Engineering Fluid Mechanics

Jet Impact on Surfaces Lab Report

Authors: Jaivir Parmar, Hugo Polo Álvarez, Luis Rodríguez Martínez, Álvaro Sanz González

Group: 38B (Subgroup 1)

Professor: Alejandro Ibáñez

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INTRODUCTION

The experiment we carried on, “Jet impact on surfaces”, explores the way in which a water jet exerts force when it strikes a surface. When the jet impacts a surface, there is a change in momentum that allows us to measure the force that is created. Measuring the force exerted on 3 different surfaces will be helpful to have a deep understanding and analysis of the fluid dynamics principles.

1 Problem Definition and Theoretical Framework

We are tasked with determining the theoretical expressions for the force exerted by a water jet on three surfaces: a flat surface, an inclined surface at 30°, and a hemispherical surface. The force is derived using the principles of conservation of mass (continuity equation) and conservation of momentum (momentum equation). The common parameters for all cases are:

- Flow rate: Q
- Jet exit area: $A = \frac{\pi d^2}{4}$
- Jet velocity: $v = \frac{Q}{A}$
- Water density: ρ

2 Case (a): Flat Surface

2.1 Control Volume and Momentum Analysis

The water jet impacts the flat surface vertically and exits radially. The incoming vertical velocity is $v_{\text{in}} = -v$, and the outgoing velocity is horizontal, with no vertical component.

2.2 Momentum Conservation in the Vertical Direction

The momentum change in the vertical direction is:

$$\Delta(\text{Momentum}_y) = \dot{m}(v_{\text{out}_y} - v_{\text{in}_y}) = \rho Q(0 - (-v)) = \rho Q v$$

The force exerted by the jet on the surface is the change in momentum:

$$F = \rho Q v = \rho Q \left(\frac{Q}{A}\right) = \frac{\rho Q^2}{A}$$

Thus, the force on the flat surface is:

$$F = \frac{\rho Q^2}{A}$$

3 Case (b): Inclined Surface at 30°

3.1 Control Volume and Momentum Analysis

In this case, the water jet impacts the surface at an angle of 30°. The incoming vertical velocity is $v_{\text{in}} = -v$, and the outgoing velocity has both horizontal and vertical components:

$$v_{\text{out}_x} = v\cos\theta \quad \text{and} \quad v_{\text{out}_y} = v\sin\theta$$

where $\theta = 30^\circ$.

3.2 Momentum Conservation in the Vertical Direction

The momentum change in the vertical direction is:

$$F_y = \rho Q(v_{\text{out}_y} - v_{\text{in}_y}) = \rho Q(v\sin\theta - (-v)) = \rho Qv(1+\sin\theta)$$

Thus, the force on the plate is:

$$F = \rho Qv(1+\sin\theta) = \rho Q\left(\frac{Q}{A}\right)(1+\sin\theta) = \frac{\rho Q^2}{A}(1+\sin\theta)$$

For $\theta = 30^\circ$, $\sin\theta = 0.5$, so:

$$F = \frac{\rho Q^2}{A}(1+0.5) = \frac{1.5\rho Q^2}{A}$$

4 Case (c): Hemispherical Surface

4.1 Control Volume and Momentum Analysis

For the hemispherical surface, the water jet is redirected directly upward, resulting in a complete reversal of velocity. The incoming vertical velocity is $v_{\text{in}} = -v$, and the outgoing velocity is $v_{\text{out}} = v$ in the upward direction.

4.2 Momentum Conservation in the Vertical Direction

The momentum change in the vertical direction is:

$$\Delta(\text{Momentum}_y) = \rho Q(v_{\text{out}_y} - v_{\text{in}_y}) = \rho Q(v - (-v)) = 2\rho Qv$$

Thus, the force on the plate is:

$$F = 2\rho Qv = 2\rho Q\left(\frac{Q}{A}\right) = \frac{2\rho Q^2}{A}$$

5 Inclusion of Atmospheric Pressure

The atmospheric pressure acts on both the incoming jet and the outgoing flow. However, since the pressures at the inlet and outlet are equal and atmospheric, their effects cancel out in the momentum equation. For the dry face of the obstacle, atmospheric pressure acts but does not contribute to the net force on the plate because it is balanced by the atmospheric pressure on the control volume boundaries.

