Mathematical Curiosities

- A Collection of Common and not-so-common Mathematical Facts -

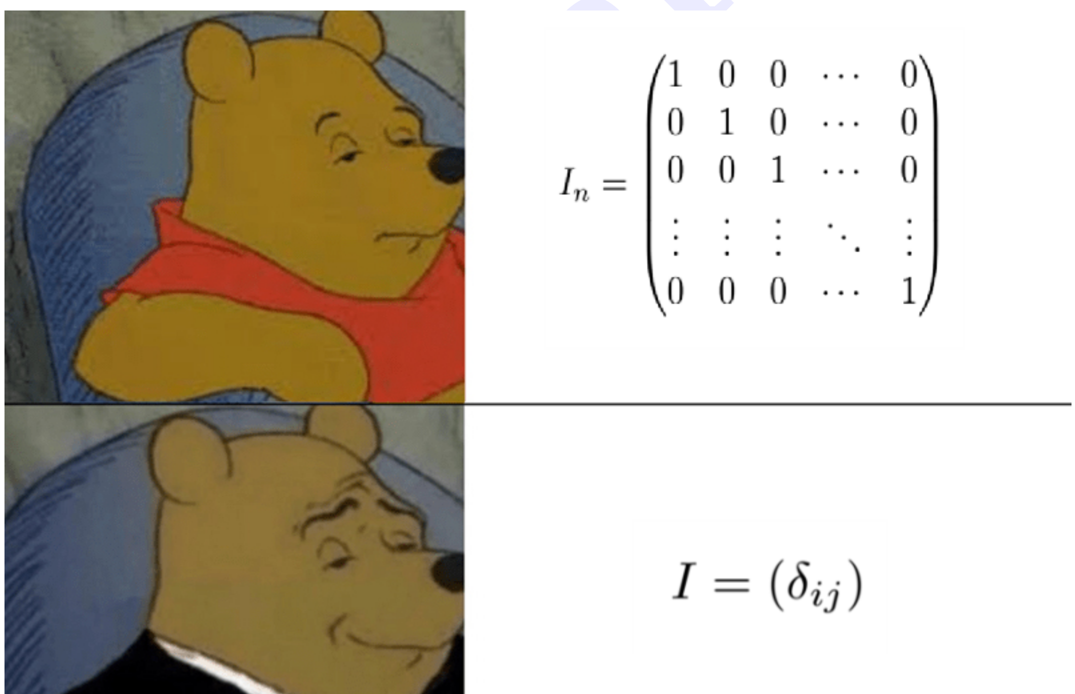

Figure 1: You before and after ~~learning about the kronecker delta~~ reading this book

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1 Analysis

2 Linear Algebra

2.1 Kronecker Delta

Everyone knows the main definition of the identity matrix, that is

Definition 1 (**Indentity matrix**).
Let $I_n \in \mathbb{R}^{n \times n}$ be a diagonal matrix whose diagonal entries are only onese and zero elsewhere, that is

$$I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \tag{1}$$

But why using this boring and rather cumbersome notation when we can use something cooler? Consider the definition of the Kronecker Delta:

Definition 2 (**Kronecker Delta**).
The kronecker delta δ_{ij} is the function $\delta : \mathbb{R} \times \mathbb{R} \to \{0, 1\}$ defined as

$$\delta_{ij} = \begin{cases} 1 & , i = j \\ 0 & , i \neq j \end{cases} \tag{2}$$

A common notation for some matrix A, say, is to define it implictly in terms of it's elements, say in this $A = \{a_{ij}\}_{i,j=1}^{n}$ or $(a_{ij})_{i,j=1}^{n}$, or for short (a_{ij}). Therefore we can define the identity matrix as $I = (\delta_{ij})$. Now you are ready to enjoy this meme:

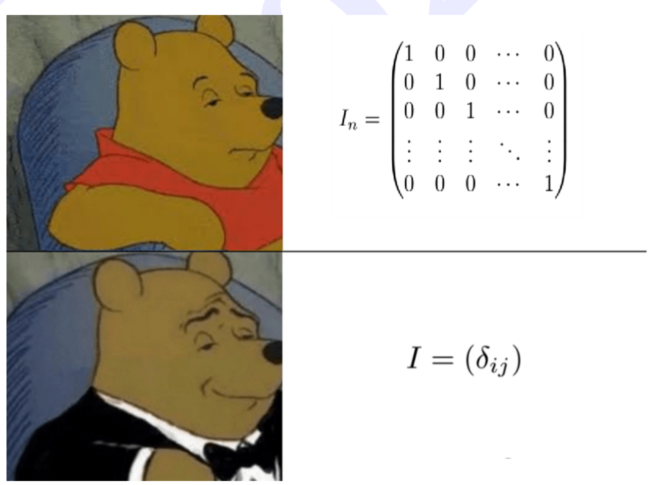

Figure 2: You before and after learning about the kronecker delta

3 Copy-paste templates (IGNORE)

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Definition 3 (null).

