Engineering Fluid Mechanics

Falling Ball Viscometer Lab Report

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1 Introduction

The aim of this lab is to determine the viscosity of three distinct liquids: glycerol, vaseline oil, and SAE 30 vacuum pump oil. We accomplish this using a falling ball viscometer, where the drag force on a sphere is determined from a simple force balance. The sphere falls at a terminal velocity, and the viscosity of the fluid can be derived using Stokes' law, which is valid for Reynolds numbers much smaller than 1 (creeping flow).

The sphere used in this experiment has a diameter of 6 mm and a density of 2475 kg/m^3. The fluids and their densities are as follows:

- Glycerol: $\rho = 1.26\,\mathrm{g/cm^3}$
- Vaseline oil: $\rho = 0.86\,\mathrm{g/cm^3}$
- SAE 30 vacuum pump oil: $\rho = 0.88\,\mathrm{g/cm^3}$

2 Results

For each fluid, the results are summarized in Table 1, and the corresponding images and plots are presented below.

Fluid	ρ [g/cm^3]	v [m/s]	μ [Pa·s]
Glycerol	1.26	0.018	1.3262
Vaseline oil	0.86	0.1225	0.2585
SAE 30 oil	0.88	0.018	0.3856

Table 1: Summary of measured and computed values.

2.1 Glycerol

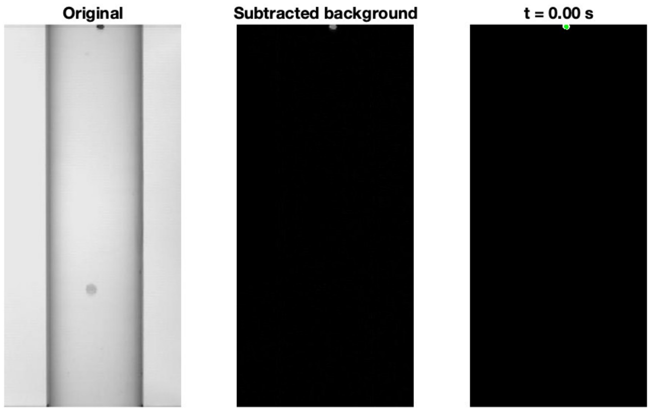

Figure 1: Original image, subtracted background, and binary image for glycerol.

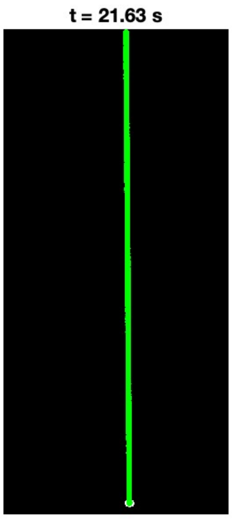

Figure 2: Highlighted path of the falling ball in glycerol.

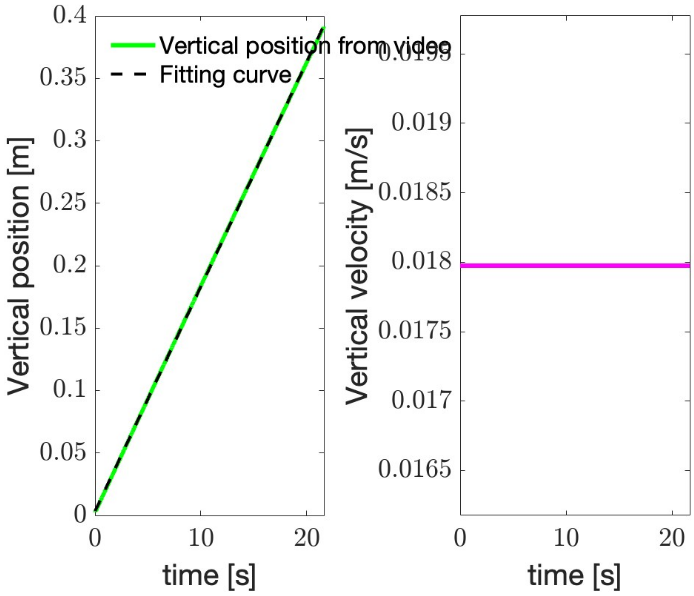

Figure 3: Vertical position vs. time and vertical velocity vs. time for glycerol.

