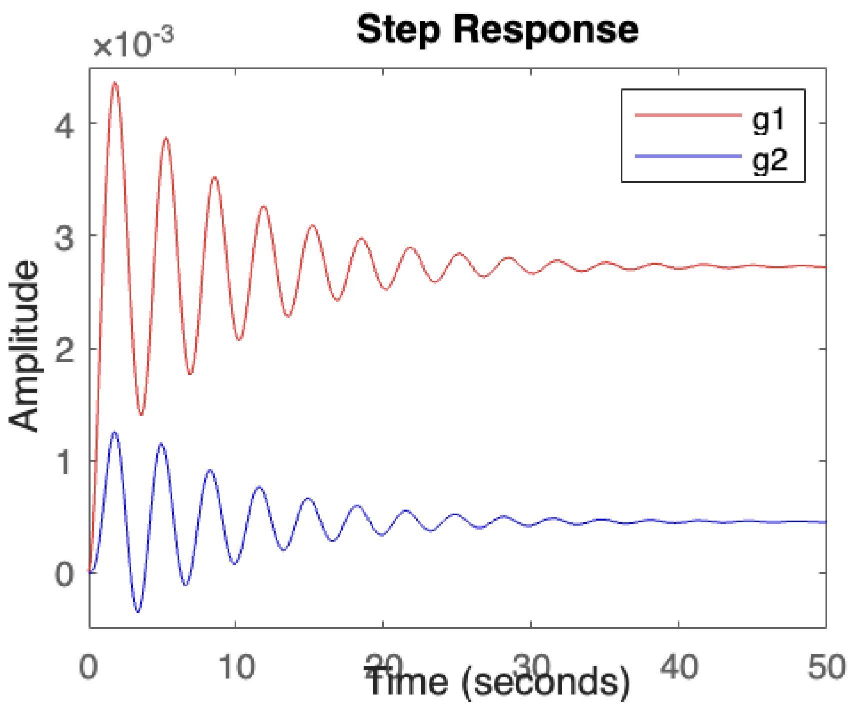

Figure 1: Mass Step Response

2.1.2 Transfer Function Denominator

Both transfer function have the following equation as the denominator (Eqn. 1):

$$(s^2 + 0.2348s + 3.612)(s^2 + 1.265s + 6.091) \tag{1}$$

If we factor the denominator, we get the following poles for the transfer function:

$$s_1 = -\frac{253}{400} \pm i\frac{\sqrt{910551}}{400}, s_2 = -\frac{0.2348}{2} \pm i\frac{\sqrt{14.39}}{2} \tag{2}$$

Having two sets of complex conjugate poles with negative real components indicates the system should be underdamped but stable. This lends itself to the oscillation into steady state reaction that we see.

2.1.3 Final Value Theorem

The final value theorem is as follows (Eqn. 3):

$$\lim_{x \to \infty} f(x) = \lim_{s \to 0} sY(s) \tag{3}$$

If we take the limit as s approaches 0 for both transfer functions, we get the following values:

$$g1 = 2.7 * 10^{-3}, g2 = 4.5 * 10^{-4} \tag{4}$$

These match the steady-state values shown in the graph, verifying the FVT for our transfer functions.

2.2 Electrical System

2.2.1 Step Input Output Voltage

The step response for the system is shown below in Fig. 2. Although the output initially jumps with the 5V input (as explained in the lab handout), the output voltage then dips as the voltage at C_2 changes over time. The response is fairly slow to match back up with the input, but eventually it should equal 5V once all the capacitors change their voltage to the correct level.

