
Mathematical Curiosities

- A Collection of Common
and not-so-common Mathematical Facts -



$$I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$



$$I = (\delta_{ij})$$

Figure 1: You before and after learning about the kronecker delta reading this book

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April 27, 2020

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1 Analysis

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2 Linear Algebra

2.1 Kronecker Delta

Everyone knows the main definition of the identity matrix, that is

Definition 1 (Identity matrix).

Let $I_n \in \mathbb{R}^{n \times n}$ be a diagonal matrix whose diagonal entries are only ones and zero elsewhere, that is

$$I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \quad (1)$$

But why using this boring and rather cumbersome notation when we can use something cooler? Consider the definition of the Kronecker Delta:

Definition 2 (Kronecker Delta).

The kronecker delta δ_{ij} is the function $\delta : \mathbb{R} \times \mathbb{R} \rightarrow \{0, 1\}$ defined as

$$\delta_{ij} = \begin{cases} 1 & , i = j \\ 0 & , i \neq j \end{cases} \quad (2)$$

A common notation for some matrix A , say, is to define it implicitly in terms of it's elements, say in this $A = \{a_{ij}\}_{i,j=1}^n$ or $(a_{ij})_{i,j=1}^n$, or for short (a_{ij}) . Therefore we can define the identity matrix as $I = (\delta_{ij})$. Now you are ready to enjoy this meme:

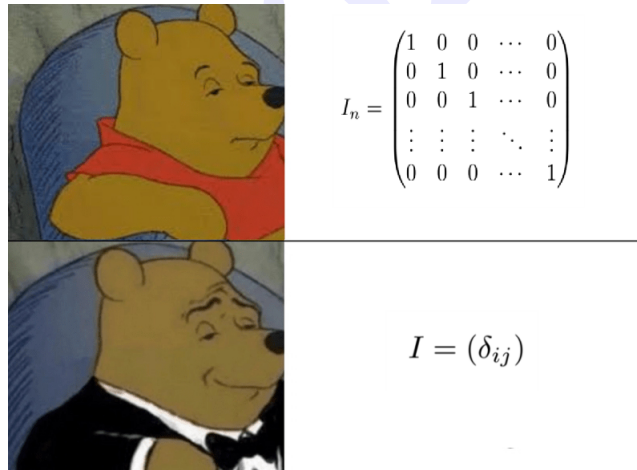


Figure 2: You before and after learning about the kronecker delta

3 Copy-paste templates (IGNORE)

\underbrace{FFF}_{text}

Definition 3 (null).

Theorem 3.1 (null).

Proposition 3.2 (ff).

Example 1 (???).

Note 1 (null).

Corollary 3.2.1 (null).

Lemma 3.3 (null).

Remark.

$$\begin{aligned} \mathbb{E}[(W_t(1 - W_{t-1})Z_t)(W_t(1 - W_{t-1})Z_t)] &= \mathbb{E}[(W_t(1 - W_{t-1})Z_t)^2] \\ &= \mathbb{E}[(W_t)^2(1 - W_{t-1})^2(Z_t)^2] \\ &= \mathbb{E}[W_t^2]\mathbb{E}[(1 - W_{t-1})^2]\mathbb{E}[Z_t^2] \\ &= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1) = \boxed{\frac{1}{4}} \end{aligned}$$

```
# estimate a linear trend
huron_linear <- tslm(LakeHuron ~ trend) ## Fit linear trend
huron_quad <- tslm(LakeHuron ~ trend + I(trend^2)) ## Fit quadratic trend

# Bind together the data and fitted trends
LakeHuron_with_fits <- cbind(LakeHuron,
                             Linear_trend = fitted(huron_linear),
                             Quadratic_trend = fitted(huron_quad))

# Construct the plot
```

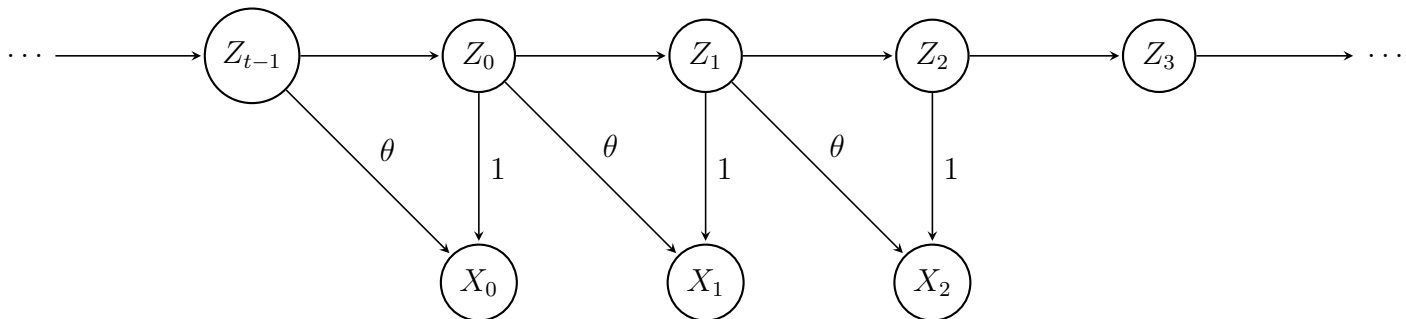
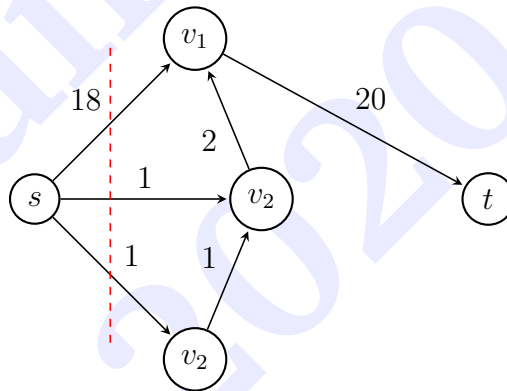
```
autoplot(LakeHuron_with_fits) +
  ylab("Water Level (in feet)") + xlab("Year") +
  ggtitle("Lake Huron water levels (1875-1972)") +
  guides(colour= guide_legend(title = "Data series")) +
  scale_colour_manual(values=c("black","red","blue"))
```

$$\begin{array}{ccc} A & \xrightarrow{\phi} & B \\ \downarrow \textcolor{red}{\eta} & & \downarrow \textcolor{red}{\psi} \\ C & \xrightarrow{\textcolor{blue}{\eta}} & D \end{array}$$



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Figure 3: Copyright



4 Copyright



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References

- [1] Rusell Steele “MATH 545: Introduction to Time Series Analysis” McGill University
Winter 2020

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