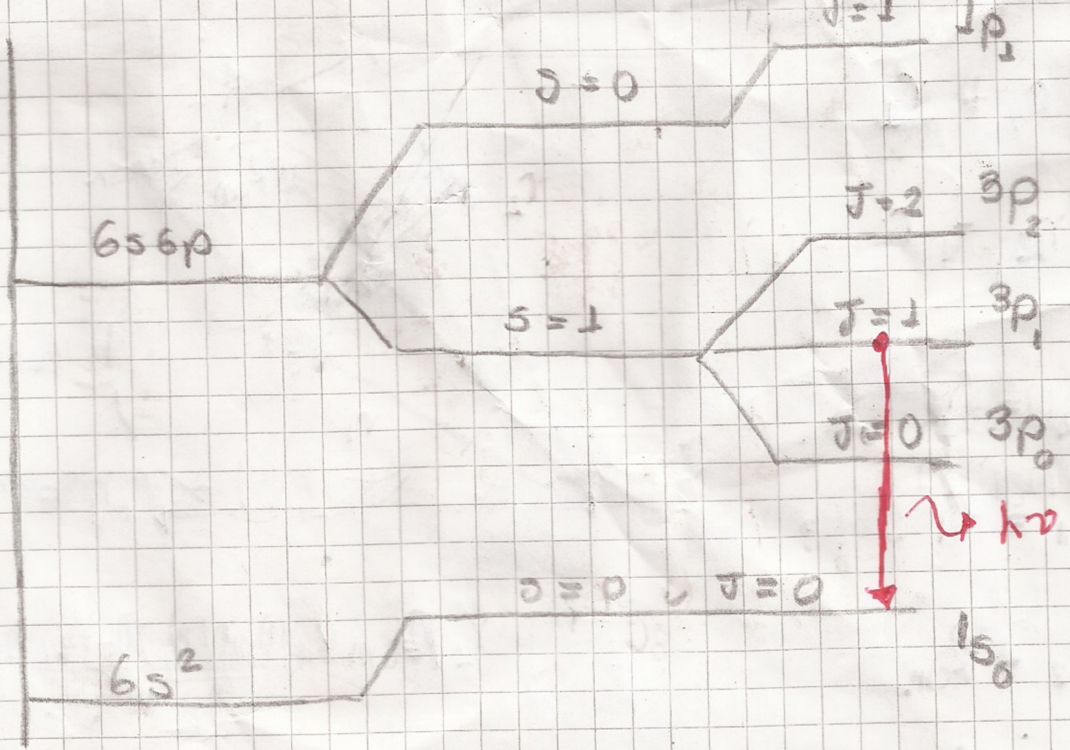


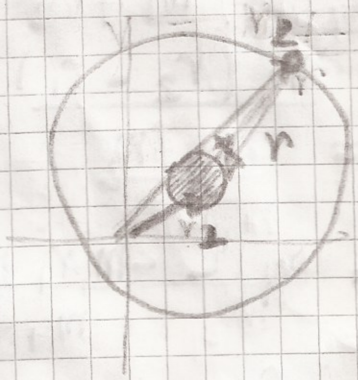
$$\hat{p} = i\hbar \frac{d}{dx} \qquad \hat{p}^2 = i^2\hbar^2\nabla^2$$

E → estados energéticos

funciones → funciones de ondas propias

$$H\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$



$$-\frac{\hbar^2}{2m_1}\left(\frac{\partial^2\psi_T}{\partial x_1} + \frac{\partial^2\psi_T}{\partial y_1} + \frac{\partial^2\psi_T}{\partial z_1^2}\right)$$

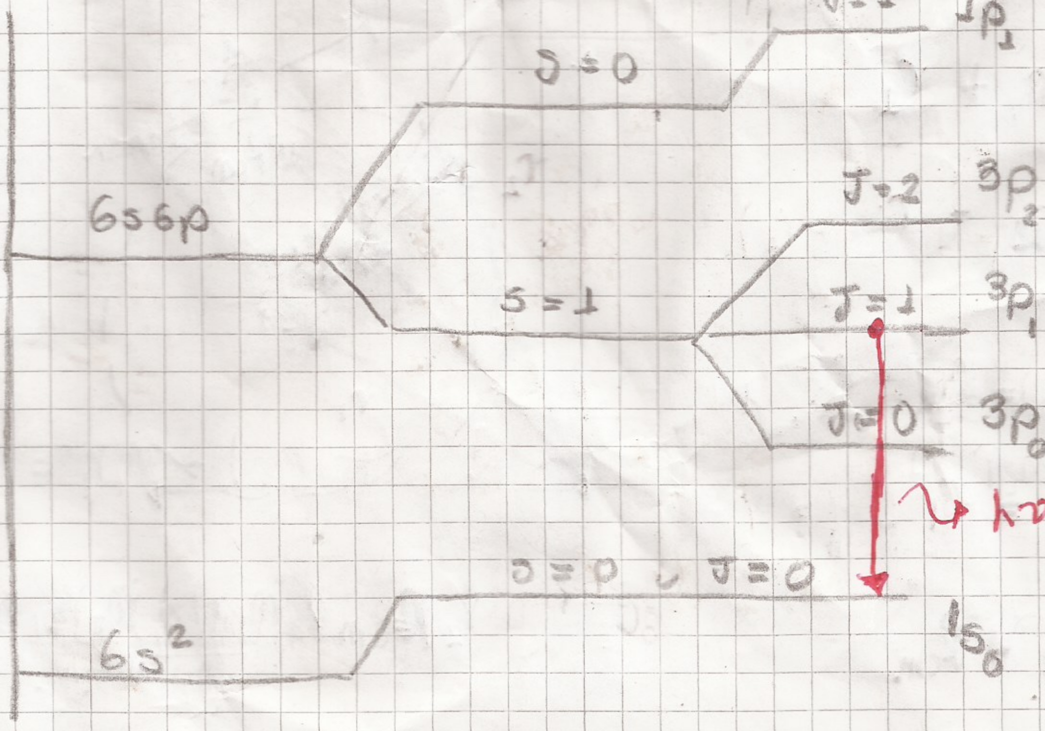
$$-\frac{\hbar^2}{2m_2}\left(\frac{\partial^2\psi_T}{\partial x_2^2} + \frac{\partial^2\psi_T}{\partial y_2^2} + \frac{\partial^2\psi_T}{\partial z_2^2}\right)$$

$$+V(r)\,\psi_T = E_T\,\psi_T$$

$$U = \frac{q_1 q_2}{r}$$

$$U = -\frac{kZe^2}{r}$$

$$\bar{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$



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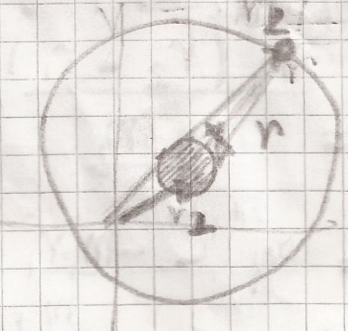

$$-\frac{\hbar^2}{2m_1}\left(\frac{\partial^2 \psi_T}{\partial x_1^2} + \frac{\partial^2 \psi_T}{\partial y_1^2} + \frac{\partial^2 \psi_T}{\partial z_1^2}\right)$$