2.2 Vaseline Oil

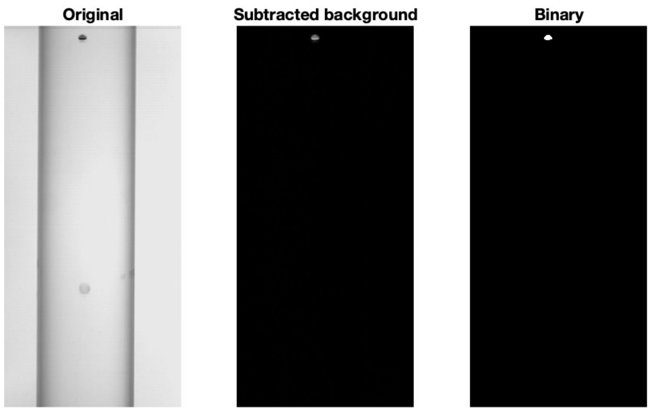

Figure 4: Original image, subtracted background, and binary image for vaseline oil.

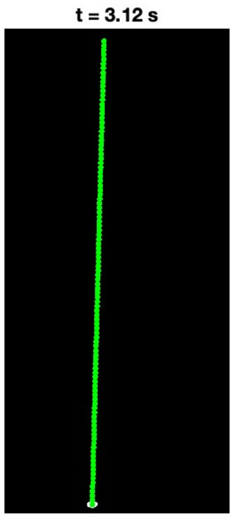

Figure 5: Highlighted path of the falling ball in vaseline oil.

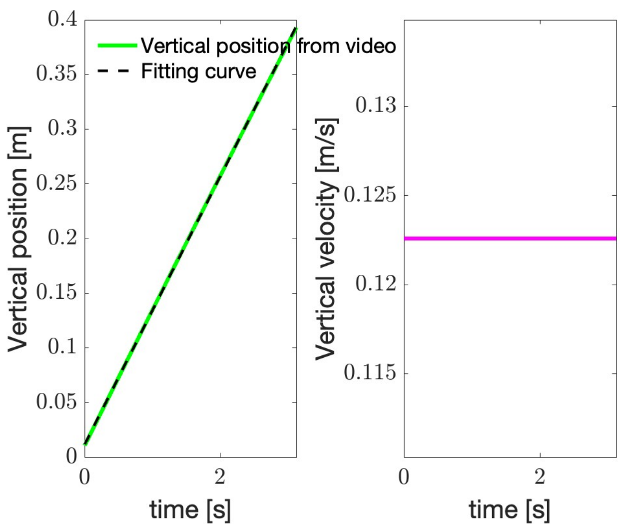

Figure 6: Vertical position vs. time and vertical velocity vs. time for vaseline oil.

2.3 SAE 30 Vacuum Pump Oil

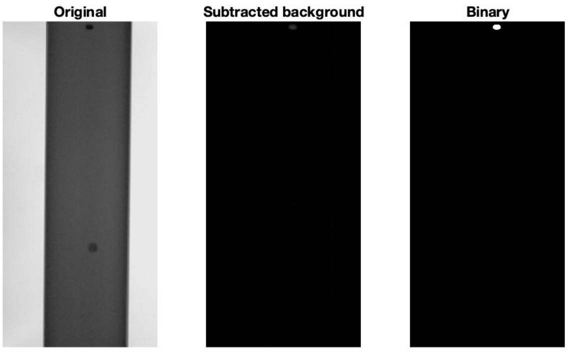

Figure 7: Original image, subtracted background, and binary image for SAE 30 oil.

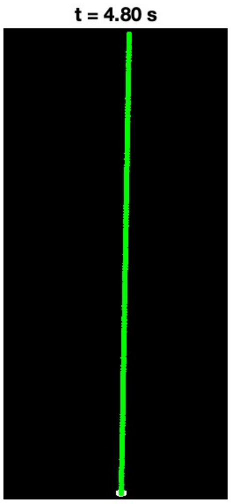

Figure 8: Highlighted path of the falling ball in SAE 30 oil.