6 Summary of Forces

The theoretical forces for each surface geometry are summarized as follows:

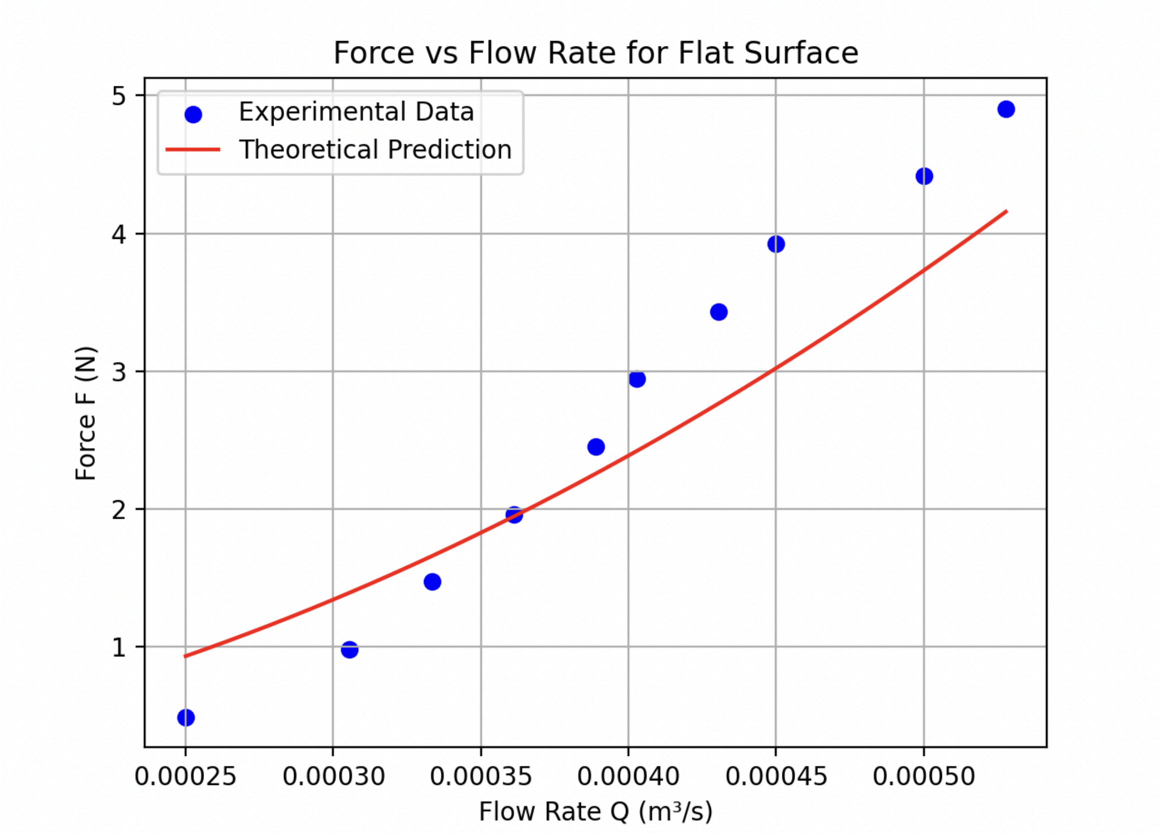
- **Flat Surface**: $F = \frac{\rho Q^2}{A}$
- **Inclined Surface (30°)**: $F = \frac{1.5\rho Q^2}{A}$
- **Hemispherical Surface**: $F = \frac{2\rho Q^2}{A}$

