1 Modeling Choices for Disturbances

1.1 Sensor Delays

1.1.1 Reason for Modeling Sensor Delays

Sensor delays are inherent in any control system due to the time required for data acquisition, processing, and transmission. In quadcopter systems, delays can arise from the inertial measurement units (IMUs), communication latency, and computational processing time. Modeling sensor delays is crucial because they introduce lag in the feedback loop, which can degrade system stability and performance if not properly accounted for.

1.1.2 Modeling Approach

The sensor delay is modeled using the Padé approximation, which provides a rational function approximation of the time delay in the Laplace domain. For a sensor delay of τ seconds, the Padé approximation of order n is given by:

$$e^{-\tau s} \approx \frac{1 - \frac{\tau s}{2} + \left(\frac{\tau s}{2}\right)^2 - \cdots + (-1)^n \left(\frac{\tau s}{2}\right)^n}{1 + \frac{\tau s}{2} + \left(\frac{\tau s}{2}\right)^2 + \cdots + \left(\frac{\tau s}{2}\right)^n} \tag{1}$$

In this simulation, a 4th-order Padé approximation is used to model a sensor delay of $\tau = 0.05$ seconds.

1.2 Wind Disturbances

1.2.1 Reason for Modeling Wind Disturbances

Wind disturbances represent one of the most common environmental factors affecting quadcopter stability. Sudden gusts or steady winds can apply external torques and forces, causing deviations from the intended flight path. Modeling wind disturbances allows for the analysis of the control system's robustness and the controller's ability to reject such disturbances.

1.2.2 Modeling Approach

The wind disturbance is modeled as an external torque input acting on the quadcopter. Specifically, it is represented as a step disturbance of magnitude 0.1 Nm occurring at $t = 1$ second. This simplification captures the sudden onset of a wind gust and its sustained effect on the system.

2 System Response and PID Controller Analysis

2.1 PID Controller's Ability to Handle Disturbances

The PID controller is expected to:

- Compensate for sensor delays by adjusting the control action based on delayed feedback.
- Reject wind disturbances by utilizing the integral and derivative actions to counteract the external torque.
- Maintain system stability and bring the quadcopter back to the desired setpoint after disturbances.

2.2 Simulation Results

The MATLAB simulation incorporates the disturbances and computes the system's step response. The response graph is presented in Figure 1, and the code used to generate it is available in Appendix Section A.

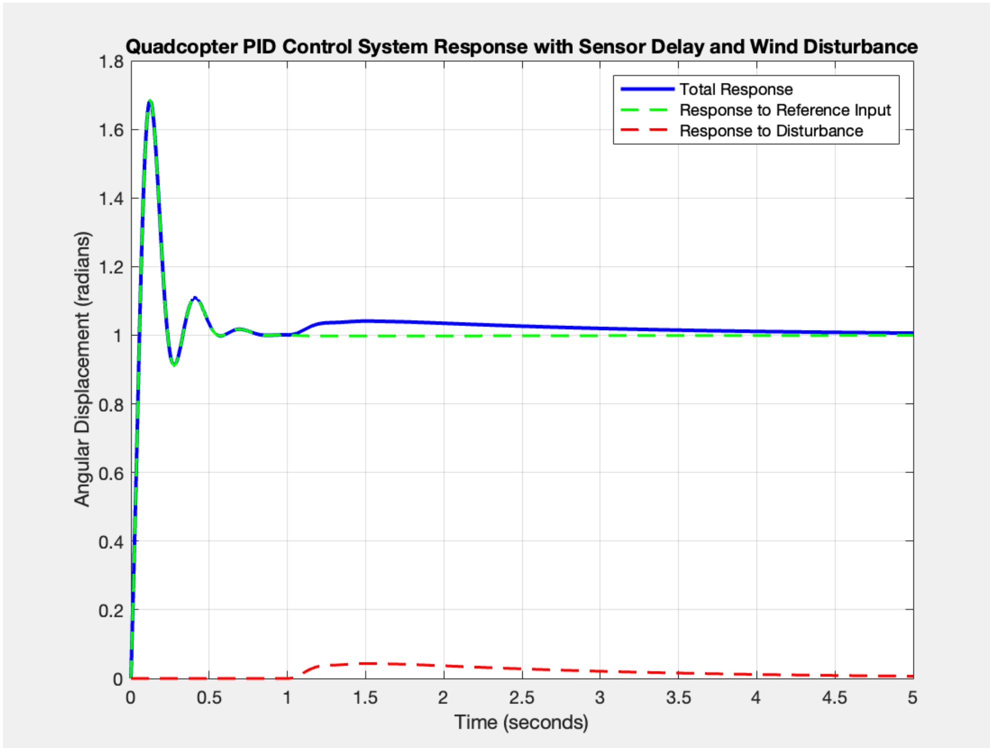

Figure 1: System Response with Sensor Delay and Wind Disturbance

The key response metrics obtained are presented in Table 1.

Table 1: Step Response Metrics with Disturbances

Metric	**Value**
Rise Time (t_r)	0.0431 seconds
Settling Time (t_s)	2.9827 seconds
Peak Time (t_p)	0.1230 seconds
Overshoot (%OS)	68.39%
Settling Min	0.9127
Settling Max	1.6839
Peak Value	1.6839
Undershoot	0%
Transient Time	2.9827 seconds

2.2.1 Interpretation of Results

Rise Time (t_r): The rise time decreased to 0.0431 seconds compared to the previous system with $I_y = 0.1$. This indicates that the system responds more quickly due to the reduced moment of inertia, which allows the quadcopter to accelerate more rapidly.