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$$U = -\frac{KZe^2}{r}$$

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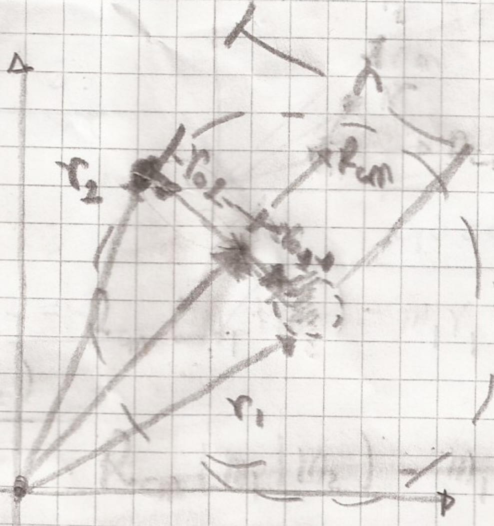
$$r = r_2 - r_1$$

$$r_2 = r + r_1$$

$$r_1 = r_2 - r$$



$$R_{cm} + r_{02} = r_2$$

$$R_{cm} + r_{01} = r_1$$

$$R_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$r_{02} = r_2 - R_{cm}$$

$$r_{01} = r_1 - R_{cm}$$

- $$r_{02} = r_2 - \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$r_{02} = \frac{m_1 r_2 + \cancel{m_2 r_2} - m_1 r_1 - \cancel{m_2 r_2}}{m_1 + m_2}$$

$$r_{02} = \frac{m_1 (r_2 - r_1)}{m_1 + m_2} = \frac{m_1 r}{m_1 + m_2}$$

- $$r_{01} = r_1 - \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$r_{01} = \frac{\cancel{m_1 r_1} + m_2 r_1 - \cancel{m_1 r_1} - m_2 r_2}{m_1 + m_2}$$

$$r_{01} = \frac{m_2 (r_1 - r_2)}{m_1 + m_2} = \frac{m_2 r}{m_1 + m_2}$$

$$E_{ks} = \frac{1}{2}(m_1+m_2)\cdot V_{cm}^2 + \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

$$E_{ks} = \frac{1}{2}(m_1+m_2)\dot{R}_{cm}^2 + \frac{1}{2} m_1 \left[\frac{m_2}{m_1+m_2}\right]^2 \dot{r}^2 + \frac{1}{2} m_2 \left[\frac{m_1}{m_1+m_2}\right] \dot{r}^2$$

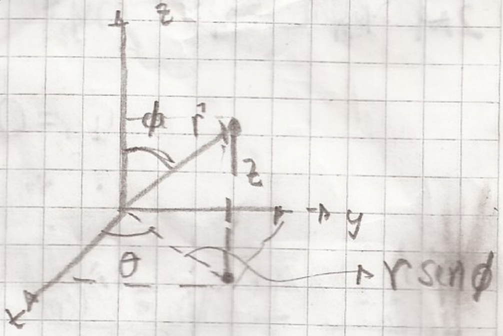
$$E_{ks} = \frac{1}{2}(m_1+m_2)\dot{R}_{cm}^2 + \frac{1}{2}\frac{m_1 m_2^2 \dot{r}^2}{(m_1+m_2)^2} + \frac{1}{2}\frac{m_2 m_1^2 \dot{r}^2}{(m_1+m_2)^2}$$

$$E_{ks} = \frac{1}{2}(m_1+m_2)\dot{R}_{cm}^2 + \frac{1}{2} m_1 m_2 \dot{r}^2 \left[\frac{m_2}{(m_1+m_2)^2} + \frac{m_1}{(m_1+m_2)^2}\right]$$

$$E_{ks} = \frac{1}{2} M \dot{R}_{cm}^2 + \frac{1}{2} m_1 m_2 \dot{r}^2 \left[\frac{\cancel{m_1+m_2}}{(m_1+m_2)^{\cancel{2}}}\right]$$

$$E_{ks} = \frac{1}{2} M \dot{R}_{cm}^2 + \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\dot{r}^2 \qquad \mu = \frac{m_1 m_2}{m_1+m_2}$$

$$E_{ks} = \frac{1}{2} M \dot{R}_{cm}^2 + \frac{1}{2}\mu \dot{r}^2$$



$$\hat{H} = \frac{\hat{P}_1^2}{2M} + \frac{\hat{P}_2^2}{2\mu} + V(r)$$

$z = r\cos\phi$

$x = r \operatorname{sen}\phi \cos\theta$

$y = r \operatorname{sen}\phi \operatorname{sen}\theta$

$$\left[-\frac{\hbar^2}{2M}\left(\frac{\partial^2 \psi_T}{\partial x_1^2} + \frac{\partial^2 \psi_T}{\partial y_1^2} + \frac{\partial^2 \psi_T}{\partial z_1^2}\right)\right] + \left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2 \psi_T}{\partial x_2^2} + \frac{\partial^2 \psi_T}{\partial y_2^2} + \frac{\partial^2 \psi_T}{\partial z_2^2}\right)\right] + V(r)\psi_T = E_T \psi_T$$

$$\psi_T(x,y,z,r,\theta,\phi) = \psi_{cm}(x,y,z)\,\psi(r,\theta,\phi)$$

$$-\frac{\hbar^2}{2M}\left(\frac{\partial^2\psi_T}{\partial x_1^2}+\frac{\partial^2\psi_T}{\partial y_1^2}+\frac{\partial^2\psi_T}{\partial z_1^2}\right)-\frac{\hbar^2}{2\mu}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_T}{\partial r}\right)+\frac{1}{r^2 sen^2\theta}\frac{\partial^2\psi_T}{\partial\phi^2}+\frac{1}{r^2 sen\theta}\frac{\partial}{\partial\theta}\left(sen\theta\frac{\partial\psi_T}{\partial\theta}\right)\right\}+V(r)\psi_T=E_T\psi_T$$

$$-\frac{\hbar^2}{2M}\psi\left[\frac{\partial^2\psi_{cm}}{\partial x_1^2}+\frac{\partial^2\psi_{cm}}{\partial y_1^2}+\frac{\partial^2\psi_{cm}}{\partial z_1^2}\right]-\frac{\hbar^2}{2\mu}\psi_{cm}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2 sen^2\theta}\frac{\partial^2\psi}{\partial\phi^2}+\frac{1}{r^2 sen\theta}\frac{\partial}{\partial\theta}\left(sen\theta\frac{\partial\psi}{\partial\theta}\right)\right\}+V(r)\psi_{cm}\psi=E_T\psi_{cm}\psi$$

Se divide la ecuación entre ψ_{cm} y ψ

$$-\frac{\hbar^2}{2M}\frac{1}{\psi_{cm}}\left[\frac{\partial^2\psi_{cm}}{\partial x_1^2}+\frac{\partial^2\psi_{cm}}{\partial y_2^2}+\frac{\partial^2\psi_{cm}}{\partial z_2^2}\right]-\frac{\hbar^2}{2\mu}\cdot\frac{1}{\psi}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2 sen^2\theta}\frac{\partial^2\psi}{\partial\phi^2}+\frac{1}{r^2 sen\theta}\frac{\partial}{\partial\theta}\left(sen\theta\frac{\partial\psi}{\partial\theta}\right)+V(r)=E_T\right.$$

$$-\frac{\hbar^2}{2M}\frac{1}{\psi_{cm}}\left[\frac{\partial^2\psi_{cm}}{\partial x^2}+\frac{\partial^2\psi_{cm}}{\partial y^2}+\frac{\partial^2\psi_{cm}}{\partial z^2}\right]=E_{cm}$$

$$-\frac{\hbar^2}{2\mu}\frac{1}{\psi}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2 sen^2\theta}\frac{\partial^2\psi}{\partial\phi^2}+\frac{1}{r^2 sen\theta}\frac{\partial}{\partial\theta}\left(sen\theta\frac{\partial\psi}{\partial\theta}\right)+V(r)=E\right.$$

$$E_{cm}+E=E_T$$

$$\frac{\hbar^2}{2M}\left[\frac{\partial^2\psi_{cm}}{\partial x^2}+\frac{\partial^2\psi_{cm}}{\partial y^2}+\frac{\partial^2\psi_{cm}}{\partial z^2}\right]=E_{cm}\psi_{cm}$$

$$-\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right)+\frac{1}{r^2 sen^2\theta}\frac{\partial^2\psi}{\partial\phi^2}+\frac{1}{r^2 sen\theta}\frac{\partial}{\partial\theta}\left(sen\theta\frac{\partial\psi}{\partial\theta}\right)\right]+V(r)\psi=E\psi$$