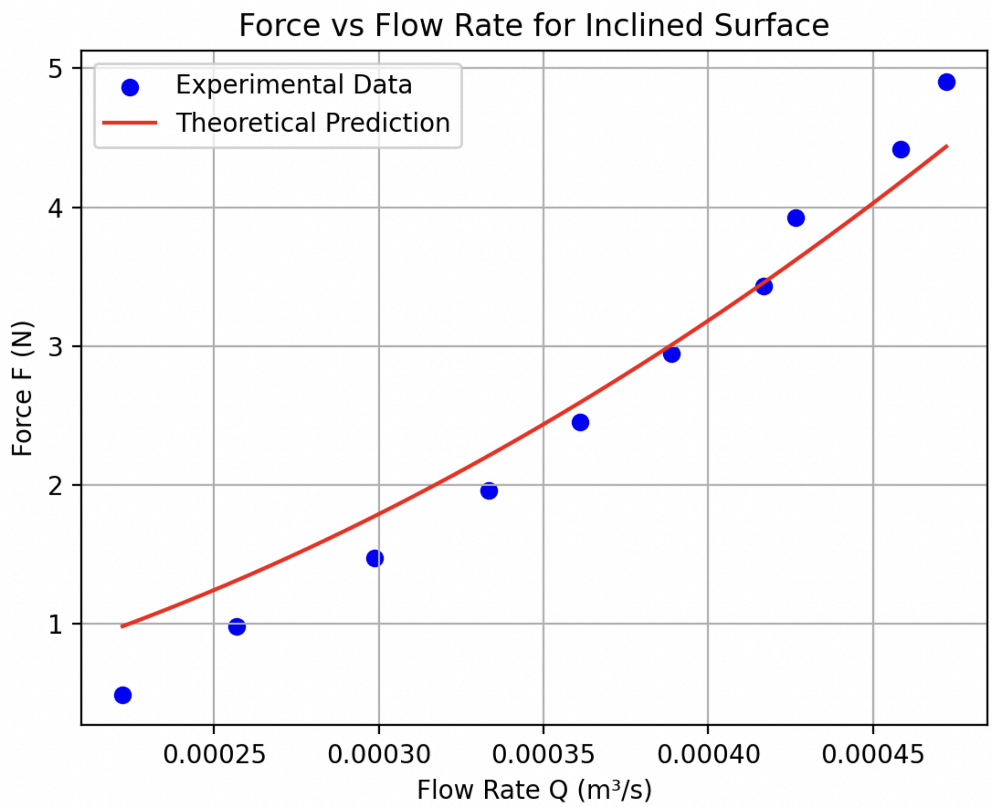
EXPERIMENTAL METHOD

The experiment was conducted to measure the force exerted by the water jet on a flat, inclined at 30º and hemispherical surfaces. An hydraulic bench was used which is able to supply a constant flow rate and a force calibration system.
The procedure followed for each surface is:

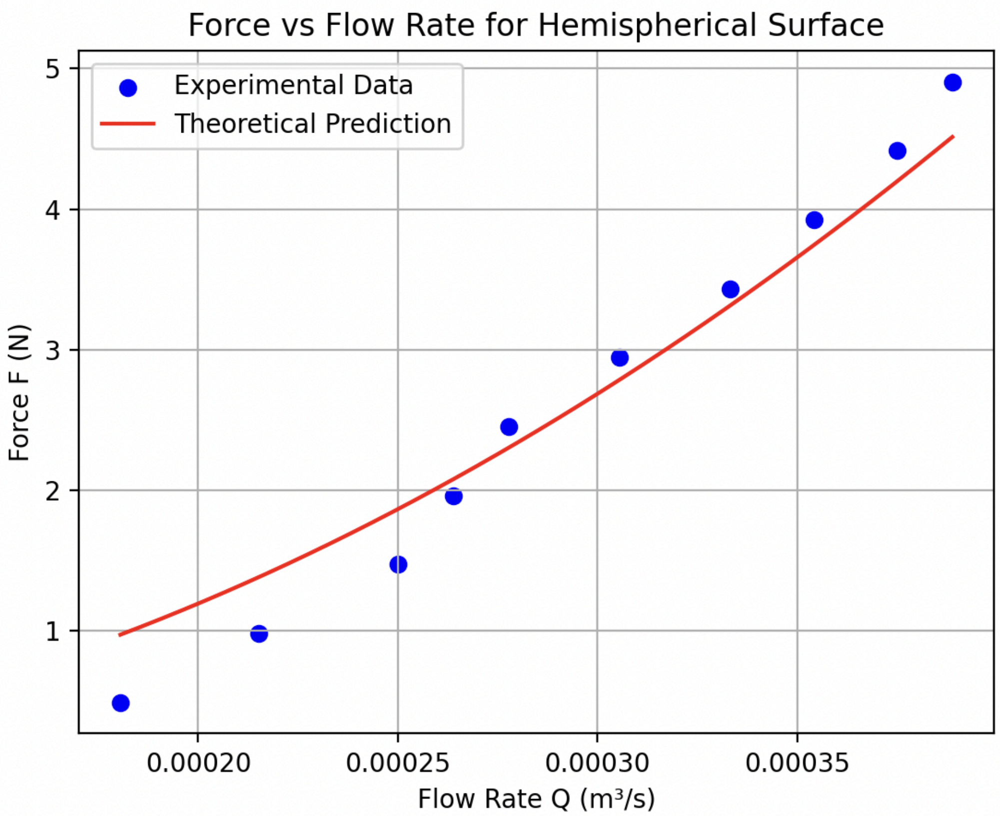
1. Setup and calibration: The cover was removed and the target surface was placed in the position where receives the impact of the water. The tank was closed and the assembly was leveled in the hydraulic bench´s channel. Finally, the dial gauge was adjusted to match the height of the auxiliary platform, making an appropriate calibration.
2. Measurement of force: Firstly, a mass was placed on the platform, the hydraulic bench´s was initiated and the flow rate of water was regulated until equilibrium was reached between the jet´s impact force and the mass, indicated by matching the height of the signal with the gauge.
3. Repetition of the process: the process was repeated with different masses and the different surfaces.

Graphs of F versus Q with Theoretical Line (Code in Appendix A)