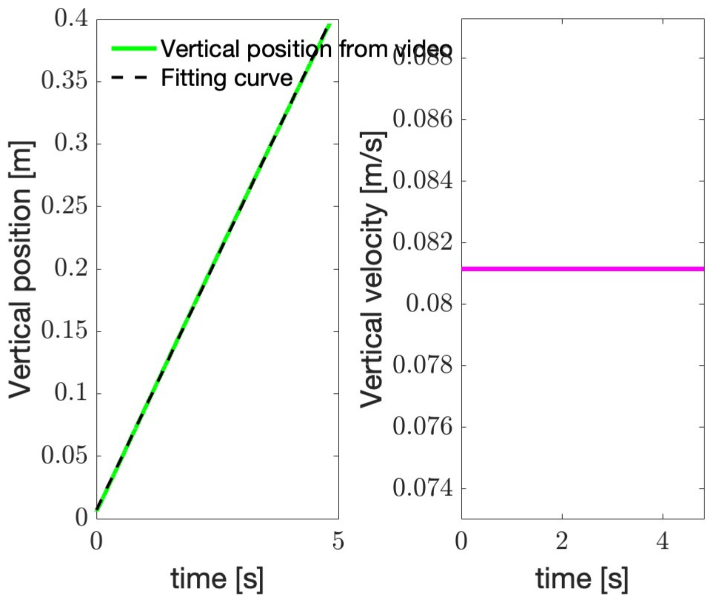

Figure 9: Vertical position vs. time and vertical velocity vs. time for SAE 30 oil.

3 Reynolds Number Calculations

The Reynolds number (Re) is a dimensionless quantity in fluid mechanics used to predict flow patterns in different fluid flow situations. It is defined as the ratio of inertial forces to viscous forces and is given by the formula:

$$Re = \frac{\rho v d}{\mu} \tag{1}$$

where:

- ρ = Fluid density (kg/m^3)
- v = Terminal velocity of the sphere (m/s)
- d = Diameter of the sphere (m)
- μ = Dynamic viscosity of the fluid (Pa·s)

3.1 Calculation of Reynolds Number for Each Fluid

Given the following parameters:

- Sphere Diameter, $d = 6\,\text{mm} = 0.006\,\text{m}$
- Sphere Density, $\rho_s = 2475\,\text{kg/m}^3$
- Acceleration due to Gravity, $g = 9.81\,\text{m/s}^2$

The terminal velocities and fluid properties are provided in Table 2.

Table 2: Properties of Fluids and Terminal Velocities

Fluid	ρ (kg/m^3)	v (m/s)	μ (Pa·s)	Re
Vaseline Oil	860	0.1225	0.2585	2.446
SAE 30 Vacuum Pump Oil	880	0.081	0.3856	1.108
Glycerol	1260	0.018	1.3262	0.1025

3.1.1 1. Vaseline Oil

Given:

- $\rho = 860\,\text{kg/m}^3$
- $v = 0.1225\,\text{m/s}$
- $\mu = 0.2585\,\text{Pa·s}$

Calculation:

$$\begin{aligned} Re &= \frac{\rho v d}{\mu} \\ &= \frac{860 \times 0.1225 \times 0.006}{0.2585} \\ &= \frac{0.6321}{0.2585} \\ &\approx 2.446 \end{aligned}$$

Interpretation:

$$Re \approx 2.446$$

Since $Re > 1$, the flow is in the **transitional flow**. This indicates that both inertial and viscous forces are significant. Inertial effects start to influence the drag force, potentially reducing the accuracy of viscosity calculations based solely on Stokes' Law.

3.1.2 2. SAE 30 Vacuum Pump Oil

Given:

- $\rho = 880\,\text{kg/m}^3$
- $v = 0.081\,\text{m/s}$
- $\mu = 0.3856\,\text{Pa·s}$

Calculation:

$$\begin{aligned} Re &= \frac{\rho v d}{\mu} \\ &= \frac{880 \times 0.081 \times 0.006}{0.3856} \\ &= \frac{0.42768}{0.3856} \\ &\approx 1.108 \end{aligned}$$

Interpretation:

$$Re \approx 1.108$$

With Re slightly above 1, SAE 30 Vacuum Pump Oil is also in the **transitional flow**. Stokes' Law provides an approximate value for viscosity, but inertial effects may introduce minor deviations.

3.1.3 3. Glycerol

Given:

- $\rho = 1260\,\text{kg/m}^3$
- $v = 0.018\,\text{m/s}$
- $\mu = 1.3262\,\text{Pa·s}$

Calculation:

$$\begin{aligned} Re &= \frac{\rho v d}{\mu} \\ &= \frac{1260 \times 0.018 \times 0.006}{1.3262} \\ &= \frac{0.13608}{1.3262} \\ &\approx 0.1025 \end{aligned}$$

Interpretation:

$$Re \approx 0.1025$$

Since $Re < 1$, the flow of Glycerol is firmly in the **creeping (laminar) flow**. Viscous forces dominate, and Stokes' Law is applicable.

3.2 Summary of Reynolds Numbers

Table 3: Reynolds Number and Flow Regime for Each Fluid

Fluid	Re	$Re < 1$?	**Flow Regime**
Vaseline Oil	2.446	No	Transitional Flow
SAE 30 Vacuum Pump Oil	1.108	No	Transitional Flow
Glycerol	0.1025	Yes	Creeping (Laminar) Flow

4 Drag Coefficient Calculations and Analysis

We calculate and analyze the drag coefficients for three distinct fluids—Vaseline Oil, SAE 30 Vacuum Pump Oil, and Glycerol—using both theoretical and empirical approaches.

