Engineering Fluid Mechanics

Water Tank Discharge Lab Report

Authors: Jaivir Parmar, Hugo Polo Álvarez, Luis Rodríguez Martínez, Álvaro Sanz González

Group: 38B (Subgroup 1)

Professor: Juan Baselga Llidó

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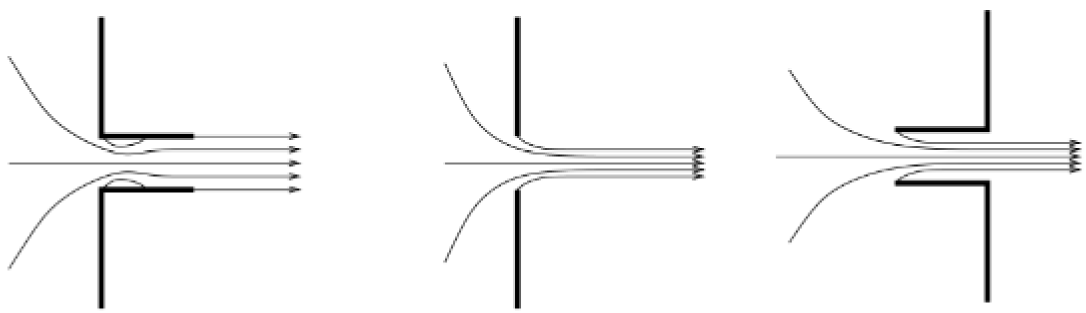
Introduction

In this experiment, the discharge of water from a tank was analyzed using different types of nozzles to investigate how nozzle geometry influences the flow rate and discharge process. The procedure involved filling cylindrical tanks with water and measuring the time it took for the water level to drop to specific heights as it flowed through nozzles of varying diameters and shapes. The three nozzles tested were an inner pipe (borda), an outer pipe with sharp edges, and a bare hole.

The theoretical approach used dimensional analysis and fluid mechanics principles, such as Bernoulli's equation; which relates the pressure, velocity, and elevation along a streamline. For an ideal fluid flow, Bernoulli’s equation, $p+g+\frac{1}{2}*v^{2}=constant$, can be applied to understand how the water's potential energy (due to its height) is converted into kinetic energy as it flows through the nozzle. This allows us to assume that the flow is quasi-steady, which means that the water height decreases slowly enough for Bernoulli’s equation to be valid at each moment.



Outer pipe Bare hole Inner pipe

Quasi Steady Flow Condition Verification

Firstly, the assumptions of quasi-steady flow will be checked. The hypothesis says that:

$$\frac{t_c}{t_d} \sim \frac{D_b^3}{D_d^2 H} \ll 1$$

Where :

D_d is the inner diameter of the water tank (150 mm)

D_b is the diameter of the nozzles

H is the height calculated as $H = z_d - z_b$

Now, $\frac{D_b^3}{D_d^2 H}$ will be calculated considering 3 references values for each nozzle type. In this way it will be checked that the hypothesis is satisfied in different heights.

	H = 75,5 cm	H = 40,5 cm	H = 25,5 cm
'Borda' nozzle: D_{b1}	$5{,}45\cdot 10^{-4}$	$1{,}02\cdot 10^{-3}$	$1{,}61\cdot 10^{-3}$
Outer pipe: D_{b2}	$2{,}64\cdot 10^{-4}$	$4{,}92\cdot 10^{-4}$	$7{,}83\cdot 10^{-4}$
Bare hole: D_{b3}	$4{,}7\cdot 10^{-4}$	$8{,}78\cdot 10^{-4}$	$1{,}4\cdot 10^{-3}$

We can observe that the obtained values are significantly smaller than 1, so we checked tha assumption of the quasi-steady flow. This can be observed in the graph displayed below, where all values are much smaller than 1:

Ideal Flow Condition

To verify that the fluid we are using acts as an ideal fluid, we will use the Reynolds formula. With this formula we can demonstrate that it acts as an ideal fluid, so that the effect of the viscosity is so small that it is negligible.