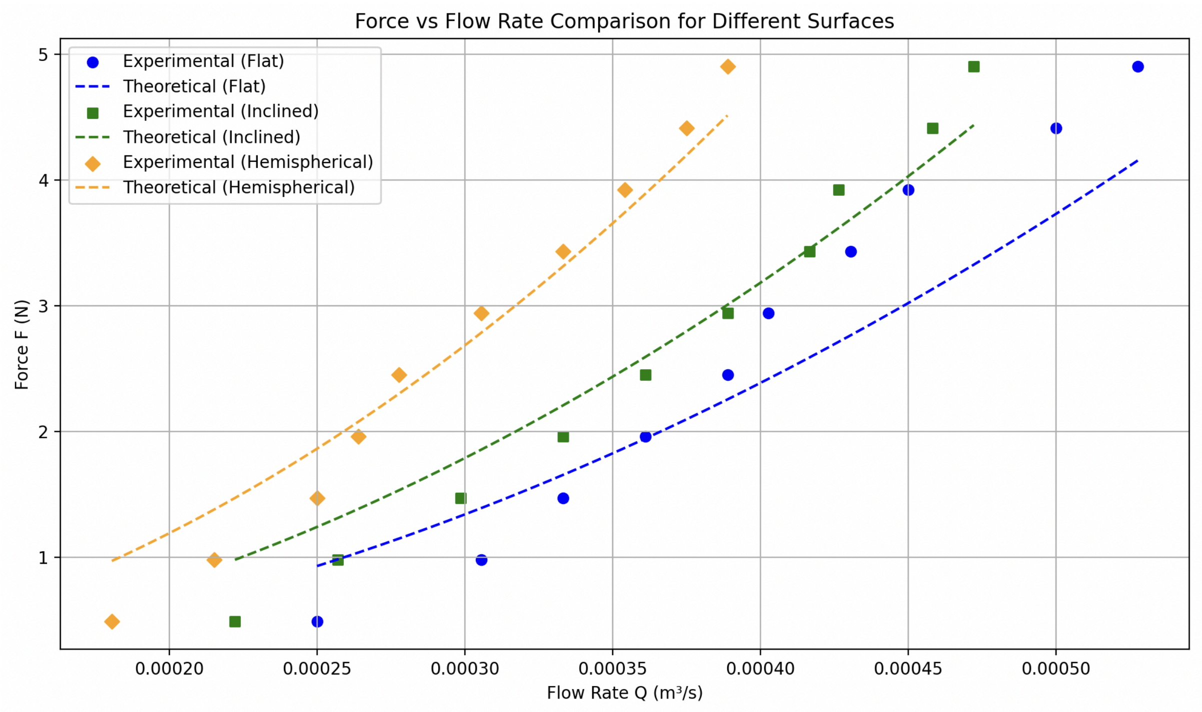


Force vs Flow Rate for Inclined Surface
Experimental Data
Theoretical Prediction
Force F (N)
5
4
3
2
1
0.00025
0.00030
0.00035
0.00040
0.00045
Flow Rate Q (m³/s)



Force vs Flow Rate for Hemispherical Surface
Experimental Data
Theoretical Prediction
Force F (N)
5
4
3
2
1
0.00020
0.00025
0.00030
0.00035
Flow Rate Q (m³/s)

Graph of Combination of Aforementioned Plots (Code in Appendix)



Calculation of C_d for each obstacle.

10 Calculation of C_d

10.1 Common Parameters

The cross-sectional area of the jet is:

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.008\,\mathrm{m})^2}{4} = 5.0265 \times 10^{-5}\,\mathrm{m}^2 \tag{2}$$

10.2 Methodology

For each trial, we perform the following steps:

1. Convert mass from grams to kilograms: $m_{\mathrm{kg}} = \dfrac{m}{1000}$.
2. Convert flow rate from liters per hour to cubic meters per second: $Q_{\mathrm{m^3/s}} = \dfrac{Q}{3,600,000}$.
3. Calculate the experimental force: $F = m_{\mathrm{kg}} \cdot g$.
4. Calculate the jet velocity: $v = \dfrac{Q_{\mathrm{m^3/s}}}{A}$.

5. Calculate the dynamic pressure force: $\frac{1}{2}\rho v^2 A$.

6. Compute C_d: $C_d = \frac{F}{\frac{1}{2}\rho v^2 A}$.

10.4 Results Tables

The calculated values of C_d for each obstacle are summarized in Tables 4, 5, and 6.

10.4.1 Flat Surface

Table 4: Calculated C_d for Flat Surface

Trial	v ($\mathrm{m\,s^{-1}}$)	F (N)	$\frac{1}{2}\rho v^2 A$ (N)	C_d
1	4.9749	0.4905	0.6231	0.787
2	6.0805	0.9810	0.9271	1.058
3	6.6332	1.4715	1.1057	1.331
4	7.1859	1.9620	1.2986	1.511
5	8.0140	2.4525	1.6118	1.521
6	8.5667	2.9430	1.8436	1.596
7	8.9565	3.4335	2.0097	1.709
8	9.3963	3.9240	2.2011	1.782
9	9.9497	4.4145	2.4960	1.769
10	10.5024	4.9050	2.8071	1.748

10.4.2 Inclined Surface

Table 5: Calculated C_d for Inclined Surface

Trial	v ($\mathrm{m\,s^{-1}}$)	F (N)	$\frac{1}{2}\rho v^2 A$ (N)	C_d
1	4.4244	0.4905	0.4905	1.000
2	5.1130	0.9810	0.6541	1.499
3	5.9422	1.4715	0.8856	1.662
4	6.6332	1.9620	1.1057	1.774
5	7.1859	2.4525	1.2986	1.889
6	7.7386	2.9430	1.5080	1.951
7	8.2914	3.4335	1.7340	1.980
8	8.4741	3.9240	1.7992	2.181
9	9.1205	4.4145	2.0839	2.119
10	9.3963	4.9050	2.2011	2.229