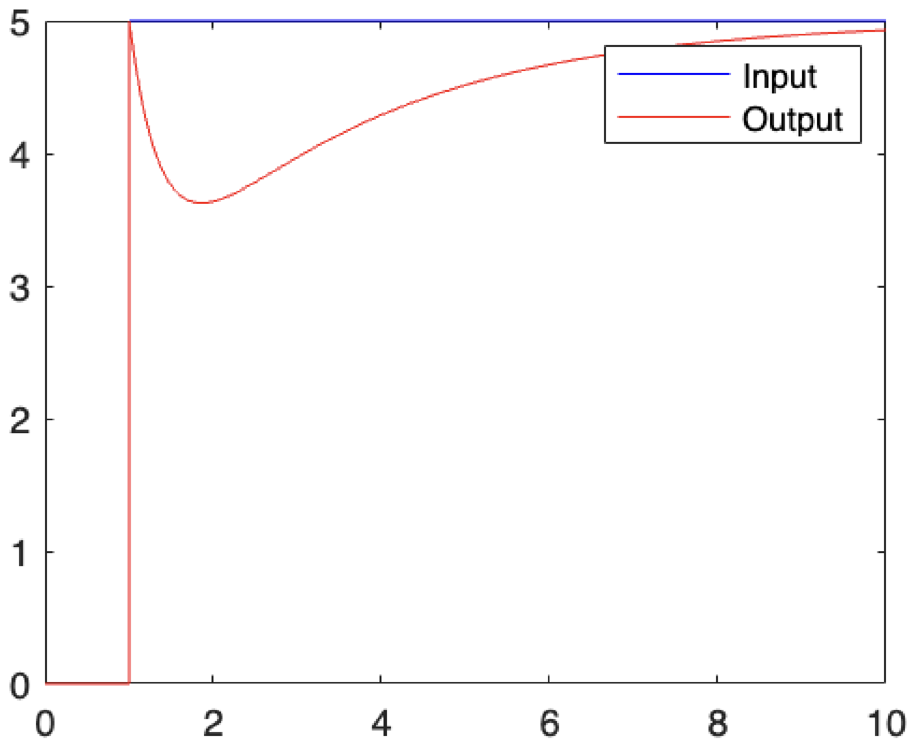

Figure 2: Output Voltage Due to Step Input

2.2.2 Ramp Input Output Voltage

The ramp response for the system is shown below in Fig. 3. The output is slow to respond. However, as time passes the output accelerates, and the slopes should eventually be equal. The horizontal difference between them should equal the time constant of the system.