Solución del la parte del electron

$$\psi(r,\theta,\phi)=R(r)\,\Theta(\theta)\,\Phi(\phi)$$

$$-\frac{\hbar^2}{2\mu}\left[\frac{\Theta(\theta)\Phi(\phi)}{r^2}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right)+\frac{\Theta(\theta)R(r)}{r^2 sen^2\theta}\frac{d^2\Phi(\phi)}{d\phi^2}+\frac{R(r)\Phi(\phi)}{r^2 sen\theta}\frac{d}{d\theta}\left(sen\theta\frac{d\Theta(\theta)}{d\theta}\right)\right]$$

$$+V(r)\psi=E\psi$$

Dividiendo entre $-\frac{R(r)\,\Theta(\theta)\,\Phi(\phi)\,\hbar^2}{2\mu}$ queda que:

$$\frac{1}{r^2R(r)}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right)+\frac{1}{r^2sen^2\theta\,\Phi(\phi)}\frac{d^2\Phi(\phi)}{d\phi^2}+\frac{1}{r^2sen\theta\,\Theta(\theta)}\frac{d}{d\theta}\left(sen\theta\frac{d\Theta(\theta)}{d\theta}\right)$$

$$\frac{2\mu}{\hbar^2}\left[E-V(r)\right]=0$$

Multiplicando por $r^2sen^2\theta$ queda:

$$\frac{sen^2\theta}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}+\frac{sen\theta}{\Theta}\frac{d}{d\theta}\left(sen\theta\frac{d\Theta}{d\theta}\right)+\frac{2\mu r^2sen^2\theta}{\hbar^2}\left[E-V\right]=0$$

* $\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}=-m^2$ $\qquad \frac{d^2\Phi}{d\phi^2}+m^2\phi=0$ + resolver

* $$\frac{sen^2\theta}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{sen\theta}{\Theta}\frac{d}{d\theta}\left(sen\theta\frac{d\Theta}{d\theta}\right)+\frac{2\mu r^2sen^2\theta}{\hbar^2}\left[E-V\right]=m^2$$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{1}{sen\theta\,\Theta}\frac{d}{d\theta}\left(sen\theta\frac{d\Theta}{d\theta}\right)+\frac{2\mu}{\hbar^2}r^2\left[E-V\right]=\frac{m^2}{sen^2\theta}$$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2\mu r^2}{\hbar}[E-V]=\frac{m^2}{\sin^2\theta}-\frac{1}{\sin\theta\,\Theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)$$

$$-\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\frac{m^2\Theta}{\sin^2\theta}=\alpha\,\Theta$$ → resolver

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2\mu}{\hbar}[E-V(r)]R=\alpha\,\frac{R}{r^2}$$ → resolver

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Se tienen que resolver estas 3 ecuaciones diferenciales

$$\frac{d^2\Phi}{d\phi^2}+m^2\Phi=0 \quad (1)$$

$$-\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\frac{m^2\Theta}{\sin^2\theta}=\alpha\Theta \quad (2)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2\mu}{\hbar}[E-V(r)]R=\alpha\frac{R}{r^2} \quad (3)$$

Solución ecuación diferencial (1)

$$\frac{d^2\Phi}{d\phi^2}+m^2\Phi=0$$

$$\Phi=e^{\lambda\phi}$$

$$\frac{d\Phi}{d\phi}=\lambda e^{\lambda\phi}$$

$$\frac{d^2\Phi}{d\phi^2}=\lambda^2 e^{\lambda\phi}$$

$$\lambda^2 e^{\lambda\phi}+m^2 e^{\lambda\phi}=0$$

$$e^{\lambda\phi}(\lambda^2+m^2)=0$$

$$\lambda^2=-m^2$$

$$\lambda=\sqrt{-m^2}$$

$$\lambda=\sqrt{(-1)m^2}$$

$$\lambda=im$$

$$\Phi(\phi)=A\,e^{\lambda\phi}$$

$$\Phi(\phi)=A\,e^{im\phi}$$

$$\int_0^{2\pi} \Phi^*(\phi)\,\Phi(\phi)\,d\phi = 1$$

$$\int_0^{2\pi} A^2 \cancel{e^{-im\phi}\, e^{im\phi}}^{\,1}\, d\phi = 1$$

$$A^2 \left[\phi\right]_0^{2\pi} = 1 \rightarrow A^2\, 2\pi = 1$$

$$A = \frac{1}{\sqrt{2\pi}}$$

$$\Phi = \frac{1}{\sqrt{2\pi}} e^{im\phi}$$

$$\Phi(0) = \frac{1}{\sqrt{2\pi}} \qquad \Phi(2\pi) = \frac{1}{\sqrt{2\pi}}\left[\cos(2\pi m) + i\,\mathrm{sen}(2\pi m)\right]$$

$$\Phi(0) = \Phi(2\pi) \qquad m = 0, 1, 2, 3, 4$$

Las Funciones que son Soluciones son:

$$\Phi_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi} \qquad m = 0, 1, 2, 3, 4$$

$$-\frac{1}{\mathrm{sen}\,\theta}\frac{d}{d\theta}\left(\mathrm{sen}\,\theta \frac{d\theta}{d\theta}\right) + \frac{m^2\,\theta}{\mathrm{sen}^2\theta} = \alpha\,\theta$$

$$z = \cos\theta \qquad \begin{cases} P(z) = \theta(\theta) \\ \downarrow \qquad\quad \downarrow \\ z = \cos\theta \end{cases}$$

$$\frac{dz}{d\theta} = -\mathrm{sen}\,\theta$$

$$z^2 = \cos^2\theta$$

$$\frac{d\theta}{d\theta} = \frac{dP(z)}{dz}\frac{dz}{d\theta} = -\frac{dP(z)}{dz}\mathrm{sen}\,\theta$$

$$\mathrm{sen}^2\theta + \cos^2\theta = 1$$

$$\mathrm{sen}^2\theta = 1 - \cos^2\theta$$

$$dz = -\mathrm{sen}\,\theta\, d\theta$$

$$\mathrm{sen}^2\theta = 1 - z^2$$

$$\frac{d}{dz} = -\frac{1}{\mathrm{sen}\,\theta}\frac{d}{d\theta}$$

$$-\frac{d}{dz}\left((1-z^2)\frac{dP(z)}{dz}\right)+\left(\frac{m^2}{1-z^2}-\alpha\right)P(z)=0$$

$$\frac{d}{dz}\left((1-z^2)\frac{dP(z)}{dz}\right)+\left(\alpha-\frac{m^2}{1-z^2}\right)P(z)=0$$

$x = 1 - z$ $\qquad R(x) = P(z)$ $\qquad z = 1$

$$\frac{dx}{dz} = -1 \qquad \frac{dP(z)}{dz} = \frac{dR(x)}{dx}\frac{dx}{dz} = -\frac{dR(x)}{dx}$$

