DUKE UNIVERSITY, PRATT SCHOOL OF ENGINEERING

ME321 - Individual Assignment 5

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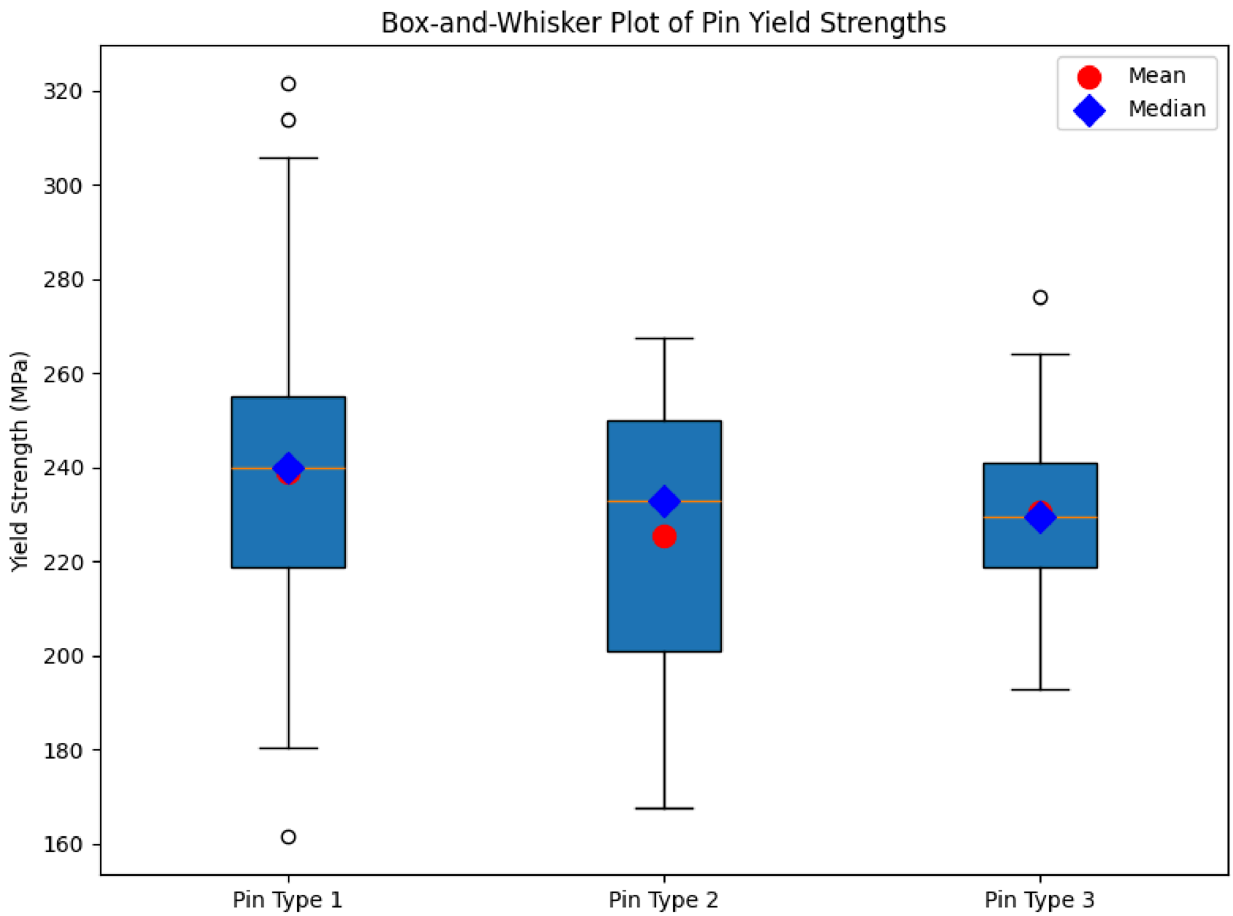
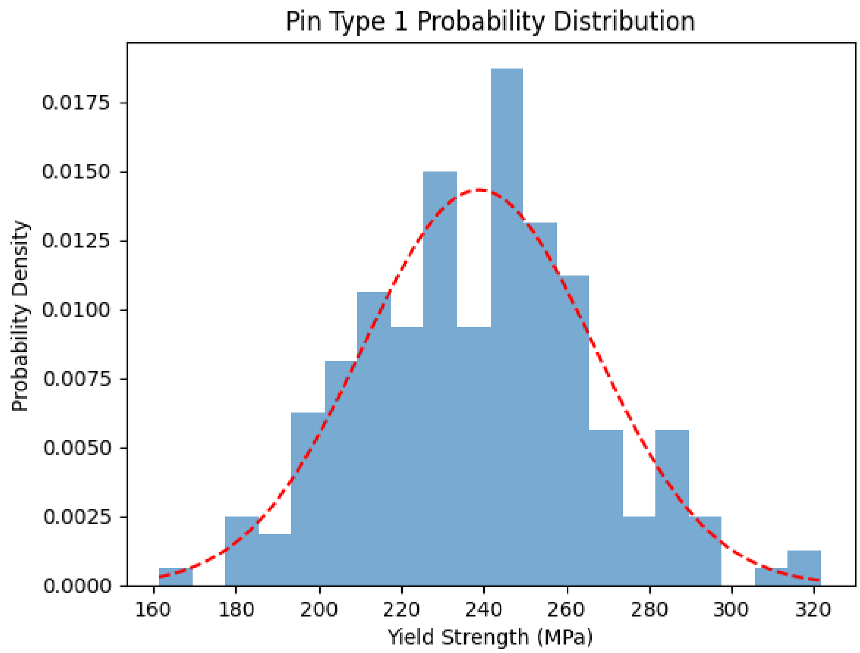
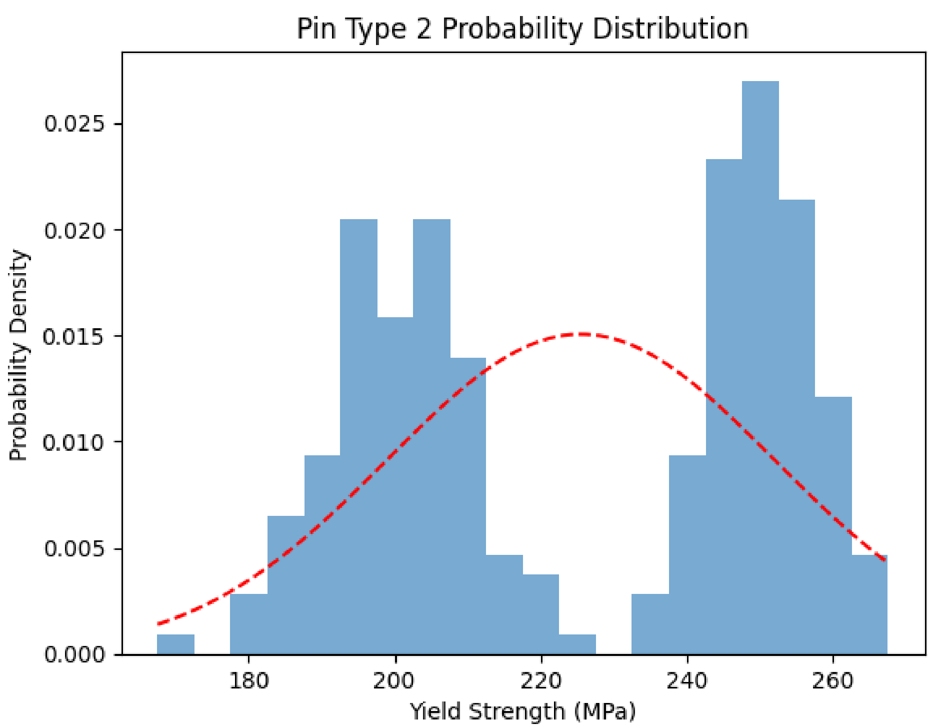
Professor . . . . Dr. Santillan