10.4.3 Hemispherical Surface

Table 6: Calculated C_d for Hemispherical Surface

Trial	v ($\mathrm{m\,s^{-1}}$)	F (N)	$\frac{1}{2}\rho v^2 A$ (N)	C_d
1	3.5947	0.4905	0.3245	1.512
2	4.2844	0.9810	0.4618	2.124
3	4.9749	1.4715	0.6231	2.361
4	5.2512	1.9620	0.6890	2.848
5	5.5274	2.4525	0.7580	3.235
6	6.0805	2.9430	0.9271	3.174
7	6.6332	3.4335	1.1057	3.104
8	7.0155	3.9240	1.2415	3.160
9	7.4684	4.4145	1.3997	3.155
10	7.7386	4.9050	1.5080	3.252

11 Results and Discussion

The C_d values vary with the type of obstacle and the flow rate:

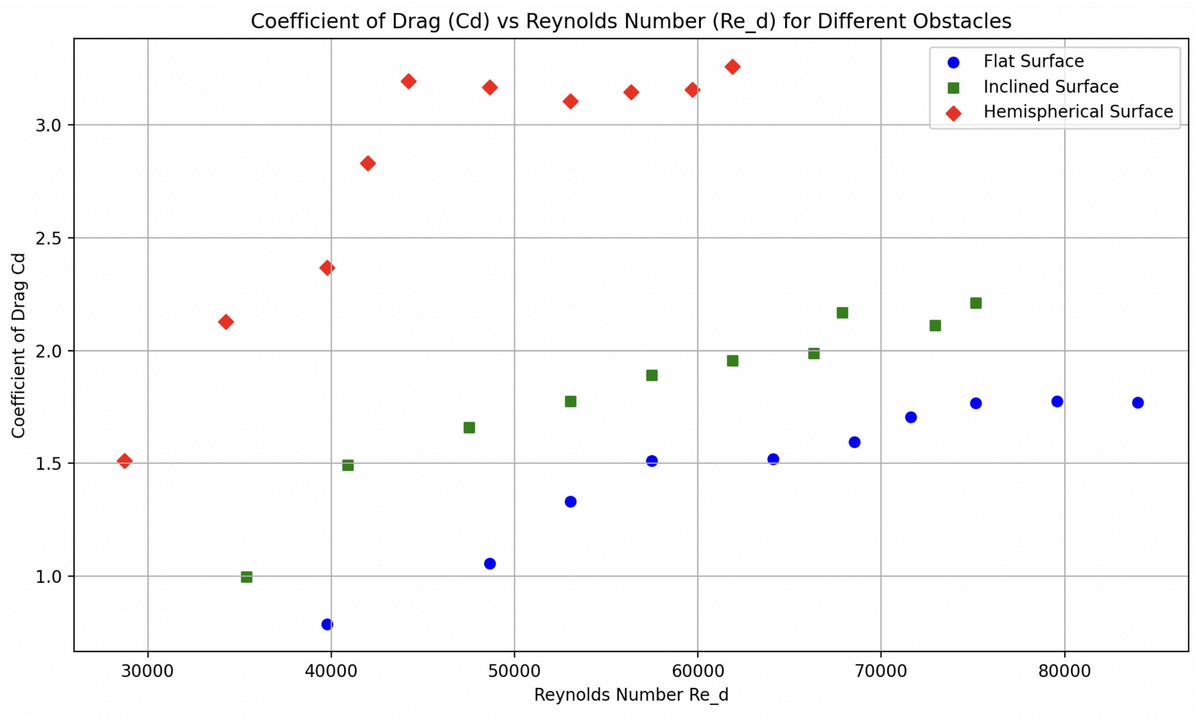
- **Flat Surface**: C_d ranges from approximately 0.79 to 1.75, indicating that the experimental force is slightly less than or comparable to the dynamic pressure force.
- **Inclined Surface**: C_d increases from 1.0 to around 2.23, reflecting the additional momentum change due to the inclination.
- **Hemispherical Surface**: C_d ranges from 1.51 to 3.25, showing a significant increase in force due to the shape causing the jet to spread in all directions.

The results demonstrate that the obstacle geometry significantly affects the coefficient C_d, with more complex shapes leading to higher values.