Theorem 3.1 (null).

Proposition 3.2 (ff).

Example 1 (???).

Note 1 (null).

Corollary 3.2.1 (null).

Lemma 3.3 (null).

Remark.

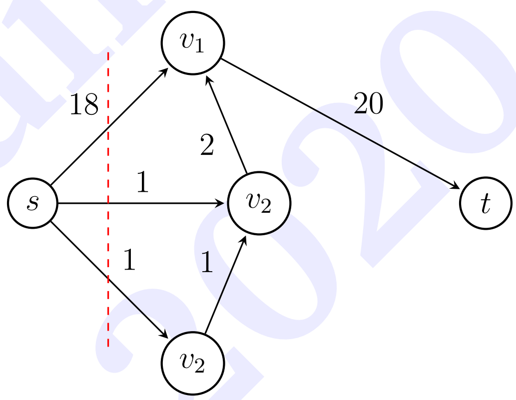
$$\begin{aligned}
\mathbb{E}[(W_t(1-W_{t-1})Z_t)(W_t(1-W_{t-1})Z_t)] &= \mathbb{E}[(W_t(1-W_{t-1})Z_t)^2] \\
&= \mathbb{E}[(W_t)^2(1-W_{t-1})^2(Z_t)^2] \\
&= \mathbb{E}[W_t^2]\mathbb{E}[(1-W_{t-1})^2]\mathbb{E}[Z_t^2] \\
&= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1) = \boxed{\frac{1}{4}}
\end{aligned}$$

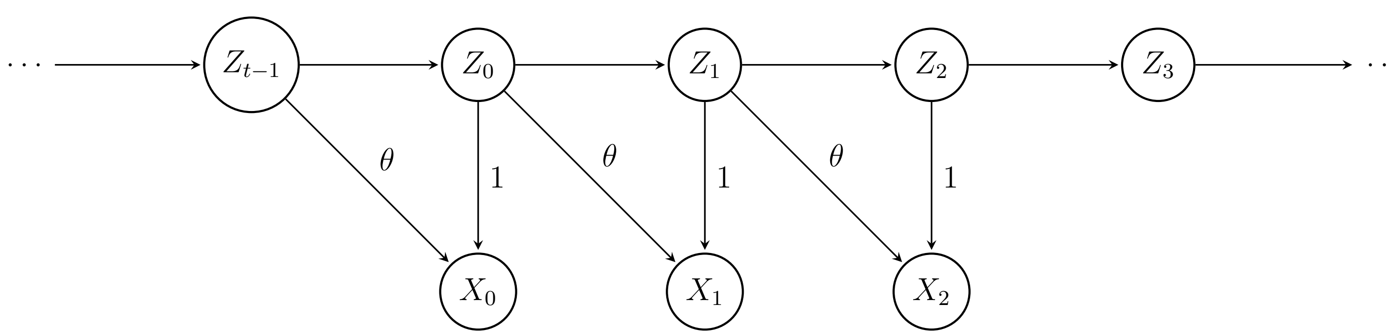
```r
# estimate a linear trend
huron_linear <- tslm(LakeHuron ~ trend) ## Fit linear trend
huron_quad <- tslm(LakeHuron ~ trend + I(trend^2)) ## Fit quadratic trend

# Bind together the data and fitted trends
LakeHuron_with_fits <- cbind(LakeHuron,
                             Linear_trend = fitted(huron_linear),
                             Quadratic_trend = fitted(huron_quad))

# Construct the plot
```

```r
autoplot(LakeHuron_with_fits) +
  ylab("Water Level (in feet)") + xlab("Year") +
  ggtitle("Lake Huron water levels (1875-1972)") +
  guides(colour= guide_legend(title = "Data series")) +
  scale_colour_manual(values=c("black","red","blue"))
```

$$\begin{tikzcd} A \arrow[r, "\phi"] \arrow[d] & B \arrow[d, "\psi"] \\ C \arrow[r, "\eta"] & D \end{tikzcd}$$

Figure 3: Copyright





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References

[1] Rusell Steele "MATH 545: Introduction to Time Series Analysis" McGill University Winter 2020