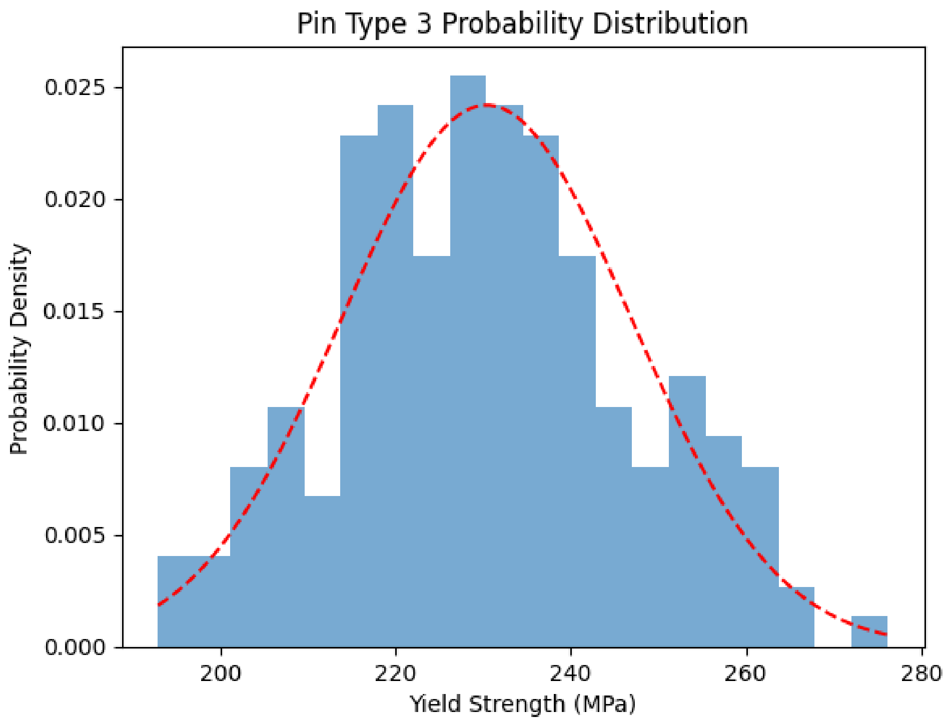
Question 1: Combined Box and Whisker Plots for 3 Data Sets