Datos:

- Zb = 9.5 cm
- Zd = 85 cm
- H = Zd - Zb = 75.5 cm
- g = 9.8 m/s²
- ν = 1.10⁻⁶ m²/s

Fórmula:

$$Re = \frac{V_c \cdot D_b}{\nu} = \frac{\sqrt{2gH} \cdot D_b}{\nu} >> 1$$

By applying this formula, we see that if the Reynolds number is much greater than 1, then it will mean that the viscosity is so small that it can be ignored.

If this is verified, then as one of the main characteristics of an ideal fluid is that the viscosity is so small that can be ineligible, we can assume that the liquid acts as an ideal fluid.

Outer pipe:

- Db = 0.0165 m

$$Re = \frac{\sqrt{2 \cdot 9.8 \cdot 0.755} \cdot 0.0165}{1 \cdot 10^{-6}} = 63472 >> 1$$

Bare hole:

- Db = 0.02 m

$$Re = \frac{\sqrt{2 \cdot 9.8 \cdot 0.755} \cdot 0.02}{1 \cdot 10^{-6}} = 76936 >> 1$$

Borda Nozzle:

- Db = 0.021 m

$$Re = \frac{\sqrt{2 \cdot 9.8 \cdot 0.755} \cdot 0.021}{1 \cdot 10^{-6}} = 80783 >> 1$$

Then in all holes, the Reynolds number is much greater than one, so we can assume that it behaves as an ideal fluid. This is further demonstrated in the graph below, showing all nozzles having a Reynold's number much greater than 1:

Restrictive Situation Analysis

The situation becomes more restrictive when the tank is close to the end of the discharge process (i.e., when H is equal to the minimum value considered of 20-25cm).

- **When the Tank is Completely Filled (H=H_0)**:
  - The initial velocity $v_c=\sqrt{(2gH_0 v)}$ is relatively high.
  - Given $H_0 \sim D_d$, the Reynolds number Re is large because v_c is substantial.
- **When the Tank is Nearly Empty ($H \approx H_{min}$)**:
  - The velocity $v_c=\sqrt{(2gH_{min}v)}$ decreases as H diminishes.
  - A smaller H leads to a lower v_c, thus reducing Re. If H_{min} is sufficiently small, Re may no longer satisfy Re$\gg$1

Initially, all primary assumptions (quasi-steady flow, negligible interior velocity, high Re) are well satisfied. Approaching the second state, the flow may transition from quasi-steady to more unsteady, and viscous effects might become non-negligible. The near-empty condition (H approaching the minimum value) is more restrictive because the assumptions of the model (such as hydrostatic pressure distribution, negligible container velocity, and high Reynolds number) are increasingly violated. Experimental data suggests that these assumptions fail to hold for approximately 17–20% of the discharge time.

Percentage of Time Near Minimum H

= (Total Discharge Time/Time Near Minimum H)

= (0.5 second average/3 second average)*100

≈ **17%.**

Contraction Coefficient Calculations

Data collected during this experiment is attached below in Appendix section C. We obtained a value for H_0 = 75,5 cm and we can do a linear regression fit ($y = mx + b$) in comparison with equation 5. Theoretical analysis is done here to establish conceptual understanding, with the calculations performed in python. From equation 5:

$$\underbrace{\sqrt{H} - \sqrt{H_o}}_{f(H)} = -\alpha \left(\frac{D_b}{D_d}\right)^2 \sqrt{\frac{g}{2}} t.$$

Therefore:

$$\alpha = \frac{-1}{\left(\frac{D_b}{D_d}\right)^2 \sqrt{\frac{g}{2}}} * \frac{(\sqrt{H} - \sqrt{H_0})}{t}$$