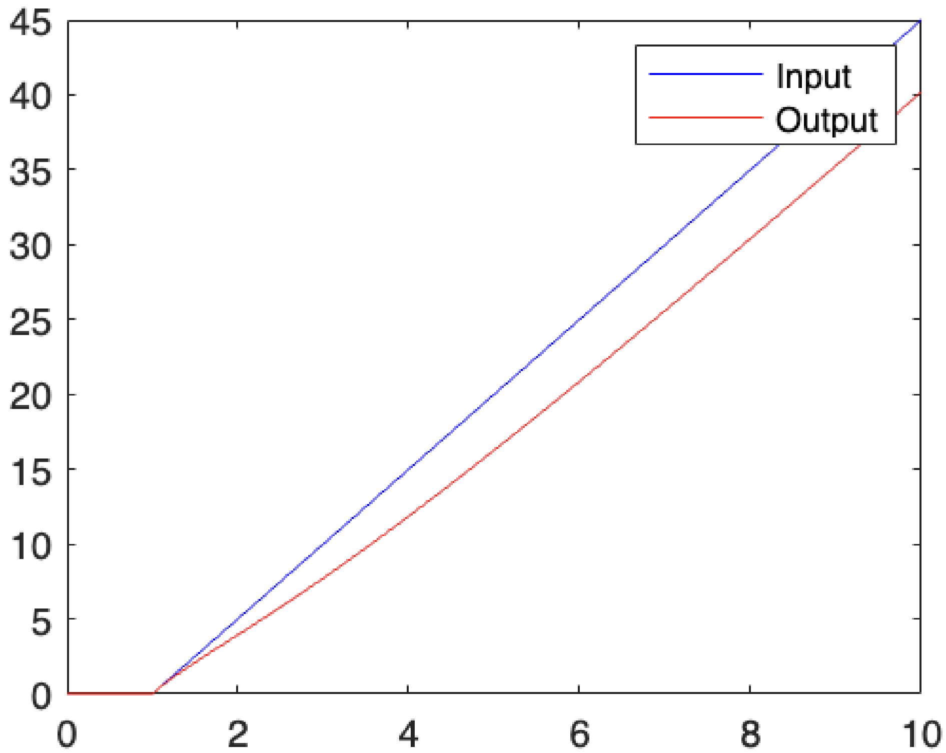

Figure 3: Output Voltage Due to Ramp Input

3 Assignment 2

3.1 Simulated and Experimental Step Response

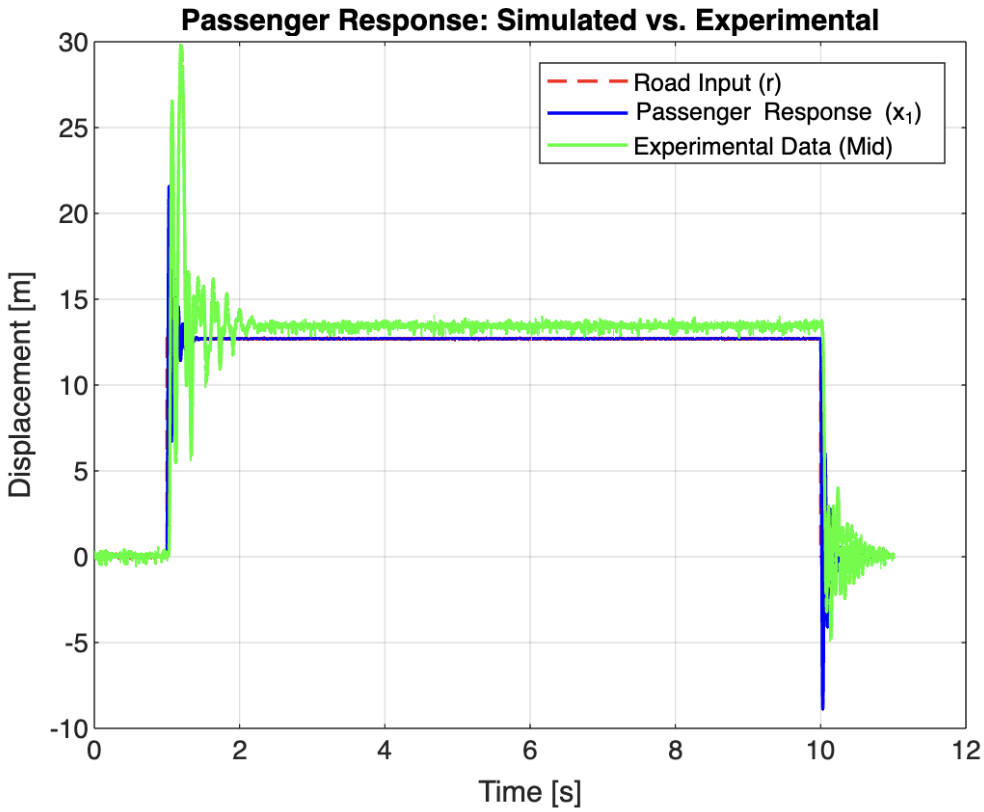

Figure 4: Simulated and experimental step response of passenger position and road profile.

3.2 Frequency

The damped frequency obtained from the simulation is $\omega_d = 99.89$ rad/s, while the experimental damped frequency is $\omega_d = 67.51$ rad/s. The discrepancy between these values can be attributed to unmodeled nonlinearities, damping variations, and external disturbances in the experimental setup.

3.3 Top Mass and Road vs Time

3.3.1 Response at 1/4 Damped Frequency

At a quarter of the damped frequency, the system exhibits prolonged oscillations with a slower decay rate, demonstrating the system's resonance effects at lower frequencies. The displacement remains significant due to reduced damping influence at this frequency range.