4.1 Calculation of Drag Coefficient (C_D)

The drag coefficient (C_D) quantifies the resistance experienced by a sphere as it moves through a fluid. It is calculated using the following formula derived from the force balance on the falling sphere:

$$C_D = \frac{4}{3} \cdot \frac{(\rho_s - \rho) \cdot g \cdot d}{\rho \cdot v^2} \tag{2}$$

where:

- ρ_s = Density of the sphere (kg/m^3)
- ρ = Density of the fluid (kg/m^3)
- g = Acceleration due to gravity ($\approx 9.81\,\mathrm{m/s^2}$)
- d = Diameter of the sphere (m)
- v = Terminal velocity of the sphere (m/s)

4.1.1 Calculation Steps for Each Fluid

1. Vaseline Oil Given:

- $\rho = 860\,\mathrm{kg/m^3}$
- $v = 0.1225\,\mathrm{m/s}$
- $\mu = 0.2585\,\mathrm{Pa{\cdot}s}$

Calculation:

$$\begin{aligned}
C_D &= \frac{4}{3} \cdot \frac{(2475 - 860) \cdot 9.81 \cdot 0.006}{860 \cdot (0.1225)^2} \\
&= \frac{4}{3} \cdot \frac{1615 \cdot 9.81 \cdot 0.006}{860 \cdot 0.01500625} \\
&= \frac{4}{3} \cdot \frac{94.8222}{12.905375} \\
&= \frac{4}{3} \cdot 7.349 \\
&\approx 9.798
\end{aligned}$$

2. SAE 30 Vacuum Pump Oil Given:

- $\rho = 880\,\mathrm{kg/m^3}$
- $v = 0.081\,\mathrm{m/s}$
- $\mu = 0.3856\,\mathrm{Pa{\cdot}s}$

Calculation:

$$\begin{aligned}
C_D &= \frac{4}{3} \cdot \frac{(2475 - 880) \cdot 9.81 \cdot 0.006}{880 \cdot (0.081)^2} \\
&= \frac{4}{3} \cdot \frac{1595 \cdot 9.81 \cdot 0.006}{880 \cdot 0.006561} \\
&= \frac{4}{3} \cdot \frac{94.1469}{5.78168} \\
&= \frac{4}{3} \cdot 16.313 \\
&\approx 21.75
\end{aligned}$$

3. Glycerol Given:

- $\rho = 1260\,\mathrm{kg/m^3}$
- $v = 0.018\,\mathrm{m/s}$
- $\mu = 1.3262\,\mathrm{Pa{\cdot}s}$

Calculation:

$$\begin{aligned}
C_D &= \frac{4}{3} \cdot \frac{(2475 - 1260) \cdot 9.81 \cdot 0.006}{1260 \cdot (0.018)^2} \\
&= \frac{4}{3} \cdot \frac{1215 \cdot 9.81 \cdot 0.006}{1260 \cdot 0.000324} \\
&= \frac{4}{3} \cdot \frac{71.649}{0.40824} \\
&= \frac{4}{3} \cdot 175.468 \\
&\approx 234.291
\end{aligned}$$

4.2 Updated Data Table

The calculated drag coefficients (C_D) are summarized in Table **??** alongside the previously provided fluid properties.

Table 4: Properties of Fluids, Terminal Velocities, and Drag Coefficients

Fluid	$\rho\,(\mathrm{kg/m^3})$	$v\,(\mathrm{m/s})$	$\mu\,(\mathrm{Pa \cdot s})$	Re	C_D
Vaseline Oil	860	0.1225	0.2585	2.446	9.798
SAE 30 Vacuum Pump Oil	880	0.081	0.3856	1.108	21.75
Glycerol	1260	0.018	1.3262	0.1025	234.291

4.3 Visualization of Drag Coefficient vs. Reynolds Number

To provide a visual representation of the relationship between the drag coefficient (C_D) and the Reynolds number (Re), we generated a log–log plot using Python.