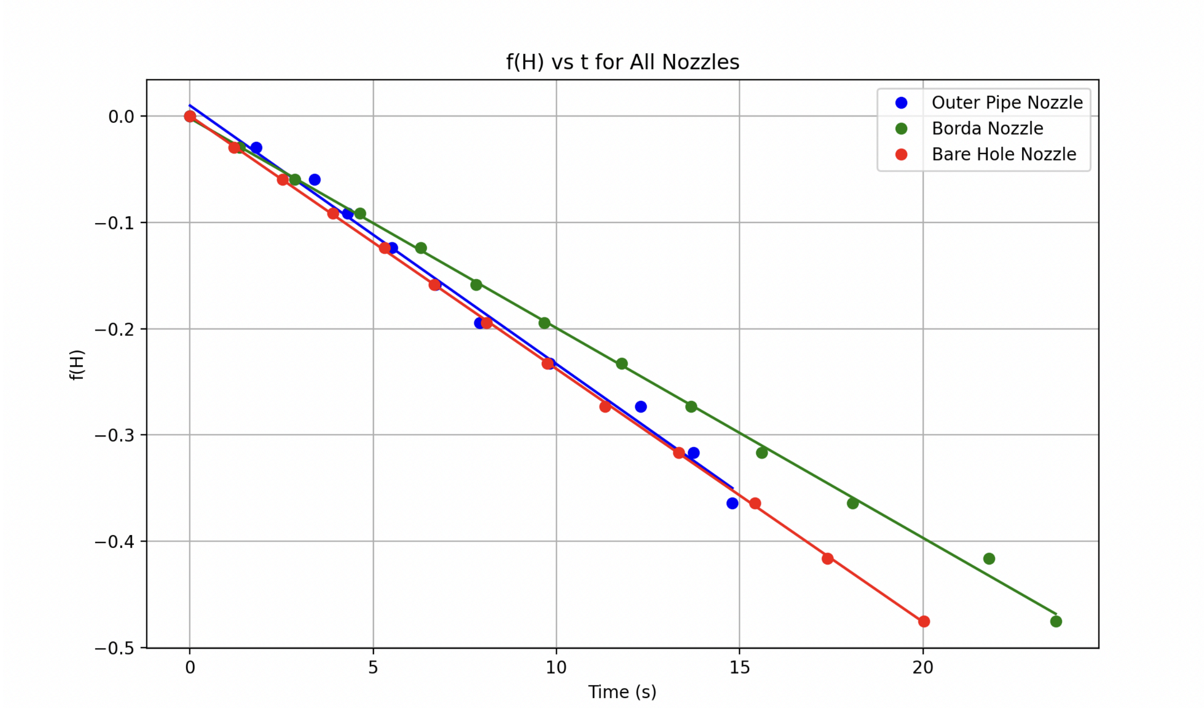
We obtain that $x = t$, $y = \sqrt{H} - \sqrt{H_0}$, $m = -\alpha\left(\frac{D_b}{D_d}\right)^2 \sqrt{\frac{g}{2}}$ and $b = 0$.

If we solve the equation for α we get that:

$$\alpha_x = - m_x \left(\frac{D_d}{D_b}\right)^2 \sqrt{\frac{2}{g}}$$

$$\text{for } m_x = \frac{(\sqrt{H} - \sqrt{H_0})}{t}$$

A graphical representation of m_x for each nozzle is displayed below:



Nozzle Type	**Slope**	**Theoretical α**	**Experimental α**	**Error**
Outer	-0.02455	1.00	0.908	9.9%
Bare	-0.01975	~0.60	0.605	0.83%
Borda	-0.02381	0.50	0.455	9.0%