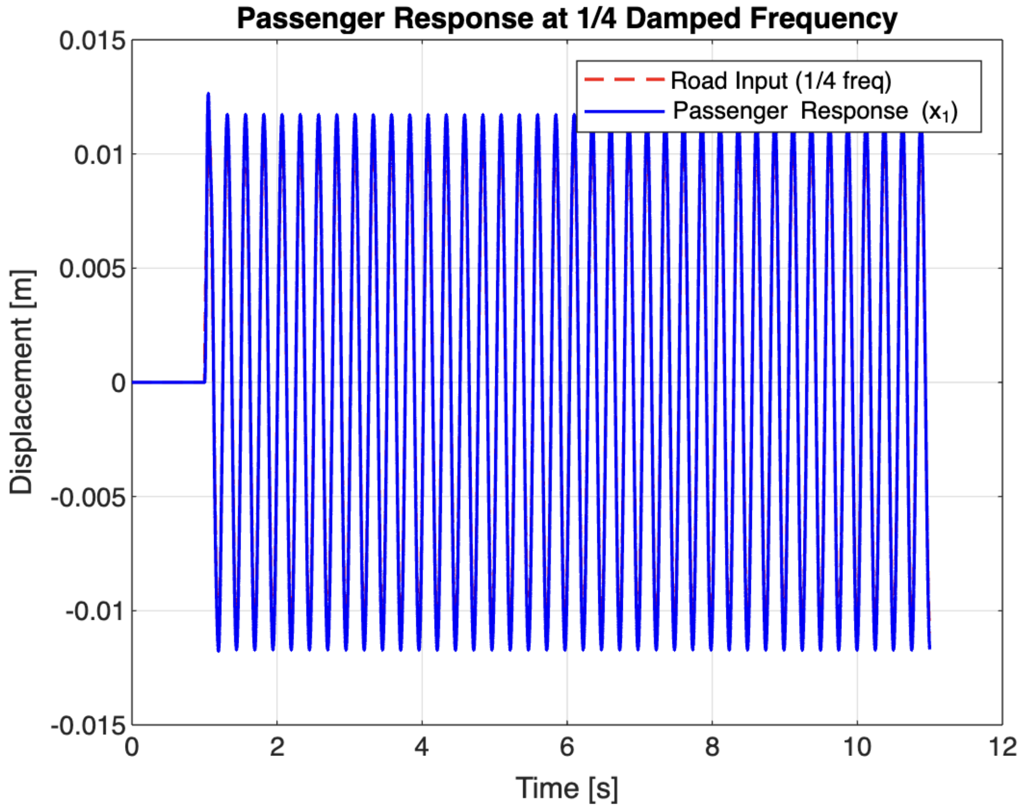

Figure 5: Passenger response at 1/4 of the damped frequency.

3.3.2 Response at Damped Frequency

At the damped frequency, the oscillations settle into a predictable steady-state response. This frequency is where the system naturally vibrates with an optimal balance of damping and amplitude.

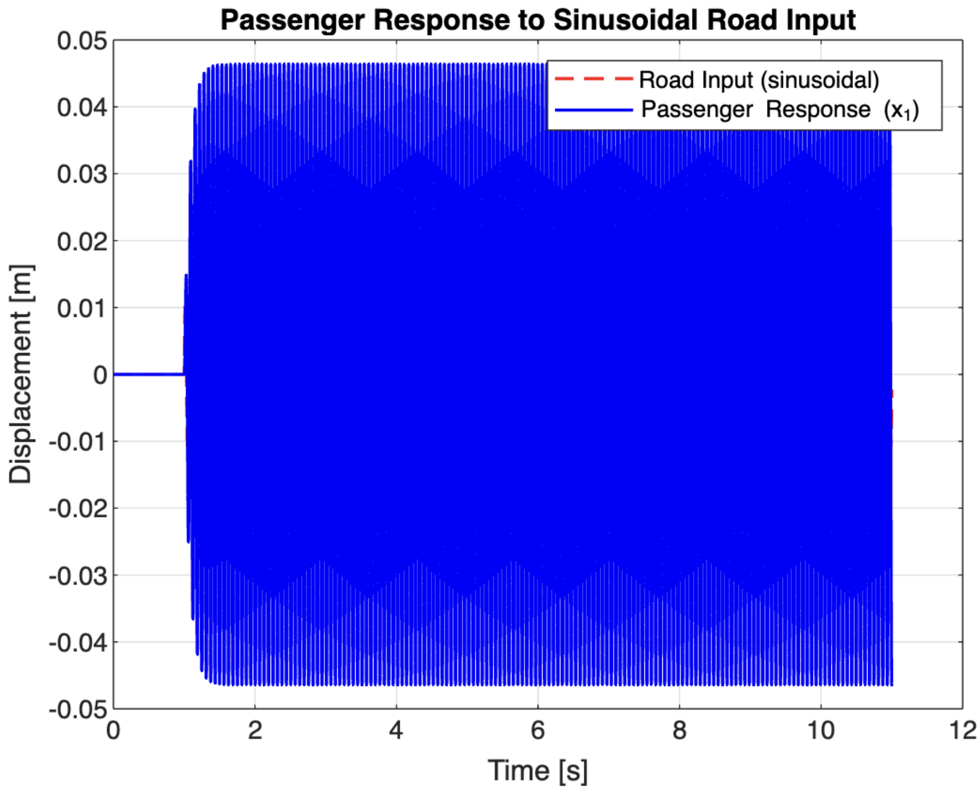

Figure 6: Passenger response at the simulated damped frequency.