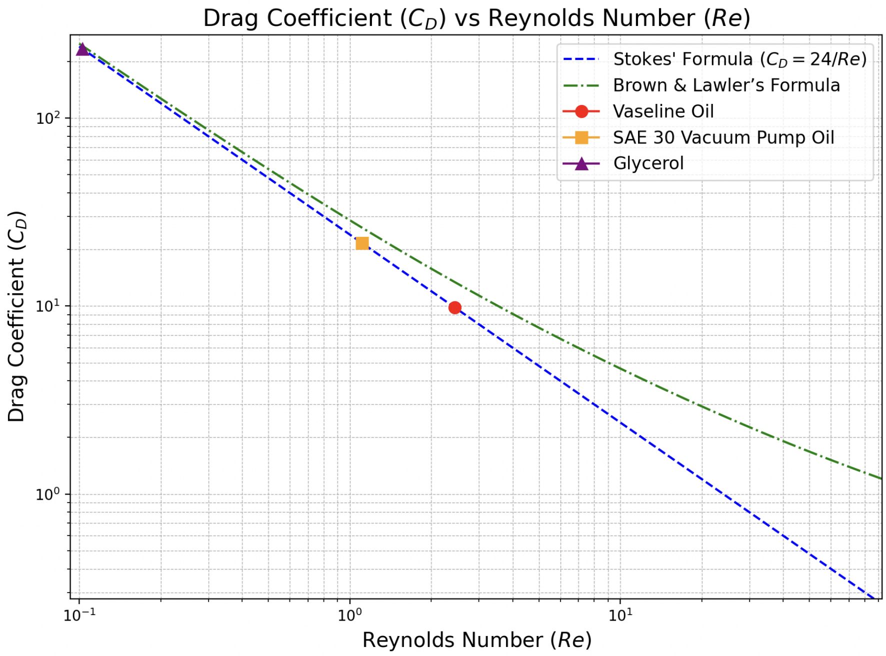

Figure 10: Drag Coefficient (C_D) vs. Reynolds Number (Re) for Various Fluids. Theoretical curves for Stokes' Formula and Brown & Lawler's Formula are also depicted for comparison.

5 Comparison with Water and Applicability of Stokes' Law

5.1 Viscosity Comparison with Water

Water has a relatively low viscosity of approximately $1 \times 10^{-3}\,\mathrm{Pa \cdot s}$ at room temperature. In comparison, the fluids examined in this experiment exhibit significantly higher viscosities:

- **Vaseline Oil**: $0.2585\,\mathrm{Pa \cdot s}$
- **SAE 30 Vacuum Pump Oil**: $0.3856\,\mathrm{Pa \cdot s}$
- **Glycerol**: $1.3262\,\mathrm{Pa \cdot s}$

This substantial increase in viscosity indicates that the studied oils and glycerol offer much greater resistance to flow compared to water. The higher viscosity values are consistent with the observed terminal velocities; fluids with higher viscosities result in slower falling sphere velocities due to increased internal friction. This relationship underscores the direct dependence of terminal velocity on fluid viscosity, as articulated by Stokes' Law.

5.2 Applicability of Stokes' Law Across Different Fluids

Stokes' Law is derived under the assumption of creeping flow conditions where the Reynolds number (Re) is much less than 1. In this experiment:

- **Glycerol**: $Re \approx 0.1025$ — *Creeping Flow*
- **Vaseline Oil**: $Re \approx 2.446$ — *Transitional Flow*
- **SAE 30 Vacuum Pump Oil**: $Re \approx 1.108$ — *Transitional Flow*

Only Glycerol is within the creeping flow area, where Stokes' Law accurately predicts the drag coefficient and viscosity. For Vaseline Oil and SAE 30 Vacuum Pump Oil, the Reynolds numbers slightly exceed 1, placing them in the transitional flow area where inertial effects begin to influence the drag force. While Stokes' Law provides a foundational approximation for these fluids, its predictions become less accurate as Re increases. However, Re is low enough for the remaining two fluids to be explained with accuracy Stoke's Law. Brown & Lawler's formula, offer improved accuracy for calculating drag coefficients in these higher Reynolds number regimes by accounting for both viscous and inertial forces.

6 Conclusion

This experiment successfully determined the viscosities of Vaseline Oil, SAE 30 Vacuum Pump Oil, and Glycerol using the falling ball viscometer method. The calculated Reynolds numbers revealed that Glycerol operates well within the creeping flow regime, validating the use of Stokes' Law for accurate viscosity measurements. In contrast, Vaseline Oil and SAE 30 Vacuum Pump Oil, situated in the transitional flow regime, demonstrated the limitations of Stokes' Law, highlighting the necessity for empirical drag force correlations to enhance accuracy. Furthermore, the viscosities of all studied fluids were found to be substantially higher than that of water, emphasizing their greater resistance to flow. These findings underscore the critical importance of selecting appropriate theoretical models based on flow regimes to ensure precise viscosity determinations in fluid mechanics studies.