$dx = -dz$

$$\frac{d}{dx} = -\frac{d}{dz}$$

$z = 1 - x$

$z^2 = (1-x)^2$

$z^2 = 1 - 2x + x^2$

$x = 0$

punto ordinario

$1 - z^2 = 1 - (1 - 2x + x^2)$

$1 - z^2 = \cancel{1-1} + 2x - x^2$

$1 - z^2 = x(2-x)$

$$-\frac{d}{dx}\left(x(2-x)-\frac{dR(x)}{dx}\right)+\left(\alpha-\frac{m^2}{x(2-x)}\right)R(x)=0$$

$$\frac{d}{dx}\left(x(2-x)\frac{dR}{dx}\right)+\left(\alpha-\frac{m^2}{x(2-x)}\right)R=0$$

$$R = x^s\sum_{\nu=0}^{\infty} a_\nu x^\nu = \sum_{\nu=0}^{\infty} a_\nu x^{\nu+s} = a_0x^s + a_1x^{s+1} + a_2x^{s+2} \cdots a_\nu$$

$$\frac{dR}{dx} = \sum_{\nu=0}^{\infty}(\nu+s)a_\nu x^{\nu+s-1} =$$

$$\frac{d^2R}{dx^2} = \sum_{\nu=0}^{\infty}(\nu+s)(\nu+s-1)a_\nu x^{\nu+s-2}$$

$$s(s-1)a_0x^{s-2}$$

$$\frac{d}{dx}\left(x(2-x)\frac{dR}{dx}\right) + \left(\alpha - \frac{m^2}{x(2-x)}\right)R = 0$$

$$(2-2x)\frac{dR}{dx} + (2x-x^2)\frac{d^2R}{dx^2} + \left(\alpha - \frac{m^2}{2x-x^2}\right)R = 0$$

$$(2-x)(2-2x)\frac{dR}{dx} + (2-x)(2x-x^2)\frac{d^2R}{dx^2} + (2-x)\left(\alpha - \frac{m^2}{x(2-x)}\right)R = 0$$

$$(2-x)(2-2x)\frac{dR}{dx} + (2-x)(2x-x^2)\frac{d^2R}{dx^2} + \left[\alpha(2-x) - \frac{m^2}{x}\right]R = 0$$

$$(4-6x+2x^2)\frac{dR}{dx} + (4x-4x^2+x^3)\frac{d^2R}{dx^2} + \left[\alpha(2-x) - \frac{m^2}{x}\right]R = 0$$

$$4\sum_{\nu=0}^{\infty}(\nu+s)a_\nu x^{\nu+s-1} - 6\sum_{\nu=0}^{\infty}(\nu+s)a_\nu x^{\nu+s} + 2\sum_{\nu=0}^{\infty}(\nu+s)a_\nu x^{\nu+s+1}$$

$$4\sum_{\nu=0}^{\infty}(\nu+s)(\nu+s-1)a_\nu x^{\nu+s-1} - 4\sum_{\nu=0}^{\infty}(\nu+s)(\nu+s-1)a_\nu x^{\nu+s}$$

$$+\sum_{\nu=0}^{\infty}(\nu+s)a_\nu x^{\nu+s+1} + 2\alpha\sum_{\nu=0}^{\infty}a_\nu x^{\nu+s} - \alpha\sum_{\nu=0}^{\infty}a_\nu x^{\nu+s+1}$$

$$-m^2\sum_{\nu=0}^{\infty}a_\nu x^{\nu+s-1} = 0$$

$$4\sum_{\nu=0}^{\infty}(\nu+s)a_\nu x^{\nu+s-1} - 6\sum_{\nu=1}^{\infty}(\nu+s-1)a_{\nu-1}x^{\nu+s-1} + 2\sum_{\nu=2}^{\infty}(\nu+s-2)a_{\nu-2}x^{\nu+s-1}$$

$$+4\sum_{\nu=0}^{\infty}(\nu+s)(\nu+s-1)a_\nu x^{\nu+s-1} - 4\sum_{\nu=1}^{\infty}(\nu+s-1)(\nu+s-2)a_{\nu-1}x^{\nu+s-1}$$

$$+\sum_{\nu=2}^{\infty}(\nu+s-2)(a_{\nu-2}x^{\nu+s-1} + 2\alpha\sum_{\nu=1}^{\infty}a_{\nu-1}x^{\nu+s-1} - \alpha\sum_{\nu=2}^{\infty}a_{\nu-2}x^{\nu+s-1}$$

$$-m^2\sum_{\nu=0}^{\infty}a_\nu x^{\nu+s-1} = 0$$

$$\sum_{\nu=2}^{\infty}\Big[4(\nu+s)a_\nu - 6(\nu+s-1)a_{\nu-1} + 2(\nu+s-2)a_{\nu-2} + 4(\nu+s)(\nu+s-1)a_\nu -$$

$$4(\nu+s-1)(\nu+s-2)a_{\nu-1} + (\nu+s-2)a_{\nu-2} + 2\alpha a_{\nu-1} - \alpha a_{\nu-2} -$$

$$m^2 a_\nu\Big]x^{\nu+s-1} + \Big[4sa_0x^{s-1} + 4(1+s)a_1x^s - 6sa_0x^s$$

$$+4s(s-1)a_0x^{s-1} + 4(1+s)s\,a_1x^s - 4s(s-1)a_0x^s +$$

$$2\alpha a_0x^s - m^2a_0x^{s-1} - m^2a_1x^s\Big] = 0$$

$$[4s + 4s(s-1) - m^2]a_0x^{s-1} = 0$$

$$[\cancel{4s} + 4s^2 \cancel{-4s} - m^2]a_0x^{s-1} = 0$$

$$4s^2 - m^2 = 0$$

$$s^2 = \frac{m^2}{4}, \quad \boxed{s = \frac{m}{2}}$$

$$P(z) = x^s y^s G(z) = x^{\frac{m}{2}} y^{\frac{m}{2}} G(z) = (1-z)^{\frac{m}{2}}(1+z)^{\frac{m}{2}} G(z)$$

$$P(z) = [(1-z)(1+z)]^{m/2} G(z) = [1-z^2]^{\frac{m}{2}} G(z)$$

$$\frac{d}{dz}\left[(1-z^2)\frac{dP(z)}{dz}\right] + \left[B - \frac{m^2}{1-z^2}\right] P(z) = 0$$

$$\frac{dP(z)}{dz} = \frac{m}{\cancel{2}}[1-z^2]^{\frac{m}{2}-1} \cdot (-\cancel{2}z) G(z) + [1-z^2]^{\frac{m}{2}} G'(z)$$

$$\frac{dP(z)}{dz} = -mz[1-z^2]^{\frac{m}{2}-1} G(z) + [1-z^2]^{\frac{m}{2}} G'(z)$$

$$\frac{d}{dz}\left[(1-z^2)\left[-mz[1-z^2]^{\frac{m}{2}-1} G(z) + [1-z^2]^{\frac{m}{2}} G'(z)\right]\right] + \left[B - \frac{m^2}{1-z^2}\right][1-z^2]^{\frac{m}{2}} G(z) = 0$$

$$\frac{d}{dz}\left[-mz[1-z^2]^{m/2} G(z) + [1-z^2]^{\frac{m}{2}+1} G'(z)\right] + \left[B(1-z^2)^{m/2} - m^2(1-z^2)^{m/2-1}\right] G(z) = 0$$

$$-m(1-z^2)^{m/2} G(z) + \frac{m^2}{\cancel{2}}[1-z^2]^{\frac{m}{2}-1}\cancel{2}z\, zG(z) - mz[1-z^2]^{m/2} G'(z)$$

$$-\left(\frac{m}{2}+1\right)(1-z^2)^{\frac{m}{2}}(2z)G'(z) + [1-z^2]^{\frac{m}{2}+1} G''(z) + B(1-z^2)^{m/2} G(z) - m^2(1-z^2)^{m/2-1} G(z) = 0$$

$$\left[m^2 z^2 [1-z^2]^{\frac{m}{2}-1} - m(1-z^2)^{m/2} + B(1-z^2)^{m/2} - m^2(1-z^2)^{\frac{m}{2}-1}\right] G(z) -$$