3.3.3 Response at 4x Damped Frequency

At four times the damped frequency, the system response is significantly reduced due to the dominance of damping forces. The high-frequency excitation is largely absorbed, leading to a minimal passenger displacement compared to lower frequencies.

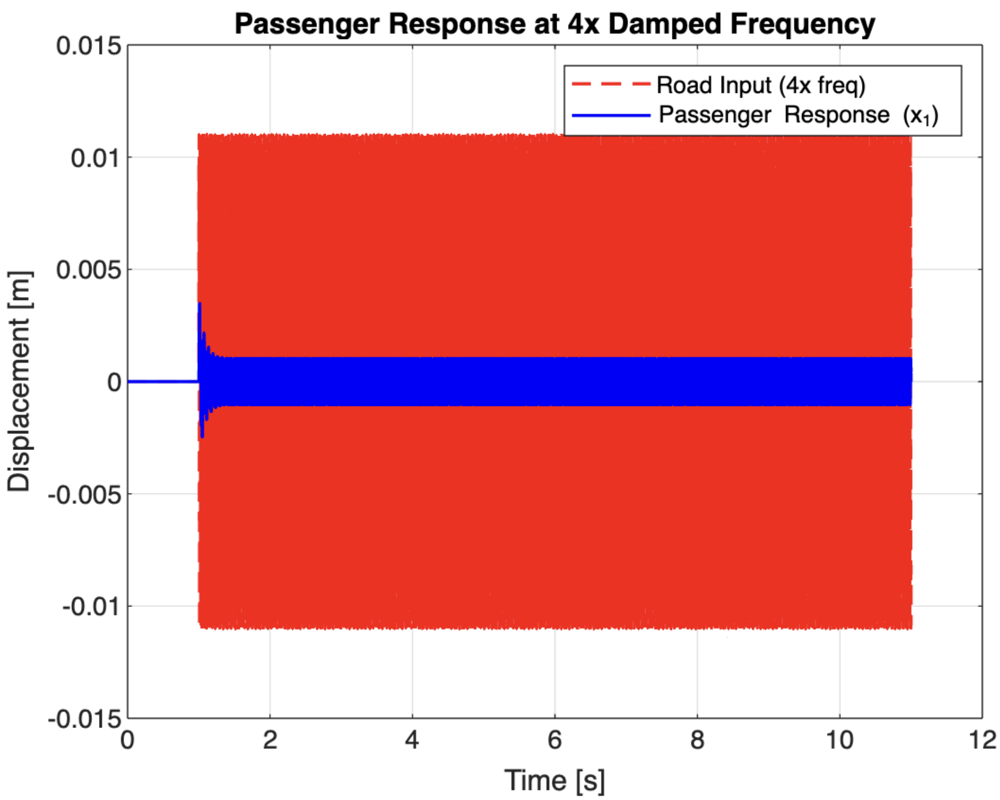

Figure 7: Passenger response at 4x the damped frequency.

4 Appendix

Listing 1: MATLAB code for Assignment 1

```
% MATLAB code for Mechanical System

m1=100;m2=200;b=100;k1=200;k2=200;k3=1000;
n1=[m2 b k2+k3];
n2=[b k2]
d=[m1*m2 b*(m1+m2) (m1*(k2+k3)+m2*(k1+k2)) b*(k3+k2) (k2*k1+k2*k3+k3*k1)];

g1=tf(n1,d);
g2=tf(n2,d);

zpk(g1)
zpk(g2)

figure
step(g1,'r')
hold on
step(g2,'b-')
legend('g1','g2')

% MATLAB code for Electrical System

r=1e4;c=100e-6;s=tf('s');
h=tf([r^2*c^2 2*r*c 1],[r^2*c^2 3*r*c 1]);

t=linspace(0,10,10000);

u1=5*(t>=1);
y1=lsim(h,u1,t);
plot(t, u1, 'b-', t, y1, 'r-');
legend('Input', 'Output');

u2=5*(t-1).*(t>=1);
y2=lsim(h,u2,t);
plot(t,u2, 'b-', t, y2, 'r-');
legend('Input', 'Output');
```