Overshoot (%OS): The overshoot increased significantly to 68.39%, reflecting the combined effect of the reduced moment of inertia, sensor delays, and wind disturbances causing the system to exceed the desired setpoint more substantially.

Settling Time (t_s): The settling time increased to 2.9827 seconds, indicating that the system takes longer to stabilize within 2% of the final value due to the higher overshoot and oscillations introduced by the disturbances.

Peak Time (t_p) and Peak Value: The peak occurs at 0.1230 seconds with a peak value of 1.6839 radians. The increased peak value and earlier peak time are consequences of the faster response and higher overshoot resulting from the reduced moment of inertia.

Steady-State Error: Despite the disturbances and changes in system dynamics, the steady-state error remains negligible due to the integral action of the PID controller, which ensures the system eventually reaches the desired setpoint.

2.3 PID Controller Performance Analysis

The PID controller demonstrates the ability to handle the disturbances and the change in moment of inertia, but with reduced performance metrics:

- The **proportional** action responds to the immediate error but is more aggressive due to the reduced inertia, leading to increased overshoot.
- The **integral** action accumulates the error over time, working to eliminate steady-state error despite the disturbances.
- The **derivative** action predicts the error trend, helping to dampen oscillations but is less effective due to the delayed feedback and faster system dynamics.

The increased overshoot and settling time suggest that the current PID gains may need adjustment to compensate for the added disturbances and the reduced moment of inertia. This could involve reducing the proportional gain or increasing the derivative gain to improve damping.

3 Conclusion

Updating the moment of inertia to $I_y = 0.03$ kg·m^2 significantly affects the quadcopter's dynamics, making it more responsive but also more susceptible to overshoot and oscillations when disturbances are present. Modeling sensor delays and wind disturbances provides a more realistic simulation of the quadcopter control system. The PID controller retains its fundamental ability to guide the system toward the desired setpoint, but its performance is degraded in terms of increased overshoot and settling time. These results highlight the importance of considering both system parameters and disturbances in controller design and tuning. Future work may involve optimizing the PID gains or implementing advanced control strategies to enhance disturbance rejection and maintain system performance under challenging conditions.

A MATLAB Simulation Code

The MATLAB code used to perform the simulation and generate the response graph is provided in appendix section c.

B Additional Figures

B.1 Response Graph without Disturbances

response.png

Figure 2: System Response without Sensor Delay and Wind Disturbance (I_y = 0.03 kg·m^2)

B.2 Response Graph with Disturbances

Figure 1 (reproduced below) shows the system response when both sensor delay and wind disturbance are included.

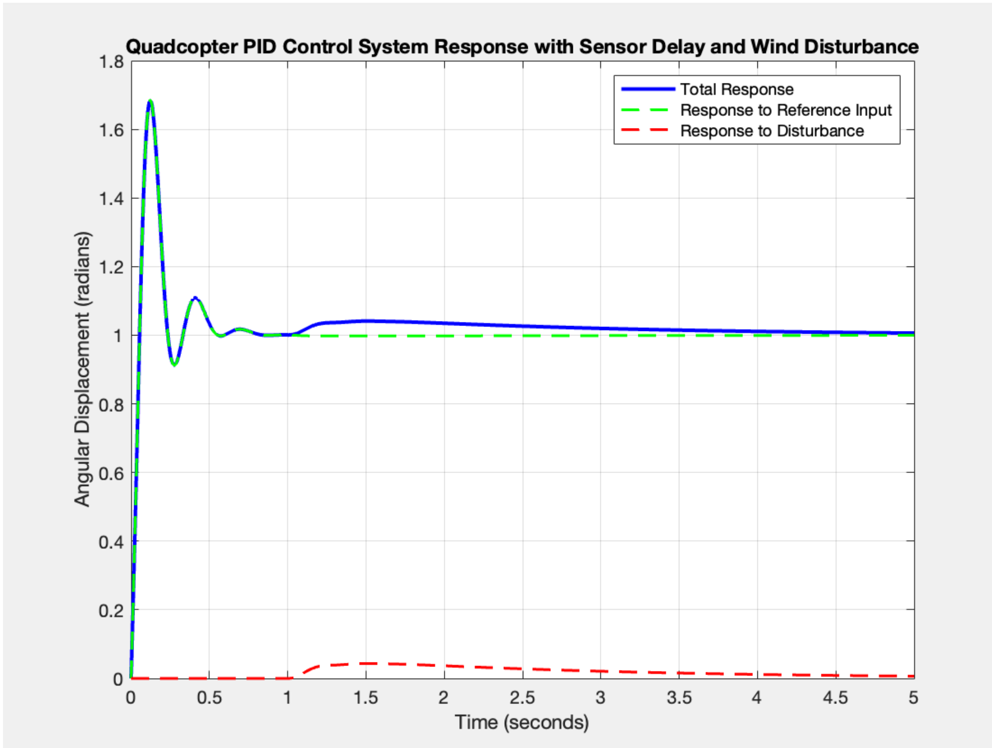

Figure 3: System Response with Sensor Delay and Wind Disturbance (I_y = 0.03 kg·m^2)

C References

References

[1] K. Ogata, *Modern Control Engineering*, 5th ed. Prentice Hall, 2010.

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[4] F. Padé, "Sur la représentation approchée d'une fonction par des fractions rationnelles," *Annales Scientifiques de l'École Normale Supérieure*, vol. 9, pp. 93–145, 1892.