$$(1-z^2)^{m/2} G'(z)\left[mz + 2z + mz\right] + [1-z^2]^{\frac{m}{2}+1} G''(z) = 0$$

$$\left[m^2 z^2 [1-z^2]^{\frac{m}{2}-1} - m(1-z^2)^{m/2} + B(1-z^2)^{m/2} - m^2(1-z^2)^{\frac{m}{2}-1}\right] G(z) +$$

$$(1-z^2)^{m/2}\left[-2z(m+1)G'(z) + (1-z^2) G''(z)\right] = 0$$

$$(1-z^2)^{m/2}\left[\frac{m^2z^2}{1-z^2}-mA+\beta-\frac{m^2}{1-z^2}\right]G(z)+$$

$$(1-z^2)^{m/2}\left[(1-z^2)G''(z)-2z(m+1)G'(z)\right]=0$$

$$(1-z^2)^{m/2}\left[\beta+\frac{m^2z^2-m^2}{1-z^2}-m_{1}\right]G(z)+(1-z^2)^{m/2}$$

$$\left[(1-z^2)G''(z)-2z(m+1)G'(z)\right]=0$$

$$(1-z^2)^{m/2}\left[\beta-\frac{m^2(\cancel{1-z^2})}{\cancel{1-z^2}}-m\right]G(z)+(1-z^2)^{m/2}\left[(1-z^2)G''(z)\right.$$

$$\left.-2z(m+1)G'(z)\right]=0$$

$$(1-z^2)^{m/2}\left[(1-z^2)G''(z)-2z(m+1)G'(z)+(\beta-m(m+1))G(z)\right]=0$$

$$(1-z^2)G''(z)-2z(m+1)G'(z)+(\beta-m(m+1))G(z)=0$$

$$G=a_0+a_1z+a_2z^2+a_3z^3+a_4z^4+a_5z^5\dots a_nz^n=\sum_{\nu=0}^{\infty}a_\nu z^\nu$$

$$G'(z)=a_1+2a_2z+3a_3z^2+4a_4z^3+5a_5z^4\dots na_nz^{n-1}$$

$$G''(z)=2a_2+6a_3z+12a_4z^2+20a_5z^3\dots n(n-1)a_nz^{n-2}$$

$$\left[2a_2+6a_3z+12a_4z^2+20a_5z^3\dots-2a_2z^2-6a_3z^3-12a_4z^4-20a_5z^5\dots\right]-$$

$$\left[2a_1z(m+1)+4a_2z^2(m+1)+6a_3z^3(m+1)+8a_4z^4(m+1)+10a_5z^5(m+1)\dots\right.$$

$$\left.\right]+[\beta-m(m+1)]a_0+[\beta-m(m+1)]a_1z+[\beta-m(m+1)]a_2z^2+[\beta-m(m+1)]a_3z^3=0$$

$2a_2 + \{B - m(m+1)\} a_0 = 0 \qquad z^0$

$6a_3 + \{B - m(m+1) - 2(m+1)\} a_1 = 0 \qquad z$

$12a_4 + \{B - m(m+1) + 4(m+1) - 2) a_2 = 0 \qquad z^2$

$20a_5 + \{B - m(m+1) + 6(m+1) - 6] a_3 = 0 \qquad z^3$

$(\nu+1)(\nu+2) a_{\nu+2} + \{(B - m(m+1)) - 2\nu(m+1) - \nu(\nu-1)] a_\nu = 0$

$(\nu+1)(\nu+2) a_{\nu+2} = [2\nu(m+1) + \nu(\nu-1) + m(m+1) - B) a_\nu$

$(\nu+1)(\nu+2) a_{\nu+2} = [2\nu m + 2\nu + \nu^2 - \nu + m^2 + m - \beta] a_\nu$

$(\nu+1)(\nu+2) a_{\nu+2} = [2\nu m + \nu^2 + \nu + m^2 + m - \beta] a_\nu$

$(\nu+1)(\nu+2) a_{\nu+2} = [(\nu+m)(\nu+m+1) - \beta] a_\nu$

$$a_{\nu+2} = \frac{(\nu+m)(\nu+m+1) - \beta}{(\nu+1)(\nu+2)} a_\nu$$

$$a_2 = \frac{m(m+1) - \beta}{2} a_0$$

$$a_3 = \frac{(1+m)(m+2) - \beta}{6} a_1$$

$$a_4 = \frac{(2+m)(m+3) - \beta}{12} a_2$$

$$a_5 = \frac{(3+m)(m+4) - \beta}{20} a_3$$

$$a_6 = \frac{(4+m)(m+5) - \beta}{30} a_4$$

Todos los coeficientes de la serie terminan dependiendo de a_0 y a_1 (serie par e impar)

$$a_7 = \frac{(5+m)(m+6) - \beta}{42} a_5$$

Para romper la serie y hacerla finita.

$$\underbrace{\beta = (\nu' + m)(\nu' + m + 1)}$$ $\nu' = 0, 1, 2, \ldots k$

$l = (\nu' + m)$

$\alpha = \beta = l(l+1) \rightarrow$ valores permitido par l

$l = m,\ l = 1 + m,\ l = 2 + m$

$l = k + m$

$\Theta(\theta) = (1 - z^2)^{m/2} G(z)$ $z = \cos\theta$

$R = x^s \sum a_\theta x^\theta$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) + \left[-\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}[E - V(r)]\right]R = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) + \left[-\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}E + \frac{2\mu z e^2}{\hbar^2 r}\right]R = 0$$

$S(\rho) = R(r)$ $\rho = 2\beta r$ $\beta^2 = -\frac{2\mu E}{\hbar^2}$

$\frac{dR}{dr} = \frac{dS}{d\rho}\frac{d\rho}{dr}$ $d\rho = 2\beta dr$

$\frac{d}{d\rho} = \frac{1}{2\beta}\frac{d}{dr}$ $\gamma = \frac{\mu z e^2}{\hbar^2 \beta}$

$\frac{dR}{dr} = \frac{dS}{d\rho} 2\beta$

$2\beta \frac{d}{d\rho} = \frac{d}{dr}$

$$\frac{4\beta^2}{\rho^2} 2\beta \frac{d}{d\rho}\left(\frac{\rho^2}{4\beta^2} 2\beta \frac{dS}{d\rho}\right)$$

$$\frac{4\beta^2}{\rho^2}\frac{d}{d\rho}\left(\rho^2 \frac{dS}{d\rho}\right) + \left[-\frac{l(l+1)}{\rho^2}4\beta^2 - \beta^2 + \frac{2\mu z e^2 2\beta}{\hbar^2 \rho}\right]$$

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{ds}{d\rho}\right)+\left[-\frac{l(l+1)}{\rho^2}-\frac{1}{4}+\frac{\mu Ze^2}{\hbar^2\beta\rho}\right]s=0$$

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{ds}{d\rho}\right)+\left[-\frac{l(l+1)}{\rho^2}-\frac{1}{4}+\frac{\gamma}{\rho}\right]s=0$$

$0 \le \rho \le \infty$

$$\frac{1}{\rho^2}\left[2\rho\frac{ds}{d\rho}+\rho^2\frac{d^2s}{d\rho^2}\right]+\left[-\frac{l(l+1)}{\rho^2}-\frac{1}{4}+\frac{\gamma}{\rho}\right]s=0$$

$$\cancel{\frac{2}{\rho}\frac{ds}{d\rho}}^{0}+\frac{d^2s}{d\rho^2}+\left[\cancel{-\frac{l(l+1)}{\rho^2}}-\frac{1}{4}\cancel{+\frac{\gamma}{\rho}}^{0}\right]s=0$$