12 Conclusion

By calculating C_d using the equation $C_d = \dfrac{F}{\frac{1}{2}\rho v^2 A}$, we have quantified the effect of obstacle shape on the force exerted by a water jet. The hemispherical surface exhibits the highest C_d values due to its geometry causing maximum momentum change. These findings are valuable for engineering applications involving fluid impact on surfaces.

Graph of R_{ed} and C_d (Code in Appendix Section C)



COMMENTS AND CONCLUSION

The experimental results confirm the theoretical predictions for the force exerted by a water jet on different surfaces. The force increases with the flow rate, and the nature of the surface significantly influences the resulting force. Among the three surfaces, the hemispherical surface experienced the highest force due to its ability to deflect the water in multiple directions, thereby maximizing momentum transfer. The graphs of Force vs Flow rate shows a closer match with the theoretical curve although in the flat and inclined surface graphs are slight deviations at certain flow rates.

The drag coefficients (C_d) for each surface increased over varying Reynolds numbers and their values ($C_d = 2$ for flat, $C_d = 3$ for inclined and $C_d = 4$ for hemispherical) but were close to the theoretical predictions. This consistency between experimental and theoretical data supports the validity of the applied fluid mechanics principles, such as the momentum conservation equation, in analyzing jet impact forces.

Overall, the experiment demonstrates the practical application of fluid dynamics concepts in determining jet forces on surfaces and highlights the importance of surface geometry in influencing fluid behavior.

Appendix A: Graph of F vs Q and Theoretical Line

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

def plot_force_vs_flow(obstacle_name, multiplier):
   # Constants
   g = 9.81
   rho = 1000
   d = 0.008
   A = np.pi * d**2 / 4

   filename = f'{obstacle_name}.csv'
   df = pd.read_csv(filename)

   df['m_kg'] = df['m (g)'] / 1000
   df['F_exp'] = df['m_kg'] * g
   df['Q_m3s'] = df['Q (l/h)'] / 1000 / 3600

   df['F_theory'] = multiplier * rho * df['Q_m3s']**2 / A

   plt.scatter(df['Q_m3s'], df['F_exp'], color='blue', label='Experimental Data')

   Q_range = np.linspace(df['Q_m3s'].min(), df['Q_m3s'].max(), 200)
   F_theory_curve = multiplier * rho * Q_range**2 / A
   plt.plot(Q_range, F_theory_curve, color='red', label='Theoretical Prediction')

   plt.title(f'Force vs Flow Rate for {obstacle_name.capitalize()} Surface')
   plt.xlabel('Flow Rate Q (m³/s)')
   plt.ylabel('Force F (N)')
   plt.legend()
   plt.grid(True)
   plt.show()

plot_force_vs_flow('flat', multiplier=1.0)
plot_force_vs_flow('inclined', multiplier=1.5)
plot_force_vs_flow('hemispherical', multiplier=2.0)
```

Appendix B: Comparison of Experimental and Theoretical Data

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

def plot_all_surfaces():
   g = 9.81
   rho = 1000
   d = 0.008
   A = np.pi * d**2 / 4

   obstacles = {
       'Flat': {'multiplier': 1.0, 'color': 'blue', 'marker': 'o'},
       'Inclined': {'multiplier': 1.5, 'color': 'green', 'marker': 's'},
       'Hemispherical': {'multiplier': 2.0, 'color': 'red', 'marker': '^'}
   }

   plt.figure(figsize=(10, 6))

   for obstacle_name, params in obstacles.items():
       multiplier = params['multiplier']
       color = params['color']
       marker = params['marker']

       filename = f'{obstacle_name.lower()}.csv'
       try:
           df = pd.read_csv(filename)
       except FileNotFoundError:
           print(f"File {filename} not found. Skipping {obstacle_name} surface.")
           continue

       df['m_kg'] = df['m (g)'] / 1000
       df['F_exp'] = df['m_kg'] * g
       df['Q_m3s'] = df['Q (l/h)'] / 1000 / 3600

       df.sort_values('Q_m3s', inplace=True)

       df['F_theory'] = multiplier * rho * df['Q_m3s']**2 / A

       plt.scatter(df['Q_m3s'], df['F_exp'], color=color, marker=marker,
label=f'{obstacle_name} Experimental')

       Q_range = np.linspace(df['Q_m3s'].min(), df['Q_m3s'].max(), 200)
       F_theory_curve = multiplier * rho * Q_range**2 / A
```

```
        plt.plot(Q_range, F_theory_curve, color=color, linestyle='--', 
label=f'{obstacle_name} Theoretical')

    plt.title('Force vs Flow Rate for Different Surfaces')
    plt.xlabel('Flow Rate Q (m³/s)')
    plt.ylabel('Force F (N)')
    plt.legend()
    plt.grid(True)
    plt.tight_layout()
    plt.show()

plot_all_surfaces()
```