Question 2: Probability Distribution Graphs for 3 Pin Types







Question 3: P(fail below 230 MPa)

To calculate the probability that the next material sample tested will fail below 230 MPa we use a CDF function in the following equation:

$$\mathrm{P}(\text{fail below } 230\,\mathrm{MPa}) = \mathrm{CDF}\left(\frac{230-\mu}{\sigma}\right)$$

Performing this calculation in python, I get the following values:

```
Pin Type 1: 37.63685%

Pin Type 2: 56.76567%

Pin Type 3: 48.98942%
```

Question 4: P(9 out of 10 fail above 240 MPa)

To calculate the probability that 9 out of the next 10 material samples tested will fail at or above 240 MPa, we use a binomial function in the following equation:

$$\text{P}(9 \ \text{ out of } \ 10 \geq 240\,MPa) = 109\left[1-\Phi\left(\tfrac{240-\mu}{\sigma}\right)\right]^{9}\left[\Phi\left(\tfrac{240-\mu}{\sigma}\right)\right]$$

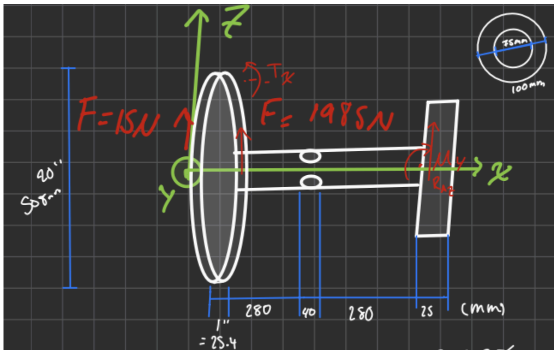
Performing this calculation in python, I get the following values:

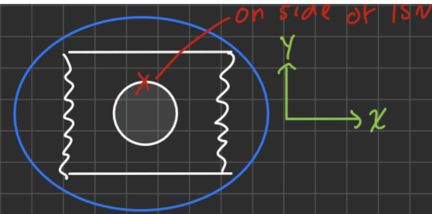
```
Pin Type 1: 0.73337%

Pin Type 2: 0.01091%

Pin Type 3: 0.00782%
```

Question 5: Pin manufacturer selection

Manufacturer 1 (Pin Type 1) is clearly the best option. Its probability of a sample failing below 230 MPa is about 37.64%, which is significantly lower than the 56.77% for Pin Type 2 and 48.99% for Pin Type 3. Additionally, the chance that 9 out of 10 samples achieve a yield strength at or above 240 MPa is roughly 0.73% for Pin Type 1, compared to just 0.01% and 0.008% for Pin Types 2 and 3 respectively.

Question 6: Critical stress location and factor of safety





$1970(0.508)$

$T = 0.254 \times (1985 - 15) = 500.38$

$M_{hole} = 0.001(280+20) \times 2000N = 600\,N$ $(625.4N)$

Bending

$\sigma_0 = M / Z_{net}$

$$\sigma = \frac{600}{\frac{\pi A}{32D}(D^4 - d^4)}$$

$$\frac{600}{\frac{\pi(0.52)}{32(0.1)}(0.1^4 - 0.075^4)} = 1.541 \times 10^7$$

$$2.16(1.719 \times 10^7) = 32.99\,MPa$$

$$\sqrt{2(3.299 \times 10^7)^2 + 6(11.81)^2}$$

$= 38.92$

$$238/38.92 = \boxed{5.962}$$

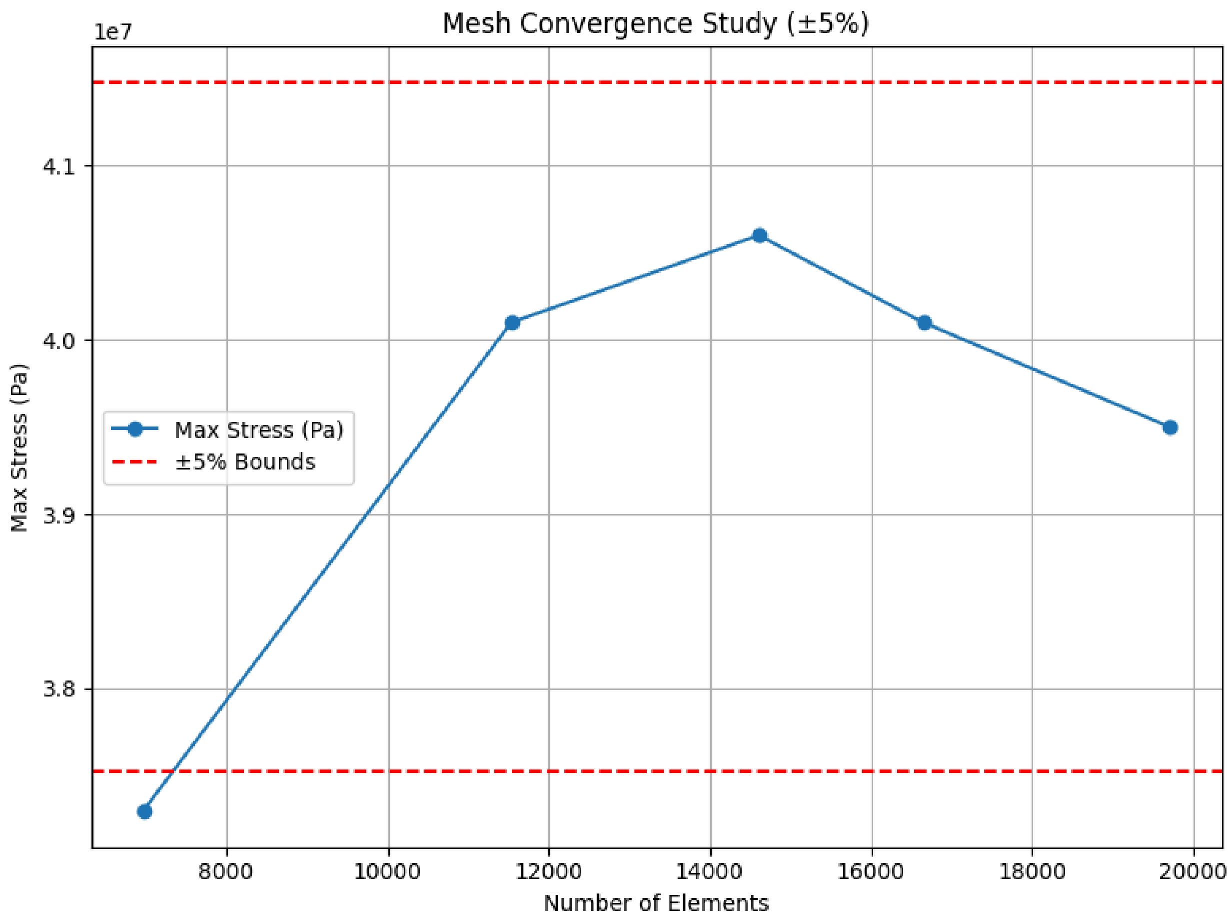
Torsional

$$\tau = \frac{TD}{2J} = \frac{(500.38(0.1))}{0.727(0.1^4 - 0.075^4)} = \frac{50.38}{1.342 \times 10^{-5}}$$