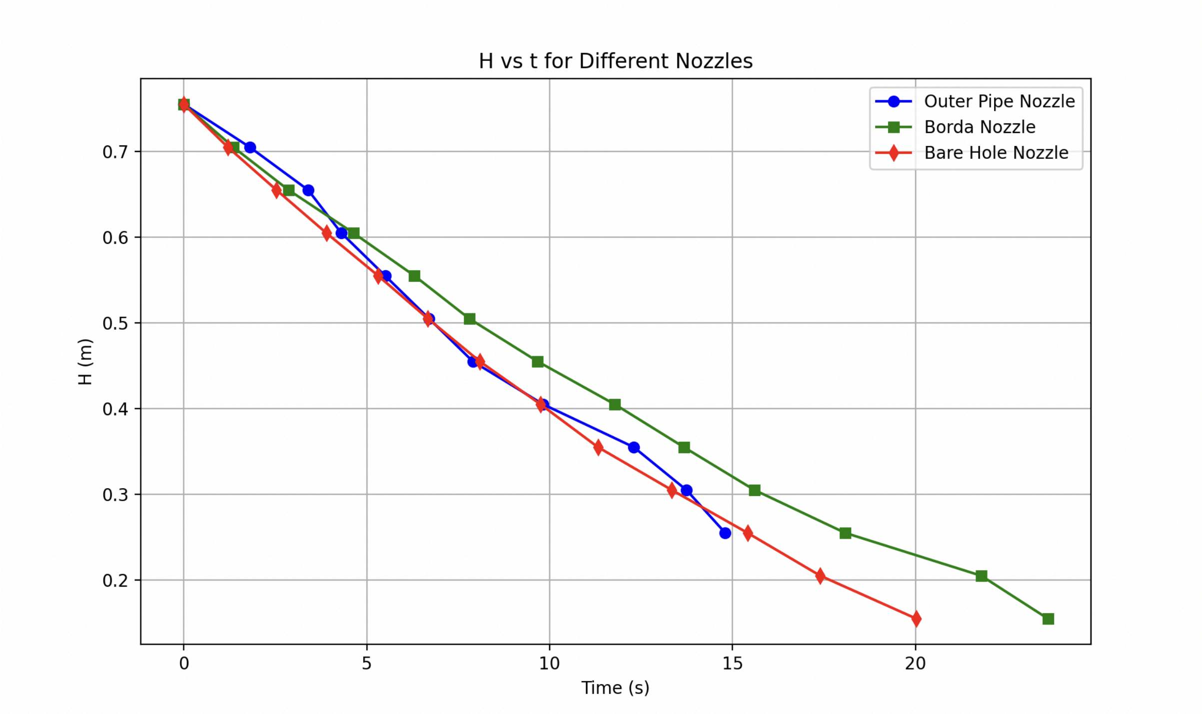
We had previously obtained an incorrect outer nozzle coefficient calculation of 1.58. This is not possible and was due to a combination of experimental and calculational errors. In particular, we had intervals where the time between trials of decreasing heights were marked with decreasing times, which is not possible. To adjust for this, intervals that seemed pragmatically intangible or prone to human error were disregarded; data that appeared error free (had increasing time between intervals) was used for analysis.

```
Nozzle: Outer Pipe Nozzle
Slope (m): -0.02433
Calculated α: 0.908
R-squared: 0.9930

Nozzle: Borda Nozzle
Slope (m): -0.01975
Calculated α: 0.455
R-squared: 0.9985

Nozzle: Bare Hole Nozzle
Slope (m): -0.02381
Calculated α: 0.605
R-squared: 0.9999
```

Plot of H vs T



We took the average from two trials with each of the three nozzles. In order to calculate the error, we have taken the numerical difference between every value that we have measured. However, even though it has been calculated we can clearly see in the table that the lines lack a clear linear/smooth pattern. This is due to the fact that it is very easy to commit errors when taking the measurements due to our reaction time and the very few repetitions of the experiment. We can conclude from the plot that the outer nozzle discharges at a similar rate. with the bare hold; the borda nozzle discharges the slowest.

Impact of Nozzle on Times

As it has been demonstrated before, the fluid acts as an ideal fluid, and the velocity of the liquid while it gets out of the tank, will follow the Torriceli's rule $v =(2gh) ^ {½}$. So v depends on the effect of the gravity and height. Although the diameter of the hole might have some influence on the velocity of discharge, it can be ignored as the variability is very small.

However, we see that the bigger the nozzle, the faster is the process of discharge of our tank. This is because the flux is increased with an increase of nozzle diameter. The amount of water per second that flows through the nozzle is larger when the nozzle has a larger diameter.

Q (velocity of discharge) = A (area of nozzle) x V(velocity)

By applying this formula, we observe that the diameter affects the velocity of discharge, or in other words, the volumetric flux.

The velocity also depends on the height (h) according to the formula $v =(2gh) ^ {½}$. As the water flows out, the height of the column of water is reduced and, as a consequence of these, the velocity decreases, reducing also the volumetric flux. This is the reason why the velocity of discharges decreases as the tank discharges.

Conclusion

This experiment successfully demonstrates the impact of nozzle contours and diameters on the flow rate of water due to the force of gravity. It justifies that the contraction coefficient impacts the discharge process. Finally, it draws conclusive results with similar measures to expected behaviors.