$\rho \to \infty$

$$\frac{d^2s}{d\rho^2}-\frac{1}{4}s=0$$

$s(\rho)=e^{\lambda\rho}$

$s'(\rho)=\lambda e^{\lambda\rho}$

$s''(\rho)=\lambda^2 e^{\lambda\rho}$

$$\lambda^2 e^{\lambda\rho}-\frac{1}{4}e^{\lambda\rho}=0$$

$$e^{\lambda\rho}\left(\lambda^2-\frac{1}{4}\right)=0$$

$$\lambda^2=\frac{1}{4}\qquad \lambda_1=\frac{1}{2}\qquad \lambda_2=-\frac{1}{2}$$

$$s(\rho)=Ae^{\lambda_1\rho}+Be^{\lambda_2\rho}$$

$$s(\rho)=Ae^{\frac{\rho}{2}}+Be^{-\frac{\rho}{2}} \longrightarrow s(\rho)=Be^{-\frac{\rho}{2}}\qquad B=1$$

$$s(\rho)=e^{-\rho/2}f(\rho)$$

$$s'(\rho)=-\frac{1}{2}e^{-\rho/2}f(\rho)+e^{-\rho/2}f'(\rho)$$

$$s''(\rho)=\frac{1}{4}e^{-\rho/2}f(\rho)-\frac{1}{2}e^{-\rho/2}f'(\rho)-\frac{1}{2}e^{-\rho/2}f'(\rho)+e^{-\rho/2}f''(\rho)$$

$$\frac{2}{\rho}\left[-\frac{1}{2}e^{-\frac{\rho}{2}}f(\rho)+e^{-\frac{\rho}{2}}f'(\rho)\right]+\left[\frac{1}{4}e^{-\frac{\rho}{2}}f(\rho)-\frac{1}{2}e^{-\frac{\rho}{2}}f'(\rho)-\frac{1}{2}e^{-\frac{\rho}{2}}f'(\rho)\right.$$

$$\left.+e^{-\frac{\rho}{2}}f''(\rho)\right]+\left[-\frac{l(l+1)}{\rho^2}-\frac{1}{4}+\frac{\gamma}{\rho}\right]e^{-\frac{\rho}{2}}f(\rho)=0$$

$$-\frac{1}{\rho}e^{-\frac{\rho}{2}}f(\rho)+\frac{2}{\rho}e^{-\frac{\rho}{2}}f'(\rho)+\frac{1}{4}e^{-\frac{\rho}{2}}f(\rho)-\frac{1}{2}e^{-\frac{\rho}{2}}f'(\rho)-\frac{1}{2}e^{-\frac{\rho}{2}}f'(\rho)$$

$$+e^{-\frac{\rho}{2}}f''(\rho)-\frac{l(l+1)}{\rho^2}e^{-\frac{\rho}{2}}f(\rho)-\frac{1}{4}e^{-\frac{\rho}{2}}f(\rho)+\frac{\gamma}{\rho}e^{-\frac{\rho}{2}}f(\rho)=0$$

$$\left[-\frac{1}{\rho}+\cancel{\frac{1}{4}}-\frac{l(l+1)}{\rho^2}-\cancel{\frac{1}{4}}+\frac{\gamma}{\rho}\right]e^{-\frac{\rho}{2}}f(\rho)+\left(\frac{2}{\rho}-\frac{1}{2}-\frac{1}{2}\right)e^{-\rho/2}f'(\rho)+e^{-\rho/2}f''(\rho)=0$$

$$\left[f''(\rho)+\left(\frac{2}{\rho}-1\right)f'(\rho)+\left[\frac{\gamma}{\rho}-\frac{1}{\rho}-\frac{l(l+1)}{\rho^2}\right]f(\rho)\right]e^{-\rho/2}=0$$

$$f''(\rho)+\left(\frac{2}{\rho}-1\right)f'(\rho)+\left[\frac{\gamma}{\rho}-\frac{1}{\rho}-\frac{l(l+1)}{\rho^2}\right]f(\rho)=0$$

$f(\rho)=\rho^s L(\rho)$. $\}$ Serie de potencia en ρ $L(\rho)$

$$L(\rho)=\sum_{\nu=0}^{\infty}a_\nu\rho^\nu, \quad a_0\neq 0$$

$$f'(\rho)=s\rho^{s-1}L(\rho)+\rho^s L'(\rho)$$

$$f''(\rho)=s(s-1)\rho^{s-2}L(\rho)+s\rho^{s-1}L'(\rho)+s\rho^{s-1}L'(\rho)+\rho^s L''(\rho)$$

$$f''(\rho)=s(s-1)\rho^{s-2}L(\rho)+2s\rho^{s-1}L'(\rho)+\rho^s L''(\rho)$$

$$\rho^s L''(\rho)+2s\rho^{s-1}L'(\rho)+s(s-1)\rho^{s-2}L(\rho)+\left(\frac{2}{\rho}-1\right)\left[s\rho^{s-1}L(\rho)+\rho^s L'(\rho)\right]$$

$$+\left[\frac{\gamma}{\rho}-\frac{1}{\rho}-\frac{l(l+1)}{\rho^2}\right]\rho^s L(\rho)=0$$

$$\left\{ \rho^{s} L''(\rho) + 2s\rho^{s-1} L'(\rho) + s(s-1)\rho^{s-2} L(\rho) + 2s\rho^{s-2} L(\rho) + 2\rho^{s-1} L'(\rho) \right.$$
$$\left. - s\rho^{s-1} L(\rho) - \rho^{s} L'(\rho) + \gamma \rho^{s-1} L(\rho) - l(l+1)\rho^{s-2} L(\rho) - \rho^{s-1} L(\rho) = 0 \right\}$$

$$\left\{ \rho^{s+2} L'' + 2s\rho^{s+1} L' + s(s-1)\rho^{s} L + 2s\rho^{s} L + 2\rho^{s+1} L' - s\rho^{s+1} L - \rho^{s+2} L' + \right.$$
$$\left. \gamma \rho^{s+1} L - l(l+1)\rho^{s} L - \rho^{s+1} L = 0 \right\}$$

$$\left\{ \rho^{s+2} L'' + 2s\rho^{s+1} L' + s(s-1)\rho^{s} L + 2s\rho^{s} L + 2\rho^{s+1} L' - s\rho^{s+1} L - \rho^{s+2} L' \right.$$
$$\left. + (\gamma - 1)\rho^{s+1} L - l(l+1)\rho^{s} L = 0 \right\}$$

$$\left\{ \rho^{s+2} \sum_{\nu=2}^{\infty} \nu(\nu-1) a_\nu \rho^{\nu-2} + 2s\rho^{s+1} \sum_{\nu=1}^{\infty} \nu a_\nu \rho^{\nu-1} + s(s-1)\rho^{s} \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu} + \right.$$
$$2s\rho^{s} \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu} + 2\rho^{s+1} \sum_{\nu=1}^{\infty} \nu a_\nu \rho^{\nu-1} - s\rho^{s+1} \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu} - \rho^{s+2} \sum_{\nu=1}^{\infty} \nu a_\nu \rho^{\nu-1}$$
$$\left. + (\gamma + 1)\rho^{s+1} \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu} - l(l+1)\rho^{s} \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu} = 0 \right\}$$

$$\left\{ \sum_{\nu=2}^{\infty} \nu(\nu-1) a_\nu \rho^{\nu+s} + 2s \sum_{\nu=1}^{\infty} \nu a_\nu \rho^{\nu+s} + s(s-1) \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu+s} + 2s \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu+s} \right.$$
$$+ 2 \sum_{\nu=1}^{\infty} \nu a_\nu \rho^{\nu+s} - s \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu+s+1} - \sum_{\nu=1}^{\infty} \nu a_\nu \rho^{\nu+s+1} + (\gamma-1) \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu+s+1}$$
$$\left. - l(l+1) \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu+s} \right\}$$