Listing 2: MATLAB code for Assignment 2 simulation

```
% MATLAB code for simulation
m1 = 0;
m2 = 0.875;
k1 = 1576;
b1 = 6.80;
k2 = 8870;
b2 = 21.6;

num = [b1*b2 (k1*b2 + k2*b1) k1*k2];
den = [m1*m2 (m1*b1 + m1*b2 + m2*b1) (m1*k1 + m1*k2 + k1*m2 + b1*b2) (k1*b2 + k2*b1) k1*
    k2];

sysX1 = tf(num, den);

dataTable = readtable('lab3data.csv');
t = dataTable.Time;
r = dataTable.Input;

x1 = lsim(sysX1, r, t);

figure;
```

```
plot(t, r, 'r--','LineWidth',1.2); hold on;
plot(t, x1, 'b-','LineWidth',1.2);
xlabel('Time [s]');
ylabel('Displacement [m]');
legend('Road Input (r)','Passenger Response (x_1)');
title('Passenger Response to Bump Using Transfer Function and lsim');
grid on;

T = 1.0784 - 1.0155;
u = sin(2*pi*(t-1)/T).*0.011.*(t>=1);
x1_sin = lsim(sysX1, u, t);

figure;
plot(t, u, 'r--','LineWidth',1.2); hold on;
plot(t, x1_sin, 'b-','LineWidth',1.2);
xlabel('Time [s]');
ylabel('Displacement [m]');
legend('Road Input (sinusoidal)','Passenger Response (x_1)');
title('Passenger Response to Sinusoidal Road Input');
grid on;

T4 = T/4;
u4 = sin(2*pi*(t-1)/T4).*0.011.*(t>=1);
x1_4 = lsim(sysX1, u4, t);

figure;
plot(t, u4, 'r--','LineWidth',1.2); hold on;
plot(t, x1_4, 'b-','LineWidth',1.2);
xlabel('Time [s]');
ylabel('Displacement [m]');
legend('Road Input (4x freq)','Passenger Response (x_1)');
title('Passenger Response at 4x Damped Frequency');
grid on;

Tquarter = 4*T;
uquarter = sin(2*pi*(t-1)/Tquarter).*0.011.*(t>=1);
x1_quarter = lsim(sysX1, uquarter, t);

figure;
plot(t, uquarter, 'r--','LineWidth',1.2); hold on;
plot(t, x1_quarter, 'b-','LineWidth',1.2);
xlabel('Time [s]');
ylabel('Displacement [m]');
legend('Road Input (1/4 freq)','Passenger Response (x_1)');
title('Passenger Response at 1/4 Damped Frequency');
grid on;

dataTable = readtable('lab3data.csv');
t = dataTable.Time;
r = dataTable.Input;

x1 = lsim(sysX1, r, t);
xExp = dataTable.Mid;

figure;
plot(t, r, 'r--','LineWidth',1.2); hold on;
plot(t, x1, 'b-','LineWidth',1.2);
plot(t, xExp-117.913, 'g-','LineWidth',1.2);
xlabel('Time [s]');
ylabel('Displacement [m]');
legend('Road Input (r)','Passenger Response (x_1)','Experimental Data (Mid)');
title('Passenger Response: Simulated vs. Experimental');
```

```
grid on;
```

5 Authorship

Christian Carbeau collected the experimental data during lab and completed Assignment 2 of the report. Winslow Griffen completed Assignment 1 of the lab report and wrote the Matlab code for the Electrical System portion of Assignment 1. Jay Parmar and Winslow Griffen wrote the MATLAB code for Assignment 1 Mechanical System, Jay Parmar completed the introduction for the lab report and edited the report.

6 Acknowledgments

We would like to acknowledge Dr. Michael Gustafson for providing the template used in this lab. Likewise, we want to acknowledge Pat McGuire for his assistance during this lab and all group members for their participation in these experiments.