The contraction coefficient values that we have obtained are similar to the real ones. We have obtained 0.605 for the bare hole (expected is 0.6), 0.455 for the inner pipe (expected is 0.5), and finally 0.908 (expected is 1) for the outer pipe. All of the errors for each nozzle type were within around 10% of the expected value. These results indicated that the outer pipe should have the fastest discharge rate.

The discrepancies in the actual vs expected times are byproducts of obvious measuring limitations. Since we used a timer and visually estimated the heights of the water, the accuracy of our data was compromised by both our vision and reaction abilities. This also affected the graphs that plot the height vs the time. If greater measuring accuracy is needed, the act of timing and measuring should be performed autonomously by a capable machine.

Appendix Section A: Contraction Coefficient Calculations & Plot (Python Code)

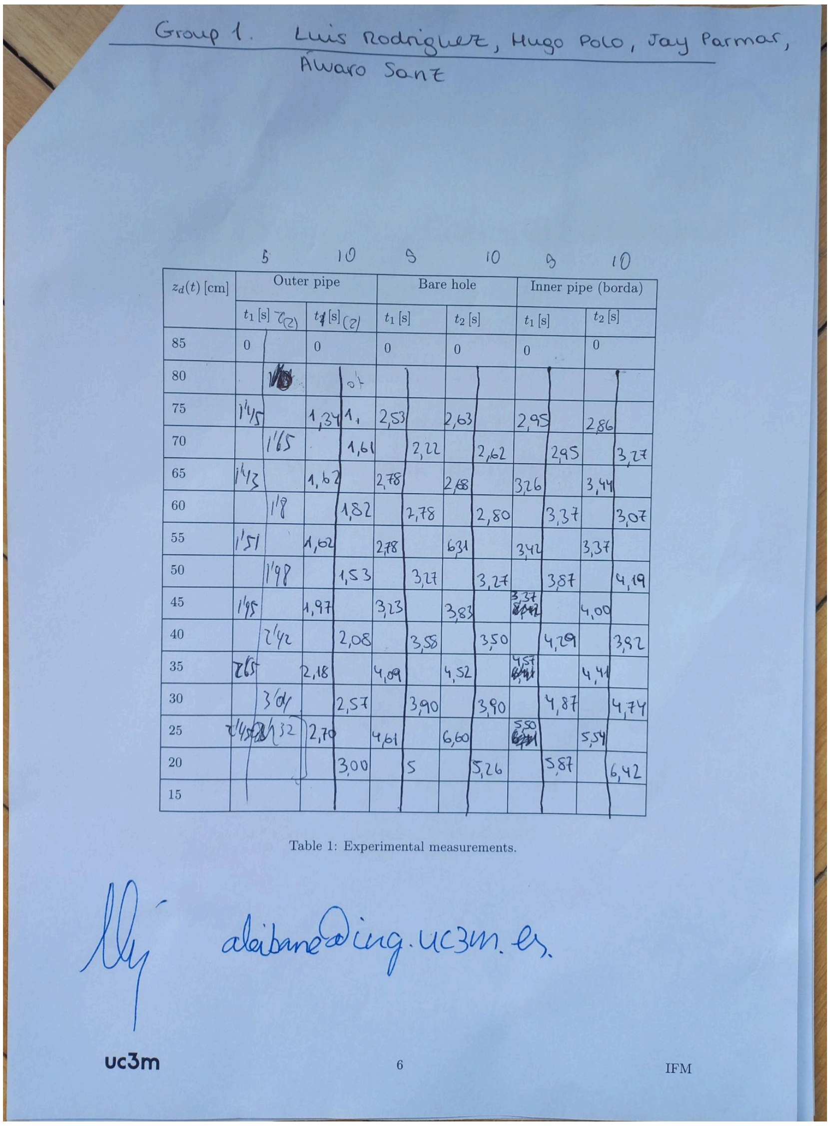
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import linregress
Dd = 0.15
zb = 0.095
g = 9.81
zd0 = 85
H0 = (zd0 / 100.0) - zb  #subtracts midplane nozzle height
Db_borda = 0.021   #borda
Db_outer_pipe = 0.0165  #outer
Db_bare_hole = 0.020  #bare hole
def run(data, time_column, nozzle, Db, color):
   zd_cm = data['Height (cm)'].values
   zd_m = zd_cm / 100.0
   H = zd_m - zb
   t = data[time_column].values
   valid_indices = (~np.isnan(t)) & (H > 0)
   H_valid = H[valid_indices]
   t_valid = t[valid_indices]
   fH = np.sqrt(H_valid) - np.sqrt(H0)
   plt.plot(t_valid, fH, 'o', color=color, label=f'{nozzle}')
   slope, intercept, r_value, p_value, std_err = linregress(t_valid, fH)
   t_fit = np.linspace(min(t_valid), max(t_valid), 100)
   fH_fit = slope * t_fit + intercept
   plt.plot(t_fit, fH_fit, '-', color=color)
   m = slope
   factor = (Db / Dd)**2 * np.sqrt(g / 2)
   alpha = -m / factor
   print(f"Nozzle: {nozzle}")
   print(f"Slope (m): {m:.5f}")
   print(f"Calculated α: {alpha:.3f}")
   print(f"R-squared: {r_value**2:.4f}\n")
   return alpha
data = pd.read_csv('lab1.csv')
plt.figure(figsize=(10, 6))
alpha_outer_pipe = run(data, 'Time Outer (s)', 'Outer Pipe Nozzle', Db_outer_pipe, 'blue')
alpha_borda = run(data, 'Time Borda (s)', 'Borda Nozzle', Db_borda, 'green')
alpha_bare_hole = run(data, 'Time Bare (s)', 'Bare Hole Nozzle', Db_bare_hole, 'red')
```