$$\left\{ \sum_{\nu=2}^{\infty} \nu(\nu-1) a_\nu \rho^{\nu+s} + 2s \sum_{\nu=1}^{\infty} \nu a_\nu \rho^{\nu+s} + s(s-1) \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu+s} + 2s \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu+s} + 2 \sum_{\nu=1}^{\infty} \nu a_\nu \rho^{\nu+s} \right.$$
$$- s \sum_{\nu=1}^{\infty} a_{\nu-1} \rho^{\nu+s} - \sum_{\nu=2}^{\infty} (\nu-1) a_{\nu-1} \rho^{\nu+s} + (\gamma-1) \sum_{\nu=1}^{\infty} a_{\nu-1} \rho^{\nu+s}$$
$$\left. - l(l+1) \sum_{\nu=0}^{\infty} a_\nu \rho^{\nu+s} \right\}$$

$$\sum_{\nu=2}^{\infty}\Big[\nu(\nu-1)a_\nu + 2s\nu a_\nu + s(s-1)a_\nu + 2sa_\nu + 2\nu a_\nu - sa_{\nu-1}$$

$$-(\nu-1)a_{\nu-1} + (\gamma-1)a_{\nu-1} - l(l+1)a_\nu\Big]\rho^{\nu+s} + \Big[2sa_1\rho^{s+1}$$

$$+ \underline{s(s-1)a_0}\,\rho^s + s(s-1)a_1\rho^{s+1} + \underline{2sa_0}\,\rho^s + 2sa_1\rho^{s+1} + 2a_1\rho^{s+1}$$

$$-sa_0\rho^{s+1} + (\gamma-1)a_0\rho^{s+1} - \underline{l(l+1)a_0}\rho^s - l(l+1)a_1\rho^{s+1}\Big] = 0$$

$$[s(s-1) + 2s - l(l+1)]a_0\rho^s \longrightarrow \text{para } \rho^s$$

$$[s^2 - s + 2s - l(l+1)]a_0 = 0 \qquad a_0 \neq 0$$

$$as^2 + bs - 4l(l+1) = 0$$

$$\frac{-1 \pm \sqrt{4l^2+4l+1}}{2} \qquad 4l^2+4l+1$$

$$16l^2+4l+4$$

$$(4l+2)(4l+2)$$

$$\frac{-1 \pm \sqrt{(2l+1)(2l+1)}}{2}$$

$$(2l+1)(2l+1)$$

$$\frac{-1 \pm (2l+1)}{2}$$

$$\longrightarrow s_2 = \frac{-1-2l-1}{2} = \frac{-2-2l}{2}$$

$$\frac{\cancel{-1}+2l\cancel{+1}}{\cancel{2}} =$$

$$s_2 = \frac{-\cancel{2}(1+l)}{\cancel{2}} = -(l+1)$$

$$l = s_1 \qquad \rho^{l+2} \qquad \rho^{l+2-(l+1)} \qquad \text{NO sirve}$$

$$F(\rho) = \rho^l L(\rho) \qquad S(\rho) = e^{-\rho/2} F(\rho)$$

$$\rho^{l+2}L'' + 2l\rho^{l+1}L' + l(l-1)\rho^l L + 2l\rho^l L + 2\rho^{l+1}L' - l\rho^{l+1}L - \rho^{l+2}L'$$

$$+(\gamma-1)\rho^{l+1}L - l(l+1)\rho^l L = 0$$

$$\rho L'' + 2lL' + \frac{l(l-1)}{\rho}L + \frac{2l}{\rho}L + 2L' - lL - \rho L'$$
$$+(\gamma-1)L - \frac{l(l+1)}{\rho}L = 0$$

$$\rho L'' + \{2l + 2 - \rho\}L' + \left\{\frac{l(l-1)}{\rho} + \frac{2l}{\rho} - l + \gamma - 1 - \frac{l(l+1)}{\rho}\right\}L = 0$$

$$\rho L'' + \{2(l+1) - \rho\}L' + \left\{\overset{0}{\cancel{\frac{l^2 - l + 2l - l^2 - l}{\rho}}} + \gamma - l - 1\right\}L = 0$$

$$\rho L'' + \{2(l+1) - \rho\}L' + \{\gamma - l - 1\}L = 0$$

$$L(\rho) = \sum_{\nu=0}^{\infty} a_\nu \rho^\nu = a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 \dots a_5\rho^5$$

$$L'(\rho) = a_1 + 2a_2\rho + 3a_3\rho^2 + 4a_4\rho^3 + 5a_5\rho^4 \dots$$

$$L''(\rho) = 2a_2 + 6a_3\rho + 12a_4\rho^2 + 20a_5\rho^3 \dots$$

$$\{2a_2\rho + 6a_3\rho^2 + 12a_4\rho^3 + 20a_5\rho^4 \dots\} + \{2(l+1) - \rho\}[a_1 + 2a_2\rho + 3a_3\rho^2 + 4a_4\rho^3$$
$$+ 5a_5\rho^4] \dots + \{\gamma - l - 1\}[a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 \dots\}$$

$$\{\gamma - l - 1\}a_0 + 2(l+1)a_1 = 0 \quad \checkmark \quad \Big\} \rho^0$$

$$\left.\begin{aligned} &\{2a_2 - a_1 + 4a_2(l+1) + a_1\{\gamma - l - 1\} = 0 \\ &\{1\cdot 2 + 2\cdot 2(l+1)\}a_2 + \{\gamma - l - 1 - 1\}a_1 = 0 \end{aligned}\right\} \rho$$

$$\{2\cdot 3 + 3\cdot 2(l+1)\}a_3 + \{\gamma - l - 1 - 2\}a_2 = 0 \quad \Big\} \rho^2$$

$$\{3\cdot 4 + 4\cdot 2(l+1)\}a_4 + \{\gamma - l - 1 - 3\}a_3 = 0 \quad \Big\} \rho^3$$

$$\{\nu(\nu+1) + 2(\nu+1)(l+1)\}a_{\nu+1} + \{\gamma - l - 1 - \nu\}a_\nu = 0 \quad \Big\} \rho^\nu$$

$$\gamma - l - 1 - n' = 0 \quad ; \quad \gamma = n \qquad n = n' + l + 1$$

Para que la serie no diverga y se rompa en n'

$$R(r) = e^{-\rho/2} \rho^{l} L(\rho)$$

$$\gamma = \frac{\mu Z e^2}{\hbar^2 \beta} \qquad \gamma^2 = \frac{\mu^2 Z^2 e^4}{\hbar^4 \beta^2}$$

$$n^2 = -\left(\frac{\cancel{\mu^2} Z^2 e^4}{\cancel{\hbar^4}}\right)\left(\frac{\cancel{\hbar^2}}{2\mu E}\right) \qquad E = -\frac{Z^2 \mu e^4}{2\hbar^2 n^2}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_1 m_2}{m_1} = m_2 = m_e$$

Solución

$$\psi_{nlm} = R_{nl}(r)\, \Theta_{lm}(\theta)\, \Phi_m(\phi)$$

$n = 1, 2, 3, 4$

$l = m, m+1, m+2$

Polinomios de Legendre.