Appendix C: R_{ed} vs C_d Plot Code

```
import numpy as np

import matplotlib.pyplot as plt


g = 9.81

rho = 1000

mu = 1e-3

d = 0.008

A = np.pi * d**2 / 4


data = {

   'flat': {

       'mass_g': [50, 100, 150, 200, 250, 300, 350, 400, 450, 500],

       'Q_lph': [900, 1100, 1200, 1300, 1450, 1550, 1620, 1700, 1800, 1900]

   },

   'inclined': {

       'mass_g': [50, 100, 150, 200, 250, 300, 350, 400, 450, 500],

       'Q_lph': [800, 925, 1075, 1200, 1300, 1400, 1500, 1535, 1650, 1700]

   },

   'hemispherical': {

       'mass_g': [50, 100, 150, 200, 250, 300, 350, 400, 450, 500],

       'Q_lph': [650, 775, 900, 950, 1000, 1100, 1200, 1275, 1350, 1400]

   }

}


colors = {'flat': 'blue', 'inclined': 'green', 'hemispherical': 'red'}

markers = {'flat': 'o', 'inclined': 's', 'hemispherical': 'D'}


plt.figure(figsize=(10, 6))


for obstacle in data:

   mass_g = np.array(data[obstacle]['mass_g'])

   Q_lph = np.array(data[obstacle]['Q_lph'])
```

```
    mass_kg = mass_g / 1000

    F_exp = mass_kg * g


    Q_m3s = Q_lph / 3_600_000

    v = Q_m3s / A


    Re_d = (rho * v * d) / mu


    F_dynamic = 0.5 * rho * v**2 * A

    Cd = F_exp / F_dynamic

        plt.scatter(Re_d, Cd, color=colors[obstacle], marker=markers[obstacle],
label=f'{obstacle.capitalize()} Surface')



plt.title('Coefficient of Drag (Cd) vs Reynolds Number (Re_d) for Different
Obstacles')

plt.xlabel('Reynolds Number Re_d')

plt.ylabel('Coefficient of Drag Cd')

plt.legend()

plt.grid(True)

plt.tight_layout()

plt.show()
```

Appendix D: Experimental Data (chart completed on pages 8 and 9):

Flat surface

	m (g)	Q (l/h)	W (N)	$\frac{1}{2}\rho Q^2/A$ (N)
1	50	1/900		
2	100	1/1100		
3	150	1/1200		
4	200	1/1300		
5	250	1/1450		
6	300	1/1550		
7	350	1/1620		
8	400	1/1700		
9	450	1/1800		
10	500	1/1900		

Oblique surface

	m (g)	Q (l/h)	W (N)	$\frac{1}{2}\rho Q^2/A$ (N)
1	50	1/800		
2	100	1/925		
3	150	1/1075		
4	200	1/1200		
5	250	1/1300		
6	300	1/1400		
7	350	1/1500		
8	400	1/1575		
9	450	1/1650		
10	500	1/1700		

Hemispherical surface

	m (g)	Q (l/h)	W (N)	$\frac{1}{2}\rho Q^2/A$ (N)
1	50	1/650		
2	100	1/775		
3	150	1/900		
4	200	1/950		
5	250	1/1000		
6	300	1/1100		
7	350	1/1200		
8	400	1/1275		
9	450	1/1350		
10	500	1/1400		

Table 1: Results tables.