```
plt.xlabel('Time (s)')
plt.ylabel('f(H)')
plt.title('f(H) vs t for All Nozzles')
plt.legend()
plt.grid(True)
plt.show()
```

Appendix Section B: Graphing Height vs Time (Python Code)

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
zb = 0.095
data = pd.read_csv('lab1.csv')
data['H (m)'] = (data['Height (cm)'] / 100.0) - zb
time_outer = data['Time Outer (s)']
time_borda = data['Time Borda (s)']
time_bare = data['Time Bare (s)']
H_outer = data['H (m)']
H_borda = data['H (m)']
H_bare = data['H (m)']
plt.figure(figsize=(10, 6))
plt.plot(time_outer, H_outer, 'o-', label='Outer Pipe Nozzle', color='blue')
plt.plot(time_borda, H_borda, 's-', label='Borda Nozzle', color='green')
plt.plot(time_bare, H_bare, 'd-', label='Bare Hole Nozzle', color='red')
plt.xlabel('Time (s)')
plt.ylabel('H (m)')
plt.title('H vs t for Different Nozzles')
plt.legend()
plt.grid(True)
plt.show()
```

Appendix Section C: Written Experimental Data



Group 1. Luis Rodriguez, Hugo Polo, Jay Parmar, Álvaro Sanz

	5		10		5		10		5		10	
$z_d(t)$ [cm]	Outer pipe				Bare hole				Inner pipe (borda)			
	t_1 [s]	τ(2)	t_2 [s]	(2)	t_1 [s]		t_2 [s]		t_1 [s]		t_2 [s]	
85	0		0		0		0		0		0	
80		[illegible]		[illegible]								
75	1'45		1,34	1,	2,53		2,63		2,95		2,86	
70		1'65		1,61		2,22		2,62		2,95		3,27
65	1'43		1,62		2,78		2,68		3,26		3,44	
60		1'8		1,82		2,78		2,80		3,37		3,07
55	1'51		1,62		2,78		6,31		3,42		3,37	
50		1'98		1,53		3,27		3,27		3,87		4,19
45	1'95		1,97		3,23		3,83		3,37		4,00	
40		2'42		2,08		3,58		3,50		4,29		3,92
35	2'65		2,18		4,09		4,52		4,57		4,41	
30		3'01		2,57		3,90		3,90		4,87		4,74
25	2'45	2'32	2,70		4,61		6,60		5,50		5,54	
20				3,00		5		5,26		5,87		6,42
15												

Table 1: Experimental measurements.

alaibane@ing.uc3m.es

uc3m 6 IFM