$$T(z,t) = \sum_{l=0}^{\infty} P_l(z)\, t^l = \frac{1}{\sqrt{1 - 2zt + t^2}}$$

$$\frac{\partial T}{\partial t} = \sum_{l=1}^{\infty} l P_l(z) t^{l-1} = -\frac{1}{2}\left[\frac{-2z + 2t}{(1 - 2zt + t^2)^{3/2}}\right] = \frac{1}{\cancel{2}} \frac{\cancel{2}(z - t)}{(1 - 2zt + t^2)^{3/2}}$$

$$\frac{\partial T}{\partial t} = \sum_{l=1}^{\infty} l P_l(z) t^{l-1} = (z - t)\left[\frac{1}{(1 - 2zt + t^2)^{1/2}(1 - 2zt + t^2)}\right]$$

$$(1 - 2zt + t^2) \sum_{l=1}^{\infty} l P_l(z)\, t^{l-1} = (z - t) \sum_{l=0}^{\infty} P_l(z)\, t^l$$

$$(1-2zt+t^2)\left[P_1+2P_2t+3P_3t^2+4P_4t^3+5P_5t^4\right]=(z-t)\left[P_0+P_1t+P_2t^2+P_3t^3+P_4t^4\right]$$

$P_1 - P_0 z = 0 \longrightarrow t^0$

$P_0 = 1 \;\longrightarrow\; P_1 = z$

$$\left.\begin{aligned}&-2zP_1+2P_2=-P_0+P_1z\\&2P_2-3zP_1+P_0=0\end{aligned}\right\} t$$

$$\left.\begin{aligned}&P_1-4zP_2+3P_3=-P_1+zP_2\\&3P_3-5zP_2-2P_1=0\end{aligned}\right\} t^2$$

$$(l+1)P_{l+1}-(2l+1)zP_l+lP_{l-1}=0$$

$-1, 1$

$z=\cos\theta$

$$P_l^m(z)=(1-z^2)^{\frac{m}{2}}\frac{d^m}{dz^m}P_l(z)$$

$$(1-z^2=1-\cos^2\theta=\operatorname{sen}^2\theta)$$

$$\int_{-1}^{1}P_l^m(z)\,P_{l'}^m(z)\,dz=\begin{cases}0 & l'\neq l\\ \dfrac{2}{2l+1}\dfrac{(l+m)!}{(l-m)!} & l'=l\end{cases}$$

$$\Theta(\theta)=\sqrt{\frac{(2l+1)}{2}\frac{(l-m)!}{(l+m)!}}\,P_l^m(\cos\theta)\Big\}\text{ función en theta.}$$

Polinomios de Laguerre

$$V(\rho,u)=\sum_{r=0}^{\infty}\frac{L_r(\rho)}{r!}u^r=\frac{e^{-\frac{\rho u}{1-u}}}{1-u}$$

$$\frac{\partial V}{\partial u}=\sum_{r=1}^{\infty}\frac{rL_r(\rho)}{r!}u^{r-1}=\frac{-\frac{\rho}{(1-u)^2}e^{-\frac{\rho u}{1-u}}\cancel{(1-u)}+e^{-\frac{\rho u}{1-u}}}{(1-u)^2}$$

$$\sum_{r=1}^{\infty} r\frac{L_r(\rho)}{r!}u^{r-1} = \frac{-\rho e^{-\frac{\rho u}{1-u}} + (1-u)e^{-\frac{\rho u}{1-u}}}{(1-u)^3}$$

$$\sum_{r=1}^{\infty} r\frac{L_r(\rho)}{r!}u^{r-1} = \frac{-\rho e^{-\frac{\rho u}{1-u}} + e^{-\frac{\rho u}{1-u}} - u e^{-\frac{\rho u}{1-u}}}{(1-u)^3}$$

$$\sum_{r=1}^{\infty} r\frac{L_r(\rho)}{r!}u^{r-1} = \frac{e^{-\frac{\rho u}{1-u}}}{1-u}\left[\frac{1-u-\rho}{(1-u)^2}\right]$$

$$1-2u+u^2 \sum_{r=1}^{\infty} r\frac{L_r(\rho)}{r!}u^{r-1} = (1-u-\rho)\sum_{r=0}^{\infty}\frac{L_r(\rho)}{r!}u^r$$

$$(1-2u+u^2)\left[L_1 + L_2 u + \frac{L_3 u^2}{2} + \frac{L_4 u^3}{6}\right]$$

$$L_{n+l}^{2l+1}(\rho) = \sum_{k=0}^{n-l-1} (-1)^{k+1} \frac{\{(n+l)!\}^2}{(n-l-1-k)!\,(2l+1+k)!\,k!}\rho^k$$

$$R_{nl}(r) = -\sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-l-1)!}{2n\{(n+l)!\}^3}}\; e^{-\rho/2}\rho^l L_{n+l}^{2l+1}(\rho)$$

$$R_{nl}(r) = e^{-\rho/2}\rho^l L(\rho) \longrightarrow R_{nl}(r) = e^{-\rho/2}\rho^l L_{n+l}^{2l+1}(\rho)$$

$$L_r^s(\rho) = \frac{d^s L(\rho)}{d_{s^2}}$$

$$\int_0^\infty e^{-\rho}\rho^{2l}\left\{L_{n+l}^{2l+1}(\rho)\right\}^2\rho^2\, d\rho \Big\}\ \text{normalizacion}$$

$$\psi_{n\ell m} = R_{n\ell}(r)\,\Theta_{\ell m}(\theta)\,\Phi_m(\phi)$$

$n=1,\ l=0,\ m=0 \qquad \psi_{100}$

K shell

$$\rho = \frac{2Zr}{na_0}$$

$$\left(\frac{Z}{a_0}\right)^{3/2} \cancel{2}\, e^{-\rho/2}\ \frac{\cancel{\sqrt{2}}}{\cancel{2}}\ \frac{1}{\cancel{\sqrt{2\pi}}}$$

$$\frac{n}{2}\rho = \frac{2r}{a_0} = \sigma$$

$$\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-\rho/2} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-\frac{2r}{a_0}}$$

L shell

$n=2,\ l=0,\ m=0 \qquad \psi_{200}$

$$\left(\frac{Z}{a_0}\right)^{3/2} \frac{(2-\rho)}{2\sqrt{2}}\, e^{-\rho/2}\ \frac{\cancel{\sqrt{2}}}{2}\ \frac{1}{\sqrt{2\pi}}$$

$$\frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left[2 - \frac{2r}{a_0}\right] e^{-\frac{2r}{2a_0}}$$

$$\frac{1}{\sqrt{\pi}}\ \frac{1}{\sqrt{a_0^3}}\ e^{-\frac{r}{a_0}} = \frac{1}{\sqrt{\pi a_0^3}}\, e^{-\frac{r}{a_0}} \qquad Z=1 \quad n=1,\ l=0,\ m=0$$

$$\psi^*_{100}\,\psi_{100} = \left(\frac{1}{\sqrt{\pi a_0^3}}\, e^{-\frac{r}{a_0}}\right)\left(\frac{1}{\sqrt{\pi a_0^3}}\, e^{-\frac{r}{a_0}}\right) = \frac{1}{\pi a_0^3}\, e^{-\frac{2r}{a_0}}$$

$$D(r)\,dr = \int_0^\infty\int_0^\pi\int_0^{2\pi} \frac{1}{\pi a_0^3}\, e^{-\frac{2r}{a_0}}\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$D(r)\,dr = \frac{1}{\pi a_0^3}\, r^2 e^{-\frac{2r}{a_0}}\, dr \int_0^\pi\int_0^{2\pi} \sin\theta\, d\theta\, d\phi$$

normalization

$$D(r)\,dr = \frac{1}{\pi a_0^3}\, r^2 e^{-\frac{2r}{a_0}}\, dr \int_0^{2\pi} d\phi \int_0^\pi \sin\theta\, d\theta = 4\pi$$

$$D(r)\,dr = \frac{4}{a_0^3}\, r^2 e^{-\frac{2r}{a_0}}\, dr \qquad D(r) = \frac{4}{a_0^3}\, r^2 e^{-\frac{2r}{a_0}}$$