$= 4.55 \times 10^7 (2.6)$

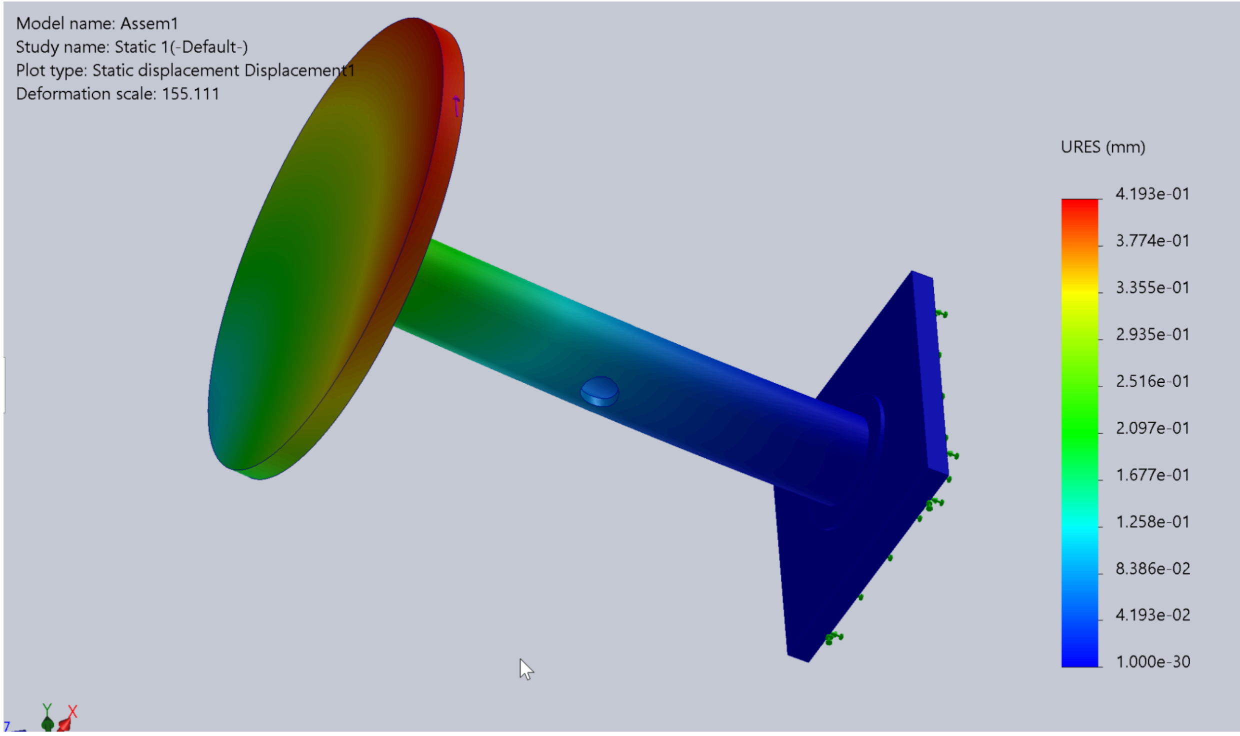
$= 11.86 \times 10^7$

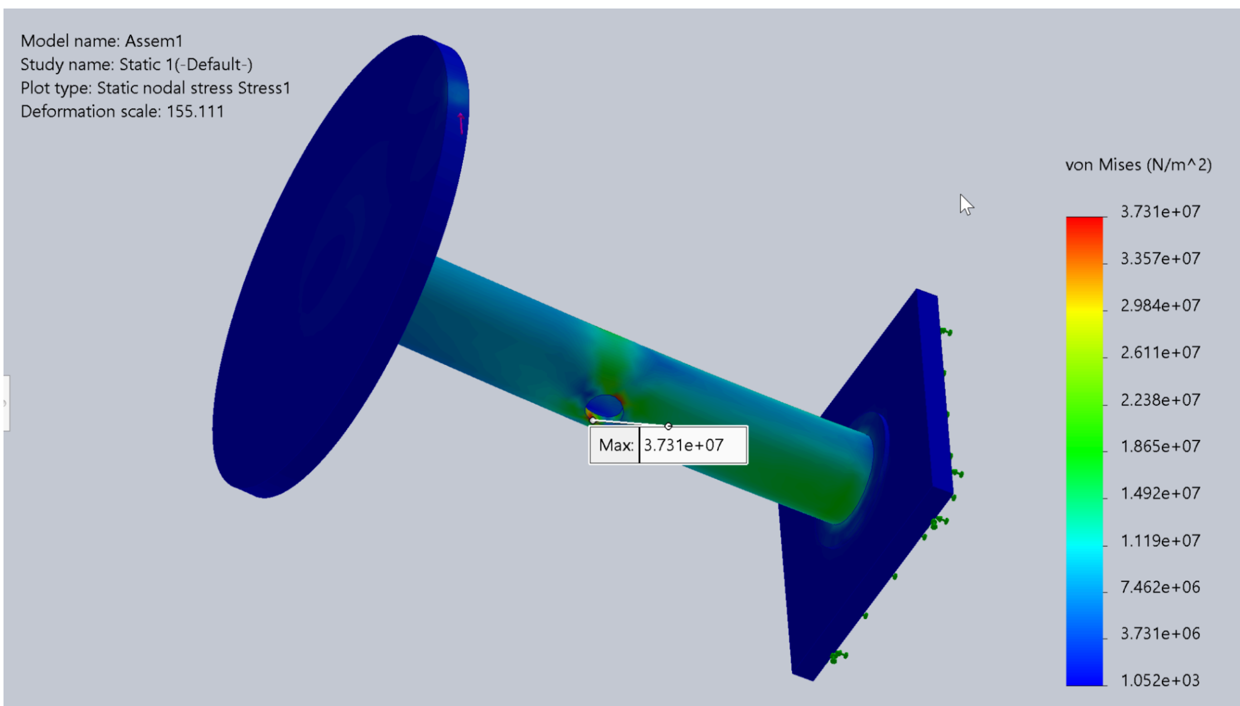
Question 7: Mesh conversion analysis



How do your FEA results compare to your analytical results?

The percent difference between my calculated stress and the highest projected stress is 1.55%, showing that the analytical solution is close to the FEA analysis.


Model name: Assem1
Study name: Static 1(-Default-)
Plot type: Static displacement Displacement1
Deformation scale: 155.111
URES (mm)
4.193e-01
3.774e-01
3.355e-01
2.935e-01
2.516e-01
2.097e-01
1.677e-01
1.258e-01
8.386e-02
4.193e-02
1.000e-30


Model name: Assem1
Study name: Static 1(-Default-)
Plot type: Static nodal stress Stress1
Deformation scale: 155.111
von Mises (N/m^2)
3.731e+07
3.357e+07
2.984e+07
2.611e+07
2.238e+07
1.865e+07
1.492e+07
1.119e+07
7.462e+06
3.731e+06
1.052e+03
Max: 3.731e+07

Source Code

```
12w3eimport pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm, binom

df = pd.read_csv('s.csv')
df['Pin Type 1 Yield Strength (MPa)'] = pd.to_numeric(df['Pin Type 1 Yield Strength (MPa)'], errors='coerce')
df['Pin Type 2 Yield Strength (MPa)'] = pd.to_numeric(df['Pin Type 2 Yield Strength (MPa)'], errors='coerce')
df['Pin Type 3 Yield Strength (MPa)'] = pd.to_numeric(df['Pin Type 3 Yield Strength (MPa)'], errors='coerce')
df.dropna(how='all', subset=['Pin Type 1 Yield Strength (MPa)', 'Pin Type 2 Yield Strength (MPa)', 'Pin Type 3 Yield Strength (MPa)'], inplace=True)

data1 = df['Pin Type 1 Yield Strength (MPa)'].dropna().values
data2 = df['Pin Type 2 Yield Strength (MPa)'].dropna().values
data3 = df['Pin Type 3 Yield Strength (MPa)'].dropna().values



data = [data1, data2, data3]

fig, ax = plt.subplots(figsize=(8, 6))
box = ax.boxplot(data, patch_artist=True, showmeans=False)
positions = [1, 2, 3]
means = [np.mean(d) if len(d) > 0 else np.nan for d in data]
medians = [np.median(d) if len(d) > 0 else np.nan for d in data]
ax.scatter(positions, means, color='red', zorder=3, label='Mean', s=100)
ax.scatter(positions, medians, color='blue', marker='D', zorder=3, label='Median', s=100)
ax.set_xticks(positions)
ax.set_xticklabels(['Pin Type 1', 'Pin Type 2', 'Pin Type 3'])
ax.set_ylabel('Yield Strength (MPa)')
ax.set_title('Box-and-Whisker Plot of Pin Yield Strengths')
ax.legend()
plt.tight_layout()
plt.show()
```

```python
for i, d in enumerate([data1, data2, data3], start=1):
   plt.figure()
   plt.hist(d, bins=20, density=True, alpha=0.6)
   mu, sigma = norm.fit(d)
   x = np.linspace(d.min(), d.max(), 100)
   p = norm.pdf(x, mu, sigma)
   plt.plot(x, p, 'r--')
   plt.title(f'Pin Type {i} Probability Distribution')
   plt.xlabel('Yield Strength (MPa)')
   plt.ylabel('Probability Density')
   plt.show()

print("\nProbability that the next sample fails below 230 MPa:")
for i, d in enumerate([data1, data2, data3], start=1):
   mu, sigma = norm.fit(d)
   prob_fail = norm.cdf(230, mu, sigma)
   print(f"Pin Type {i}: {prob_fail:.5%}")

print("\nProbability that 9 out of 10 samples are >= 240 MPa:")
for i, d in enumerate([data1, data2, data3], start=1):
   mu, sigma = norm.fit(d)
   p_above = 1 - norm.cdf(240, mu, sigma)
   prob_9_of_10 = binom.pmf(9, 10, p_above)
   print(f"Pin Type {i}: {prob_9_of_10:.5%}")

elements = np.array([6973, 11539, 14609, 16650, 19711])
stress   = np.array([37300000, 40100000, 40600000, 40100000, 39500000])

reference_stress = stress[-1]
upper_bound = reference_stress * 1.05
lower_bound = reference_stress * 0.95

plt.figure(figsize=(8,6))
plt.plot(elements, stress, 'o-', label='Max Stress (Pa)')
plt.axhline(upper_bound, color='r', linestyle='--', label='±5% Bounds')
plt.axhline(lower_bound, color='r', linestyle='--')
plt.xlabel('Number of Elements')
plt.ylabel('Max Stress (Pa)')
plt.title('Mesh Convergence Study (±5%)')
plt.grid(True)
```

```
plt.legend()
plt.tight_layout()
plt.show()
```